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Scientific paper

Prediction of Drying Shrinkage Cracks of Steel Chip Reinforced Polymer Cement Mortar

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Abstract

This paper describes experimental and analytical study of drying shrinkage crack behavior of steel chip reinforced polymer cement mortar (SCRPCM) and polymer cement mortar (PCM). (1) Drying shrinkage test is conducted with four restrained wall specimens of 2500 mm length and 150×300 mm cross section. The drying shrinkage strains, the number of cracks and the crack patterns of the specimens are observed. (2) (a) flexural creep test, (b) pull-out bond test and (c) bond creep test are carried out to evaluate the bond between the SCRPCM/PCM and the steel bar as well as creep characteristics. The shrinkage strains and creep strains of SCRPCM/PCM, and the bond stress-slip curve and bond creep of steel bars are modelled partially according to CEB-FIP Model Code. These models are incorporated with bond computation between the SCRPCM and the steel bar to predict effective strain. The bond stress distribution is computed using analytical solutions of the differential equation of the bond problem, and crack numbers are predicted. (3) 2D finite element analyses are conducted for the four restrained wall specimens of SCRPCM/PCM subjected to drying shrinkage to practically simulate the crack behaviors. The analyzed crack patterns, number of cracks and crack widths are compared with the result of drying shrinkage test.

1. Introduction

Since the adoption of Kyoto Protocol in 1997, reducing of CO₂ emission became important tasks in the entire industry (AIJ 2008). Among others, construction industry represents one of the major constituents of industrial waste (ex. Keoleian *et al.* 2005; Sahmaran *et al.* 2008). Therefore, the reuse of steel materials contributes to a reduction in the environmental load.

Numerous attempts have been made to reuse the waste produced by the iron and steel industry. For example, there is an idea of using steel chips as a replacement material for fine aggregate in concrete (Zainab and Enas 2008; Alwaeli and Nadziakiewicz 2012). As part of research on using steel chips as a reinforcing material, there is a study on the shape memory alloy machining chips used in reinforcing smart composites (Watanabe *et al.* 2005). However, their application to building structures is not economically practical. Therefore, the use of steel chips, which are waste products of the iron and steel

industry, is economically efficient and environmentally sound because it reduces the total amount of industrial waste.

Recently, many studies related to fiber reinforced cementitious composites (FRCC) have been conducted, and various structures have been built with FRCC. The fibers used for FRCC, which improve the bond strength of mortar or concrete, tend to be expensive. As a result, the authors attempted to develop and implement steel chip reinforced cementitious composite (SCRCC), which is reinforced with steel chips instead of conventional steel fibers. The steel chips are produced when steel plates are precisely machined on numerically controlled lathes.

If the use of SCRCC is economically practical, it will also contribute to reducing the environmental load by reusing steel chips, which are currently being disposed of as industrial waste. In addition, SCRCC can be applied to building, which require very long-term durability, such like storage facilities of radioactive wastes.

In a previous study (Kanada *et al.* 2011), fundamental material tests clarified that SCRCC has properties that are similar to FRCC. Based on these results, we set out to examine the drying shrinkage cracking behavior of SCRCC to determine its long-term durability.

To enhance strength, adhesion, bond, waterproofness and durability of building structures, the polymer cement mortar (PCM) is one of the promising solutions (Ohama 1987). Hence, this research investigates the drying shrinkage properties and cracking characteristics of the newly developed steel chip reinforced polymer cement mortar (SCRPCM) with wall specimens (Koyanagi *et al.* 1990; Kheder 1997).

An experiment based on the research of Koyanagi *et al.*

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and Kheder was conducted in an attempt to predict the shrinkage strain and cracking behavior of SCRCC (Hong *et al.* 2015). It was found that the drying shrinkage and cracking characteristics of SCRCC were influenced by tensile creep caused by restraint of the reinforcing bar. As a result, the drying shrinkage cracking behavior was predicted analytically by considering tensile creep based on studies by Kumano *et al.* (Kumano *et al.* 1999), See *et al.* (See *et al.* 2003), Kojiima and Nakamura (2008), and Ranaivomanana *et al.* (2013). In this paper, the drying shrinkage cracks of the four restrained wall specimens made of PCM and SCRPCM are estimated by the same manner.

Then a simplified method to predict the drying shrinkage cracks is proposed to incorporate with 2D finite element analyses. In this method, the effective drying shrinkage strain is evaluated by multiplying a coefficient to consider the influence of the tensile creep strain. The significance of this research is that crack widths can be explicitly output by the FEM. The input strain data is simply but rationally given as the effective drying shrinkage. The FEM results output not only the widths but also numbers and patterns of the cracks.

2. Drying shrinkage test

2.1 Materials

Steel chips (Kanada *et al.* 2011) are produced when a steel plate is precisely machined on a numerically controlled lathe (Fig. 1). Now, all these steel chips occurred during machining are being buried as wastes. Ordinary steel chips are lubricated with oil during precise machining. And the shape of steel chip is not uniform as shown in left picture. But the steel chips used in this study (right) are almost equal in shape, and are not lubricated with oil during precise machining work. And the steel grade is only 235 N/mm²-class (density: 7.86 g/cm³). Thus, they have the effect of stable reinforcement in the cementitious composites.

The mix proportions of cementitious composites used in this study are given in Table 1. Polymer cement mortar was used as binder. Ordinary Portland cement was used as a cement. Polymeric admixture used is ethylene vinyl acetate (EVA) emulsion which is a fluid milk-white solution with a solid content of 45% and the density 1.07

Fig. 1 Ordinary steel chips (left) vs steel chips (right).

Table 1 Mix proportion of PCM/SCRPCM.

	PCM	SCRPCM
W/B (%)	30	30
Water (kg)	24	24
Cement (kg)	80	80
Fine aggregate (kg)	160	142.2
Polymer (kg)	8	8
Steel chip (Vol.%)	0	2
Antifoaming Agent (kg)	0.08	0.08

g/cm³. An antifoaming agent was used to control the entrained air with 1% by mass on polymer. River sand was used as a fine aggregate. Steel chip contents of 2% by volume were used in this study.

The compressive and splitting tensile strengths of cementitious composites are listed in Table 2. The compressive strengths of PCM and SCRPCM are slightly higher than 40 N/mm². In contrast to the tensile strength of PCM at 3.6 N/mm², that of SCRPCM is 4.9 N/mm². The results of compression test and three-point flexural test are shown in Fig. 2. Relationships between tensile stress and crack width are derived from the result of three-point flexural tests by inverse analyses (Fig. 3). Comparing to PCM, SCRPCM presents higher strength and ductility both in the compression and the tension side because of the bridge effect of the steel chips.

2.2 Drying shrinkage test

When concrete is restrained, drying shrinkage contributes to nearly all the cracking observed in concrete members before loading. A free shrinkage test cannot

(a) Compressive stress – strain relationships

(b) flexural load – crack width relationships

Fig. 2 Mechanical properties of PCM and SCRPCM.

demonstrate the true potential of fiber reinforcement to resist restrained shrinkage stresses and to control shrinkage cracking (Swamy and Stavrides 1979). Specimens for drying shrinkage tests were made in two sizes. Four large wall specimens (PCM4, SCRPCM4, PCM10, and SCRPCM10) of size 150×300×2500 mm were prepared for restrained drying shrinkage tests with parameters of reinforcement (four or ten steel bars 6.4 mm in diameter; reinforcement ratio $\rho = 0.29\%$ or 0.72%) and materials (PCM, SCRPCM), as shown in **Fig. 4**. The material property of the steel bar is listed in **Table 2**. To guarantee certain cracking behavior, the restrained shrinkage specimens were prepared larger than normal size based on a study by Koyanagi *et al.* (Koyanagi *et al.* 1990)

Specimens for the free shrinkage tests were made into beam specimens of size 100×100×500 mm. Four specimens (PCM-In, SCRPCM-In, PCM-Out, SCRPCM-Out) were prepared with the same parameters of the materials and curing conditions. The parameters of the eight specimens are listed in **Table 3**. Each of the three-beam specimens was made with two types of materials (PCM, SCRPCM).

The molding process of the restrained wall specimens is as follows. First, the restraining block (450×600×300 mm) was cast with ordinary concrete. Then, each block was fixed with four prestressed bolts of 32 mm diameter,

Table 2 Mechanical properties of materials.

(a) Cementitious composite		
	PCM	SCRPCM
Compressive strength f_{c28} (N/mm ²)	41.8	44.8
Strain corresponding to f_{c28} : $\varepsilon_c (\times 10^{-3})$	3.33	3.25
Splitting tensile strength f_{t28} (N/mm ²)	3.62	4.89
(b) Steel bar (6.4 mm diameter)		
Yield stress f_{sy} (N/mm ²)	425.9	
Tensile strength f_{su} (N/mm ²)	531.6	
Elastic modulus E_s (N/mm ²)	189,000

Fig. 3 Relationships between tensile stress and crack width.

Fig. 4 Restrained wall specimens of drying shrinkage test.

Table 3 Parameters of drying shrinkage specimens.

Specimen	Condition	Restrain	Cementitious composite	Bar (ρ)
PC4	Outside	Rest.	PCM	4D6 (0.29%)
SPC4	Outside	Rest.	SCRPCM	4D6 (0.29%)
PC10	Outside	Rest.	PCM	10D6 (0.72%)
SPC10	Outside	Rest.	SCRPCM	10D6 (0.72%)
PC-In	20°C, 60% R.H	Free	PCM	-
SPC-In	20°C, 60% R.H	Free	SCRPCM	-
PC-Out	Outside	Free	PCM	-
SPC-Out	Outside	Free	SCRPCM	-

to which 250 kN tensile force was applied. Second, the center part (150×300×2500 mm) was cast with PCM or SCRPCM. Third, after the casting form was removed (curing: five days), beam specimens (100×100×500 mm) for free shrinkage and wall specimens (150×300×2500 mm) for restrained drying shrinkage were cured for five days under each of the curing conditions. After that, measuring targets were bonded to the surface of each specimen with 100 mm spacing. The initial measurements were made after curing for seven days. Strains

were measured and crack patterns were observed every seven days.

2.3 Curing and drying conditions

Two beam specimens for the free shrinkage test (PCM-In and SCRPCM-In) were subjected to a constant temperature and humidity condition of 20°C and 60% R.H. On the other hand, two beam specimens (PCM-Out and SCRPCM-Out) and four wall specimens (PCM4, SCRPCM4, PCM10, and SCRPCM10) were subjected to outside though protected from the rain. The average daily temperature and average daily humidity outside the laboratory that the specimens were exposed to during the drying period are shown in Fig. 5.

2.4 Free shrinkage

Figure 6 represents the relationship between the drying shrinkage of the free shrinkage specimens and the drying period. The effect of the curing conditions on drying shrinkage was that the drying shrinkage of the specimens left outside was less than that of the indoor specimens. The explanation for this is that the humidity outdoors was usually higher than 60% while the humidity indoors was constantly 60%. In this study, autogenous shrinkage strain is not subtracted from the measured drying shrinkage strain. Significant difference of drying shrinkage strain was not observed between PCM and SCRPCM.

2.5 Restrained shrinkage and cracking characteristics

Figures 7 to 10 show the drying shrinkage crack patterns for three sides (top and sides) of each specimen and the distributions of the crack widths along the longitudinal direction of the specimens. The crack widths are the maximum values attained during the drying period.

Figure 11 shows the relationships between the equivalent number of cracks and drying period. An equivalent number of cracks is defined as the total lengths of the cracks on the top surface of the specimen divided by the width (300 mm). The equivalent number of cracks is used since only a few cracks penetrated the entire width of any specimen.

Figure 12 shows the relationships between the average crack width and drying period and Fig. 13 the relationships between the maximum crack width and drying period. The latter widths are also the maximum attained values.

Fig. 5 Average daily temperature and relative humidity.

Fig. 6 Relationships between drying shrinkage strain and drying period.

Fig. 7 Drying shrinkage crack patterns and widths of PCM4 by test (left) and FEM (right).

Fig. 8 Drying shrinkage crack patterns and widths of SCRPCM4 by test (left) and FEM (right).

Fig. 9 Drying shrinkage crack patterns and widths of PCM10 by test (left) and FEM (right).

Fig. 10 Drying shrinkage crack patterns and widths of SCRPCM10 by test (left) and FEM (right).

In Figs. 11 and 12, the equivalent number of cracks of SCRPCM4 is slightly larger than that of PCM4 and the crack width of SCRPCM4 is also larger than that of PCM4. Difference of the equivalent number of cracks between SCRPCM10 and PCM10 is insignificant but the crack width of SCRPCM10 is smaller than that of PCM10. Generally, the number of cracks of all specimens increased with the drying period and also with the number of reinforcing bars. The numbers cracks in-

creased and the crack widths decreased as the amount of reinforcing bars increased. On the other hand, the steel chip reinforcement did not present significant influence on the numbers of cracks. The influence of the steel chip on the maximum crack widths depended on the amount of reinforcing bars: The maximum crack width of SCRPCM4 was larger than that of PCM4 while the width of SCRPCM10 was smaller than that of PCM10. It is concluded that the bridging effect of the steel chips indicated in Fig. 3 can reduce the crack width if it was accompanied with ten reinforcing bars ($\rho = 0.72\%$), but it is ineffective if only four reinforcing bars ($\rho = 0.29\%$) were provided.

The reason why the bridge effect is insignificant in the specimens with four bars ($\rho = 0.29\%$) is that the crack opening is concentrated at the boundaries between the central test part and the side fixing part (i.e. cracks at 0 mm and 2,500 mm in Figs. 7 to 10). Initial cracks were induced to these boundaries because of the stress concentration due to the change of cross sectional area. The widths of the boundary cracks of four-bar specimens continue to increase while those of ten-bar specimens are restrained by the bars. The bridge effect of the steel chip was insufficient to prevent the widening of the boundary cracks while it contributed to decrease the numbers and widths of cracks in the central part.

3. Tests for creep, bond and bond creep

3.1 Flexural creep test

18 block specimens of 100 mm square cross section and 500 mm long were subjected to flexural creep tests. The specimens are listed in Table 4. A load was applied at

Fig. 11 Relationships between equivalent number of cracks and drying period.

Fig. 12 Relationships between average crack width and drying period.

Fig. 13 Relationships between maximum crack width and drying period.

either 7, 14, or 28 material day after which creep was observed for 60, 56, 14 days, respectively. Loading was applied to only two of the three specimens; the third was compared to the other two.

The loading condition was consisted of a 48 kg steel block hanging from the end of a loading arm, so three-point loading was applied to each specimen, as shown in **Fig. 14**.

The average load and flexural stress of the center part of each specimen was 1.65 N/mm^2 . The creep test was conducted under a constant temperature and humidity (20°C , 60% R.H.). The measuring point of 100 mm was fixed on the center of the top and bottom of each specimen, and the change in the length was measured with a contact gauge. **Figure 15** shows the relationships between specific flexural tensile creep and drying period.

3.2 Pull-out bond test

Six pull-out bond specimens were made to evaluate the bond characteristics between the PCM/SCRPCM and the steel bar. **Figure 16** shows the specimens and **Table 5** lists the specimens.

Each specimen was a cylinder 180 mm long and 65 mm in diameter made of PCM or SCRPCM, in which a steel bar of 6.4 mm in diameter was embedded. This bar is the same as those used in the restrained drying shrinkage specimens. Tensile force was applied to the bar while a side of the cylinder was fixed by a steel plate. To evaluate bond stress, strain gauges were installed at three points on the bar (SG1, SG2, and SG3).

All the specimens failed when the steel bar broke. The maximum bond stresses observed for each specimen are listed in **Table 5**. The bond stress is derived from the difference of the strains of two neighboring strain gauges. **Figure 17** shows the typical relationships between tensile force and displacement. **Figure 18** shows the relationships between bond stress and slip. The bond stress shown in **Fig. 18** is evaluated by the strain difference of strain gauges SG1 and SG2.

3.3 Bond creep test

Four bond creep specimens were made to evaluate the bond characteristics between the PCM/SCRPCM and the steel bar under sustained load. **Figure 19** shows the specimens and **Table 6** lists the specimens. Each specimen was a block of size $100 \times 200 \times 60 \text{ mm}$ in which a steel bar of 6.4 mm in diameter was embedded. This bar is the same as those used in the restrained drying shrink-

Table 4 Flexural creep test.

Cementitious composite	Specimen	Loading day	Flexural stress (N/mm^2)
PCM	Cr-PC-7a	-	0
	Cr-PC-7b	7	1.66
	Cr-PC-7c	7	1.64
	Cr-PC-14a	-	0
	Cr-PC-14b	14	1.66
	Cr-PC-14	14	1.64
	Cr-PC-28a	-	0
	Cr-PC-28b	28	1.66
	Cr-PC-28c	28	1.64
SCRPCM	Cr-SPC-7a	-	0
	Cr-SPC-7b	7	1.66
	Cr-SPC-7c	7	1.62
	Cr-SPC-14a	-	0
	Cr-SPC-14b	14	1.66
	Cr-SPC-14	14	1.62
	Cr-SPC-28a	-	0
	Cr-SPC-28b	28	1.66
	Cr-SPC-28c	28	1.62

Fig. 15 Relationships between specific flexural tensile creep and drying period.

Fig. 14 Flexural creep test.

age specimens. The measuring points were fixed on the each 100×200 mm surface facing each other and strain gauges were installed (Fig. 19).

The bond creep test was conducted under a constant temperature and humidity (20°C, 60% R.H.). The loading condition was consisted of a 48 kg steel block hanging from the end of a loading arm, so sustained stresses of 174.4 N/mm² (PCM) or 228.7 N/mm² (SCRPCM) were applied to the steel bar of each specimen, as shown in Fig. 20. A load was applied at 14 material day after which bond creep was observed for 24 days. Loading was applied to one of the two specimens and compared to the

Fig. 16 Pull-out bond test specimen.

Fig. 17 Relationships between tensile force and displacement of pull-out bond test specimens.

Table 5 Pull-out bond test.

Cementitious composite	Specimen	Maximum bond stress (N/mm ²)
PCM	B-PCa	3.93
	B-PCb	3.74
	B-PCc	4.29
SCRPCM	B-SPCa	6.05
	B-SPCb	7.83
	B-SPCc	5.67

Table 6 Bond creep test.

Cementitious composite	Specimen	Steel bar stress (N/mm ²)
PCM	B-Cr-PCa	0
	B-Cr-PCb	174.4
SCRPCM	B-Cr-SPCa	0
	B-Cr-SPCb	228.7

Fig. 18 Relationships between bond stress and slip of pull-out bond test specimens.

Fig. 19 Bond creep specimen.

Fig. 20 Bond creep test instrumentation.

other. The bond slip was evaluated from difference between the measured elongations of the bar and the PCM/SCRPCM. The specific bond slip creep is defined as the increment of bond slip divided by the instantaneous bond slip at the initial loading. **Figure 21** shows the relationships between specific bond slip creep and drying period.

4. Prediction of number of shrinkage cracks

4.1 Drying shrinkage strain

In this section, the numbers of drying shrinkage cracks of the restrained wall specimens are predicted based on modelling of the material behavior according to the results of the drying shrinkage tests and creep tests.

Based on the CEB-FIP equation, curve approximation was performed on the drying shrinkage strain of free shrinkage specimens PCM-Out and SCRPCM-Out, which were placed in conditions to similar outside without rainfall. Shrinkage strain (ε_{sc}) is expressed by Eq. (1) with the coefficient C_1 of CEB-FIP Model Code 1990 (CEB-FIP 1990) equation modified. The relationships between drying shrinkage strain and drying period, and the result of curve approximation by Eqs. (1) and (2) are shown in **Fig. 22(a)**.

$$\varepsilon_{sc} = \left[\{160 + 10 \cdot C_1 \cdot (9 - f_{c28})\} \cdot 10^{-6} \cdot \beta_{RH} \right] \cdot \left[t / \left\{ 350 \cdot \left(\frac{2A_c}{u} / 100 \right)^2 + t_0 \right\} \right]^{0.5} \quad (1)$$

Fig. 21 Relationships between specific bond slip creep and drying period.

(a) Observed strains and fitted curves

$$\beta_{RH} = -1.55 \cdot \{1 - (RH/100)^3\} \quad (2)$$

where A_c = cross-sectional area of member (mm^2), C_1 = 3.16 (PCM), 3.37 (SCRPCM), f_{c28} = 28-day compressive strength (N/mm^2), RH = relative humidity during drying period (64.4%), t = age (days), t_0 = age when loaded (days) and u = length of part in contact with outside air (mm).

Figure 22(b) compares the curve-fitted drying shrinkage strains considering the cross section of specimens ($A_c = 100 \times 100$, $u = 400$ for free shrinkage specimens, $A_c = 300 \times 150$, $u = 900$ for restrained wall specimens). This modeled drying shrinkage strain is used in calculation of number of cracks.

4.2 Bond

(1) Bond stress-slip model

Figure 23 shows the assumed relationships between bond stress and the slip of the interface between the cementitious composite and the steel bar. This relationship is based on the fib Model Code 2010 (fib 2010) with slight modification considering the creep.

$$S_1 = 0.1 \cdot \varphi_{bs} \quad (\text{mm}) \quad (3)$$

$$S_2 = 1 \cdot \varphi_{bs} \quad (\text{mm}) \quad (4)$$

$$\tau_0 = \Omega_y \cdot \tau_1 \cdot S / S_1 \quad (\text{N/mm}^2) \quad \text{for } 0 \leq S < S_1 \quad (5)$$

Fig. 23 Relationships between bond stress and slip for bond between PCM/SCRPCM and steel bar.

(b) Correction based on cross section

Fig. 22 Curve fittings of relationships between drying shrinkage and drying period.

$$\tau_0 = \Omega_y \cdot \{\tau_1 + (\tau_2 - \tau_1) \cdot (S_2 - S) / (S_2 - S_1)\} \quad (N/mm^2) \quad (6)$$

for $S_1 \leq S < S_2$

$$\tau_0 = \Omega_y \cdot \tau_2 \quad (N/mm^2) \quad \text{for } S_2 \leq S \quad (7)$$

$$\tau_1 = 1.25\sqrt{f_{cm}} \quad (N/mm^2) \quad (8)$$

$$\tau_2 = 2.5\sqrt{f_{cm}} \quad (N/mm^2) \quad \text{for } S_2 \leq S \quad (9)$$

$$\Omega_y = 1 \quad \text{for } \varepsilon_s \leq \varepsilon_{sy} \quad (10)$$

$$\Omega_y = 1 - 0.85 \cdot \left[1 - \exp \left[-5 \cdot \left\{ \frac{\varepsilon_s - \varepsilon_{sy}}{\varepsilon_{su} - \varepsilon_{sy}} \right\}^2 \cdot \left(\frac{f_{su}}{f_{sy}} \right)^2 \right] \right] \quad (11)$$

for $\varepsilon_{sy} < \varepsilon_s \leq \varepsilon_{su}$

$$\varphi_{bs} = c_{bs} \times \{\ln(t - t_0 + e) - 1\} \quad (12)$$

where c_{bs} = bond slip creep coefficient: 0.075 (PCM), 0.182 (SCRPCM),

f_{cm} = compressive strength (N/mm²), f_{su} = tensile strength of steel bar (N/mm²), f_{sy} = yield stress of steel bar (N/mm²), S = bond slip (mm), S_1 = slip at first branch (mm), S_2 = slip at second branch (mm), t = age (day), t_0 = age when loaded (day), τ_0 = bond stress for elastic steel bar (N/mm²), τ_1 = bond stress at first branch (N/mm²), τ_2 = bond stress at second branch (N/mm²), ε_s = strain of steel bar, ε_{su} = strain corresponding to tensile strength of steel bar, ε_{sy} = yield strain of steel bar, φ_{bs} = specific bond slip creep and Ω_y = bond stress reduction factor for yielded steel bar.

Figure 17 shows the relationships between tensile force and the displacement of the pull-out test specimens. The analyzed relationships, which are derived from the numerical integration of above defined bond stress-slip relationship, agree well with the test results. The bond stress of “analysis” in this figure is the average value between strain gauges SG1 and SG2 (see **Fig. 16**).

The bond slip creep is considered by φ_{bs} defined in Eq. (12). **Figure 21** compares the modeled and experimentally obtained relationships between specific bond slip creep and drying period.

(2) Second order differential equation for bond

Using this bond stress-slip model, the calculation is performed by numerical integration of ordinary differential equations, which are shown here from the research by Rehm (Rehm 1961) and Yannopoulos *et al.* (Yannopoulos *et al.* 1991).

The compatibility condition is given by Eq. (13).

$$\frac{ds}{dx} = \varepsilon_s - \varepsilon_c - \varepsilon_{sc} - \varepsilon_{crp} \quad (13)$$

where S = bond slip (mm), x = coordinate along bar axis (mm), ε_s = strain of steel bar, ε_c = strain of PCM/SCRPCM, ε_{sc} = drying shrinkage strain of PCM/SCRPCM and ε_{crp} = creep strain of

PCM/SCRPCM.

The second order ordinary differential equation for the bond is given by Eq. (14).

$$\frac{d^2S}{dx^2} = \frac{4}{d_b} \cdot \left(\frac{1}{E_s} + \frac{\rho}{E_c} \right) \cdot \tau \quad (14)$$

where d_b = diameter of steel bar (mm), E_s = elastic modulus of steel bar (N/mm²), E_c = elastic modulus of PCM/SCRPCM (N/mm²), ρ = reinforcement ratio and τ = bond stress (N/mm²).

The tensile stress increment of PCM/SCRPCM $\Delta\sigma_c$ is obtained by integrating cross sectional area A_c of the PCM/SCRPCM divided by bond stress ($n_s \cdot \pi \cdot d_b \cdot \tau$) in the x direction. Thus, Eq. (15) is established.

$$\sigma_c(x) = - \int_0^x \frac{n_s \pi d_b \tau}{A_c} dx + \sigma_c(x=0) \quad (15)$$

where n_s = number of steel bars, $\sigma_c(x)$ = stress of PCM/SCRPCM (N/mm²) and $\sigma_c(x=0)$ = stress of PCM/SCRPCM at the center of between two adjacent cracks (N/mm²).

The tensile stress of the PCM/SCRPCM becomes greatest at the center between two adjacent cracks, and cracking occurs when the stress reaches the tensile strength f_t .

4.3 Modeling of specific creep

As described in Eq. (13), the real strain that causes tensile stress in the PCM/SCRPCM is the sum of drying shrinkage strain ε_{sc} (negative value) and creep strain ε_{crp} (positive value). A formula for calculating specific creep based on CEB-FIP Model Code 1990 is given here.

$$\varphi_{CEB} = \left\{ 1 + \frac{1 - RH/100}{0.46 \cdot \left(\frac{A_c}{50 \cdot u} \right)^{1/3}} \right\} \cdot \frac{C_2}{\sqrt{f_{c28}/11}} \cdot \frac{1}{0.1 + t_0^{0.2}} \cdot \left(\frac{t - t_0}{\beta_H + t - t_0} \right)^{0.3} \quad (16)$$

$$\beta_H = 150 \cdot \left\{ 1 + \left(1.2 \cdot \frac{RH}{100} \right)^{18} \right\} \cdot \frac{A_c}{50 \cdot u} + 250 \leq 1500 \quad (17)$$

where A_c = cross-sectional area of member (mm²), u = length of part in contact with outside air (mm) and t_0 = age when loaded (day).

A coefficient C_2 is 5.3 in the original equation, but the coefficient is set based on the results of the calculation, as 0.21 (PCM); 0.79 (SCRPCM).

Furthermore, it is necessary to model the changes caused by compressive strength f_{cm} , tensile strength f_t , and age of elastic modulus E_c in the calculation of Eqs. (1), (2) and (16). Based on the experiment, compressive strength, tensile strength, and elastic modulus are given by Eqs. (18), (19), and (20), respectively.

$$f_{cm} = f_{c28} \cdot \left\{ \frac{t}{28 \cdot (1 - C_3) + C_3 \cdot t} \right\} \quad (18)$$

$$f_t = f_{t28} \cdot \left(\frac{t}{28} \right)^{C_4} \quad (19)$$

$$E_c = E_{c28} \cdot \left[\exp \left[C_5 \cdot \left\{ 1 - \left(\frac{28}{t} \right)^{1/2} \right\} \right] \right]^{1/2} \quad (20)$$

where f_{t28} = 28-day tensile strength (N/mm²), E_{c28} = 28-day elastic modulus (N/mm²), C_3 = 0.901 (PCM), 0.902 (SCRPCM), C_4 = 0.108 (PCM), 0.094 (SCRPCM) and C_5 = 0.254 (PCM), 0.351 (SCRPCM).

The relationships between specific creep and drying period calculated by Eqs. (16) to (20) are compared with experimental curves in **Fig. 15**. The modeled curves run the lower bounds of widely scattered experimental data since it is considered that the embedded steel bars in the drying shrinkage specimens may prevent excessive creep strain. However, the modeled curve considerably underestimates the experimental data of PCM of loading day = 7. This discrepancy will be discussed in future studies.

In addition, the relationships between drying shrinkage strain (negative value), creep strain (positive value), sum of both values calculated by Eqs. (16) to (20), and drying period are represented in **Fig. 24**. Creep strain was drawn by combining the creep strain of each day caused by the daily increasing tensile stress. The sum of shrinkage strain and creep strain shown in **Fig. 24** is effective strain, which causes tensile stress in the PCM/SCRPCM. By applying this strain, the calculation of the number of cracks is conducted.

4.4 Calculation of number of cracks

The analyzed relationships between the equivalent number of cracks considering the influence of creep and the drying period is shown in **Fig. 11** (indicated by

“Cal.”). The cracking stress at 28 material day is assumed as 1.05 N/mm² (PCM); 2.99 N/mm² (SCRPCM). Eq. (15) is numerically computed allowing an error of 10%. The analysis values correspond closely with the test results. **Figure 25** shows distributions of tensile stresses of PCM/SCRPCM of restrained wall specimens. The origin of the horizontal axis represents the center point between two cracks while the right point to which the tensile stress drops corresponds to the crack location. The distribution therefore represents a half part between two cracks. The crack spacing decreases as the drying period proceeds from 5 week up to 24 week. The tensile stress at crack is less than 1.0 N/mm² for the PCM specimens while it is enhanced to around 3.0 N/mm² for the SCRPCM specimens because of the bridge effect at crack shown in **Fig. 3**. The crack spacing of Specimen SCRPCM4 at 24 week is smaller than PCM4 while that of SCRPCM10 is equivalent to PCM10. The steel chip enhances not only the ductility at the crack opening but also the tensile strength of uncracked part. Because of this balance, the content of steel chip does not always reduce the crack spacing as a comparison between PCM10 and SCRPCM10 indicates.

5. Finite element analysis with simplified effective drying shrinkage strain model

2D finite element analyses are conducted for the four restrained wall specimens. The finite element program used is FINAL developed by Naganuma *et al.* (2004). The specimens are modelled by four-node plane stress elements each 25 mm square. The right and left sides of the wall is fixed. **Figure 26** shows the mesh division. One analysis step corresponds to one material day and the analysis is conducted for 168 material days.

The finite element program is incorporated with the discrete-like crack simulation method developed by one

Fig. 24 Relationships between drying shrinkage strain, creep strain, and drying period.

Fig. 25 Distributions of tensile stresses of PCM/SCRPCM of restrained wall specimens.

of the authors (Sato and Naganuma 2007). The discrete-like crack simulation method was developed by aiming at an explicit computation of crack widths, which used to be impossible by using the conventional smeared crack-based finite element method. **Figure 27(a)** shows a crack pattern analyzed by the conventional method. Too many cracks are induced all over the specimen since stress redistributions are inadequately conducted. On the other hand, the crack pattern is realistically improved by the discrete-like crack simulation method by considering the bond between steel bars and the mortar as shown in **Fig. 27 (b)**.

In a previous research (Sato and Naganuma 2012), the strains estimated by the method described in Section 4.4 were directly implemented into the finite element algorithm. For the interest of practical application, the summed strain ε_{sc_e} of the shrinkage strain and the creep strain is simply evaluated by Eq. (21) in this paper.

$$\varepsilon_{sc_e} = K \varepsilon_{sc} \quad (21)$$

where

K = coefficient considering the influence of creep; PCM: 0.94, SCRPCM: 0.79; and

ε_{sc} = drying shrinkage strain of test.

The analyzed relationship between the equivalent number of cracks and the drying period is shown in **Fig. 11**, and the analyzed crack patterns and the maximum crack widths are compared in **Figs. 7 to 10**. Again, the analyzed crack widths are the maximum values along the specimen attained during the drying period. Some discrepancies are found in the crack patterns between the test results and analyses because three dimensional effects such like unequal drying rate depending on the surfaces (i.e. top / bottom / side) cannot be considered by the 2D finite element analyses although the equivalent number of cracks are largely coincided as shown in **Fig. 11**.

Figures 12 and 13 compares the experimental and analyzed crack widths. The former shows the average widths while the latter the maximum. The analyzed crack

Fig. 26 Finite element modeling of restrained wall specimen.

Fig. 27 Stress redistribution by discrete-like crack simulation method (PCM10 168-day).

widths largely agree with the test results of SCRPCM specimens while the widths of PCM specimens are overestimated. As mentioned in Section 2.5, relatively large crack opening occurs at the boundary between the central and side parts of specimens. The estimation of the crack width at the cross-section changing boundary must be considered by 3D analyses in future study.

6. Conclusions

This study examined the drying shrinkage cracking behavior of steel chip reinforced polymer cement mortar (SCRPCM) using steel chips as an eco-friendly recycled material. To evaluate the drying shrinkage cracking behavior of SCRPCM considering tensile creep with steel bars inside a member, creep test was simultaneously performed. For the four restrained wall specimens of SCRPCM and PCM subjected to shrinkage, the bond stress distribution is computed using analytical solutions of the differential governing equation of the bond problem, and equivalent numbers of cracks are predicted. In addition, 2D finite element analyses are conducted to practically simulate the crack patterns. The analyzed

equivalent numbers and widths of cracks are compared with the test results. The conclusions obtained from this study are summarized below.

- (1) Significant difference of drying shrinkage strain was not observed between PCM and SCRPCM.
- (2) In the restraint wall specimens, the maximum crack width was reduced by the bridge effect of steel chips if it was accompanied with ten reinforcing bars ($\rho = 0.72\%$), but it was ineffective if only four bars ($\rho = 0.29\%$) were provided.
- (3) The numbers of cracks are predicted by analyzing the bond behavior of the steel bars and binder in the member using the specific creep model and drying shrinkage strain model.
- (4) A simplified method to evaluate the effective drying shrinkage strain is proposed to incorporate with the FEM. These results nearly matched the crack widths obtained by the experiments.

Some discrepancies are found in the crack patterns and widths between the test results and the 2D finite element analyses. For these problems, further research will be conducted by employing 3D finite element modeling.

References

- AIJ, (2008) "Recommendations for environmentally conscious practice of reinforced concrete buildings." Architectural Institute of Japan, (in Japanese).
- Alwaeli, M. and Nadziakiewicz, J., (2012). "Recycling of scale and steel chips waste as a partial replacement of Sand in Concrete." *Construction and Building Materials*, 28(1), 157-163.
- CEB-FIP, (1990). "Model code for concrete structures." Comité Euro-International du Béton- Fédération Internationale de la Précontrainte, Thomas Telford, London.
- fib. (2010). "fib Model code 2010." Fédération Internationale de la Précontrainte, Lausanne, Switzerland, 233-239.
- Hong, S., Sato, Y., Kaneko, Y. and Kimura, S., (2015). "Prediction of drying shrinkage cracking of eco-efficient steel chip reinforced cementitious composite considering tensile creep." *Journal of Advanced Concrete Technology*, 13, 393-402.
- Kanada, M., Sato, Y. and Kaneko, Y., (2011). "Seismic retrofit of exposed column basement using steel chip reinforced cementitious composite." *Proceedings of Constructional Steel*, 19, 155-162. (in Japanese)
- Keoleian, G. A., Kendall, A., Dettling, J. E., Smith, V. M., Chandler, R., Lepech, M. D. and Li, V. C., (2005). "Life cycle modeling of concrete bridge design: comparison of engineered cementitious composite link slabs and conventional steel expansion joints." *Journal of Infrastructure Systems*, 11(1), 51-60.
- Kheder, G. F., (1997). "A new look at the control of volume change cracking of base-restrained concrete walls." *ACI Structural Journal*, 94(3), 262-270.
- Kojima, K. and Nakamura, N., (2008). "Experiment of tensile creep and tensile relaxation for high strength mortar in early age." *Proceedings of the Japan Concrete Institute*, 30(1), 441-446. (in Japanese)
- Koyanagi, M., Masuda, Y. and Nakane, S., (1990). "A study on shrinkage cracks in reinforced concrete walls (Part 4) -Prediction analysis of cracking widths due to restrained volume change in one-way concrete members-." *Report of Obayashi Corporation Technical Research Institute*, 41, 73-79. (in Japanese)
- Kumano, T., Nishibayashi, S., Inoue, S. and Yoshino, A. (1999). "A study on the prediction model of tensile creep strain of concrete based on micro pore behavior." *Journal of Japan Society of Civil Engineers*, 42(613), 121-135.
- Naganuma, K., Yonezawa, K., Kurimoto, O. and Eto, H., (2004). "Simulation of nonlinear dynamic response of reinforced concrete scaled model using three-dimensional finite element method." *Proceedings of the 13th World Conference on Earthquake Engineering (WCEE13)*, Vancouver, BC, Canada, 586.
- Ohama, Y., (1987). "Principle of latex modification and some typical properties of latex-modified mortars and concretes." *ACI Materials Journal*, 84, 511-518.
- Ranaivomanana, N., Multon, S. and Turatsinze, A., (2013). "Tensile, compressive and flexural basic creep of concrete at different stress levels." *Cement and Concrete Research*, 52, 1-10.
- Rehm, G., (1961). Über die Grundlagen des Verbundes zwischen Stahl und Beton," *Deutscher Ausschuss für Stahlbeton*, Nr. 138.
- Sahmaran, M., Li, V. C. and Andrade, C., (2008). "Corrosion resistance performance of steel reinforced engineered cementitious composite beams." *ACI Materials Journal*, 105(3), 243-250.
- Sato, Y. and Naganuma, K., (2007). "Discrete-like crack simulation by smeared crack-based FEM for reinforced concrete." *International Journal for Earthquake Engineering and Structural Dynamics*, 36, 2137-2152.
- Sato, Y. and Naganuma, K., (2012). "Discrete-like crack simulation by smeared crack-based FEM --Reinforced concrete wall subjected to shrinkage." *Proceedings of Bond in Concrete 2012*, Vol. 1, Brescia, Italy, 193-200.
- See, H. T., Attiogbe, E. K. and Miltenberger, M. A., (2003). "Shrinkage cracking characteristics of concrete using ring specimens." *ACI Materials Journal*, 100(3), 239-245.
- Swamy, R. N. and Stavrides, H., (1979). "Influence of fiber reinforcement on restrained shrinkage and cracking." *ACI Journal*, 94(3), 262-270.
- Watanabe, Y., Wakatsuki, T., Sato, H. and Maruyama, T., (2005). "Development of Fe-Mn-Si-Cr shape memory alloy fiber reinforced plaster-based smart composites." *Materials Science Forum*, 475, 2063-2066.
- Yannopoulos, P. J. and Tassios, T. P., (1991). "Reinforced concrete axial elements analyzed under monotonic and cyclic actions." *ACI Structural Journal*, 8(1), 3-11.
- Zainab, Z. I. and Enas, A. A., (2008). "Reuse of waste iron as a partial replacement of sand in concrete." *Waste Management*, 28(11), 2048-2053.