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Behavior of Confined High Strength Concrete Columns under Axial Compression
Umesh K. Sharma, Pradeep Bhargava, S. K. Kaushik
Journal of Advanced Concrete Technology, volume 3 (2005), pp. 267-281

Modeling of Reinforcement Buckling in RC Columns Confined with FRP
Yuichi Sato, HuneBum Ko
Journal of Advanced Concrete Technology, volume 6 (2008), pp. 195-204
### Scientific paper

**Finite Element Analysis of Actively Confined Concrete Using Shape Memory Alloys**

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**Abstract**

The novel technique of using Shape Memory Alloy (SMA) transverse reinforcement to confine concrete has proven its success in increasing the strength and ductility of concrete elements significantly. This new technique exploits the thermally activated shape memory feature of SMAs to apply high and permanent confinement pressure on concrete. Despite the experimental work done in this topic over the last few years, the numerical/analytical efforts are still lacking. This paper utilizes the analytical framework of the damaged plasticity model along with the finite element method (FEM) to predict and study the behavior of SMA confined concrete under uniaxial compression load. The flow rule and hardening/softening function adopted in the models are developed based on previous test results. The results prove that the finite element models can successfully capture the behavior of SMA confined concrete.

1. Introduction

Typical plain concrete is characterized by a brittle behavior under uniaxial compressive stress. Lateral concrete confinement is commonly used to delay the failure of concrete and improve its ductility; a feature that is critically important for structures subjected to extreme loads such as earthquakes. The pioneer work of Richart et al. (1928), which aimed at studying the concrete behavior under multi-axial stresses, demonstrated the effectiveness of lateral confinement on concrete behavior. This early research encouraged many researchers to explore the behavior of concrete under lateral confinement and to investigate different methods to apply lateral confining pressure in a practical and efficient manner. There are mainly two types of lateral confinement techniques, namely passive confinement and active confinement. In the case of passively confined concrete, the confining pressure develops gradually as a result of concrete dilation as the concrete is loaded axially. Passive confinement, which is the more commonly used technique, is applied in new concrete structures using internal transverse steel reinforcement (e.g. spirals, hoops, stirrups). For old structures with insufficient ductility, supplementary passive confinement is often applied using external steel jackets or fiber reinforced polymer (FRP) jackets. Many studies focused on exploring the behavior of concrete confined by transverse reinforcement (Sheikh et al. 1982; Scott et al. 1982; Mander et al. 1988). Mander et al. (1988) conducted study on concrete columns with circular and square sections and rectangular wall sections confined with spirals or hoops under monotonic and cyclic loading. They proposed a stress-strain model for confined concrete under monotonic compression, which was widely used by many researchers. More recently, many researchers were interested in investigating the behavior of concrete confined by FRP jackets (Mirmiran and Shahawy 1997; Fam and Rizkalla 2001; Harries and Kharel 2002; Jiang and Teng 2007; Yu et al. 2010a, 2010b). Teng et al. (2007) proposed a uniaxial stress-strain model to predict the behavior of FRP confined concrete by modifying the model by Mander et al. (1988). The advantages of FRP confined concrete are not only the efficacy in increasing the strength and ductility of structures, but also its light weight and ability to resist corrosion compared to steel jackets. On the other hand, in the case of active confinement, the lateral confining pressure is applied to concrete prior to loading. Due to the initial confining pressure before loading, active confinement can effectively delay the dilation of concrete and hence is found to be more efficient in increasing concrete compressive strength and ultimate strain than passive confinement. Most of the studies on active confinement investigated the behavior of concrete using triaxial testing devices such as triaxial pressure vessel (Richart et al. 1928; Imran and Pantazopoulou 1996; Candappa et al. 2001). Researches on applying active confinement in structural concrete practically are much fewer than those on passive confinement. Moghaddam et al. (2010) used prestressed metal strips to actively confine concrete. Yamakawa et al. (2004) used pre-tensioned aramid FRP belts to retrofit damaged reinforced concrete columns. Due to the limitation and challenges (requires too much labor, time and hence increases the cost) associated with

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2. SMA Concrete confinement

2.1 Shape memory alloys

SMA is a class of metallic alloys characterized by the following typical properties: superelasticity (SE) and shape memory effect (SME); the material possesses a wide thermal hysteresis such that the transformation between austenite and martensite phases is governed by four transformation temperatures, namely martensite start temperature (Ms), martensite finish temperature (Mf), austenite start temperature (As), and austenite finish temperature (Af). If the SMA wire is restrained, hence not able to recover its original (undeformed) shape upon unloading. Many recent studies have investigated the application of active confinement in structures is limited. SE is a phenomenon where a mechanically loaded SMA wire is heated. To satisfy this condition, the NiTiNb alloy is utilized in this study due to its wide thermal hysteresis. Figure 1 illustrates the phenomenon of SME; the temperature to above Af fixes the prescribed strain. The results illustrated that this new technique holds promise for both new and old structures. Dommers and Andrawes (2012) conducted experiments to examine the behavior of SMA wires using SME. Krstulovic-Opara and Thiedeman (2000) utilized this type of on concrete through increasing the temperature. Their results illustrated that this new technique holds promise for both new and old structures. Dommers and Andrawes (2012) conducted experiments to examine the behavior of SMA wires using SME. Krstulovic-Opara and Thiedeman (2000) utilized this type of self-stressing composite to apply active confining pressure to concrete (SMA) spirals to actively confine concrete through applying mechanical prestress in steel and FRP. The paper.}

Q. Chen and B. Andrawes / Journal of Advanced Concrete Technology Vol. 12, 520-534, 2014

**Fig. 1** Shape memory effect of SMA (a) unrestrained ends (F is applied force); (b) restrained ends (F is recovery force).

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The behavior of SMA confined concrete is different from purely active or purely passive confinement in the sense that prior to concrete loading, the spiral is not restrained and is free to recover its original length; rather a large recovery stress in the spiral actively confines concrete. During loading however, concrete dilates laterally and the constraint provided by the concrete element prevents SMA from austenite to recover its original shape when heated to a temperature above Af. The SMA wire is heated. To satisfy this condition, the NiTi alloy is utilized in this study due to its wide thermal hysteresis. Figure 1 illustrates the phenomenon of SME; the temperature to above Af fixes the prescribed strain. The results illustrated that this new technique holds promise for both new and old structures. Dommers and Andrawes (2012) conducted experiments to examine the behavior of SMA wires using SME. Krstulovic-Opara and Thiedeman (2000) utilized this type of self-stressing composite to apply active confining pressure to concrete (SMA) spirals to actively confine concrete through applying mechanical prestress in steel and FRP. The paper.
additional passive confining pressure is exerted on the concrete element. Figure 3 displays the recovery stress test results of a restrained SMA wire with 6.4% pre-strain from Shin and Andrawes (2010). Before the cyclic loading was applied, recovery stress was induced in the wire through heating. When the SMA wire was subjected to cyclic loading after the recovery stress became stable, additional stress developed in the wire. Due to this unique characteristic, the models available in the literature for concrete confined either passively or actively would not be suitable for describing the behavior of SMA confined concrete.

2.2 SMA Confining pressure calculation

When using SMA spirals for confinement of concrete elements with circular sections such as columns, the total confining pressure comprises an active and a passive component. In general, the lateral confining pressure applied on circular section from transverse reinforcement can be expressed using the following formula (Mander et al. 1988):

$$ f_l = 2f_A A_{sp} / s d_s $$  \hspace{1cm} (1)

where $f_l$ is lateral confining pressure, $f_A$ is transverse reinforcement hoop stress; $A_{sp}$ is transverse reinforcement cross section area; $d_s$ is section diameter; $s$ is pitch of spiral. Applying Eq. 1 for SMA confinement, one can compute the effective active component of the overall confining pressure from SMA spirals by using SMA recovery stress as the hoop stress. In the equation, however, it should be noted that the maximum recovery stress obtained from a recovery stress test using a straight wire cannot be fully achieved due to prestress losses (Shin and Andrawes 2010). Therefore, active hoop stress along spirals is equal to the residual recovery stress after prestrain loss. The passive confining pressure component, on the other hand is calculated using hoop stress developing in the SMA spirals during loading. In order to obtain the hoop stress along spirals when concrete dilates, tensile test results of SMA wires from Dommer and Andrawes (2012) were used.

2.3 Axial stress-strain curve and lateral vs. axial strains

Many researchers proposed empirical equations that were calibrated based on experimental data to predict the axial stress-strain relationship of either purely actively confined or purely passively confined concrete. Eq. 2 was proposed by Popovics (1973) and adopted by Mander et al. (1988) to predict the axial stress-strain relation of transverse steel reinforced concrete. Mander et al. (1988)’s model is one of the widely used axial stress-strain model for confined concrete in the literature.

$$ f_c = f_c' \left( \varepsilon / \varepsilon_c' \right) r $$  \hspace{1cm} (2)

where $f_c'$ is the peak strength of confined concrete and $\varepsilon_c'$ is the concrete axial strain corresponding to peak strength $f_c'$; $f_c$ is the axial stress of concrete and $\varepsilon_c$ is axial strain corresponding to $f_c$; $r$ is defined by Eq. 3.

$$ r = \frac{E_c}{E_c - f_{co}'/\varepsilon_{co}'} $$  \hspace{1cm} (3)

where $E_c$ is the elastic modulus of concrete. Mander et al. (1988) proposed to use Eq. 4 and Eq. 5 to predict the peak stress $f_c''$ and the corresponding strain $\varepsilon_c''$ at peak stress of confined concrete. Eq. 4 was calibrated by using the triaxial tests results from Schickert and Winkler (1977).

$$ f_c'' = f_{co}'' \left( -1.254 + 2.254 \sqrt{ 1 + \frac{7.94f_f}{f_{co}''} - 2 \frac{f_f}{f_{co}''} } \right) $$  \hspace{1cm} (4)

$$ \varepsilon_c = \varepsilon_{co} \left[ 1 + 5 \left( \frac{f_c''}{f_{co}''} - 1 \right) \right] $$  \hspace{1cm} (5)

where $f_f$ is confining pressure; $f_{co}''$ is unconfined concrete strength; $\varepsilon_{co}$ is axial strain corresponding to unconfined concrete strength.

Besides axial stress-strain model, some researchers also investigated the relation between axial strain and lateral strain of concrete (Elwi and Murray 1979; Mir-miran and Shahawy 1997; Teng et al. 2007). Among
these, Teng et al. (2007) proposed Eq. 6 to describe the lateral-axial strain relationship of unconfined, actively confined, and FRP confined concrete.

\[
\frac{\varepsilon_l}{\varepsilon_{co}} = 0.85 \left( 1 + 8 \frac{f_l}{f_{f,co}} \right) \left[ 1 + 0.75 \left( -\frac{\varepsilon_l}{\varepsilon_{co}} \right) \right]^{0.7} - \exp \left[ -7 \left( -\frac{\varepsilon_l}{\varepsilon_{co}} \right) \right]
\]

(6)

In this equation, \( \varepsilon_l \) is lateral strain. Previous researchers modeled FRP confined concrete stress-strain curves based on a series of active confinement stress-strain curves (Chun and Park 2002; Teng et al. 2007), with an assumption that each point on the stress-strain curve of FRP confined concrete is corresponding to a point on the active confinement stress-strain curve with the same lateral confining pressure as provided by FRP jacket. This paper extends this concept to SMA confined concrete. Figure 4 shows the intersection of SMA confined concrete stress-strain test result (Shin and Andrawes 2010) and a set of actively confined concrete stress-strain curves with different confining pressures (based on Mander et al. 1988). The figure illustrates that the initial response of SMA confined concrete follows the path of actively confined concrete stress-strain curve with confining pressure similar to the initial active confining pressure from SMA spirals, which was equal to 1.42 MPa for this test based on Eq. 1. During loading, as concrete dilates, SMA confined concrete stress-strain curve intersects those actively confined concrete stress-strain curves sequentially as the confining pressure increases. This validates the assumption that each point on the axial stress-strain curve of SMA confined concrete corresponds to a point on the actively confined concrete axial stress-strain curve with the same lateral confining pressure as provided by the SMA spirals.

### 3. Finite element analysis

To validate the damaged plasticity model adopted in this study, Concrete Damaged Plasticity Model within the framework of finite element program ABAQUS was utilized to model the three 152mm×305mm concrete cylinders with different amount of SMA confinement and different concrete strengths. The characteristics of these three specimens are summarized in Table 1 and Fig. 5 also shows photos of the three specimens. SP-1 and SP-3 were tested by Shin and Andrawes (2010) under uniaxial compression: SP-1 was confined with SMA spirals with 13 mm pitch spacing and SP-3 was confined with 4 layers of Glass-FRP (GFRP) and 13 mm pitch spacing SMA spirals wrapped on the top of the GFRP. One more cylinder (SP-2) confined with SMA spirals.

*Fig. 4 Intersection of several actively confined concrete stress-strain curves with SMA confined concrete test result.*

**Table 1 Characteristics of the SMA confined concrete specimens in the model.**

<table>
<thead>
<tr>
<th>Label</th>
<th>Confinement Type</th>
<th>Concrete Strength (MPa)</th>
<th>SMA Residual Recovery stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP-1</td>
<td>13 mm spacing SMA</td>
<td>39.2</td>
<td>447</td>
</tr>
<tr>
<td>SP-2</td>
<td>25.4 mm spacing SMA</td>
<td>56.8</td>
<td>600</td>
</tr>
<tr>
<td>SP-3</td>
<td>4 layers GFRP and 13 mm spacing SMA</td>
<td>39.2</td>
<td>427</td>
</tr>
</tbody>
</table>

*Fig. 5 SMA Confined Specimens: (1) SP-1; (2) SP-2; (3) SP-3.*
spirals with 25.4 mm pitch spacing was tested by the authors. Each layer of GFRP sheet had a thickness of 0.11 mm. The elastic modulus and ultimate strain of GFRP were 19000 MPa and 0.018 mm/mm, respectively. SMA wires from all these three specimens had the same diameter (2 mm), but have different residual recovery stresses after prestrain losses as shown in Table 1, since different batches of SMA wires were used. The constitutive behavior of SMA hoops, which is shown in Fig. 6, is based on SMA wire tensile coupon test results after stress recovery (Dommer and Andrawes 2012), with modification based on different recovery stresses in SP-1, SP-2 and SP-3.

3.1 Finite element model

A vertical slice at the mid-height of the cylinder with height of one pitch spacing is modeled for all three specimens. An assumption is made that the constraints at both ends of the cylinder have little effect on the behavior at mid-height of the cylinder and concrete slice is free to dilate laterally. For simplicity and since the spiral pitch is considered small, the spiral was modeled as a series of hoops. Figure 7 displays the finite element mesh of concrete with a SMA hoop at the mid-height of the slice (Fig. 7a) and GFRP jacket (Fig. 7b). For all three specimens, concrete is modeled by 8-node solid elements. SMA hoops are modeled by 2-node truss elements. 4-node shell elements are used to model the GFRP jacket. GFRP jacket is assumed to have linear elastic behavior until rupture. The failure surface is represented by Von Mises Criterion with failure stress calculated from the tensile rupture strain. The interactions between concrete and GFRP jacket, between concrete and SMA hoops, and between GFRP jacket and SMA hoops are modeled by tie constraint in ABAQUS, which ties the nodes from one surface to the corresponding node in the contact surface, and the two tied nodes maintain the same displacements. This assumes slippage between two contact surfaces to be negligible. As for the loading it was applied on two phases; in the first phase, SMA wires are activated (prestressed), while in the second phase displacement-controlled downward loading is applied on the top surface of the model. During the activation of SMA, both top and bottom surfaces of the model are free to move vertically and laterally. However, during the axial loading phase, the bottom surface of the model is restricted from moving in the vertical direction.

3.2 Damaged plasticity model

In order to incorporate the material constitutive properties of SMA confined concrete and GFRP-SMA confined concrete in the FE model, damaged plasticity model was used. Plasticity models are characterized by three important components, namely, yield criterion, hardening/softening function and flow rule. Yield criterion describes yield surfaces by using stress invariants. When material reaches the first yield surface, plastic deformation is initiated. Hardening/softening of material determines the subsequent yield surfaces after first yield. Flow rule determines the direction and magnitude of plastic deformation. Many studies have been done on developing plasticity models for confined concrete. Oh (2002) calibrated a flow rule and a hardening/softening function based on Drucker-Prager model. Wolf (2008) developed a plasticity model including the effect of concrete damage based on the framework of Malvar’s plasticity model for LS-DYNA3D finite element program (Malvar et al. 1994). Lubliner et al. (1989) proposed a plastic-damage model for concrete to include the effect of stiffness degradation in the nonlinearity of concrete behavior, which was modified by Lee and Fenves (1998) to account for different stiffness degradations in tension and in compression. This model is adopted in ABAQUS as Concrete Damaged Plasticity Model. In the present paper, numerical modeling of SP-1, SP-2 and SP-3 is conducted within the framework of Concrete Damaged Plasticity Model in ABAQUS, with an assumption that the nonlinearity of concrete is due to plasticity only and no stiffness degradation occurs, i.e. damage variable is equal to zero.

3.2.1 Yield criterion

![Fig. 6 SMA constitutive relation in FE model.](image)

![Fig. 7 Finite element mesh for (a) concrete and (b) GFRP jacket.](image)
The yield criterion described in Lubliner et al. (1989) and modified in Lee and Fenves (1998) was adopted in this paper. This yield function reduces to Drucker-Prager yield function when concrete is in triaxial compression (Yu et al. 2010b) as given by the following function:

\[ \sqrt{J_2 + \theta I_1} - k \]  

where, \( J_2 \) and \( I_1 \) are second deviatoric stress invariant and first stress invariant, respectively; \( \theta \) is frictional angle and its calculation was described in Oh (2002); \( k \) is hardening/softening function. Drucker-Prager yield criterion has been widely used by many researchers (Mirmiran et al. 2000; Oh 2002; Yu et al. 2010a) to model the behavior of confined concrete, due to its ability to simulate the effect that concrete shear strength increases as hydrostatic pressure increases. One of the input parameters in the damaged plasticity model in ABAQUS is parameter \( K_c \), which is defined as the ratio of second stress invariant on the tensile meridian to that on the compressive meridian at initial yield for a given first stress invariant (Lubliner et al. 1989). \( K_c \) represents the effectiveness of lateral confining pressure in improving shear strength and it decreases as the effectiveness of confining pressure increases. Different tests revealed different values for \( K_c \) (0.69 from Richart et al. 1928 using active confinement; 0.64 from Schickert and Winkler 1977 using active confinement; 0.725 from Teng et al. 2007 using FRP confinement). A \( K_c \) factor of 0.64 is used for SP-1 and SP-2 based on Schickert and Winkler (1977) test result for active confinement. Although the true behavior of SMA confined concrete is not entirely active, the contribution from active confining pressure is dominant, which will be discussed later and hence a \( K_c \) value corresponding to active confinement is used. However, more experimental work should be done in order to investigate a \( K_c \) factor that can better represent the behavior of SMA confined concrete which is the combination of active and passive confinement. For SP-3 (GFRP-SMA confined concrete), a modified \( K_c \) should be utilized, as a result of the presence of GFRP jacket between SMA hoops and concrete, which reduces the effectiveness of SMA confinement as verified by the experimental results of Shin and Andrawes (2010). In their experiments, SMA confined concrete using SMA spirals with 13 mm pitch spacing resulted in a peak stress of 47.3 MPa, while GFRP-SMA confined concrete using hybrid wraps with 4 layers of GFRP and 13 mm pitch spacing SMA wrapped on the top of the GFRP ended up with a peak stress of 42.8 MPa. Therefore, the introducing of GFRP jacket between concrete and SMA spirals reduced the peak stress by 9.5%. Using trial and error and the experimental results of Shin and Andrawes (2010), a value of 0.85 was adopted for \( K_c \) before GFRP ruptures.

### 3.2.2 Hardening/softening function

Hardening/softening functions feature the yield surfaces evolution as plastic strain increases. Previous experimental results demonstrated that hardening/softening function of confined concrete depends on concrete strength, lateral confining pressure and plastic deformation (Oh 2002; Yu et al. 2010). In the proposed model, the same assumption was adopted so that the hardening/softening function \( k = f(f_s', f_m', \varepsilon_p') \) is a function of current confining pressure \( f_s' \), concrete strength \( f_m' \) and plastic strain \( \varepsilon_p' \). Two series of hardening/softening functions with different confining pressures are calculated using \( k = \sqrt{J_2 + \theta I_1} \) and the triaxial stress status generated by using Mander et al. (1988) model in order to represent the hardening/softening functions for SP-1, SP-2 and SP-3. It is worth noting that in the proposed numerical model, the peak strain and the slope of the descending branch are closely related to the peak strain value proposed by Mander et al. (1988) in Eq. 8.

\[ \varepsilon_c = \varepsilon_{cm}[1 + k_c(f_s'/f_m' - 1)] \]  

where \( \varepsilon_c \) is axial strain; \( \varepsilon_{cm} \) is strain corresponding to unconfined peak concrete stress; \( f_s' \) is unconfined concrete strength; \( f_m' \) is confined concrete peak stress; and \( k_c \) is a constant parameter that can vary between 1.7 and 5 (Chang and Mander 1994). Figure 8 shows the theoretical results of concrete axial stress-strain curves based on Mander et al. (1988)’s model with varying \( k_c \) values and this figure demonstrates that smaller \( k_c \) value results in smaller peak strain and steeper descending branch. In the model proposed by Mander et al. (1988), \( k_c \) was equal to 5. After several trials, \( k_c \) values of 4.5 and 5 were adopted in this study for SMA (SP-1 and SP-2) and GFRP-SMA (SP-3) confined concrete, respectively.

Figure 9 displays the two series of hardening/softening functions normalized by \((3^{0.5} - \theta)\) with different confining pressures \( f_s' \) : \( k_c \) values in Fig. 9(a) are used in the simulation of SP-1 and SP-3, while \( k_c \) values in Fig. 9(b) are used in the simulation of SP-2. The reason that the calculated \( k_c \) values are normalized by \((3^{0.5} - \theta)\) is because this normalized \( k_c \) is the required input in ABAQUS for the damaged plasticity model. Since SP-

![Fig. 8 Theoretical stress-strain curves based on Mander et al. (1988) with varying \( k_c \).](image-url)
1/SP-3 and SP-2 had different unconfined concrete strength and the proposed hardening/softening function is a function of current confining pressures (lateral stresses), concrete strength and plastic strain, the hardening/softening values $k$ in Fig. 9(a) and 9(b) appear to be different when they are plotted as a function of plastic strain in 2-D plots. One should note that the hardening/softening function proposed in this paper is the same for all the specimens and they are all calculated based on the triaxial stresses from Mander et al. (1998)’s model. Isotropic hardening is assumed in the following discussion. Interpolation is adopted among hardening/softening functions with different confining pressures.

### 3.2.3 Flow rule

Flow rule illustrates the plastic strain deformation rule. Flow rule is described by potential flow function. If the plastic potential function is the same as the yield criterion, it is called associated flow rule. Otherwise, it is called non-associated flow rule. Associated flow rule is widely used for metals, but it overestimates plastic expansion of concrete, soil and rock. Therefore, a non-associated flow rule is adopted for concrete plastic flow, which is a function of concrete strength, plastic deformation and lateral confining pressure (Oh 2002; Yu et al. 2010). The following Drucker-Prager type potential function is used in the current model:

$$G = \sqrt{3J_2} + \left(\frac{\tan \gamma}{3}\right)I_1$$

(9)

where $\gamma$ is dilation angle function. In order to describe the potential flow for SP-1, SP-2 and SP-3, two series of confining pressure dependent dilation angle functions are calculated based on axial stress-strain relation from Mander et al. (1988) model, axial-lateral strain relation from Eq. 6, using Prandtl-Reuss equations (Eq. 10) and Eq. 11 (Yu et al. 2010).

$$\varepsilon_p^e = \varepsilon_t - \left(\frac{1-v}{E} \sigma_t - v \sigma_r\right)$$

(10a)

$$\tan \gamma = \frac{3}{2} \frac{\Delta \varepsilon_p^e + 2 \Delta \varepsilon_r^e}{\Delta \varepsilon_r^e - \Delta \varepsilon_p^e} = \sqrt{\frac{3}{2}} \frac{\Delta I_1^p}{\Delta J_2^p} = \sqrt{\frac{3}{2}} \alpha$$

(11)

where,

$$\Delta I_1^p = \Delta \varepsilon_p^e + 2 \Delta \varepsilon_r^e$$

$$\Delta J_2^p = \sqrt{\frac{1}{3} (\Delta \varepsilon_p^e - \Delta \varepsilon_r^e)^2}$$

and $\Delta \varepsilon_p^e$ is incremental axial plastic strain; $\Delta \varepsilon_r^e$ is incremental lateral plastic strain; $\varepsilon_p^e$ is axial plastic strain; $\varepsilon_r^e$ is lateral plastic strain; $\varepsilon_t$ is total axial strain; $\varepsilon_r$ is total lateral strain; $\sigma_t$ is axial stress; $\sigma_r$ is lateral stress; $E$ is elastic modulus and $\nu$ is Poisson’s ratio, and $\alpha$ is the dilation rate, which is defined as the ratio between incremental volumetric plastic strain and incremental deviatoric plastic strain. If $\alpha$ is negative, that means the volume of the concrete specimen decreases (volumetric compaction); if $\alpha$ is positive, that means the volume increases (volumetric dilation). Figure 10 displays the two series of dilation rate functions $\alpha$ : functions in Fig. 10(a) are used in the simulation of SP-1 and SP-3, while functions in Fig. 10(b) are used in the simulation of SP-2. Note that similar to the hardening/softening function, the dilation angle function proposed in this paper is the same for all the specimens and because SP-1/SP-3 and SP-2 had different unconfined concrete strength, the dilation rate functions appear to be different in Fig. 10(a) and 10(b) when they were plotted as a function of plastic strain in 2-D plots. One might notice from Fig. 10 that the $\alpha$ increases steeply at the beginning and the increasing rate becomes much smaller after a transition point. This transition point happens when concrete reaches its peak stress, which indicates that after reaching the peak stress, the plastic strain accumulation rate decreases. Interpolation is adopted among dilation angle functions with different confining pressures.
3.3 Validation of the proposed model

Since the material parameters for the damaged plasticity model proposed in the previous section were calibrated from the stress-strain relation of Mander et al. (1988) and Eq. 6, in this section, numerical simulations are conducted to verify that the proposed material model can closely predict the experimental results of specimens SP-1, SP-2 and SP-3. Figure 11(a) shows a comparison of the stress-strain behavior of SMA confined concrete with spirals spacing 13 mm (SP-1) between the experimental result and numerical prediction from the proposed damaged plasticity model. The proposed model can capture the peak stress and the softening branch of the axial stress-strain curve in an acceptable accuracy. The prediction of the peak stress is 47.2 MPa, while the experimental result is 47.4 MPa (0.4% difference). The residual stress, which is defined as the axial stress of SMA confined concrete right before the failure of the specimen (i.e. SMA hoop fractures), in the proposed model at 0.038 mm/mm is 22.9 MPa, while the
experimental result is 26.2 MPa (12.6% difference). Figure 11(b) shows a comparison of the stress-strain behavior of SMA confined concrete with spirals spacing 25.4 mm (SP-2) between the experimental result and numerical prediction from the proposed damaged plasticity model. Similar to SP-1, the proposed model is able to closely predict the behavior of SP-2. The prediction of the peak stress is 63.4 MPa, while the experimental result is 60.1 MPa (5.2% difference). The residual stress in the proposed model at 0.057 mm/mm is 19.6 MPa, while the experimental result is 16.7 MPa (17.4% difference). Figure 11(c) shows axial stress-strain curve of GFRP-SMA confined concrete (SP-3) test result compared with the proposed damaged plasticity model. The proposed model can capture peak stress and closely predict the rupture of GFRP jacket. The peak stress from the proposed model is 42.82 MPa, while the experimental result is 42.77 MPa. In the proposed model, GFRP ruptures at a concrete axial stress of 34.7 MPa and axial strain of 0.0091 mm/mm; while in the experiment, GFRP ruptured at a concrete stress of 32.6 MPa and a strain of 0.0093 mm/mm. The differences are 6.4% and 2.2%, respectively. After GFRP ruptures, experimental axial stress drops abruptly and increases gradually afterward, while the FE simulation shows a slightly decreasing branch. However, the difference between the experimental and numerical results is deemed minor and the proposed model is found to be capable of closely predicting the behavior of GFRP-SMA confined concrete. In the GFRP-SMA hybrid confinement case, one might notice that after reaching the peak stress, in both the experimental result and the numerical prediction, the axial stress decreased slowly and approximately reached a plateau; while compared to the pure SMA confinement case, the axial stress of concrete dropped faster than the case with GFRP jacket between concrete and SMA spiral. This is attributed to the passive confining pressure developed and applied by both SMA spiral and GFRP jacket as concrete dilates, which can further improve the strength of concrete and compensates the stress degradation due to the cracks development and propagation compared to the pure active confinement case in which the passive confining pressure is only provided by SMA spiral. Furthermore, since hardening/softening function and flow rule are calibrated from a series of actively confined concrete stress-strain and lateral-axial strain curve under different confining pressures, one can conclude that the assumption that each point on the axial stress-strain curve of SMA confined concrete corresponds to a point on the actively confined concrete axial stress-strain curve with the same lateral confining pressure provided by SMA hoops is valid.

3.4 FE Data analysis

Figure 12 compares the numerical average confining pressures applied on specimens SP-1 and SP-3, since they had the same SMA spirals spacing, unconfined concrete strength and used the same batch of SMA, in order to find out the effect of GFRP jacket on the confining pressure. Step 0-1 represents the initialization of active confining pressure (i.e. prestressing of SMA hoops), while step 1-2 represents the application of the axial compressive load. The average active confining pressure in SP-1 model was 1.41 MPa, while in SP-3 model it was 1.34 MPa. This difference is mainly due to the difference in the recovery stresses of SMA in the two models (see Fig. 6). Therefore, the presence of GFRP jacket between SMA wires and concrete does not affect the active confining pressure and Eq. 1 can still be used to calculate the active confining pressure. During axial loading (Step 1-2), passive confining pressure exerted by both GFRP jacket and SMA hoops causes the average confining pressure in the case of GFRP-SMA confined specimen (SP-3) to be greater than that in the case of SMA confined concrete (SP-1) before GFRP jacket ruptures. The maximum average confining pressure in the concrete before GFRP ruptures was 2.96 MPa. After GFRP ruptures, average confining pressure applied on SP-3 reduced to a similar level to that of SP-1 (around 2.4 MPa). Figure 13 shows the evolution of confining pressure of concrete in specimen SP-1 model after active confinement is initiated and while the axial load is being applied. Confining pressure is uniformly distributed in the circumferential direction. Before concrete dilates, surface concrete under SMA hoop exhibits higher confining pressure than core concrete; while confining pressure of surface concrete between SMA hoops appears to be lower than core concrete. According to the FE results, the variation of active confining pressure throughout the height of surface concrete converges to the average value (1.41 MPa) at a point about 12.7 mm from the surface. As the applied axial load increases, confining pressure starts to increase slightly. However, when the confining pressure under SMA hoop becomes large enough to cause crushing of concrete under the hoop, concrete under SMA hoop begins to lose the ability to sustain larger confining pressure from the SMA hoop. Therefore, it shows in Fig. 13(c) that the confining pressure under SMA hoop starts to decrease and in Fig. 13(d) the diametric stress under SMA hoop is...
greater than zero but is lower than tensile strength; while at the same time the confining pressure between SMA hoops is higher than that under SMA hoop. However, the variation of confining pressure on the surface concrete still converges to a certain value as the distance to the surface becomes larger. **Figure 14** displays the evolution of GFRP hoop stress in SP-3 model after active confinement initiation and while the axial load is being applied. It is shown that before axial load is applied, the hoop stress in the GFRP is in compression (negative) due to the prestress from the SMA hoop. As axial load increases, the compressive GFRP hoop stress under the spiral increases at the beginning, because the change of GFRP hoop stress due to concrete lateral expansion (results in tensile GFRP hoop stress) is less than that due to the increase of SMA hoop stress (results in compressive GFRP hoop stress). Then, as concrete starts to dilate rapidly, the tensile hoop stress in the GFRP keeps increasing until GFRP ruptures; after that, the GFRP hoop stress reduces to almost zero. One should notice that the GFRP tensile stress under the SMA hoop appears to be less than that between the SMA hoops, because, without the SMA hoop on top of the GFRP, the expansion of the GFRP between SMA is larger.

### 3.5 Dilation ratio variation

In order to understand behavior of concrete having different confinement types and the passive confining pressure generated due to the dilation of concrete, it is important to investigate the dilation ratio (Poisson’s ratio), which is defined as the ratio of lateral strain to axial strain. Harries and Kharel (2002) studied the dilation ratio of concrete subjected to FRP confinement and claimed that the initial dilation ratio is approximately equal to the elastic Poisson’s ratio, and it increases rapidly as concrete dilates, and reaches a plateau when it exceeds a certain axial strain. Their experimental results revealed that when the confining pressure is low, the dilation ratio can be as high as 2.4. **Figure 15** compares the evolution of dilation ratio from the unconfined con-
crete, SP-1, SP-2 and SP-3 until 0.04 mm/mm axial strain. At the beginning of loading, all four cases start with a dilation ratio of 0.2 and follow a similar path until they reach a dilation ratio of 0.5. Compared to SP-1, SP-2 and SP-3, unconfined concrete expands more quickly, because the confinement from SMA hoops and GFRP jackets can effectively help in delaying the concrete dilation, hence increase the ductility of concrete. Comparing SP-1, SP-2 and SP-3, the dilation ratio in SP-2 is larger than those from both SP-1 and SP-3, and SP-3 is larger than that from SP-1, because first the spirals spacing from SP-2 is twice of those from SP-1 and SP-3, although the initial recovery stress from SP-2 is about 1.4 times of those from SP-1 and SP-3, which results in the dilation in SP-2 is slightly greater than those in SP-1 and SP-3; second, the initial recovery stress in SP-1 is slightly higher than that in SP-3; and third, the presence of GFRP jacket between concrete SMA hoops reduces the efficiency of SMA confinement, which makes the concrete dilates more in SP-3 than that in SP-1. The dilation ratios from all three cases reach plateaus at an approximate value of 2. The validated numerical models were then used in the rest of this paper to investigate important design parameters including the effects of spiral spacing and thickness of GFRP jacket on the overall behavior of confined concrete.

4. Effect of spiral spacing

Numerical simulations were conducted using the validated finite element model to investigate the SMA confined concrete behavior for different hoop spacing. Three spacing values were considered in the study, namely 13 mm, 26 mm, and 39 mm. The same SMA residual recovery stress (447 MPa) was assumed for all three cases. In the SP-1 test, concrete cylinder failure was defined at the moment of SMA hoops fracture, at which concrete axial strain was 0.038 mm/mm. From FE simulation using the proposed model, the axial stress in the SMA hoops at 0.038 mm/mm axial concrete strain was 864 MPa with a corresponding SMA strain of 0.074 mm/mm, which was assumed to be the fracture strain of SMA wires in this study for pure SMA confinement case. Figure 16 compares the axial stress-axial strain and axial stress-lateral strain relationships of SMA confined concrete with different hoop spacing (13 mm, 26 mm and 39 mm) using the validated model. The peak stresses for hoop spacing values of 13 mm, 26 mm and 39 mm are 47.4 MPa, 43.1 MPa and 41.5 MPa, respectively, while the unconfined concrete strength is 39.2 MPa. Therefore, the peak stress of concrete increased by 20.9 %, 9.9 %, and 5.9% using the 13 mm, 26 mm, and 39 mm SMA hoop spacing, respectively. After SMA fractures, the residual axial stresses of concrete for hoop spacing values of 13 mm, 26 mm and 39 mm were 22.9 MPa, 13.0 MPa and 10.2 MPa, respectively. The ability of concrete to maintain high residual stress even at high levels of deformation is an important characteristic to define the ductility of structural systems. From Fig. 16 it can be observed that the axial stress of concrete at the ultimate point (point of SMA fracture) is 48.3%, 30.2% and 24.6% of the peak stress for the cases with hoop spacing of 13 mm, 26 mm, and 39 mm, respectively. The ultimate axial strains for same hoop spacing are 0.0382 mm/mm, 0.0374 mm/mm and 0.0377 mm/mm, respectively. As for the lateral strain, the ultimate average lateral strains of surface concrete are 0.0761 mm/mm, 0.0763 mm/mm and 0.0774 mm/mm, respectively. This shows that changes in SMA hoop spacing exert obvious effect on the peak stress and residual stress of the concrete, while the influence on ultimate axial and lateral strain is relatively minor. Since the new confinement technique is considered a hybrid technique which comprises active and passive confinement pressures, it was essential to shed the light on the percentage of the active and passive pressures in the overall pressure exerted on the concrete. Figure 17 compares the numerical average active, passive and total confining pressures for cases with various hoop spacing at the ultimate point. The average active confining pressures were found to be 1.41 MPa, 0.70 MPa and 0.47 MPa for the cases with 13 mm, 26 mm and 39 mm hoop spacing, respectively. When the hoop spacing of 13 mm is doubled and tripled, the average active confining pressure decreased by 50% and 66.7%, respectively. This is consistent with Eq. 1 which demonstrates that the average
active confining pressure is inversely proportional to the transverse reinforcement spacing. As for the average maximum passive confining pressures induced during loading, they are 1.02 MPa, 0.47 MPa and 0.30 MPa for the cases with hoop spacing of 13 mm, 26 mm and 39 mm, respectively. Again, as the hoop spacing of 13 mm is doubled and tripled, the average maximum passive confining pressure exerted on the concrete prior to SMA hoop fracture reduced by 53.9% and 70.6%, respectively. This reduction is believed to be not only due to the different hoop spacing, but also is attributed to the dilation of concrete and cracking in surface concrete which reduce the amount of confining pressures that concrete can undertake during loading. In addition, the average active confining pressure is 58%, 59.8% and 61% of the average total confining pressure in the cases with 13 mm, 26 mm and 39 mm hoop spacing, respectively. Hence, the contribution of active confining pressure to the total confining pressure is more than that from passive confining pressure in all three cases. As the hoop spacing increases, the contribution of active confining pressure to the total confining pressure also increases slightly. However, average active, passive and total confining pressures are not linearly related to SMA hoop spacing.

5. Effect of FRP thickness

In this section, numerical simulations are conducted to investigate the differences in GFRP-SMA confined concrete behavior for different GFRP jacket thickness. Three thicknesses were considered in the study, namely, 0.44 mm, 0.66 mm, and 0.88 mm. The same SMA hoop spacing of 13 mm and SMA residual recovery stress of 427 MPa were assumed for all three cases. In the SP-3 experimental results, concrete cylinder failure was defined at the moment of SMA wires fracture, at which concrete axial strain was 0.04 mm/mm. From FE simulation using the proposed model, the axial stress in the SMA hoops at 0.04 mm/mm axial concrete strain was 862 MPa with a corresponding fracture strain of 0.08 mm/mm, which was assumed as the fracture strain of SMA hoops in this study for GFRP-SMA hybrid confinement case. Note that the actual fracture strain of SMA spirals is different from the fracture strain that is obtained from a uniaxial tensile coupon test, due to the curvature of the concrete cylinder, the friction between the SMA spirals and GFRP jacket, and the geometry imperfection of the specimens. Therefore, 0.08 mm/mm was assumed in this paper to represent the facture strain of SMA spirals that has already taken into account all these above mentioned effects. Figure 18 compares the axial stress-axial strain and axial stress-lateral strain relationships of GFRP-SMA confined concrete with different GFRP jacket thickness values of 0.44 mm, 0.66 mm and 0.88 mm using the validated model. The GFRP thickness does not affect the elastic branch of the stress-strain relation since the SMA hoop spacing and SMA residual recovery stress are the same for all three cases. The peak stresses for specimens with different GFRP jacket thickness values of 0.44 mm, 0.66 mm and 0.88 mm are 42.8 MPa, 43.4 MPa and 45.3 MPa, respectively, while the unconfined concrete strength is 39.2 MPa. Thus, the peak stress of concrete increased by 9.2%, 10.7% and 15.6%, respectively for the three cases. The axial stress in concrete when GFRP jacket ruptures for different GFRP jacket thickness of 0.44 mm, 0.66 mm and 0.88 mm are 32.6 MPa, 34.2 MPa and 34.3 MPa

![Graph comparing active, passive, and total confining pressures at SMA fracture for different hoop spacings](image1.png)

**Fig. 17** Comparison of average active, passive and total confining pressure at SMA fracture for different hoop spacing using FE simulation.

![Graph comparing axial stress vs. axial and lateral strain for different GFRP jacket thickness](image2.png)

**Fig. 18** Numerical axial stress vs. axial and lateral strain relations for GFRP-SMA confined concrete with different GFRP jacket thickness.
respectively. The residual axial stresses at the ultimate point (i.e. when SMA hoops fracture) for GFRP jacket thickness of 0.44 mm, 0.66 mm and 0.88 mm are 22.6 MPa, 22.7 MPa and 22.7 MPa. As for the lateral strain, the ultimate average lateral strains of surface concrete are all 0.08 mm/mm, which is consistent with the rupture strain of SMA hoops. For the axial ultimate strain, all three cases show a strain of 0.04 mm/mm. From the figure it is clear that peak stress increase with the increase of GFRP thickness. However, the residual stress appears to be identical for all three cases, because after GFRP jacket ruptures, SMA hoop spacing and SMA residual recovery stress are the dominant factors that affect the stress-strain behavior. Since the SMA hoop spacing and SMA residual recovery stress are the same for all three cases, it is reasonable that the residual stresses in all three cases are very similar. To assess the percentage of active versus passive confinement applied on the concrete in each case, Fig. 19 compares the numerical average active, maximum passive (i.e. right before GFRP fracture) and ultimate passive (i.e. at the fracture of SMA hoops) confining pressures for the three studied cases. The average active confining pressures are found to be all equal to 1.34 MPa for GFRP thickness of 0.44 mm, 0.66 mm and 0.88 mm. This shows that the thickness of GFRP jacket does not affect the average active confining pressure. For the maximum passive confining pressure induced during loading, as the thickness of the GFRP increases from 0.44 mm to 0.88 mm, the maximum passive pressure increases by 110%. The figure also shows that the thickness of the GFRP jacket has little effect on the ultimate passive confining pressure, which is primarily controlled by the SMA hoop spacing and SMA residual recovery stress after heating. It is worth noting that the active confining pressure is 45.3%, 33.4% and 28.0% of the maximum total confining pressure (right before GFRP fracture) in the cases with 0.44 mm, 0.66 mm and 0.88 mm GFRP thickness, respectively; while it is 55.6%, 55.4% and 55.4% of the ultimate total confining pressure (at SMA hoops fracture), respectively. Therefore, in the pre-GFRP fracture phase the contribution of active confinement to the total confinement is less than that from passive confinement and it decreases as GFRP thickness increases.

6. Conclusions

This study focused on analyzing the behavior of SMA confined concrete and GFRP-SMA confined concrete using FEM within the framework of damaged plasticity model in ABAQUS. The results show that the proposed damaged plasticity model can closely predict the stress-strain relation of SMA confined concrete and GFRP-SMA confined concrete, including peak stress, ascending/descending branch and the rupture of GFRP jacket. The numerical results for peak stress of SMA confined concrete, SP-1 and SP-2, and GFRP-SMA confined concrete, SP-3, were 0.4%, 5.2% and 0.1% different from the experimental results. The numerical GFRP rupture stress and strain for the GFRP-SMA confined concrete case were 6.4% and 2.2% different from the experimental results, respectively. The FE analysis showed that the confining pressure of concrete was uniformly distributed in the circumferential direction, but varied through the height on surface concrete. However, the variation along the height converged to a constant value as the distance from surface concrete increased. The FE analysis also shed the light on the variation of concrete dilation ratio with loading. Initially, dilation ratio was approximately equal to the elastic Poisson’s ratio (about 0.2); however, it increased rapidly with axial loading until it reached a plateau at a value of 2. The parametric study which was carried out to investigate the effect of hoop spacing on SMA confined concrete and the effect of GFRP thickness on GFRP-SMA confined concrete revealed that smaller hoop spacing results in higher active confining pressure, higher concrete peak stress and higher residual stress for SMA confinement. For the range of 13-39 mm hoop spacing, the variation of peak stress and residual strength were 12.4% and 55.5%, respectively. As hoop spacing increases from 13 mm to 39 mm, the active to passive confining pressure ratio increased from 1.38 to 1.61. The study also showed that for GFRP-SMA confinement, peak concrete stress increases as GFRP jacket thickness increased, but residual concrete strength remained the same when the hoop spacing and residual recovery stress were the same. This proves that the thickness of GFRP jacket has little effect on the average active confining pressure and average ultimate confining pressure, but affects the average maximum passive confining pressure. As GFRP thickness increased from 0.44 mm to 0.88 mm, the average maximum passive confining pressure increased by 110%. Finally, it is worth noting that the hardening/softening function and flow rule adopted in the FE models were both developed based on Mander et al. (1988) model, which assumes purely actively confined concrete, however, SMA.
confinement is a combination of both active and passive confinement. Therefore, more experimental work is needed to develop a more generalized model for SMA confined concrete.

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