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Compressive Stress-Strain Relationship of High Strength Steel Fiber Reinforced Concrete

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Abstract

Many stress-strain models of high strength steel fiber reinforced concrete (SFRC) were proposed to account for major characteristics of SFRC; however, the presence of bond strength between steel fibers and matrix was not considered in most studies. In this study, the bond strength is considered in the proposed stress-strain model. The empirical expressions for determining the proposed stress-strain model are obtained by regressing 61 of stress-strain curves of SFRC. The compression tests on SFRC specimens are also conducted to verify the proposed stress-strain model. In addition, the comparison study between pullout energy obtained from fiber pullout tests and that obtained by using empirical equation are presented, since the bond strength is an important parameter to describe fiber characteristics. By treating steel fibers as confinements, the mechanical properties of SFRC are expressed in terms of reinforcing index, equivalent bond strength, and ultimate compressive stress. The proposed stress-strain model has good agreements with the experimental stress-strain curves obtained either in this study or by other researchers. Furthermore, by considering the bond strength between fibers and matrix and treating steel fibers as confinements, the post-peak behavior of SFRC can be well described and avoid either overestimating or underestimating the post-peak behavior.

1. Introduction

A demand for high strength concrete is increasing over-time since the member section can be further reduced in buildings. In addition, durability of concrete can be enhanced owing to lower water-to-cementitious materials ratio of high strength concrete. However, concrete turns more brittle if its compressive strength increases. This brittle response can be improved by adding steel fibers (Fanella and Naaman 1985; Hsu and Hsu 1994; Mansur *et al.* 1999) because the steel fibers provide bridging action across the microcracks in the matrix and improve resistance to crack opening (Naaman and Najm 1991; Thomas and Ramaswamy 2007). The mechanism of steel fibers in bridging microcracks exists mainly due to the existence of shear stress at the interface between the steel fibers and the surrounding matrix (bond strength between steel fibers and matrix), where this shear stress is transmission of force between fibers and matrix through interfacial bond (Naaman and Najm 1991). Furthermore, the bond strength can exist because of the presence and the combination action of bond components such as physical and chemical adhesion between fibers and matrix, the mechanical component of bond (type of fiber such as hooked-end, crimped, or straight),

and fiber-to-fiber interlock (Naaman and Najm 1991). The mechanical behavior of SFRC is also influenced by the characteristic of each component, synergistic interaction, and mix proportion in the mixture (Bencardino *et al.* 2008). Moreover, the other important characteristics of steel fiber are shape of steel fiber, length of steel fiber, diameter of steel fiber, aspect ratio of steel fiber, and fiber volume fraction (Bencardino *et al.* 2008). The compressive behavior of SFRC can mainly described by its compressive stress-strain relationship.

A number of theoretical models for determining the compressive stress-strain relationship of SFRC have been proposed to account for major characteristics of SFRC (Ezeldin and Balaguru 1992; Hsu and Hsu 1994; Soroushian and Lee 1989; Ou *et al.* 2012). However, the presence of bond strength between steel fibers and matrix was not considered in most studies. In this study, the compressive stress-strain relationship of SFRC is proposed by considering the bond strength of steel fibers and matrix. Sixty-one stress-strain curves of SFRC of compressive strength f'_{cf} of 70 to 115 MPa with fiber volume fraction V_f of 0.5% to 1.5% are regressed. The compression tests on SFRC specimens with compressive stress f'_{cf} of 80 MPa with different fiber volume fraction V_f are also conducted to verify the proposed stress-strain model. Furthermore, the comparison study between pullout energy $E_{pullout}$ obtained from fiber pullout test and that obtained from micromechanical model for steel fiber in cement based composite (Xu *et al.* 2010) are presented, since the bond strength is important parameter to describe fiber pullout behavior. The proposed stress-strain model is not only compared with experimental curves in this study, but also experimental curves obtained by other researchers, and stress-strain

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models proposed by other researchers.

2. Literature review

Four compressive stress-strain models of high strength SFRC proposed by other researchers are discussed in the following sections.

2.1 Ezeldin and Balaguru (1992)

Ezeldin and Balaguru (1992) undertook research study regarding normal and high strength of SFRC in compression. The normal strength and high strength of plain concrete were 35 MPa and 75.9 MPa, respectively. The parameters investigated were water/binder ratio (0.35 and 0.46), weight of fiber in 1 m³ of concrete (30 kg/m³, 45 kg/m³, and 60 kg/m³), and fiber aspect ratio (60, 75, and 100). The compressive stress-strain relationship proposed by Ezeldin and Balaguru (1992) is given in Eq. (1). It is worth mentioning that this equation is exactly the same with that proposed by Carreira and Chu (1985). However, Carreira and Chu (1985) only proposed stress-strain relationship for plain concrete.

$$\frac{f_c}{f'_{cf}} = \frac{\beta \left(\frac{\varepsilon_c}{\varepsilon'_{cf}} \right)}{\beta - 1 + \left(\frac{\varepsilon_c}{\varepsilon'_{cf}} \right)^\beta} \quad (1)$$

Where f_c = stress of SFRC, ε_c = strain of SFRC, β = material parameter, f'_{cf} = SFRC compressive strength (peak stress of SFRC) and ε'_{cf} = strain corresponding to f'_{cf} .

The following equations were obtained using the regression analysis performed using their experimental data:

$$f'_{cf} = f'_c + 3.51(RI_w) \quad (MPa) \quad (2)$$

$$\varepsilon'_{cf} = \varepsilon'_c + 446 \times 10^{-6} (RI_w) \quad (3)$$

$$\beta = 1.093 + 0.7132(RI_w)^{-0.926} \quad (4)$$

where f'_c = plain concrete compressive strength (peak stress of plain concrete), ε'_c = strain corresponding to f'_c ($\varepsilon'_c = 0.002$) and RI_w = reinforcing index which is calculated based on the weight fiber fraction.

$$RI_w = W_f \cdot \frac{L_f}{d_f} \quad (5)$$

where w_f = fiber weight fraction ($w_f \approx 3V_f$, V_f = fiber volume fraction), L_f = length of fiber and d_f = diameter of fiber.

2.2 Hsu and Hsu (1994)

Lin Showmay Hsu and Cheng-Tzu Thomas Hsu (1994) used steel fibers with three volume fractions (0.5%, 0.75%, and 1%) to investigate the effect of fiber volume fraction V_f in high strength concrete. The compressive

Table 1 Material parameter n for high-strength steel fiber concrete (Hsu and Hsu 1994).

V_f , percent	$0 \leq (\varepsilon/\varepsilon'_c) \leq 1$	$1 < (\varepsilon/\varepsilon'_c) \leq x_d$
0.5	$n=1$	$n=1.0$, if $f_{cf} < 11.5$
		$n=1.5$, if $11.5 \leq f_{cf} < 12$
		$n=2$, if $12 \leq f_{cf}$
0.75	$n=1$	$n=1.0$, if $f_{cf} < 11.5$
		$n=1.5$, if $11.5 \leq f_{cf} < 12.5$
		$n=2$, if $12.5 \leq f_{cf}$
1.5	$n=1$	$n=1.0$, if $f_{cf} < 12$
		$n=1.5$, if $12 \leq f_{cf}$

Table 2 Parameters in Equation 8 and 9 for high strength SFRC (Hsu and Hsu 1994).

V_f (%)	a_1	C_1	a_2	C_2
0.5	0.000142	0.001837	43.66	3629.24
0.75	0.000118	0.002172	35.51	3792.86
1.0	0.000178	0.001645	33.77	3792.59

strength of plain concrete designed was 10,000 psi (68.9 MPa). The steel fibers used in this study were made from low-carbon steel, cold-drawn steel wire, hooked at both ends, glued together at their sides, and aspect ratio of 60. The complete compressive stress-strain relationship is represented by the following equations:

$$\frac{f_c}{f'_{cf}} = \frac{n\beta \left(\frac{\varepsilon_c}{\varepsilon'_{cf}} \right)}{n\beta - 1 + \left(\frac{\varepsilon_c}{\varepsilon'_{cf}} \right)^{n\beta}} \quad \text{for } 0 \leq \frac{\varepsilon_c}{\varepsilon'_{cf}} < x_d \quad (6)$$

$$\frac{f_c}{f'_{cf}} = \eta_d \exp \left(-k_d \left(\frac{\varepsilon_c}{\varepsilon'_{cf}} - x_d \right)^a \right) \quad \text{for } x_d \leq \frac{\varepsilon_c}{\varepsilon'_{cf}} \quad (7)$$

where n = modification parameter for β (shown in Table 1), x_d = ratio of post-peak strain corresponding to $0.6f_{cf}$ to strain corresponding to the peak stress and f_{cf} = maximum compressive strength of SFRC (expressed in ksi).

a , k_d , and η_d are constant factors which are equal to 0.8, 0.7, and 0.6, respectively. A regression analysis was conducted to create a relationship between peak stress f_{cf} and corresponding strain ε'_{cf} or tangent modulus E_{it} at different fiber volume fraction V_f , as shown in the following expressions:

$$\varepsilon'_{cf} = a_1 f'_{cf} + C_1 \quad (8)$$

$$E_{it} = a_2 f'_{cf} + C_2 \quad (9)$$

where E_{it} = tangent modulus of SFRC and a_1 , a_2 , C_1 , C_2 = constant factors as summarized in Table 2.

Table 3 Mixture proportions of SFRC specimens (kg/m³).

ID	28-day compressive strength target value (MPa)	Fiber volume fraction (%)	Cement	Slag	Silica fume	Water	Course Aggregate	Fine Aggregate	Steel Fiber	SP
H0	80	0.00	365.0	285.0	50.0	163.8	866.0	647.0	0.0	4.5
H0.75		0.75	362.3	282.9	49.6	162.6	859.5	642.2	58.9	4.5
H1		1.00	361.4	282.2	49.5	162.2	857.3	640.5	78.5	4.5
H1.5		1.50	359.5	280.7	49.2	161.3	853.0	637.3	117.8	4.5

The relationship between peak stress f'_{cf} and material parameter β is given in Eq. (10).

$$\beta = \left(\frac{f'_{cf}}{A} \right)^3 + C \quad (10)$$

Both parameters A (ksi) and C (dimensionless) can be determined by using the following equations:

$$A = 1.717V_f^3 + 8.501 \quad (11)$$

$$C = -0.26V_f + 2.742 \quad (12)$$

2.3 Soroushian and Lee (1989)

The compression stress-strain model proposed by Soroushian and Lee (1989) are given in Eqs. (13) and (14).

$$f_c = -f'_{cf} \left(\frac{\varepsilon_c}{\varepsilon'_{cf}} \right)^2 + 2f'_{cf} \left(\frac{\varepsilon_c}{\varepsilon'_{cf}} \right) \quad \text{for } \varepsilon_c \leq \varepsilon'_{cf} \quad (13)$$

$$f_c = z(\varepsilon - \varepsilon'_{cf}) + f'_0 \geq f_0 \quad \text{for } \varepsilon_c > \varepsilon'_{cf} \quad (14)$$

where f_0 = the residual stress and z = the slope of the descending of branch.

The following expressions (Bencardino *et al.* 2008) for determining mechanical properties (peak stress f'_{cf} , strain corresponding to peak stress ε'_{cf} , slope of the descending of branch z , residual stress f_0) of SFRC were obtained using the least-squares curve fitting the experimental results obtained by Soroushian and Lee (1989).

$$f'_{cf} = f'_c + 3.6RI_V \quad (15)$$

$$f_0 = 0.12f'_{cf} + 14.8RI_V \quad (16)$$

$$z = -343f'_c(1 - 0.66\sqrt{RI_V}) \leq 0 \quad (17)$$

$$\varepsilon'_{cf} = 0.0007RI_V + 0.0021 \quad (18)$$

RI_V is defined as reinforcing index calculated based on the fiber volume fraction V_f , as shown in Eq. (19).

$$RI_V = \frac{V_f \cdot L_f}{d_f} \quad (19)$$

Peak strain of plain concrete ε'_c was fixed as 0.0021. By contrast, the ultimate strain ε_{ucf} was not fixed.

2.4 Ou *et al.* (2012)

Ou *et al.* (2012) investigated the complete compressive stress-strain relationship of SFRC by involving high reinforcing index RI_V . The following parameters were varied in experimental program: aspect ratio of steel fiber in concrete (50, 60, 70, 100, hybrid), fiber volume fraction V_f from 0% to 3.4%. The type of steel fiber used was hooked-end steel fiber having tensile strength of 1000 MPa. The stress-strain model proposed by Ou *et al.* (2012) was similar to that proposed by Ezeldin and Balaguru (1992); however, Ou *et al.* (2012) proposed the following equations to determine peak stress f'_{cf} , corresponding strain ε'_{cf} , and material parameter β .

$$f'_{cf} = f'_c + 2.35(RI_V) \text{ (MPa)} \quad (20)$$

$$\varepsilon'_{cf} = \varepsilon'_c + 0.0007(RI_V) \quad (21)$$

$$\beta = 0.71(RI_V)^2 - 2.00(RI_V) + 3.05 \quad (22)$$

3. Experimental program

Both compression test and fiber pullout test are presented and discussed in the following section.

3.1 Compression test

The compression test was carried out on SFRC with fiber volume fraction V_f of 0%, 0.75%, 1%, and 1.5%. The number of specimens for each fiber volume fraction V_f was three specimens, and these specimens were subjected to concentric axial load. Four mix proportions are summarized in **Table 3**, where the plain concrete was designed for 28-day cylinder strength of 80 MPa. These compression tests were undertaken to verify the proposed stress-strain model, and these test results were not involved in regression analysis. The following component materials were used in experimental program: Portland cement type 1 with specific gravity of 3.15, granulated blast furnace (GGBC) slag with specific gravity of 2.82, silica fume with specific gravity of 2.21, coarse aggregates with nominal maximum size of 9.5 mm, natural river sands as fine aggregate, and superplasticizer with specific gravity of 1.09. The hooked-end steel fiber with ultimate tensile strength of 2300 MPa, 0.38 mm in diameter, 30 mm in length, specific gravity of 7.85, and elastic modulus of 200 GPa was used in this study.

The compression tests on standard cylindrical specimens 10 cm x 20 cm were carried out in accordance with ASTM C39 (ASTM 2003) procedure at 28-day of

Table 4 Mixture proportions of fiber pullout test specimens (kg/m³).

ID	28-day compressive strength target value (MPa)	cement	slag	fly ash	sand	coarse aggregate	water	SP	steel fibers
C30	30	233	233	200	878	413	280	4	118
C40	40	266	200	200	899	423	266	4	118
C50	50	366	200	100	933	439	266	5	118
C60	60	366	200	100	977	455	240	5	118

age, and conducted with displacement control. The displacement rate employed was 0.01 mm/sec. Two LVDTs at both side of concrete specimen were installed to measure axial displacement as shown in **Fig. 1**. The distance between each LVDT with the center of concrete cylinder was 10 cm.

3.2 Fiber pullout test

In order to verify the micromechanical model proposed by Xu *et al.* (2010) for determining pullout energy E_{pull} .

out, the fiber pullout tests with different concrete compressive strength were conducted in this study. There were 3 specimens for each mix proportion. Every specimen had 9 hooked-end steel fibers embedded in the middle section, and was casted in special shape mold. This test was conducted with servo-hydraulic closed-loop testing machine with capacity of 30 kN and constant rate of 0.01 mm/sec. Two LVDT's were provided to obtain the complete fiber pullout load and slip relationship. Four mix proportions are summarized in **Table 4**. Specimen geometry, schematic test setup, and test setup documentation are shown in **Fig 2**.

4. Test results

4.1 Compression test

The compressive stress-strain curves with different fiber volume fractions are shown in **Fig. 3**. It can be observed that the influence of steel fibers on the concrete peak stress f_c is minor. This result was also found in previous research studies (Ezeldin and Balaguru 1992; Hsu and Hsu 1994; Mansur *et al.* 1999; Susetyo 2009). Although the fiber volume fraction is increased up to 1.5%, the increase of peak stress f_c is less than 1.2%. The peak strain ϵ'_{cf} increases while adding fibers. The increase of peak strain ϵ'_{cf} by 7.6% can be observed in the test result of specimen with fiber volume fraction V_f of 0.75%. Furthermore, the use of steel fibers in concrete does not affect tangent modulus notably. The mechanical proper-

Fig. 1 Compression cylinder specimen test setup.

Fig. 2 (a) The geometry of specimen; (b) Schematic test setup, and; (c) Test setup documentation.

Table 5 Summary of Compression Test Results.

ID	Fiber Volume Fraction (%)	Fiber Aspect ratio (L_f/Φ_f)	Reinforcing index ($V_f L_f/\Phi_f$)	Average Concrete Compressive Strength f'_c (MPa)	Average Concrete Strain	Tangent Modulus E_{it} (MPa)
H0	0.00	79	0.00	86.8	0.0029	34639.2
H0.75	0.75		0.59	90.7	0.0031	34649.8
H1	1.00		0.79	90.7	0.0033	34592.8
H1.5	1.50		1.19	92.0	0.0034	34381.5

Fig. 3 Stress-strain curves of high strength concrete with different fiber volume fractions.

Fig. 5 Steel fiber pullout load-slip curves.

Fig. 4 The failure modes of cylindrical specimens with: (a) V_f of 1.5%; (b) V_f of 1%; (c) V_f of 0.75%; (d) V_f of 0%.

ties obtained from compression test are summarized in Table 5. The stress-strain relationships shown in Fig. 3 illustrate that the addition of steel fibers improves the post-peak behavior of concrete in compression. The failure modes of cylindrical specimens are shown in Fig. 4. In case of plain concrete, the type of damage is combination of shear cone and splitting. Meanwhile, the SFRC specimens did not show shear cone or splitting and a large number of cracks appeared near the top and middle zone of specimen.

4.2 Fiber pullout test

The fiber pullout load-slip relationships are shown in Fig. 5. The complete mechanism of bond-slip of steel fiber in matrix has formerly been studied by Naaman *et al.* (1991, 1999). The first mechanism is observed from linear part of pullout-slip curves which remain linear up to about 20 percent of its ultimate load. Naaman (1999)

believed that the first mechanism might describe the end of debonding along the fiber during which adhesive bond was broken and some frictional bond was engaged. In Fig. 5, the second mechanism can be observed when the pullout load larger than 20 percent of the ultimate load, and the slope of the ascending branch changes. The second mechanism occurred due to the hook tending to straighten up when the hook was being pulled out from the matrix. The third mechanism was defined as the stage of pullout load beginning to drop (post-peak pullout load) because the hook was partially straightened and debonding occurred along the entire fiber. In Fig. 5, the third mechanism can be observed from the pullout load continuous to drop, as the fiber slips further in the matrix tunnel. The fourth mechanism is represented when the pullout load drops to about 25 percent of its ultimate load, and the curve shows pullout load to approach a constant value. It can be explained that the fourth mechanism occurred due to the decrease of frictional force and the embedment length.

The pullout energy $E_{pullout}$ can be defined as the area under the pullout load-slip curve. The equivalent bond strength τ_{eq} is a function of pullout energy $E_{pullout}$, diameter of steel fiber, the embedded length of steel fiber in matrix, and constant ϕ . The equivalent bond strength τ_{eq} is expressed in Eq. (23) (Kim *et al.* 2007).

$$\tau_{eq} = \frac{2E_{pullout}}{\pi d_f L_e^2} \quad (23)$$

where E_{pullout} = pullout energy, d_f = diameter of steel fiber and L_e = embedded length of steel fiber in matrix ($=0.5L_f$).

Besides providing fiber pullout test to obtain equivalent bond strength τ_{eq} , the micromechanical model for determining pullout energy E_{pullout} of hooked-end steel fiber in cement based composite proposed by Xu *et al.* (2010) is also presented and discussed. **Figures 6(a)** and **6(b)** show the schematic geometry of hooked-end steel fiber and the pullout process of a hooked-end steel fiber, respectively. In order to model the pullout behavior of a hooked-end steel fiber, Xu *et al.* (2010) adopted the micromechanical frictional pulley model (**Fig. 6(c)**) proposed by Alwan and Naaman in 1999. Moreover, the energy dissipation of a hooked-end steel fiber during the pullout stages 1 to 3 (**Fig. 6(b)**) was realized by adopting the superposition principle shown in **Fig. 7**. **Figure 7(a)** shows the frictional pullout energy of single hooked-end steel fiber, and the maximum frictional pullout force $P_{\text{max-f}}$ can be calculated with Eq. (a) (Xu *et al.* 2010).

$$P_{\text{max-f}} = \frac{\mu \cdot \sigma_y \cdot \pi \cdot r_f^2 \cdot \cos \varphi \cdot (2 - \mu \cdot \cos \varphi)}{3 \cdot \cos \alpha \cdot (1 - \mu \cdot \cos \varphi)^2} + \pi \cdot \tau_f \cdot d_f \cdot L_e \quad (\text{a})$$

The pullout energy g_f of a hooked-end steel fiber contributed by the frictional effects can be determined with Eq. (b) (Xu *et al.* 2010).

$$g_f = \frac{k \cdot P_{\text{max-f}}}{L_e} \cdot \left(L_e \delta - \frac{1}{2} \delta^2 \right) \quad (\text{b})$$

where μ = Coulomb friction coefficient ($= 0.5$), σ_y = yield stress of steel fiber, r_f = radius of steel fiber ($= 0.5 d_f$), τ_f = frictional interfacial stress, $\alpha = 0.25\pi$ and $\varphi = 0.5(\pi - \alpha)$.

Equation (c) through (j) were proposed by Xu *et al.* (2010) to calculate the energy contribution from the plastic straightening mechanism of the steel fiber hooked-end, as shown in **Fig. 7(b)**. The segment of hooked-end was considered as a beam, and the constitutive behavior of the steel fiber was simplified as elastic-perfectly plastic curve, as shown in **Fig. 7(d)**. Moreover, the arched components of the hooked-end C_1 (the arched segment connecting S_1 and S_2 as shown in **Fig. 6(a)**) and C_2 (the other arched segment connecting S_2 and the main straight fiber shaft) were part of circles such that the curvatures ρ_1 and ρ_2 of C_1 and C_2 were constant. The following equations were provided (Eq. (c) through (f)) to calculate the moment-curvature rela-

Fig. 6 (a) The detail geometry of hooked-end steel fiber (Xu *et al.* 2010); (b) The pullout process of hooked-end steel fiber in cement-based matrix (Xu *et al.* 2010); (c) The micromechanical frictional pulley model (Xu *et al.* 2010).

Fig. 7 (a)-(c) The superposition principle for analyzing the pullout energy of a hooked-end steel fiber (Xu *et al.* 2010); (d) the elastic-perfectly plastic model for the hooked-end steel fiber (Xu *et al.* 2010); (e) the stress distribution of the hooked-end segment profile in the elastic-plastic stage (Xu *et al.* 2010).

relationship of fiber profile in elastic stage, elastic-plastic stage, and plastic stage:

Elastic stage:

$$M_e = \frac{\pi \cdot E_f \cdot r_f^4}{4 \cdot \rho} \quad (c)$$

Elastic-plastic stage:

$$M_e = 4 \cdot \sigma_y \cdot r_f^3 \cdot \left[\frac{1}{\sin \theta_e} \left(\frac{\theta_e}{8} - \frac{\sin(4\theta_e)}{32} \right) + \frac{\cos^3(\theta_e)}{3} \right] \quad (d)$$

Plastic stage:

$$M_p = \frac{4}{3} \cdot \sigma_f \cdot r_f^3 \quad (e)$$

where E_f = young's modulus of steel fibers,

$$\theta_e = \arcsin \left(\frac{\sigma_y \cdot \rho}{E_f \cdot r_f} \right) \quad (\text{Fig. 7(e)}) \quad (f)$$

M_e , M_{e-p} , M_p = corresponding moments of the elastic, elastic-plastic, and plastic section

Figure 8(a) shows the schematic of moment-curvature relationship of fiber profile when the hooked-end segment slips into or out of the arched channel (C_1

and C_2). The following equations present the plastic deformation energy-displacement E_p - δ relationship of a unit length of hooked-end segment slipping into or out of the arched channel:

a unit length of hooked-end passing through C_1 (elastic stage):

$$E_{p1} = (A_{11} + A_{12} + A_{21} + A_{22})\delta, \quad 0 \leq \delta < l_{h1} \quad (g)$$

a unit length of hooked-end passing through C_2 (elastic-plastic stage):

$$E_{p2} = (A_{11} + A_{12})l_{h1} + (A_{21} + A_{22})\delta, \quad l_{h1} \leq \delta < l_{h1} + l_{h2} \quad (h)$$

In plastic stage, the pullout behavior of a hooked-end steel fiber is treated as the same as that of a straight fiber:

$$E_{p3} = (A_{11} + A_{12} + A_{21} + A_{22})l_{h1} + (A_{21} + A_{22})l_{h2}, \quad l_{h1} + l_{h2} \leq \delta \leq l_e \quad (i)$$

Figure 8(b) shows the E_p - δ relationship during the fiber pullout process. The total pullout energy g_{tp} can be determined by the following equation:

$$g_{tp} = g_f + E_{pj} \quad (j)$$

where j = the subscript in E_{pj} for three pullout stage ($j=1,2,3$) and A = area under moment-curvature curve

Table 6 Summary of fiber pullout test.

ID	concrete compressive strength f'_{cf} (MPa)	Pullout energy $E_{pullout}$ (N-mm)			Equivalent bond strength τ_{eq} (MPa)		
		Test Result	Analytical Result g_{tp}	Error (%)	Test Result	Analytical Result	Error (%)
C30	37.0	599.2	676.8	12.9	4.5	5.0	13.0
C40	45.4	643.2	725.3	12.8	4.8	5.4	12.8
C50	54.3	695.7	739.6	6.3	5.2	5.5	6.4
C60	66.2	739.9	766.2	3.6	5.5	5.7	3.6

Fig. 8 (a) The schematic of moment-curvature relationship of fiber profile when the hooked-end segment slips into or out of the arched channel (Xu *et al.* 2010); (b) The energy-displacement relationship during the fiber pullout process (Xu *et al.* 2010).

Fig. 9 Proposed stress-strain model for SFRC.

[Fig. 8(a)]

In order to obtain equivalent bond strength τ_{eq} , the energy pullout g_{tp} can be used in Eq. (23) instead of $E_{pullout}$. Both energy pullout and equivalent bond strength obtained from fiber pullout tests and analytical models are summarized in **Table 6**. The maximum percentage error in energy pullout and equivalent bond strength are 12.94% and 13.02%, respectively. Accordingly, the empirical model proposed by Xu *et al.* (2010) is valid for estimating the equivalent bond strength τ_{eq} without providing fiber pullout test. Moreover, the matrix strength improves the pullout load, as shown in **Fig. 5**. For equal slips, the pullout load increases with matrix strength either on the ascending branch or the descending branch of curve. The complete mechanism of bond-slip and the effect of fiber and matrix parameters can be found in experimental study conducted by Naaman *et al.* (1991, 1999).

5. Proposed model

The complete stress-stain relationship of high strength

SFRC is essential for analysis and design of SFRC member. As seen in **Fig. 3**, the stress-strain curve of SFRC consists of three parts. The first part is the ascending branch, and started from strain of zero until peak strain ϵ'_{cf} . The second part (the first part of descending branch) is started from post-peak strain up to strain ϵ_2 . The strain ϵ_2 corresponds with stress f_2 on the descending branch of stress-strain curve. The f_2/f'_{cf} ratio is about 0.5 ($f_2/f'_{cf} \approx 0.5$). The third part (the residual stress) is started from post-strain ϵ_2 up to the ultimate strain ϵ_{ucf} . The proposed stress-strain model is illustrated in **Fig. 9**.

5.1 Part 1: The Ascending Branch ($0 \leq \epsilon_c \leq \epsilon'_{cf}$)

As given in Eq. (1), the compressive stress-strain model proposed by Carreria and Chu (1985) is employed to determine the stress-strain relationship on the ascending branch of curve, since the presence of steel fibers does not significantly influence the ascending branch of stress-strain curve. The mechanical properties such as peak stress f'_{cf} , strain corresponding to peak stress ϵ'_{cf} , and tangent modulus E_{it} of SFRC shall be obtained first in order to fit the stress-stain relationship. The peak stress f'_{cf} can be determined from either compression test or empirical expression proposed by other researchers (Ezeldin and Balaguru 1992, Soroushian and Lee 1989, and Ou *et al.* 2012). The comparison among empirical expressions for determining f'_{cf}/f'_c in terms of R/V is shown in **Fig. 10**. As seen in **Fig. 10**, the presence of steel fibers does not significantly improve the peak stress f'_{cf} . By contrast, it adequately influences the peak strain ϵ'_{cf} , as presented in **Table 2**. In this study, the presence of steel fibers is treated as confinement. For concrete confined by transverse reinforcement, the con-

Fig. 10 The relationship of f'_{cf}/f'_c and RI_v with constant f'_c .

finement index is a function of transverse reinforcement ratio, stress on transverse reinforcement, and concrete compressive strength (Cusson and Paultre 1993). Therefore, the parameters of confinement index in SFRC are reinforcing index RI_v , equivalent bond strength τ_{eq} , and concrete compressive strength f'_c instead of transverse reinforcement ratio, stress on transverse reinforcement, and plain concrete compressive strength, respectively. In order to provide equation of peak strain ϵ'_{cf} , the relationship between $\frac{\epsilon'_{cf}}{\epsilon'_c}$ and $\left(RI_v \times \frac{\tau_{eq}}{f'_{cf}}\right)$ is plotted in

Fig.11. Important mechanical properties of SFRC were taken from experimental data (Ezeldin and Balaguru 1992; Hsu and Hsu 1994; Mansur *et al.* 1999; Susetyo 2009; Authors 2013). The micromechanical model proposed by Xu *et al.* (2010) can be applied to determine equivalent bond strength τ_{eq} . **Figure11** shows that the value of $\frac{\epsilon'_{cf}}{\epsilon'_c}$ ranges from 0.934 to 1.345 for concrete with fiber volume fraction of 0.5% to 1.5%. Eq. (24) is provided to obtain the peak strain ϵ'_{cf} .

$$\epsilon'_{cf} = 1.113\epsilon'_c \quad \text{for } V_f > 0 \quad (24)$$

The peak strain ϵ'_{cf} can be taken as the same as ϵ'_c for concrete with V_f of 0. The compressive stress-strain relationship model for ascending branch is given in Eq. (25).

$$\frac{f_c}{f'_{cf}} = \frac{\beta \left(\frac{\epsilon_c}{\epsilon'_{cf}} \right)}{\beta - 1 + \left(\frac{\epsilon_c}{\epsilon'_{cf}} \right)^\beta} \quad \text{for } 0 \leq \epsilon_c \leq \epsilon'_{cf} \quad (25)$$

The following equations are employed to calculate

Fig. 11 The relationship of $\epsilon'_{cf}/\epsilon'_c$ and three main fiber factors $RI_v \times \tau_{eq}/f'_{cf}$.

the material parameter β and the tangent modulus E_{it} .

$$\beta = \frac{1}{1 - \frac{f'_{cf}}{\epsilon'_{cf} E_{it}}} \quad (26)$$

$$E_{it} = \frac{24.82}{\epsilon'_{cf}} + \frac{0.92 \cdot f'_{cf}}{\epsilon'_{cf}} \quad (MPa) \quad (27)$$

5.2 Part 2: the descending branch ($\epsilon'_{cf} < \epsilon_c \leq \epsilon_2$)

The descending branch of stress-strain curve can be separated into two parts. The first part of the descending branch ranges from post-peak strain up to strain ϵ_2 , while the second part ranges from post-strain ϵ_2 up to ultimate strain ϵ_{ucf} . The strain ϵ_2 corresponds with stress f_2 on the descending branch of stress-strain curve. In order to fit the descending branch, Eq. (6) and Eq. (7) are modified. The following equation is proposed to fit the stress-strain relationship for the first part of descending branch.

$$\frac{f_c}{f'_{cf}} = \frac{n_1 \cdot \beta \left(\frac{\epsilon_c}{\epsilon'_{cf}} \right)}{n_1 \cdot \beta - 1 + \left(\frac{\epsilon_c}{\epsilon'_{cf}} \right)^{n_1 \cdot \beta}} \quad \text{for } \epsilon'_{cf} < \epsilon_c \leq \epsilon_2 \quad (28)$$

where n_1 = the modification factor for β .

The following expression is provided to calculate n_1 :

$$n_1 = -0.151 \cdot \ln \left[\left(RI_v \times \frac{\tau_{eq}}{f'_{cf}} \right) + 1 \right] + 1 \quad (29)$$

where τ_{eq} is expressed in MPa.

The modification factor n_1 has the same definition with modification parameter n proposed by Hsu and Hsu (1994), i.e., as modification factor for material parameter β . According to Hsu and Hsu (1994), the improvement of fiber volume fraction V_f changes the slope of the descending branch of curve. As shown in **Table 1**, the value of n depends on fiber volume fraction V_f and peak stress f'_{cf} , and n is defined as 1 to develop the ascending branch. In this study, modification factor n_1 is not involved in Eq. (25), since the presence of steel fibers has no influence on the ascending branch, as shown in **Fig. 3**. In order to fit the descending branch of curve, the modification factor n_1 is given in Eq. (28).

In this study, modification factor n_1 is defined as the ratio of material parameter β of SFRC to material parameter β of plain concrete (β_{SFRC}/β_{PC} ratio), and material parameter β is calculated with Eq. (26). A propose equation modification factor n_1 shall be expressed in term of reinforcing index RI_v , equivalent bond strength τ_{eq} , and peak stress f'_{cf} , since the steel fibers are treated as confinement. The relationship between modification factor n_1 and $\left(RI_v \times \frac{\tau_{eq}}{f'_{cf}}\right)$ is plotted in **Fig. 12**. In order

to propose equation of modification factor n_1 , as shown in Eq. (29), the analytical study was conducted first by adopting Hsu and Hsu's equation. The result of this analytical study showed that the post-peak behavior could be well-fitted if the modification factor n_1 was smaller than 1 ($n_1 < 1$). For the plain concrete, modification factor n_1 is 1.

5.3 Part 3: the descending branch – residual stress ($\varepsilon_2 < \varepsilon_c \leq \varepsilon_{ucf}$)

The stress f_2 is the first point of stress on the third part. According to experimental data, the f_2/f'_{cf} ratio ranged from 0.35 to 0.96, as shown in **Fig. 13**. A proposed equation of f_2/f'_{cf} ratio is expressed in term of

Fig. 12 The relationship of material parameter n_1 and three main fiber factors $RI_v \times \tau_{eq} / f'_{cf}$.

$\left(RI_v \times \frac{\tau_{eq}}{f'_{cf}}\right)$. The stress f_2 and stress-strain model for the second part of descending branch are given in Eq. (30) and Eq. (31), respectively.

$$\frac{f_2}{f'_{cf}} = 0.244 \ln \left(RI_v \times \frac{\tau_{eq}}{f'_{cf}} + 1 \right) + 0.1 \quad (30)$$

$$f_c = f_2 \exp \left(-0.46 \left(\frac{\varepsilon_c}{\varepsilon'_{cf}} - \frac{\varepsilon_{c2}}{\varepsilon'_{cf}} \right)^{0.8} \right) \quad \text{for } \varepsilon_2 < \varepsilon_c \leq \varepsilon_{ucf} \quad (31)$$

The ultimate strain ε_{ucf} is fixed as 0.015. The stress f_2 can be defined as the ultimate stress of plain concrete for concrete with fiber volume fraction V_f of zero. The ultimate stress of plain concrete is about 0.1 of its peak stress (Wight and MacGregor 2012). Accordingly, stress f_2 can be equal to 10% of peak stress f'_c ($0.1f'_c$) for plain concrete. The value of strain ε_2 can be obtained by substituting stress f_2 into Eq. (28). Moreover, fiber volume fraction V_f is expressed in percent for calculating reinforcing index RI_v .

6. Comparison of stress-strain curves

A cross comparison study between the stress-strain curves from compression tests and those from various stress-strain models is presented in this section. **Figure 14** shows the comparison study between experimental stress-strain curves from this study and various stress-strain models. Meanwhile, the comparison among the stress-strain models versus test result from Hsu and Hsu (1994), Mansur *et al.* (1999), Susetyo (2009), and authors (2013), are presented in **Fig. 15**. The test data corresponding to experimental curves in **Fig. 15** are identified in **Fig. 11** through **Fig. 13**. As seen in **Figs. 14** and **15**, by selecting the peak stress and peak strain from

Fig. 13 The relationship of f_2/f'_{cf} and three main fiber factors $RI_v \times \tau_{eq} / f'_{cf}$.

compression test, every peak point of stress-strain model is identical to that of experimental curve. Therefore, this cross comparison study can more clearly illustrate the difference of both pre-peak and post peak behavior between the stress-strain curve from theoretical model and stress-strain curve from compression test.

Figures 14 and 15 show that the stress-strain model proposed by Ezeldin and Balaguru (1992) and Ou *et al.* (2012) may overestimate both the initial stiffness of concrete and the stresses on the descending branch. Meanwhile, the stress-strain model proposed by Soroushian and Lee (1989) can have adequate agreement with the experimental stress-strain curves of SFRC with V_f of 0.75% and 1%, although it may still overestimate the initial stiffness of concrete. However, the model proposed by Soroushian and Lee (1999) overestimates the

stresses on descending branch for SFRC with V_f of 1.5%, as illustrated in Fig. 14(c) and Fig. 15(c). In general, the stress-strain model proposed by Soroushian and Lee (1989) is more conservative than that proposed either by Ezeldin and Balaguru (1992) or Ou *et al.* (2012).

The stress-strain model proposed by Hsu and Hsu (1994) does not overestimate both the initial stiffness and the stresses on the descending branch. The value of modification factor n is assumed as 1.5 for SFRC with V_f of 1.5%, because Hsu and Hsu (1994) only provided the modification factor n for V_f less than or equal to 1% ($V_f \leq 1\%$), as summarized in Table 1. Figures 14 and 15(b) through 15(d) illustrate that the stress-strain model proposed by Hsu and Hsu (1994) tends to underestimate the stresses on the descending branch of experimental curve obtained by other researchers.

Fig. 14 Comparison between the present experimental curve and the stress-strain models for SFRC with: (a) fiber volume fraction of 0.75%; (b) fiber volume fraction of 1%; and (c) fiber volume fraction of 1.5%.

The complete equations of proposed stress-strain model for high strength SFRC have been given in Eq. (24) through Eq. (31). Equation (25) is provided to determine the stress-strain relationship on the ascending branch, while Eqs. (28) and (31) are provided to determine the stress-strain relationship on the descending branch. The modification factor n_1 and stress f_2 influence the shape of descending branch of stress-strain curve. The modification parameter n_1 is provided to fit the first part of descending branch of SFRC, while the stress f_2 is the first point of residual stress on the descending branch (the second part of descending branch). The modification factor n_1 and stress f_2 are expressed in

term of $\left(RI_V \times \frac{\tau_{eq}}{f_{cf}'} \right)$, since the steel fibers are treated as confinement. Accordingly, considering the bond strength between steel fiber and matrix is necessary. Equations (23) and (a) through (j) are given to calculate the equivalent bond strength τ_{eq} . By separating stress-strain model into three parts, the shape of every part of proposed stress-strain model can be similar to that of experimental curve. It can be observed in **Figs. 14** and **15** that the ascending branch of proposed stress-strain model is very similar to that of experimental curve. Furthermore, the stress ranges from post-peak stress to about $0.5f_{cf}'$ on the descending branch of proposed stress-strain model is not significantly different from

Fig. 15 Comparison among the stress-strain models versus experimental curve from: (a) Hsu and Hsu (1994) with fiber volume fraction of 1%; (b) Mansur et al (1999) with fiber volume fraction of 1%; (c) Susetyo (2009) with fiber volume fraction of 1.5%; and (d) Authors (2013) with fiber volume fraction of 0.75%.

that of experimental curve.

In general, no stress-strain model can have perfect agreement with experimental curve, because every stress-strain model is developed by regression analysis where there must be an error between prediction value and real value. Overall, the stress-strain model proposed in this study can be executed having good agreement with experimental results.

7. Conclusion

Based on the test results and cross comparisons of this study, the following conclusions can be drawn:

- (1) In this study, the complete stress-strain relationship is developed by considering not only the common steel fiber characteristics but also the bond strength between steel fiber and matrix. The development process involves 61 stress-strain curves of SFRC. The proposed stress-strain model can be applied to predict stress-strain relationship of high strength SFRC with compressive strength f_{cf} ranging from 70 to 115 MPa.
- (2) The addition of steel fibers to high strength concrete increases strain at peak stress ε'_{cf} , however, it does not significantly improve the concrete compressive strength f_{cf} as well as tangent modulus E_{it} of concrete. Furthermore, the addition of steel fibers to high strength concrete can enhance the post peak response due to bridge effect of fibers. From the compression test, the damage occurred on plain concrete in compression was combination of shear cone and splitting. By contrast, the SFRC specimens in compression showed many cracks near failure zone and reduced concrete spalling.
- (3) The pullout energy obtained from micromechanical model proposed by Xu *et al.* (2010) can have good agreement with pullout test result. Therefore, the analytical model is valid for further estimating energy pullout $E_{pullout}$ and equivalent bond strength τ_{eq} even no pullout tests provided.
- (4) The proposed model consists of three parts. The first part is on ascending branch of stress-strain curve; both the second part and the third part are on descending of branch of stress-strain curve. The stress-strain model of plain concrete proposed by Carreira and Chu (1985) as given in Eqs. (1) and (25) can be used to determine stress-strain relationship on ascending branch. Two stress-strain equations proposed by Hsu and Hsu (1994) are modified to obtain the stress-strain relationship on descending branch. In addition, both modification parameter n_1 as used in Eq. (28) and stress f_2 as used in Eq.

(31) are expressed in term of $\left(RI_v \times \frac{\tau_{eq}}{f'_{cf}} \right)$, since the

steel fibers are treated as confinements instead of transverse reinforcements.

- (5) Good agreement is reached for the stress-strain

curves between the proposed stress-strain model and the experimental results either from this study or by other authors.

- (6) By considering the bond strength between fibers and matrix and treating steel fibers as confinement, the post-peak behavior of SFRC can be well-described. Either overestimating or underestimating the post-peak behavior can be avoided.

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