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Scientific paper

Shear Strength of Reinforced Concrete Beams: Concrete Volumetric Change Effects

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Abstract

Volumetric changes in concrete may affect the structural performance of reinforced concrete structures and their durability. It has been revealed that autogenous shrinkage of high-strength concrete decreases reinforced concrete beam's shear strength compared with low-shrinkage concrete. High-drying shrinkage of concrete may also decrease the shear strength of reinforced concrete beams using normal-strength concrete. In this study, the effects of drying shrinkage and expansion of normal-strength concrete on the shear strength of reinforced concrete beams with/without stirrups were investigated. Six concrete mix proportions, using different aggregates and admixtures, were used to control concrete volumetric changes. The used concrete showed a large range in volumetric change and high-shrinkage concrete caused shrinkage-induced cracking in the beams before loading. Loading test results showed that drying shrinkage and expansion decreased and increased the shear strength, respectively. The decrease or increase in measured shear strength as a result of volumetric changes was well reproduced by the proposed equation. The equation included the new concept of strain change in tension reinforcement caused by concrete shrinkage or expansion, which was proposed previously for autogenous shrinkage of high-strength concrete.

1. Introduction

Concrete volume can change with time. Autogenous and drying shrinkage result from drying with cement hydration and moisture evaporation, respectively. Autogenous shrinkage is significant in high-strength concrete (i.e., low water-to-cement ratio concrete), whereas drying shrinkage is affected by the materials used and the environmental conditions. Aggregate type is a key factor that controls concrete shrinkage; some types increase shrinkage because of their own shrinkage (Carlson 1938; Hansen and Nielsen 1965; Hobbs 1974; Goto and Fujiwara 1979; Fujiwara 2008; Imamoto and Arai 2008).

Several Japanese reports have shown recent increases in drying shrinkage of concrete and shrinkage-induced damage of concrete structures in Japan. For example, the Architectural Institute of Japan (2006) reported that measured drying shrinkage rose, on average, by 100×10^{-6} from 672×10^{-6} in 1976 to 774×10^{-6} in the late 1990s. The concrete committee of the Japan Society of Civil Engineers (2005) also reported significant shrinkage cracks and excessive deflection of a partially prestressed concrete girder because of extensive concrete shrinkage. Recent increases in drying shrinkage in Japan

have most likely resulted from changes in aggregate quality associated with shortages of natural resources.

Countermeasures have been developed to overcome the negative effects of excessive shrinkage. For example, concrete expansion can be initiated chemically by the reaction of expansive additives. The use of expansive concrete can reduce concrete shrinkage (shrinkage-compensating) and establish internal compressive concrete stresses (chemical pre-stressing) in concrete structures (Klein *et al.* 1961; Muguruma 1968; Okamura and Tsuji 1972; Tsuji 1973; 1980; Nagataki and Gomi 1998; Collepardi *et al.* 2005).

The effect of volumetric changes in concrete materials on concrete structure performance needs to be discussed. To date, many studies have been published on internal stress, shrinkage cracking, crack control, and flexural stiffness change (Rüsch *et al.* 1983; Bischoff 2001; Sakata and Shimomura 2004; Tanimura *et al.* 2005; 2007; Sato *et al.* 2007; Gribniak 2008). Furthermore, some studies on structural strength have been published. Sato and Kawakane (2008) revealed that the autogenous shrinkage of high-strength concrete reduced the shear strength (diagonal cracking strength) of reinforced concrete (RC) beams without stirrups that were formed using high-strength concrete based on experimental comparisons between high- and low-shrinkage concretes. They also showed that the reduction in shear strength caused by autogenous shrinkage could be reproduced by their proposed method based on the concept of strain change in tension reinforcement. An increase in shear strength by chemical prestressing has been reported for expansive concrete (Okamura and Tsuji 1972). In addition, Tsuji and his colleagues proposed a method to evaluate chemical prestressing based on the concept of the

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Table 1 Concrete mix proportions.

d = 190 mm		Unit mass (kg/m ³)										
Name	W/B (%)	W	C1	L	Ex	S1	G1	G2	G3	SRA	SCA	AE
A	60	315	525	200	0	350	682	0	0	0	2	0.16
B	60	315	525	200	0	350	0	692	0	0	2	0.16
C	60	315	505	200	20	350	0	0	735	0	2	0.16
D	60	299	505	200	20	350	0	0	735	16	2	0.26
E	60	294	485	200	40	350	0	0	735	21	2	0.20
F	57	283	472	203	61	356	0	0	747	21	2	0.20
d = 500 mm		Unit mass (kg/m ³)										
Name	W/B (%)	W	C2	L	Ex	S2	G4	G5	SRA	SCA	AE	
A	60	315	525	200	0	351	679	0	0	2	0.16	
C	60	315	505	200	20	351	0	730	0	2	0.16	
D	60	299	505	200	20	351	0	730	16	2	0.26	

W: water, B: binder, C: ordinary Portland cement (3.16 g/m³), L: limestone powder (2.71 g/m³, 3,533 cm²/g), Ex: ettringite–lime composite expansive additive (3.10 g/m³, 2,820 cm²/g), S1: normal fine aggregate (2.72 g/m³, 1.33%), S2: normal fine aggregate (2.69 g/m³, 1.38%), G1: high-shrinkage coarse aggregate (2.66 g/m³, 0.98%), G2: high-shrinkage coarse aggregate (2.70 g/m³, 0.84%), G3: normal coarse aggregate (2.87 g/m³, 0.81%), G4: normal coarse aggregate (2.67 g/m³, 1.02%), G5: normal coarse aggregate (2.87 g/m³, 0.81%), SRA: shrinkage-reducing agent, SCA: segregation-controlling admixture, AE: air-entraining agent

Table 2 Volumetric ratios of materials in concrete.

Name	W/B (%)	Volumetric ratio (vol%)					
		Liquid (W + SRA)	Binder (B = C + Ex)	Powder (C + Ex + L)	S	G	Aggregate (S + G)
A–E	60	31.5	16.7	24.1	12.9	25.6	38.5
F	57	30.0	16.7	24.1	12.9	25.6	38.5

chemical energy conservation law of expansive concrete and discussed increases in the shear strength of RC beams (Tsuiji 1973, 1980; Suhara *et al.* 2009).

Few studies exist on quantitative investigations into the effect of drying shrinkage on the shear performance of RC beams. In 1957, Elstner and Hognestad tried to explain shear failure in a rigid frame warehouse as being caused by axial tensile stress related to shrinkage and temperature drops. Collins and Kuchima (1999), however, estimated that failure occurred because of size effects. After Elstner and Hognestad's work (1957), the effect of axial force has been studied to improve the design equations (Mattock 1969; Bhide and Collins 1988; Tamura *et al.* 1991). However, almost no studies exist that treat the effect of drying shrinkage directly. As one of the few exceptions, Vecchio and Shim (2004) suggest that micro-cracking caused by drying shrinkage decreased the beam shear strength with a decrease in tension-to-compression strength ratio.

A recent possibility of excessive drying shrinkage with higher aggregate shrinkage highlights the possible importance of studying the effect of drying shrinkage on the shear strength of RC structures quantitatively. The effects of internal tensile stress and initial cracking caused by drying shrinkage and the applicability of the concept proposed by Sato and Kawakane (2008) to the evaluation of these effects have been investigated previously (Morito and Nakarai 2009, 2010; Matsushita and Nakarai 2013). Mitani *et al.* (2011) also performed experimental studies on the influence of drying shrinkage on the shear performance of RC beams with different effective depths. Gebreyouhannes *et al.* (2014) published their numerical study to simulate the shrinkage effect by multi-scale

analysis with linkages to material science and structural mechanics.

Details of the study to investigate the effect of drying shrinkage on the shear strength of RC beams with/without stirrups are presented in this paper. An investigation of the effect of shrinkage cracking caused by drying is also included, because drying shrinkage in normal-strength concrete often causes cracks in concrete structures. The investigation was extended to the shear performance of chemically prestressed concrete beams by expansive additive to evaluate completely the effect of concrete volumetric change from shrinkage to expansion. Part of the experimental work in this paper is an English translation from the authors' papers that were published in Japanese (Morito and Nakarai 2009, 2010; Matsushita and Nakarai 2013). In this paper, former papers' contents are reanalyzed in a detailed discussion and new experimental works on relatively large beams are added to extend the discussion.

2. Experimental outline

2.1 Concrete

Table 1 shows the concrete mix proportions used in RC beams for two effective depths of 190 mm and 500 mm. Six mix proportions were prepared to control concrete volumetric changes. Constant volumetric ratios of the liquid (water, including the shrinkage-reducing agent (SRA)) and aggregate contents and a constant water-to-binder ratio (W/B) were used in all concrete samples, as shown in **Table 2**, to assess the effect of aggregate interlocking on tensile shear failure of RC beams. A calculation error led to the unit amount of water and W/B

Table 3 Design values for RC beam specimens.

Beam type	Size	h (mm)	b_w (mm)	d (mm)	A_s (mm ²)	A_{sw} (mm ²)	f_{cd} (N/mm ²)	P_{Vcd} (kN)	P_{Vsd} (kN)	P_{Vnd} (kN)	P_{Md} (kN)
N	Small	250	200	190	1588.8	0	30	134	0	134	340
S		250	200	190	1588.8	63.34	30	134	83	217	340
LN	Large	600	200	500	4560	0	30	282	0	282	1128

h : height of beam, b_w : web width, d : effective depth, A_s : area of longitudinal tensile reinforcements, A_{sw} : area of web reinforcements in single cross-section, f_{cd} : designed compressive strength of concrete, P_{Vcd} : designed load for shear carried by concrete (diagonal cracking load) calculated using Niwa's equation (1986), P_{Vsd} : designed load for shear carried by stirrup calculated by truss analogy, P_{Vnd} : designed nominal load for shear failure ($= P_{Vcd} + P_{Vsd}$), P_{Md} : designed nominal load for bending failure.

in mixture F being slightly smaller than the other mixtures. All concrete mixtures were designed to contain extremely large amounts of water to promote drying shrinkage, and contained mixed limestone powder and segregation controlling admixture to reduce concrete segregation. The maximum aggregate size was 20 mm.

So-called high-shrinkage aggregates were used in mixtures A and B (high-shrinkage concrete) to increase concrete shrinkage and shrinkage cracking of RC beams. In mixture C (normal-shrinkage concrete), 20 kg/m³ of the cement was replaced with an expansive additive (standard amount) to reduce early shrinkage cracking. Both of expansive additive (20 kg/m³) and SRA (16 kg/m³) were added to mixture D (low-strength concrete) to compensate for the drying shrinkage of concrete. Mixtures E and F (expansive concrete) were designed to generate chemical prestressing in the RC beam even under drying conditions by mixing large amounts of expansive additive (40, 60 kg/m³) and SRA (21 kg/m³). The targeted compressive strength of concrete at 28 days was 30 N/mm².

2.2 Specimens

Table 3 and Fig. 1 show the RC beam specimen design values and a schematic of their structure, respectively. In the beams, concrete listed in the previous section was casted vertically. Table 4 shows properties of the reinforcements. In this study, two kinds of RC beam specimens were prepared; those without stirrups (N and LN) and those with stirrups (S). Although the effective depth of the RC beams that were most commonly used was 190 mm (N and S), relatively large RC beams were also prepared with an effective depth of 500 mm (LN) only for beams without stirrups and for concrete mixtures A, C, and D. Two beams each were prepared except for mixtures B and F. The specimen name indicates the beam type (N, S, LN), concrete mixture (A–F), and specimen number (1, 2). All beams were designed to fail in shear. The longitudinal tensile reinforcement was a high-strength deformed bar (KSS785D16 for N and S, USD685AD38 for LN) and its ratio was set to be high ($p_s=4.18\%-4.56\%$, where p_s is the longitudinal tensile reinforcement ratio) to increase the internal confinement

Fig. 1 Schematic of RC beam specimens.

Table 4 Properties of reinforcing bars in RC beams.

Bar type	Beam type	Diameter (mm)	Yield strength (N/mm ²)	Elastic modulus (kN/mm ²)	Reinforcement ratio (%)
Longitudinal bar (tension)	N, S	16	1039	175	4.18
Longitudinal bar (compression)	N, S	16	1039	175	2.09
Stirrup	S	6	357	190	0.35
Longitudinal bar (tension)	LN	38	700	185	4.56
Longitudinal bar (compression)	LN	38	700	185	2.28

of concrete shrinkage and examine the effect of shrinkage cracking. The stirrup was a normal strength-deformed bar (SD295D6) and its ratio was 0.35%. The design capacities of the RC beams were calculated from these material properties.

Beam specimens were sealed for a few days after casting; 1 day sealing was used for N and S, 2 days for LNA, and 3 days for LNC and LND. The specimens were exposed to air in a room without controlled environmental conditions such as temperature and humidity. Friction between the specimens and the floor was eliminated by using rollers. Changes in strain of the longitudinal reinforcements and the stirrups were measured immediately after casting at the location shown in Fig. 1, to evaluate progress in concrete shrinkage or expansion in the RC beam specimens.

Cylindrical specimens of 100 mm diameter and 200 mm height were also prepared to measure the compressive strength and elastic modulus of the concrete used. Compressive tests were performed based on the Japan Industrial Standard (JIS) A 1108 at the loading test age. Other cylindrical specimens of 150 mm diameter and 150 mm height were prepared to measure the splitting tensile strength based on JIS A 1113. Prism specimens of 100 × 100 × 400 mm plain concrete were prepared to measure the free length change in concrete material. The change in length was measured by contact-type strain gauge using method JIS A 1129-2 after removal of the frame. Cylindrical and prism specimens were cured under the same conditions as the beams.

2.3 Loading tests

Figure 1 shows a schematic of the loading tests for the RC beam specimens. All RC beam specimens were simply supported and loaded with two concentrated loads. The total length, full span, and shear span of the N and S beams were 2,400 mm, 1,600 mm, and 650 mm, and 6,000 mm, 4,500 mm, and 1,750 mm in the LN

Fig. 2 Overview of loading test (LN).

beams, respectively. The ratio of shear span to effective depth was 3.42 or 3.50 to initiate diagonal tension failure of the slender beams. During the loading test, the load, displacements, and strains of the reinforcements were measured. Vertical displacements in the shear span were measured using π -shaped displacement transducers to detect the generation of significant diagonal cracking. In beams N and S, four π -shaped transducers with a reference length of 200 mm were used for each shear span at 130 mm intervals. In the LN beams, six π -shaped transducers with a reference length of 500 mm were used at 250 mm intervals as shown in Fig. 2.

3. Experimental results and discussion

3.1 Compressive strength and Young's modulus of concrete

Table 5 shows results for the compressive strength, Young's modulus, and tensile strength of concrete measured at the loading test age. They showed almost the same level of compressive strength (~ 30 N/mm²) and tensile strength (~ 2.5 N/mm²). For mixtures E and F, however, a slightly lower strength resulted because the large expansion of expansive concrete was not confined in the material tests.

3.2 Free length change in plain concrete

Figure 3 shows changes in the expansion or shrinkage strains measured on plain concrete prism specimens without reinforcements. Concrete in mixtures A and B showed a high shrinkage of over $-1,000 \times 10^{-6}$. For mixture C, the shrinkage was $\sim 500 \times 10^{-6}$, which was slightly smaller than the average shrinkage of normal concrete in Japan. Mixture D showed a small length change. The concrete in E and F showed a large expansion of approximately $+600$ – $+900 \times 10^{-6}$. Expansive additive expanded the concrete and the SRA reduced the drying shrinkage after concrete expansion. This indicated that concrete prepared in this study exhibited a large variety of volumetric changes.

3.3 Strain of reinforcement and concrete in RC beams before loading tests

Figure 4 shows measured strains of longitudinal reinforcing bars in RC beam specimens with time. These were average values for each mix proportion, which were calculated from all measured strains on the longitudinal reinforcements of beam specimens. Table 5 shows measured strains of the three-level longitudinal reinforcements (tensile bars at the bottom and middle and

Table 5 Measured concrete properties and initial strains of reinforcements and calculated stresses of concrete at loading age.

Name	Compressive strength of concrete (N/mm ²)	Young's modulus of concrete (kN/mm ²)	Splitting tensile strength of concrete (N/mm ²)	Strain of upper longitudinal rebar ($\times 10^{-6}$)	Strain of middle longitudinal rebar ($\times 10^{-6}$)	Strain of bottom longitudinal rebar ($\times 10^{-6}$)	Strain of stirrup ($\times 10^{-6}$)	Longitudinal stress of bottom concrete (N/mm ²)	Vertical stress of web concrete (N/mm ²)
NA1	30.2	19.0	2.58	-199	-128	-101	—	3.26*	—
NA2	30.8	18.8	2.60	-183	-132	-131	—	3.40*	—
NB1	29.7	17.5	2.39	-203	-130	-118	—	3.54*	—
NC1	29.5	20.8	2.34	-103	-105	-101	—	2.32	—
NC2	28.1	19.8	2.57	-124	-97	-79	—	2.26	—
ND1	26.8	19.7	2.41	16	25	23	—	-0.43	—
ND2	30.4	19.7	2.44	19	2	-4	—	-0.14	—
NE1	25.2	21.1	2.56	105	79	75	—	-1.97	—
NE2	25.2	21.1	2.54	97	66	53	—	-1.64	—
NF1	23.4	21.5	—	244	243	218	—	-5.28	—
SA1	29.5	19.0	2.47	-181	-133	-134	-284	3.41*	0.19
SA2	29.8	18.6	2.69	-204	-141	-96	-326	3.34*	0.22
SB1	29.1	18.6	2.33	-187	-127	-103	-283	3.16*	0.19
SC1	29.1	20.5	2.21	-126	-97	-98	-86	2.44	0.06
SC2	27.2	18.2	2.40	-126	-78	-67	-79	2.07	0.05
SD1	27.1	18.4	2.27	16	21	16	105	-0.40	-0.07
SD2	29.7	19.7	2.28	10	7	-7	149	-0.07	-0.10
SE1	25.2	21.1	—	87	45	44	337	-1.36	-0.23
SE2	25.2	21.1	—	114	87	90	251	-2.21	-0.17
SF1	23.4	21.5	—	296	281	242	737	-6.14	-0.49
LNA1	32.6	18.3	2.59	-203	-87	-82	—	4.27*	—
LNA2	32.6	18.3	2.59	-210	-77	-67	—	4.08*	—
LNC1	31.5	20.5	2.60	-79	-28	-46	—	1.78	—
LNC2	31.5	20.5	2.60	-93	-49	-54	—	2.24	—
LND1	30.0	20.0	2.38	-5	23	0	—	-0.14	—
LND2	30.0	20.0	2.38	-17	28	-25	—	0.27	—

* Unconfirmed values because of shrinkage cracking.

— Not measured items.

(a) Concrete for beam types N and S

(b) Concrete for beam type LN

Fig. 3 Volumetric changes in plain concrete specimens.

(a) Beam types N and S (d = 190 mm)

(b) Beam type LN (d = 500 mm)

Fig. 4 Changes in average strain of longitudinal reinforcement of RC beams.

(a1) Beam types N and S (d = 190 mm, full span)

(a2) Beam types N and S (d = 190 mm, half span (shear-failed side))

(b) Beam types LN (d = 500 mm, full span)

Fig. 5 Crack patterns of RC beams immediately before loading tests and after failure.

compressive bars at the top) and stirrups in the RC beams and the calculated tensile stresses of the concrete at different loading test times. They are measured immediately before the loading tests as initial strains in the loading tests. The maximum compressive strain was -150×10^{-6} as measured in mixtures A and B and the maximum tensile strain was $+250 \times 10^{-6}$ as measured in mixture F. Longitudinal concrete stresses at the extreme bottom fiber were calculated from measured strain distributions of longitudinal reinforcements and vertical stresses were calculated from the average strains of stirrups based on the Navier hypothesis and assuming concrete elasticity. Calculated longitudinal stresses of concrete for mixtures A and B are unconfirmed because initial cracks occurred in the beams because of high shrinkage before the loading tests.

Concrete in mixtures A, B, and C applied compressive strain on the reinforcements because of concrete shrinkage. In cases A and B, the calculated longitudinal concrete stress exceeded the measured tensile concrete strength ($\sim 2.5 \text{ N/mm}^2$). Concrete in mixture D caused small strain and stress on the reinforcements and concrete because of a lower volumetric change. Concrete in mixtures E and F caused tensile strain on the reinforcements and chemical pre-strain on the concrete itself. Increases in compressive strains of the reinforcements in the larger beam LN were smaller and slower than those in the smaller beam N and S.

3.4 Shrinkage cracking in RC beams before loading tests

Figure 5 shows crack patterns observed before and after the loading tests. The initial cracks (shrinkage-induced cracks) before loading include surface cracking caused by rapid shrinkage of surface concrete confined by interior concrete. Extensive shrinkage cracking occurred for mixtures A and B. The maximum shrinkage crack widths were 0.04 mm in beams N and S and 0.06 mm in beam LN. The specimen crack pattern, however, varied because of differences in local environmental conditions during curing, even for the same beam conditions. Mixture C showed relatively low concrete shrinkage; few shrinkage cracks existed in small beams (NC and SC) whereas many shrinkage cracks were observed in the large beams (LNC). Shrinkage cracks in beam LNC appeared to be surface cracks. In other cases, cracks before loading tests were not observed. Cracks during loading tests will be discussed with other results in the next section.

3.5 Results of loading test of RC beams

3.5.1 Relationship between load and deflection

Figure 6 shows examples of the relationship between measured load and RC beam deflection. The bending elasticity of the beams decreased with increase in shrinkage. After bending cracking, all beams showed diagonal cracks that were initiated by bending cracks. In the small beams without stirrups (beam type N), small

decreases in load resulted in significant diagonal cracking under loads of $\sim 100\text{--}150 \text{ kN}$ and then the load was increased to final failure. The large increase in load after diagonal cracking may be explained by the strong restraint provided by the large amount of longitudinal reinforcements in the bottom and top areas in the small beam. In the beams with stirrups (beam type S), the load was increased almost constantly regardless of the generation of diagonal cracks because of the function of the stirrups. The ultimate load was recorded after yielding of the stirrups. In the large beam without stirrups (beam type LN), the load was decreased immediately after significant diagonal cracking. Shear failure modes occurred for all beams (tensile shear or compressive shear failure without yielding of the longitudinal reinforcements).

The initial bending elasticities of the expansive con-

(a) N ($d = 190 \text{ mm}$, without stirrups)

(b) S ($d = 190 \text{ mm}$, with stirrups)

(c) LN ($d = 500 \text{ mm}$, without stirrups)

Fig. 6 Examples of relationship between load and deflection of RC beams.

crete beams with stirrups (SE and SF) showed quite higher values in this study. Since the reason was not clear, the effect of confinement effect needs to be investigated in future study.

3.5.2 Cracking

Figure 5 shows the crack patterns after loading tests with the initial shrinkage-induced cracks. All beams showed typical shear failure. For the high-shrinkage concrete, shrinkage-induced cracking as initial cracks before the loading test may have influenced the development of diagonal cracking because initial cracks connected the diagonal cracking during loading tests. In some previous studies on the effect of initial cracks on the shear capacity of RC beams, it was reported that initial vertical cracks decreased the flexural stiffness but increased the shear strength because of a positive interaction between initial and diagonal cracks (Niwa *et al.* 1986, Pimanmas *et al.* 2001). In this study, however, the increase in shear strength of the RC beams with initial cracks was not observed clearly. On the contrary, the possibility of a decrease in shear strength will be discussed in Section 4.2.1 with the calculated values.

3.5.3 Shear strength

The shear strength of the RC beam was calculated from a significant diagonal cracking load, which is referred to the discussion by Sato and Kawakane (2008). The significant diagonal cracking load was determined from sudden increased points of measured vertical deformation, the stirrup strain in the shear span of the beam, and

visual crack observation during the loading test. **Figure 7** shows examples of measured vertical displacements and strains to determine significant diagonal cracking loads. In beams without stirrups such as NA1 and LNA1, sudden large increases in vertical displacements were clearly visible. In beams with stirrups such as SA1, increases in vertical displacements were smaller than those without stirrups because the stirrups prevent large shear deformation after diagonal cracking. Even in beams with stirrups, a diagonal cracking load could be detected by comparison of changes in vertical displacement and strain. Increases in vertical deformation and stirrup strain by bending cracking were neglected for decisions on diagonal cracking load.

Table 6 summarizes experimental results from the loading tests. **Figure 8** shows the relationship between the modified shear strength and the average initial tensile strain of the longitudinal reinforcements. Because the compressive strength varied slightly, the shear strength at significant diagonal cracking was modified to indicate a concrete strength of 30 N/mm². In this modification, it was assumed that the RC beam shear strength is a function of one-third of the concrete compressive strength based on the study by Niwa *et al.* (1986). The initial strain shows the average value of the three layers (upper, middle, and bottom) at the center of the longitudinal reinforcement immediately before the loading test. The results showed that the shear strength increased as the tensile strain of the reinforcement was increased by an increase in expansion or decrease in concrete shrinkage. The effect of stirrups on the shear strength is unclear

Fig. 7 Examples of determination of significant diagonal cracking load.

Table 6 Summary of experimental loading test results.

Names	P_{fc} (kN)	τ_{c_exp} (N/mm ²)	$\tau_{c_exp30}^*$ (N/mm ²)	P_u (kN)	τ_{u_exp} (N/mm ²)
NA1	112	1.47	1.47	151	1.99
NA2	96	1.26	1.25	167	2.20
NB1	97	1.28	1.28	133	1.75
NC1	125	1.64	1.65	170	2.24
NC2	123	1.62	1.65	157	2.07
ND1	124	1.63	1.69	218	2.87
ND2	131	1.72	1.72	162	2.13
NE1	132	1.74	1.84	217	2.86
NE2	136	1.79	1.90	207	2.72
NF1	151	1.98	2.15	316	4.16
SA1	110	1.45	1.46	255	3.36
SA2	100	1.32	1.32	260	3.42
SB1	102	1.34	1.36	258	3.39
SC1	122	1.61	1.62	283	3.72
SC2	117	1.54	1.59	267	3.51
SD1	129	1.70	1.76	281	3.70
SD2	136	1.79	1.80	297	3.91
SE1	141	1.86	1.97	348	4.58
SE2	152	2.00	2.12	337	4.43
SF1	177	2.33	2.53	392	5.16
LNA1	224	1.12	1.09	224	1.12
LNA2	230	1.15	1.13	230	1.15
LNC1	233	1.17	1.17	233	1.17
LNC2	206	1.03	1.01	268	1.34
LND1	238	1.19	1.19	241	1.21
LND2	254	1.27	1.27	254	1.27

P_{fc} : measured significant diagonal cracking load, τ_{c_exp} : measured shear strength at significant diagonal cracking, $\tau_{c_exp30}^*$: modified shear strength by compressive strength ($= \tau_{c_exp} / f_c^{1/3} \times 30(\text{N/mm}^2)^{1/3}$) at significant diagonal cracking, P_u : measured maximum load, τ_{u_exp} : measured ultimate shear strength at maximum load

because changes in the vertical stress in the concrete caused by the stirrup's restraint of the concrete volume change were small. Among the small beams in this study, the minimum modified shear strength measured in NA2 was only 49% of the maximum strength measured in SF1. The shear strength of the large beams was smaller than that of the small beams because of the size effect of shear strength. When the average modified shear strength of high-shrinkage concrete beams was compared with that of low-shrinkage concrete beams, the reduction ratios were 22% and 10% for the small and large beams, respectively. These reduction ratios of beams were larger

or almost similar values as the results by Sato and Kawakane (2008), which showed 5% and 12% reductions by autogenous shrinkage in their high-strength beams with the effective depth of 250mm and 500mm.

3.5.4 Ultimate shear strength

Figure 9 shows the relationship between ultimate shear strength and average tensile strain of the longitudinal reinforcements. The ultimate shear strength was calculated from the measured maximum load. The results also show the tendency for an increase in ultimate shear strength with increase in reinforcement tensile strain.

Fig. 8 Relationship between modified shear strength and initial tensile strain of longitudinal reinforcement of RC beams.

Fig. 9 Relationship between ultimate shear strength and initial tensile strain of longitudinal reinforcement of RC beams.

The internal stress caused by a volumetric change in concrete could affect the ultimate failure of RC beams in this study. In the small beam, the ultimate shear strength became much higher than the shear strength at diagonal cracking. The large increase in the beam with stirrups could be explained by the contribution of the stirrups; the increase in the beam without stirrups may be explained by the tied-arch system with a large amount of longitudinal reinforcement. Large beams showed brittle failure with diagonal cracking because they contained a relatively large volume of plain concrete without reinforcement.

3.5.5 Stirrup stress

Figure 10 shows examples of changes in average stress of stirrups of RC beams during loading tests and an estimation by truss analogy. The stirrup stress indicated by the solid line was calculated from the measured strain during loading tests by considering the initial strain associated with volumetric changes in concrete. The average value was calculated from the results of stirrups, which yielded to ultimate failure at the shear span. The estimated value indicated by the dashed line was calculated using Eqs. (1) – (3). The contribution of the stirrups was calculated using Eq. (3) based on the truss analogy. In this study, the simplified assumption that the angle of the diagonal cracks is 45° in the truss analogy was used as several codes like Japanese and Amirian codes although variable angle truss model is also used in Eurocode. Then, it was added to the internal resisting shear provided by the concrete from the measured significant diagonal cracking load, as shown in Eqs. (1) and (2). This is also the simplified assumption adopted in several codes in Japan, North America and other countries.

$$V_n = V_c + V_s \quad (1)$$

$$V_c = \frac{P_{Vc}}{2} \quad (2)$$

$$V_s = A_{sw} \frac{\sigma_{sw}}{1000} \frac{z}{s} \quad (3)$$

where V_n : nominal shear force (kN), V_c : internal resisting

shear by concrete (kN), V_s : internal resisting shear by stirrups (kN), P_{Vc} : measured significant diagonal cracking load (kN), A_{sw} : area of stirrups in one cross section (mm^2), σ_{sw} : stress of the stirrups (N/mm^2), z : distance between internal compressive and tensile forces on a cross section ($= 7/8d$, mm), d : effective depth of beam (mm) and s : spacing of the stirrups measured along the longitudinal axis of the beam (mm).

The dots in Fig. 11 show the increase in shear force after significant diagonal cracking when the average stirrup stress reached 100, 200, and 300 N/mm^2 . The dashed line shows the internal resisting shear by stirrups, which was calculated from Eq. (3) based on the truss analogy.

Results in Figs. 10 and 11 show that the measured increase in shear force for a small stress level almost agree with the calculated internal resisting shear by stirrups based on the truss analogy. This means that volumetric changes in concrete do not affect the shear resistant contribution by stirrups and differences in the concrete contribution affected by volumetric changes did not disappear after significant diagonal cracking. The measured increase in shear force became larger than the calculated value. A similar tendency has also been reported in experiments using high-strength concrete by Kawakane *et al.* (2007, 2009).

4. Evaluation of shear strength of RC beams

4.1 Evaluation methods

4.1.1 Shear strength carried by concrete at significant diagonal cracking

In the civil engineering field in Japan, the equation proposed by Niwa *et al.* (1986) is used widely to evaluate shear capacity of slender RC beams without stirrups:

$$\tau_{c_cal} = 0.2 \sqrt[3]{f'_c} \sqrt[3]{100 p_s} \sqrt[4]{1000/d} \left(0.75 + \frac{1.4}{a/d} \right) \quad (4)$$

where τ_{c_cal} : shear strength of the RC beam (N/mm^2), f'_c : measured concrete compressive strength (N/mm^2), p_s : longitudinal tensile reinforcement ratio ($= A_s/b_w/d$), a : shear span of the beam (mm), A_s : area of longitudinal

Fig. 10 Changes in average stirrup stress during loading test and truss analogy.

Fig. 11 Increase in shear force with increase in average stirrup strain.

tensile reinforcement (mm^2) and b_w : web width of the beam (mm).

The design equation in the standard specification for concrete structures published by the Japan Society of Civil Engineers (JSCE) was established based on Niwa's equation and neglects the term for shear span to effective depth ratio (a/d). Niwa's equation was formulated based on a statistical analysis of the experimental results (~300 results) of RC beams with normal-strength concrete. Here, the effect of the shrinkage was not considered explicitly.

To consider the effect of autogenous shrinkage of high-strength concrete on the shear strength of RC beams, Sato and Kawakane (2008) proposed a new evaluation method based on the concept of strain change in tension reinforcement caused by concrete shrinkage. Here, they used the following equivalent tensile reinforcement ratio:

$$p_{s,e} = \frac{\varepsilon_s}{\varepsilon_s - \varepsilon_{s0,def}} p_s \quad (5)$$

where $p_{s,e}$: equivalent tensile reinforcement ratio, ε_s : tension reinforcement strain at section $1.5d$ distant from the loading section in the shear span at the diagonal cracking and $\varepsilon_{s0,def}$: tension reinforcement strain when the concrete stress at a depth of reinforcement is zero, which is positive in tension and negative in compression.

Sato and Kawakane (2008) included this concept in the equation to evaluate the shear strength of RC beams with high-strength concrete, which was formulated based on fracture energy. Then, they showed that their proposed new evaluation method reproduced the shear strengths of two types of high-strength RC beams made with high- and low-autogenous shrinkage types of concrete.

In this study, our target is RC beams with normal-strength concrete that shows a volumetric change by drying shrinkage or expansive additive. Therefore, it was proposed that the concept of Sato and Kawakane (Eq. (5)) is included in Niwa's equation (Eq. (4)) by replacing the definition of tensile reinforcement ratio as follow:

$$\tau_{c_cal'} = 0.2 \sqrt[3]{f'_c} \sqrt[3]{100 p_{s,e}} \sqrt[4]{1000/d} \left(0.75 + \frac{1.4}{a/d} \right) \quad (6)$$

where $\tau_{c_cal'}$: shear strength of RC beams (N/mm^2) by the proposed equation.

4.1.2 Ultimate shear strength of RC beams with stirrups

In many design codes such as JSCE, ACI and Canadian codes, to evaluate the ultimate shear strength of the RC beams with stirrups, the shear strength carried by the stirrups is added to the shear strength carried by concrete as discussed in Section 3.5.5. The shear strength carried by the stirrups can be calculated based on the truss analogy; the shear strength carried by the concrete was calculated using Eqs. (4) or (6), as explained above. Then, two nominal ultimate shear strengths (τ_{n_cal} and $\tau_{n_cal'}$) were obtained in this study.

4.2 Applicability of evaluation methods considering effect of volumetric change

4.2.1 Shear strength

Table 7 and Fig. 12 compare the measured shear strength (τ_{c_exp}) and the calculated shear strength (τ_{c_cal} and $\tau_{c_cal'}$) obtained using Niwa's equation (Eq. (4)) and the proposed equation (Eq. (6)). The measured shear strength was obtained from experimental results of significant diagonal cracking loads as discussed in Section 3.5.3. Table 8 lists average values and standard deviations of ratios of measured to calculated shear strength in Table 7 to indicate the accuracy of the equations.

Figure 12(a) shows that Niwa's equation underestimates the shear strength with concrete expansion and overestimates shear strength with concrete shrinkage. The measured strengths of the high-shrinkage concrete beams were 71%–84% of the calculated strength in the small beams (NA, NB, SA, and SB) and 70%–79% in the large beams (LNA). The significant decrease in shear strength by drying shrinkage was not covered by the conventional equation. In the small beams, the measured beam strength with lower volumetric change concrete (ND and SD) almost agreed with the calculated value.

(a) Calculations using Niwa's equation (Eq. (4))

(b) Calculations using proposed equation (Eq. (6))

Fig. 12 Relationship between measured and calculated shear strength at significant diagonal cracking and average initial strain of longitudinal reinforcement of RC.

Table 7 Measured and calculated shear strength at point of significant diagonal cracking.

Names	τ_{c_exp} (N/mm ²)	τ_{c_cal} (N/mm ²)	$\tau_{c_exp}/\tau_{c_cal}$	$\varepsilon_{s0,def}(\times 10^{-6})$	$p_{s,e}$ (%)	$\tau_{c_cal'}$ (N/mm ²)	$\tau_{c_exp}/\tau_{c_cal'}$
NA1	1.47	1.76	0.84	-173.4	3.13	1.60	0.92
NA2	1.26	1.77	0.71	-200.3	3.00	1.59	0.80
NB1	1.28	1.75	0.73	-193.4	3.03	1.57	0.81
NC1	1.64	1.75	0.94	-152.0	3.23	1.60	1.03
NC2	1.62	1.72	0.94	-131.7	3.32	1.59	1.02
ND1	1.63	1.69	0.96	36.1	4.47	1.73	0.94
ND2	1.72	1.77	0.98	-1.7	4.17	1.76	0.98
NE1	1.74	1.66	1.05	112.9	5.21	1.79	0.97
NE2	1.79	1.66	1.08	87.1	4.95	1.75	1.02
NF1	1.98	1.62	1.23	336.6	8.58	2.06	0.96
SA1	1.45	1.75	0.83	-202.7	2.98	1.56	0.93
SA2	1.32	1.75	0.75	-180.8	3.09	1.59	0.83
SB1	1.34	1.74	0.77	-175.6	3.11	1.58	0.85
SC1	1.61	1.74	0.92	-144.4	3.26	1.60	1.00
SC2	1.54	1.70	0.90	-111.5	3.43	1.59	0.97
SD1	1.70	1.70	1.00	28.5	4.41	1.73	0.98
SD2	1.79	1.75	1.02	-0.1	4.18	1.75	1.02
SE1	1.86	1.66	1.12	65.0	4.74	1.73	1.07
SE2	2.00	1.66	1.21	129.8	5.39	1.81	1.11
SF1	2.33	1.62	1.44	381.7	9.55	2.13	1.09
LNA1	1.12	1.45	0.77	-134.2	3.32	1.30	0.86
LNA2	1.15	1.45	0.79	-109.6	3.51	1.33	0.87
LNC1	1.17	1.43	0.81	-57.6	3.96	1.37	0.85
LNC2	1.03	1.43	0.72	-82.1	3.73	1.34	0.77
LND1	1.19	1.41	0.84	21.5	4.82	1.44	0.83
LND2	1.27	1.41	0.90	4.7	4.62	1.42	0.90

Table 8 Accuracy of calculated shear strength at point of significant diagonal cracking.

(a) Average and standard deviation of shear strength ratio (experimental/calculated) in beam type N and S (d = 190 mm)

Equation	Average		Standard deviation	
	All (A-F)	Without severe shrinkage cracks (C-F)	All (A-F)	Without severe shrinkage cracks (C-F)
Niwa	0.97	1.06	0.18	0.14
Proposed	0.97	1.01	0.09	0.05

(b) Average and standard deviation of shear strength ratio (experimental/calculated) in beam type LN (d = 500 mm)

Equation	Average		Standard deviation	
	All (A-F)	Without severe shrinkage cracks (C, D)	All (A-F)	Without severe shrinkage cracks (C, D)
Niwa	0.81	0.82	0.06	0.07
Proposed	0.85	0.84	0.04	0.05

These results suggest (i) that the experimental results used to formulate the equation were not affected significantly by drying shrinkage or (ii) that the concrete in this study reduced the overall shear strength because it contained fewer aggregates (38.5 vol% shown in **Table 2**). The later possibility means that the experimental results used for the equation included a shrinkage effect. The measured strength in the large beams using lower volumetric change concrete (LND) was smaller than the calculated value. This could also be explained by the reduced aggregate contents. In either case, an appropriate modification was necessary to evaluate the shear strength of RC beams with a consideration for the effect of the concrete volumetric change.

Figure 12(b) and **Table 8** show that the proposed modifications with equivalent tensile reinforcement ratio increases the accuracy of the calculation of shear strength. For example, the ratios of measured to calculated strengths of the high-shrinkage concrete beams were

improved to 80%–93% from 71%–84% in the small beams and to 86%–87% from 70%–79% in the large beams. These are similar tendencies as the study by Sato and Kawakane (2008). In all small beams including expansive concrete beams in this study, the average value of the measured/calculated ratio was still close to 1.0 and the standard deviation was reduced from 0.18 to 0.09. The accuracy was improved further when the results of RC beams without initial shrinkage cracks (NA, NB, SA and SB) were excluded. This occurs because the new method did not consider the loss of shrinkage-induced internal stress by shrinkage cracking. In addition, shrinkage-induced cracking may affect the progress of shear cracks during loading. In large beams, the accuracy was also improved by using the proposed method so that the standard deviation decreased. The average shear strength ratio, however, was small (~0.85) because the effect of reduced aggregate contents became significant in large beams with brittle failure.

Table 10 Accuracy of calculated ultimate shear strength of RC beams with stirrups.

Equation	Average			Standard deviation		
	All (A-F)	Without expansion (A-D)	With expansion (E, F)	All (A-F)	Without expansion (A-D)	With expansion (E, F)
Niwa + Truss analogy	1.40	1.26	1.73	0.23	0.07	0.13
Proposed + Truss analogy	1.39	1.31	1.58	0.13	0.04	0.04

Table 9 Measured and calculated ultimate shear strength of RC beams with stirrups.

Names	τ_{u_exp} (N/mm ²)	τ_{n_cal} (N/mm ²)	$\tau_{u_exp} / \tau_{n_cal}$	$\tau_{n_cal'}$ (N/mm ²)	$\tau_{u_exp} / \tau_{n_cal'}$
SA1	3.36	2.84	1.18	2.65	1.26
SA2	3.42	2.85	1.20	2.68	1.28
SB1	3.39	2.83	1.20	2.67	1.27
SC1	3.72	2.83	1.31	2.69	1.38
SC2	3.51	2.79	1.26	2.69	1.31
SD1	3.70	2.79	1.32	2.82	1.31
SD2	3.91	2.84	1.37	2.84	1.37
SE1	4.58	2.75	1.66	2.82	1.62
SE2	4.43	2.75	1.61	2.90	1.53
SF1	5.16	2.71	1.90	3.22	1.60

τ_{n_cal} : nominal ultimate shear strength calculated using the truss analogy with Niwa's equation, $\tau_{n_cal'}$: nominal ultimate shear strength calculated using the truss analogy with the proposed equation.

4.2.2 Ultimate shear strength with stirrups

Table 9 and Fig. 13 compare the measured ultimate shear strength (τ_{u_exp}) and the calculated nominal ultimate shear strength (τ_{n_cal} or $\tau_{n_cal'}$) using the truss analogy with Niwa's equation (Eq. (4)) or the proposed equation (Eq. (6)). The measured shear strength was obtained from results of the maximum loads in the experiments. Table 10 lists the average values and standard deviations of the ratios of measured to calculated ultimate shear strength in Table 9.

The ultimate shear strength ratios increased with increase in initial strains in both calculations. The calculated nominal shear strength was always smaller than the measured ultimate shear strength because the truss analogy is a simplified method that neglects many factors

Fig. 13 Relationship between measured/calculated ultimate shear strength and average initial strain of longitudinal reinforcement of RC beams.

such as the strength increase after stirrup yielding. These results suggest that calculations with the truss analogy could assess the ultimate shear strength conservatively, even for the RC beam with high-shrinkage concrete. The detailed comparison also indicated that the proposed calculation with equivalent tensile reinforcement ratio increased the accuracy slightly, where the standard deviation of the ultimate shear strength ratio was improved from 0.23 to 0.13. The ultimate shear strength ratio of the chemically prestressed concrete (SE and SF) was significantly higher than the others (SA, SB, SC, and SD). This special behavior of chemically prestressed concrete needs to be considered in future studies.

5. Conclusions

The effect of volumetric changes in normal-strength concrete on the shear strength of RC beams was investigated. Volumetric changes in concrete were controlled by changing materials such as aggregates and admixture. The used concrete showed a large range of volumetric change from $-1,000 \times 10^{-6}$ for shrinkage to $+900 \times 10^{-6}$ for expansion in plain concrete without reinforcements. In the loading tests, the shear strength and ultimate shear strength of the RC beams were determined from a significant diagonal cracking load and the maximum load, respectively. Results of the loading tests indicate the followings.

- 1) Volumetric changes in concrete induced internal stress and strain in the RC beams. In this study, the measured strain of longitudinal reinforcement at the loading test age varied from -150×10^{-6} for compressive strain by concrete shrinkage to $+250 \times 10^{-6}$ for tensile strain by expansion. The high-shrinkage concrete caused shrinkage-induced cracking in the RC beams before the loading tests.
- 2) Drying shrinkage and expansion of the concrete decreased and increased the RC beam shear strength, respectively, when measured under significant cracking loads. The minimum shear strength measured in the high-shrinkage concrete beam with the shrinkage-induced cracks was 49% of the maximum strength measured in the expansive concrete beam. The ultimate shear strength, which was recorded at the maximum load after significant diagonal cracking, also decreased and increased with the volumetric changes in the concrete. The increase in shear force after significant diagonal cracking, which was measured in the RC beams with stirrups, almost agreed with the calculated internal resisting shear by stirrups based on the truss analogy.

- 3) The conventional design equation proposed by Niwa *et al.* (1986) underestimated the shear strength with concrete expansion and overestimated the shear strength with concrete shrinkage. For high-shrinkage concrete beams, the measured strength was 70%–84% of the calculated strength.
- 4) A modified equation using the equivalent tensile reinforcement ratio established by Sato and Kawakane (2008) was proposed to consider the effect of volumetric change on the shear strength. Comparisons with the experimental results showed the improved accuracy of the calculations. For example, the shear strength ratio (experimental/calculated) of high-shrinkage concrete beams increased from 70%–84% to 86%–93% and the standard deviation of the shear strength ratio of all beams decreased from 0.18 to 0.09. Shrinkage-induced cracking additionally reduced the shear strength by affecting the progress of the shear cracks during loading.
- 5) To determine the ultimate shear strength of RC beams with stirrups, calculations using the truss analogy with the conventional and proposed calculations of the shear strength carried by concrete were performed to provide a conservative evaluation. The proposed calculation improved the standard deviation of ultimate shear strength ratio (experimental/calculated) from 0.23 in the conventional calculation to 0.13. The chemically prestressed concrete beam showed a large ultimate shear strength ratio compared with other normal RC beams.

These findings were obtained from the experiments with special conditions such as high longitudinal tensile reinforcement ratio (4.18%–4.56%), few coarse aggregate concrete (38.5 vol%), and short curing periods (1day–3days), in order to emphasize the effect of the drying shrinkage on the shear strength. The general versatility of these conclusions needs to be verified in future study.

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Notation list

- a : shear span of beam (mm)
 A_s : area of longitudinal tensile reinforcement (mm^2)
 A_{sw} : area of stirrups in single cross-section (mm^2)
 b_w : web width of beam (mm)
 d : effective depth (mm)
 E_c : measured Young's modulus of concrete (kN/mm^2)
 f'_c : measured compressive strength of concrete (N/mm^2)
 f'_{cd} : designed compressive strength of concrete (N/mm^2)
 h : height of beam (mm)
 p_s : longitudinal tensile reinforcement ratio ($=A_s/b_w/d$)
 P_{Md} : designed nominal load for bending failure (kN)
 P_{Vc} : measured significant diagonal cracking load (kN)
 P_{Vcd} : designed load for shear carried by concrete calculated using Niwa's equation (kN)
 P_{Vsd} : designed load for shear carried by stirrup calculated using the truss analogy (kN)
 P_{Vnd} : designed nominal load for shear failure ($=P_{Vcd} + P_{Vsd}$) (kN)
 P_u : measured maximum load (kN)
 s : spacing of stirrups measured along the longitudinal

- axis of the beam (mm).
- V_n : nominal shear force (kN)
- V_c : internal resisting shear produced by concrete (kN)
- V_s : internal resisting shear produced by stirrups (kN)
- z : distance between internal compressive and tensile forces on a cross-section ($= 7/8d$, mm)
- σ_{sw} : stress of the stirrups (N/mm^2),
- τ_{c_exp} : measured shear strength at point of significant diagonal cracking (N/mm^2)
- $\tau_{c_exp30}^*$: modified shear strength multiplied by compressive strength ($= \tau_{c_exp} / f'_c{}^{1/3} \times 30(\text{N/mm}^2)^{1/3}$) at point of significant diagonal cracking
- (N/mm^2)
- τ_{c_cal} : shear strength of the RC beams by Niwa's equation (N/mm^2)
- τ_{c_cal} : shear strength of RC beams by the proposed equation (N/mm^2)
- τ_{n_cal} : nominal ultimate shear strength calculated using the truss analogy with Niwa's equation (N/mm^2)
- τ_{n_cal} : nominal ultimate shear strength calculated using the truss analogy with the proposed equation (N/mm^2)
- τ_{u_exp} : measured ultimate shear strength at maximum load (N/mm^2)