Journal of Advanced Concrete Technology

Materials, Structures and Environment

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Scientific paper

Elastoplastic-Damage Compression Constitutive Model for Cementitious Material Subjected to Alkali-Silica Reaction

Yingdi Liao¹; Da Chen²; Zihan Liu³; Feng Ouyang⁴ and Lijun Hou⁵

Received 13 November 2013, accepted 18 May 2014

doi:10.3151/jact.12.158

Abstract

In this paper, a multi-axial compression constitutive model of cement-based material subjected to combined alkali-silica reaction (ASR) and mechanical loading is proposed based on the plastic and damage theories. In this model, the elastoplastic behavior is coupled with the mechanical damage due to external loading and chemical damage due to ASR expansion. The linear swelling rate of prism specimen is used to represent the ASR damage. A modified plastic yielding criterion and plastic potential function are introduced, taking into account the ASR damage on compression and tension properties. The proposed model is further verified by the tested stress-strain relationship under different confining pressures reported in a companion paper, and a good agreement is obtained.

1. Introduction

The alkali-silica reaction (ASR) in concrete is a chemical reaction between the silica-rich aggregate and the alkali in pore solution. It is a type of chemical corrosion which results in the deterioration of concrete durability. It is generally accepted that the swelling action of the ASR is derived from the hydrated alkali-silica gel, but currently there is no acceptable uniform swelling mechanism (Bangert *et al.* 2004). Dron and Brivot (1993) hypothesized that the fluid alkali-silica gel diffused into the pores near the aggregate and further reacted with Ca ions, resulting in the expansion and cracking of concrete. Bažant and Steffens (2000) thought that the alkali-silica gel was confined by hardening cement paste around the aggregate, and the increasing compressive stress led to the cracking of the cement paste and concrete. However, Ichikawa and Miura (2007) proposed a model in which a reaction rim, more rigid and denser than the cement paste and aggregate, formed around the reactive aggregate due to the reaction between alkali-silica gel and Ca ions dissolved in the pore solution. This reaction rim prevented the cohesive alkali-silica gel from passing through, whereas the alkaline solution could still freely diffuse into the aggregate. With the development of the

ASR, the cracking of cement paste and aggregate occurred. Moreover, Dunant and Scrivener (2010) found the gel pocket expansion is presented in the reactive aggregates through the microscopic observation, and is responsible for the damage and cracking of concrete. Further, based on the gel pocket expansion mechanism, a micro-mechanical model was proposed to simulate the ASR expansion with the reaction fraction and the induced degradation of mechanical properties. Ponce and Batic (2006) presented that the mineralogy and fabric of the rocks involved are responsible for different manifestations of the ASR. For concrete containing rapid-reacting aggregates, the ASR-induced reactive rim and “dissolution texture” due to the dissolution, reaction, displacement and loss of the binding material are responsible for the cracking inside the aggregates and cement paste. However, for concrete containing slow-reacting aggregates, the ASR occurs only at some local sites, and the reaction texture is merely related with the internal cracks in the aggregate. This crack is filled with the gel from the ASR product inside the aggregate and no reaction rims can be observed.

Although the characteristics of the swelling mechanism are not clear, it goes without saying that the ASR only takes place under conditions with adequate humidity, alkaline, and reactive aggregate. The review on ASR by Lindgård *et al.* (2012) indicated that the most important influence factors in lab measure of ASR are the internal humidity in the test specimens, the extent of alkali leaching and the storage temperature. Ahmed *et al.* (2003) found that the mechanical properties decreased depending on the alkali reactivity of aggregate. The experiment study carried out by Multon and Toutlemonde (2006) showed that ASR volumetric imposed strain can be considered as constant regardless of the applied stress state and ASR-induced strain can be transferred in the less compressed direction when subjected to different stress level in different direction. Dunant and Scrivener (2012) further revealed that the

¹Ph.D, Key Laboratory of Coastal Disaster and Defence, Ministry of Education, Hohai University, Nanjing, China; and College of Harbour, Coastal and Offshore Engineering, Hohai University, Nanjing, China.

²Professor, College of Harbour, Coastal and Offshore Engineering, Hohai University, Nanjing, China.

³Master candidate, College of Harbour, Coastal and Offshore Engineering, Hohai University, Nanjing, China.

⁴Ph.D candidate, College of Harbour, Coastal and Offshore Engineering, Hohai University, Nanjing, China.

⁵Ph.D, College of Harbour, Coastal and Offshore Engineering, Hohai University, Nanjing, China.

E-mail: ljhou_hhu@126.com

expansion of ASR reactive concrete under the uniaxial stress was related to the orientation and development of micro-cracks in the aggregates and paste. In addition, the following methods were demonstrated to be effective in relieving the passive effects of ASR: e.g. the reduction of alkaline and relative humidity; the addition of silica fume, fly ash, slag, pozzolana materials etc.; the use of fiber; and combinations of these measures Xu *et al.* 1995; Song *et al.* 2008; Yi and Ostertag 2005; Yazıcı 2012).

It is noted that the swelling induced by ASR is one of major factors considered in reactive cementitious material. Based on chemical and damage mechanics theories, a series of models was proposed to predict the expansion with the evolution of ASR (Poyet *et al.* 2007; Multon *et al.* 2009; Bazant and Steffens 2000). In these models, the parameters considered mainly included alkali and reactive silica contents, aggregate sizes, mechanical properties of the mortar, temperature and humidity, etc. Further, Dunant and Scrivener (2012) and Comi *et al.* (2009) took into account the effect of applied stress on the ASR swelling, and further proposed corresponding prediction models. Moreover, for experimentally examining the evolution of volume expansion with ASR, Scanning Electron Microscopy Image Analysis (SEM-IA) was used by Haha *et al.* (2007) to quantify the degree of alkali silica reaction in affected cementitious prisms and the degree of reaction showed a good correlation with the macroscopic expansion for three types of different aggregates studied.

On the other hand, the influence of ASR swelling on the mechanical properties is another major consideration in durability design of engineering structures. The studies conducted by Ahmed *et al.* (2003), Multon *et al.* (2008), Giaccio *et al.* (2008), and Larive *et al.* (1996) indicated that the degree of reactivity and the aggregate size had an effect on the mechanical properties of cement-based material. However, the effect of expansion due to ASR differed for different mechanical properties. The tensile strength, flexural strength and elastic modulus appeared to reduce with the development of ASR. However, the compressive strength generally showed a smaller reduction than the stiffness, on some occasions presenting no reduction or an increment. Furthermore, Pignatelli *et al.* (2013) proposed a coupled mechanical and chemical damage model for concrete affected by ASR which can be used to predict the mechanical behavior of concrete under uniaxial compression load.

The study by Yurtdas *et al.* (2013) showed that no matter whether to be subjected to ASR, the multiaxial mechanical behaviors were closely related to the plastic deformation, particularly under a high confining pressure. Therefore, it is necessary to consider the coupled elastoplastic and damage effect on the mechanical behavior of cement-based materials, where the damage includes the mechanical damage due to external load and the chemical damage due to the ASR swelling. In the present paper, an elastoplastic damage constitutive model for

compression of cementitious material is proposed, and is further verified through the experimental investigation presented in a companion paper (Yurtdas *et al.* 2013).

2. Elastoplastic damage constitutive model

In the case of mortar made of alkali reactive aggregate, the chemical damage occurs due to the swelling induced by ASR. In addition, the mortar subjected to a compression load shows a plastic deformation accompanied by the mechanical damage in the meantime. Therefore, the combined chemical and mechanical damage both affect the elastic and plastic properties of materials. It is noted that with the increase in confining pressure, the plastic deformation, ultimate strength and strain etc., are enhanced greatly, presenting more ductile characteristics under a triaxial compression.

In this section, an elastoplastic damage constitutive model based on the laws of thermodynamics and plastic theory is proposed to describe the mechanical behavior of cement-based materials subjected to damage. In this model, the increment elastoplastic constitutive model is used to represent the influence of loading history on the mechanical behavior of mortar. Further, the damage due to ASR is coupled with the mechanical damage in this proposed model, to predict the stress-strain relationship of cement-based materials subjected to ASR.

The small deformation assumption is adopted in the following analysis. Both the strain $\underline{\underline{\varepsilon}}$ and strain increment $d\underline{\underline{\varepsilon}}$ of materials are composed of elastic and plastic parts:

$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^e + \underline{\underline{\varepsilon}}^p, \quad d\underline{\underline{\varepsilon}} = d\underline{\underline{\varepsilon}}^e + d\underline{\underline{\varepsilon}}^p \quad (1)$$

where $\underline{\underline{\varepsilon}}^e$ and $\underline{\underline{\varepsilon}}^p$ denote the elastic strain tensor and plastic strain tensor, respectively; and $d\underline{\underline{\varepsilon}}^e$ and $d\underline{\underline{\varepsilon}}^p$ are the elastic strain increment and plastic strain increment, respectively.

2.1 Thermodynamics framework

The laws of thermodynamics are the basis of the proposed elastoplastic damage constitutive model. It is assumed that the damage of concrete is isotropic (Poyet *et al.* 2007). The damage consists of tensile and compression damages, whose mechanisms and effects are different from each other. The tensile damage results from the increase in tensile strain and propagation of cracks, whereas the compression damage is related to frictional sliding along the closed crack surface. In addition, the degraded elastic stiffness under tension loading may be partially or totally recovered, due to the closed cracks when subjected to the compression loading. Based on these points, two damage variables, the tension damage variable ω_t and the compression damage variable ω_c , are introduced to represent this phenomenon. The total damage ω of material is then obtained by:

$$\omega = (1 - \alpha_t)\omega_c + \alpha_t\omega_t \quad (2)$$

where α_i is a parameter related to the stress state, and is further defined as:

$$\alpha_i = \frac{\|\sigma^+\|}{\|\sigma\|} \quad (3)$$

where σ^+ represents the positive cone of stress tensor σ , obtained from the spectral decomposition based on eigenvalues and vectors (Pietruszczak *et al.* 1988). In particular, $\alpha_t=1$ represents the pure tensile stress state, and $\alpha_c=0$ denotes the pure compressive stress state.

From the first law of thermodynamics, the thermodynamic energy is equal to the sum of the elastic and plastic free energy. Thus the free energy of materials at constant temperature can be expressed as follows:

$$\Psi(\underline{\varepsilon}^e, \omega, \gamma_{ps}) = \Psi_e(\underline{\varepsilon}^e, \omega) + \Psi_p(\gamma_{ps}, \omega) \quad (4)$$

where Ψ_p represents the plastic energy of materials, given mechanical damage and plastic strain. Plastic strain is a function of the plastic hardening parameter γ_{ps} and mechanical damage ω . Ψ_e denotes the elastic free energy under given mechanical damage and elastic strain, and can be written as follows:

$$\begin{aligned} \Psi_e(\underline{\varepsilon}^e, \omega) &= \frac{1}{2} \underline{\varepsilon}^e : \mathbf{C}(\omega) : \underline{\varepsilon}^e \\ &= \frac{1}{2} (\underline{\varepsilon} - \underline{\varepsilon}^p) : \mathbf{C}(\omega) : (\underline{\varepsilon} - \underline{\varepsilon}^p) \end{aligned} \quad (5)$$

where $\mathbf{C}(\omega)$ is the fourth-order elastic coefficient tensor considering the damage effect on elastic parameters, and can be written as in Eq. (6), based on Hill's lemma.

$$\mathbf{C}(\omega) = 3k(\omega)\mathbf{J} + 2\mu(\omega)\mathbf{K} \quad (6)$$

where $\mathbf{J} = \frac{1}{3} \delta_{ij} \delta_{kl}$ and $\mathbf{K} = \mathbf{I} - \mathbf{J}$, in which $\mathbf{I}$ is the fourth-order identity symmetric tensor $\mathbf{I} = \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$.

$k(\omega)$ and $\mu(\omega)$ represent the volume modulus and shear modulus of damaged cementitious material, respectively, which can be defined by:

$$k(\omega) = [1 - H(\varepsilon_v)\omega]k_0, \quad \mu(\omega) = (1 - \omega)\mu_0 \quad (7)$$

where k_0 and μ_0 denote the initial volume and shear modulus, respectively. $H(x)$ is the Heaviside function: $H(x)=0$ for $x \leq 0$, $H(x)=1$ for $x > 0$.

The constitutive relationship of cement-based material subjected to mechanical damage is defined as:

$$\underline{\underline{\sigma}} = \frac{\partial \Psi}{\partial \underline{\underline{\varepsilon}}} = \mathbf{C}(\omega) : (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) \quad (8)$$

Then, the incremental constitutive model is further obtained by:

$$d\underline{\underline{\sigma}} = \mathbf{C}(\omega) : (d\underline{\underline{\varepsilon}} - d\underline{\underline{\varepsilon}}^p) + \frac{\partial \mathbf{C}(\omega)}{\partial \omega} : (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) d\omega \quad (9)$$

In the irreversible thermodynamics framework, the damage evolution is controlled by the damage criterion, which is determined by an energy dissipation potential based on the thermodynamics. In general, the scalar-valued conjugated damage force Y_t and Y_c , for damage variables ω_t and ω_c , respectively, are established by:

$$Y_t = \frac{\partial \Psi}{\partial \omega_t}, \quad Y_c = \frac{\partial \Psi}{\partial \omega_c} \quad (10)$$

However, it is difficult to make an analytical solution of Eq. (10). As highlighted previously, the tension damage is related to the tensile strain, and the compression damage is associated with frictional sliding between the crack surfaces. Accordingly, two equivalent deformations are introduced to represent the conjugated damage force in tension and compression:

$$Y_t = \max(\varepsilon_{eq}^t, Y_0^t), \quad \varepsilon_{eq}^t = \sqrt{\sum_{i=1}^3 \langle \varepsilon_i \rangle^2} \quad (11)$$

$$\begin{aligned} Y_c &= \max(\varepsilon_{eq}^c, Y_0^c), \quad \varepsilon_{eq}^c = \sqrt{\frac{2}{3} e_{ij} e_{ij}}, \quad e_{ij} \\ &= \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \end{aligned} \quad (12)$$

where Y_0^c and Y_0^t are the compression and tension damage threshold values during loading, respectively. Referring to the investigation by Mazars (1984), the tension and compression damage can be taken into consideration through the following damage criteria:

$$f_{\omega_t} = \omega_t - \beta_t \left(1 - \frac{1}{\exp(B_t Y_t)} \right) = 0 \quad (13)$$

$$f_{\omega_c} = \omega_c - \beta_c \left(1 - \frac{1}{\exp(B_c Y_c)} \right) = 0 \quad (14)$$

where parameters β_c and β_t are the maximum damage value of compression and tension, respectively; and B_c and B_t are parameters controlling the compression and tension damage evolution, respectively.

2.2 Plastic characteristics

It is generally accepted that plastic behavior is closely related to material damage. To effectively describe the plastic development process, it is indispensable to establish the plastic yielding surface, failure surface, plastic hardening criterion, and plastic potential function. It is well known that both the yielding surface and failure surface of most frictional material like mortar are closely related to the confining pressure (Chen *et al.* 2007). With the increase in confining pressure, the mortar strength gradually rises and the failure is greatly delayed. Therefore, for the mortar under a confining pressure, a curved surface yielding criterion seems to be more applicable than the linear Mohr-Coulomb criterion and Drucker-Prager criterion. Based on the yielding criterion

proposed by Pietruszczak *et al.* (1988), the following yielding criterion in the p - q plane is used:

$$f_s(\underline{\sigma}, \gamma_{ps}, \omega) = q + \alpha_s(p - C_s) \left(\frac{A}{1 - B \frac{p}{p_r}} \right) = 0 \quad (15)$$

where α_s is the plastic hardening function; p is the hydrostatic pressure, $p = \frac{1}{3} \sigma_{kk}$; q is the equivalent deviatoric stress, $q = \sqrt{\frac{3s_{ij}s_{ij}}{2}}$, where $s_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$; C_s

is the tensile strength under hydrostatic pressure, which is defined by the intersection between the failure surface and the p axis; parameters A and B control the curvature of the failure surface; and p_r is the reference pressure, and is taken as 1 MPa to ensure the non-dimensionalization of parameters A and B . When parameter B is equal to 0, Eq. (15) is degraded to be the linear Drucker-Prager criterion.

In this model, it is assumed that the softening characteristic of concrete depends on the damage development due to cracks, and thus the deviatoric stress loading surface is influenced by damage. According to analysis of the experimental data, the plastic hardening function α_s can be expressed by:

$$\alpha_s = (1 - \omega) \bar{\alpha}_s = (1 - \omega) \left[1 - (1 - \alpha_s^0) e^{-b_1 \gamma_{ps}} \right] \quad (16)$$

where α_s^0 is the initial plastic hardening index; and b_1 is the plastic parameter, reflecting the influence of confining pressure on the plastic hardening rate. γ_{ps} is the generalized plastic deformation, which is defined by:

$$\gamma_{ps} = \int d\gamma_{ps}, \quad d\gamma_{ps} = \sqrt{\frac{2}{3} d\epsilon^p : d\epsilon^p} \quad (17)$$

The plastic hardening parameter α_s has values in the range of α_s^0 to 1.0. When $\alpha_s = 1.0$, the plastic yielding surface f_s reaches the failure surface.

In the case of quasi-brittle materials, the triaxial compression test (Yurtdas *et al.* 2013) indicated that the specimen volume during loading presents a transition, from plastic compression in the initial stage to expansion with a larger load. Herein, a plastic flow law is introduced in the present model to take this transition into account. However, an associated flow criterion cannot reflect this plastic evolution. Therefore, a non-associated plastic potential function is applied, based on the plastic model proposed by Pietruszczak *et al.* (1988):

$$Q_s(\underline{\sigma}, \gamma_{ps}, \omega) = q + \mu_s(p - C_s) \ln \left(\frac{p - C_s}{I_0} \right) = 0 \quad (18)$$

where I_0 is defined as the interaction between the plastic potential surface and the p axis in the p - q coordinate

system; and μ_s is the slope of the transition curve from plastic compression to expansion, which can be obtained from the test data. When $\partial Q_s / \partial p = 0$, the plastic volume compression begins to transform to volume dilatancy. It is found, from experimental data, that this transition surface can be simplified as a linear function:

$$\bar{f}_{cs} = q - \mu_s(p - C_s) = 0 \quad (19)$$

The plastic strain increment can be obtained based on the non-associated plastic flow criterion shown below:

$$d\epsilon^p = d\lambda_s \frac{\partial Q_s}{\partial \underline{\sigma}} \quad (20)$$

where the plastic factor $d\lambda_s$ can be obtained through plastic consistent conditions,

$$d\lambda_s = \frac{\frac{df_s}{d\underline{\sigma}} : \underline{C} : d\underline{\epsilon}}{\frac{df_s}{d\underline{\sigma}} : \underline{C} : \frac{dQ_s}{d\underline{\sigma}} - H_s} \quad (21)$$

where H_s denotes the plastic hardening modulus associated with the plastic shear mechanism, and is defined by:

$$H_s = \frac{\partial f_s}{\partial \alpha_s} \frac{d\alpha_s}{d\gamma_{ps}} = \sqrt{\frac{2}{3}} \left(\underline{K} : \frac{\partial Q_s}{\partial \underline{\sigma}} \right) : \left(\underline{K} : \frac{\partial Q_s}{\partial \underline{\sigma}} \right), \quad (22)$$

and $\frac{\partial Q_s}{\partial \underline{\sigma}}$ and $\frac{\partial f_s}{\partial \underline{\sigma}}$ are obtained from Eq. (15) and (18).

$$\begin{aligned} \frac{\partial f_s}{\partial \underline{\sigma}} &= \frac{\partial f_s}{\partial q} \frac{dq}{d\underline{\sigma}} + \frac{\partial f_s}{\partial p} \frac{dp}{d\underline{\sigma}} \\ &= \frac{3}{2q} s + \frac{1}{3} \alpha_s P_r A \frac{P_r - BC_s}{(P_r - BP)^2} \delta \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{\partial Q_s}{\partial \underline{\sigma}} &= \frac{\partial Q_s}{\partial q} \frac{\partial q}{\partial \underline{\sigma}} + \frac{\partial Q_s}{\partial p} \frac{\partial p}{\partial \underline{\sigma}} \\ &= \frac{3}{2q} s + \frac{1}{3} \mu_s \left[1 + \ln \left(\frac{p - C_s}{I_0} \right) \right] \delta \end{aligned} \quad (24)$$

2.3 Coupled plasticity and ASR damage

It is well known that, if the aggregate used in concrete shows high alkali reactivity, then the ASR will take place in an alkaline concrete environment. This chemical reaction results in a volume expansion, and the effect becomes more severe with the development of the reaction. The linear expansion rate seems to be applicable to enable evaluation of the degree of ASR. In the process of ASR, the material damage increases gradually due to the cumulated expansion effect. To represent this damage phenomenon, a new chemical damage variable, ω_g , is introduced:

$$\omega_g = \bar{\omega}(1 - e^{-bt}) \quad (25)$$

where $\bar{\omega}$ is the critical damage value; ω_g denotes the expansion damage with the ASR development; parameter b controls the development speed of the expansion, which can be obtained through regression analysis of the tested expansion-reaction time evolution curve; and t is the reaction time. The change of $\bar{\omega}$ and b results in the difference in the maximum damage level and its development rate. These two parameters are both closed related with the aggregate reactivity, environment condition and alkalinity of concrete.

The strength of the material without damage is taken as the reference value to define the strength at different stages of the ASR. Assume that the ASR damage has a negative effect on the tensile and compressive strength. Based on this, the related parameters in the plastic model for concrete subjected to ASR can be expressed by:

$$C_s = C_s^0(1 - \omega_g), \quad A = A_0(1 - \omega_g) \quad (26)$$

where C_s^0 denotes the initial tensile strength before ASR; C_s represents the tensile strength after ASR; and A_0 and A are material parameters associated with the compressive strength before and after ASR, respectively. Substituting parameters C_s and A of Eq. (26) into Eq. (15), the plastic yielding criterion coupled with ASR damage is further recast as

$$f_s(\underline{\sigma}, \gamma_{ps}, \omega, \xi) = q + \alpha_s \left[p - C_s^0(1 - \omega_g) \right] \left(\frac{A_0(1 - \omega_g)}{1 - B \frac{p}{p_r}} \right) = 0 \quad (27)$$

Likewise, the plastic potential function is also rewritten.

$$Q_s(\underline{\sigma}, \gamma_{ps}, \omega) = q + \mu_s \left[p - C_s^0(1 - \omega_g) \right] \ln \left(\frac{p - C_s^0(1 - \omega_g)}{I_0} \right) = 0 \quad (28)$$

2.4 Computation procedure

Iterative computation, including elastic prediction and plastic correction, is used at each strain increment step based on the proposed elastoplastic chemical damage model above. In this model, mechanical damage is merely related to the total strain, which is calculated using the strain obtained in the previous step and the strain increment in the current step. Chemical damage ω_g can be obtained by Eq. (25) at the different ASR period. For the k^{th} step, the general computation procedure is presented as follows:

- (1) At the end of the $(k-1)^{\text{th}}$ step, the following parameters are obtained: stress $\underline{\sigma}^{(k-1)}$, total strain $\underline{\varepsilon}^{(k-1)}$, plastic strain $\underline{\varepsilon}^{p(k-1)}$, plastic hardening variable $\gamma_{ps}^{(k-1)}$, damage variable $\omega^{(k-1)}$ and chemical

damage ω_g . The strain increment input $\Delta \underline{\varepsilon}^{(k)}$ is given in the current step;

- (2) Update the total strain, through $\underline{\varepsilon}^{(k)} = \underline{\varepsilon}^{(k-1)} + \Delta \underline{\varepsilon}^{(k)}$, and then calculate the nominal strain $\varepsilon_{eq}^{t(k)}, \varepsilon_{eq}^{c(k)}$. Based on the proposed damage criterion, compute the corresponding damage variables $\omega_t^{(k)}, \omega_c^{(k)}$;
- (3) Assuming the strain increment is fully elastic, calculate the stress increment and total stress in this loading step through these equations: $\Delta \underline{\sigma}^{(k)} = \mathbf{C}(\omega^{(k)}) : \Delta \underline{\varepsilon}^{(k)}$ and $\underline{\sigma}^{(k)} = \underline{\sigma}^{(k-1)} + \Delta \underline{\sigma}^{(k)}$;
- (4) Verify whether the plastic yielding relationship, $f_s(\underline{\sigma}^{(k)}, \gamma_{ps}^{(k-1)}) > 0$, is satisfied.

If satisfied, the plastic yielding occurs and the plastic correction needs to be conducted. The iterative calculation is utilized to determine the corresponding plastic strain increment $d\underline{\varepsilon}^{p(k)}$. Then, the incremental plastic hardening parameter $d\gamma_{ps}$ is calculated via Eq. (17), and the stress increment $\Delta \underline{\sigma}^{(k)}$ is obtained by Eq. (9);

Else, the plastic yielding does not occur; proceed directly to step (5);

- (5) Update the corresponding variables for the current step.

3. Experiment verification on proposed model

In this section, we verify the proposed elastoplastic damage constitutive model by using the experimental study presented in a companion paper (Yurtdas *et al.* 2013). For this end, it is needed to make a simply summarization on the experimental program and results. The detailed information can be referred to in the previous investigation (Yurtdas *et al.* 2013). Then, the general methodology for the determination of parameters used in the proposed model is outlined. Finally, the real verification of a model is to compare the predictions of the model with the experimental data.

3.1 Experimental program and results

In Yurtdas *et al.* (2013) study, the mortar with proportions of sand:cement:water=3:1:0.5 was tested to investigate the influence of ASR on material properties. Non-reactive and reactive sand were utilized to make two types of mortar, named non-reactive motor (NR) and reactive motor (R), respectively. After 28 days of standard curing with relative humidity (HR) above 95% and a temperature of $20 \pm 2^\circ\text{C}$, a part of specimens was further cured in a box with constant temperature of 60°C and HR of 95% for 100 days to accelerate the ASR. The specimen subjected to accelerated ASR was labeled 'A', while the standard curing specimen was named 'B'. Therefore, the mortar specimen that was made of reactive sand and subjected to accelerated ASR for 100 days was denoted by RA. Likewise, the other type of specimen was labeled RB, NRA and NRB, respectively.

Fig. 1 Relationship between linear expansion rate and reaction time.

Fig. 2 Tested uniaxial stress-strain curves.

The compression tests of two sets of cylindrical samples with diameter 36 mm×72 mm were carried out on a triaxial rheological test machine. The confining pressure was the varied parameter, and was set to values of 0 MPa, 5 MPa, and 15 MPa. One set of samples was subjected to accelerated ASR for 100 days as described previously, whereas the other set was only cured for 28 days under standard curing conditions. The linear expansion test was performed on prism specimens with dimensions 40 mm×40 mm×160 mm. The reference length of specimens was measured at 28 days of standard curing. And during accelerated ASR, the specimen length was then measured every 6~7 days to evaluate the evolution of the linear expansion rate.

Figure 1 shows the change of the linear expansion rate with the increase in reaction time. It is found from **Fig. 1**

Fig. 3 Tested deviatoric stress-strain curves under confining pressure of 15 MPa.

that sample RA shows apparent expansion deformation, as expected, and the maximum expansion rate is high up to 0.38%. As shown in **Fig. 1**, the expansion deformation increased rapidly in the first 25 days, showing an active ASR in the initial period. However, when the reaction time was in excess of 50~60 days, the expansion rate became almost stable.

Figure 2~3 show the stress-strain curves obtained under confining pressures of 0 MPa and 15 MPa in reference (Yurtdas *et al.* 2013). The applied confining pressure greatly improved some mechanical behaviors, including the ultimate strain, the peak stress and corresponding strain, and strain-softening behavior in the descending branch of constitutive curves. **Table 1** further summarizes the experimental results reported in Yurtdas *et al.* (2013) study. As shown in **Table 1**, the higher is the confining pressures, the more apparent is the strength improvement. In detail, with the increasing confining pressure from 5 MPa to 15 MPa, the deviatoric strength increased by about 44%~105% for R series and by 41%~110% for NR series.

As shown in **Table 1**, the compressive strength increased by 6%~14% for NR series samples, mainly due to the continued hydration of cement. However, the comparison of compressive strength between RA and RB series reveals that an improvement of 25%~42% is obtained for RA series after the accelerated ASR for 100 days. The experimental results reported by Ahmed *et al.* (2003) and Giaccio *et al.* (2008) indicated that the compressive strength of mortar subjected to ASR may show an increase to some extent compared with 28-day

Table 1 Summarization of tested results.

	f_{c0} /MPa	f_{c5} /MPa	f_{c15} /MPa	f_{c5}/f_{c0}	f_{c15}/f_{c0}
RA	54.30	78.10	98.86	1.44	1.82
RB	38.30	56.59	78.60	1.48	2.05
f_{RA}/f_{RB}^*	1.42	1.38	1.26	—	—
NRA	36.25	51.87	75.96	1.43	2.10
NRB	41.34	58.32	80.43	1.41	1.95
f_{NRA}/f_{NRB}^*	1.14	1.12	1.06	—	—

Note: f_{c0} , f_{c5} and f_{c15} denote the compressive strength at confining pressure of 0 MPa, 5 MPa and 15 MPa, respectively; f_{RA}/f_{RB} denotes the ratio of compressive strength between RA and RB series; f_{NRA}/f_{NRB} denotes the ratio of compressive strength between NRA and NRB series; * denotes dimensionless parameter.

strength, but the final strength was supposed to lower than that of normal specimen without ASR. Larive (1996) found that the ASR reactive and nonreactive samples had almost the same increase in compressive strength after 2 years curing at 38°C and HR of 95% compared with the strength at 15-day age. Besides, Cope and Slade (1992) reported that the ASR reactive sample showed tested compressive strength about 1.5 times larger than the 28-day value. On the contrary, Multon (2004) found a decrease in compressive strength when the specimens were stored in water at 38 °C for 2 years. Based on these points, the compressive strength may be a complex and undermined index for representing the effect of ASR. In the initial period of ASR (e.g. first 2~3 months), although the expansion damage may lead to a few microcracks in the specimen, some of the internal holes are filled by the alkali-silica gel production. And thus, a more compact micro-structure of cementitious material is obtained, which may improve the compressive strength in the initial period of ASR. Moreover, when cementitious material was subjected to the ASR, the internal swelling stress was applied as a kind of prestress. During the compression test of the ASR-affected mortar, the compressive stress is first applied to resist the swelling stress, and thus the tested material strength was improved. However, it can be imagined that if the ASR is continued, it is probable to show a drop in compressive strength due to the accumulated ASR swelling in the long term.

3.2 Parameter determination

Numerical simulations are presented to verify the feasibility of the proposed model. The parameters involved in the present model can be obtained from the expansion and triaxial compression tests with different confining pressure.

Based on three tested reference points on the failure surface under different confining pressures, parameters A_0 and B , controlling the curvature of the failure surface, can be established in the p - q coordinate system for each sample series. These two parameters were obtained through the regression analysis of the failure surface function. The tested points and regression curves are shown in Fig. 4 and Fig. 5. In general, the tensile strength can be taken as 1/10 of the compressive strength. Correspondingly, the parameters related to tension damage are $\beta_t = 0.5$, $B_t = 2000$, and $Y_t^0 = 0$. The elastic parameters, such as elastic modulus, Poisson's ratio, and compressive strength, are established from the uniaxial compression test. Parameters associated with compression damage (β_c , B_c , and Y_c^0) can be obtained based on the experimental

Fig. 4 Failure curve of R series before and after the ASR.

Fig. 5 Failure curve of NR series before and after the ASR.

data. The plastic parameter α_s^0 is determined through the tested α_s - γ_{ps} curve under the uniaxial compression, and parameter b_1 is established through the comparison between the α_s - γ_{ps} curves under confining pressures. The complex process of parameter determination is not given here, but the reader can refer to the study by Chen (2005). Moreover, the ASR-induced chemical parameters $\bar{\omega}$ and b in Eq. (25) can be taken as 0.38 and 0.085, respectively, through the regression analysis of the linear swelling experiment results. Table 2 summarizes the parameters used here in detail.

3.3 Model verification

Figures 6~8 show the comparison between the tested data shown in reference (Yurtdas *et al.* 2013) and numerical simulation for R and NR series samples under different confining pressures. The stress-strain relationship before and after ASR are also presented. It is found that the simulated results are well in agreement with the tested data. Representative features due to the combined ASR and confining pressure are reasonably well re-

Table 2 Model parameters used in numerical calculation.

series	E /GPa	ν	f_{c0} /MPa	A_0	C_s^0	B	α_s^0	b_1	μ_s	$\bar{\omega}$	b
NRB	17.73	0.19	36.25	1.99	7.2	0.0057	0.60	210	-1.75	0	0
NRA	17.73	0.19	41.25	2.11	6.8	0.006	0.60	270	-1.75	0	0
RB	17.73	0.19	38.25	2.00	8.6	0.006	0.65	200	-1.75	0	0
RA	16.17	0.23	54.25	2.35	7.3	0.006	0.65	300	-1.75	0.38	0.085

Fig. 6 Comparison between simulations and tested data under uniaxial compression..

Fig. 7 Comparison between simulations and tested data under triaxial compression with confining pressure of 5 MPa.

flected in the present model. For example, the improvement of compressive strength due to ASR is shown in simulated curves. Moreover, this model reproduces the transformation from brittle behavior to ductile behavior owing to increasing confining pressure. Under confining pressure, the plastic deformation capacity and the deviatoric strength are both greatly enhanced. In addition, the apparent softening characteristic under uniaxial compression is also shown by taking into account the damage effect. In summary, this proposed model is capable of making a reliable prediction on the compression behavior of cementitious material subjected to ASR.

However, it needs to be noted that the mechanical behavior of cementitious materials is closely related to the composition of the materials, the water-cement ratio, and other factors. The proposed constitutive model can further be applied in an engineering environment, as long as the model parameters are determined from compression experiments. In the meantime, the evolution of mechanical properties of cement-based materials subjected to ASR can also be evaluated.

4. Conclusions

This paper presents an elastoplastic damage constitutive

Fig. 8 Comparison between simulations and tested data under triaxial compression confining pressure of 15 MPa.

model for describing the compression behavior of reactive cementitious materials subjected to ASR and mechanical loading. In this model, the plastic behavior is coupled with chemical and mechanical damage. The simulated stress-strain relation shows a good agreement with the test data. This model can capture the behavior of the main mechanical characteristics under combined ASR and applied load. The proposed model provides an effective method to predict the constitutive curve of damaged cementitious materials.

Further, in the case of cementitious materials in an engineering environment, the mechanical behaviors are in close relation to the parameters of material and environment. However, the mechanical parameters (strength, elastic modulus, Poisson's ratio, etc.) need to be obtained experimentally for different kinds of cementitious material. After the model parameters have been obtained from experimental study of the materials, the degradation of the mechanical properties of ASR-attacked concrete can also be assessed using the tested parameters and the proposed model. The treatment of ASR in the present paper is proposed on the basis of continuum mechanics and thermodynamics. How to describe the microscopic characteristics such as the composition of materials, the water-cement ratio, and other factors by multiscale simulation method is an interesting issue under investigation.

Acknowledgements

This research was supported by the National Natural Science Foundation of China (Grant No. 51137002), the Ministry of Water Resources' Special Funds for Scientific Research on Public Causes (Grant No. 20130152 and the 111 Project (Grant No. B12032)).

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