Nonlinear Gel Migration in Cracked Concrete and Broken Symmetry of Corrosion Profiles

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Nonlinear Gel Migration in Cracked Concrete and Broken Symmetry of Corrosion Profiles

Esayas Gebreyouhannes\textsuperscript{1} and Koichi Maekawa\textsuperscript{2*}

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Abstract

The initiation and subsequent propagation of corrosion-induced cracks is simulated by multi-phase mechanics where the coupled corrosion gel migration and crack-based fracture can be coherently taken into consideration. A key focus is given to the nature of corrosion products and their dependency on the corrosion rate and the anisotropic stiffness and permeability caused by cracking. Migration of corrosion gel to both micro-pores of cementitious composites and crack gaps are explicitly considered under the gradient of gel pressure, which is newly defined on the corrosion gel phase. The effects of cover depth, corrosion rate and the mix-proportion of concrete on the crack initiation and propagation are discussed with experimental facts. The broken symmetry of corrosion profile around reinforcing bars is successfully simulated as a nature even though the geometry of analysis domains would be in perfect symmetry. This scheme which allows kinematics of produced gel is indispensable for a versatile framework of durability mechanics.

1. Introduction

Losses due to ageing of civil Infrastructures is one of the major dares that the global societies are facing. Scores of reinforced concrete (RC) are suffering from rapid deterioration such as corrosion, alkali silicate reaction (ASR), freeze/thaw, fatigue and/or their combination. Especially, those built in severe environments may suffer seriously from corrosion of reinforcing bars. Corrosion in concrete induces expansive pressure causing tensile stresses in surrounding concrete. Its effect on RC includes loss of functions, reduction and eventual loss of bond between concrete and corroding steel and damaged cover concrete. These may affect the serviceability and capacity of RC leading to accelerated ageing as well.

Rational maintenance strategies based on realistic understanding of deterioration mechanisms may play a crucial role in easing the existing challenges of ageing. For this purpose, codes and standard specifications denote some limit states based on the past engineering experiences. There are some stage gates of limit state; as the initiation of corrosion (Maage et al. 1996); as the first cracking of cover concrete (Liu and Weyers 1998; Torres-Acosta and Martinez-Madrid 2003); based on a certain limit of crack width (Andrade et al. 1993; Stewart 2001). While each of these definitions has its own rational, end outcomes in terms of maintenance cost may significantly vary.

After the initiation of corrosion, expansive strains induced on cover concrete may be relaxed due to the migration of corrosion products to micro-pores and crack gaps around reinforcing bars, thereby delaying the initiation of cracks and its propagation. After the occurrence of cracking, it takes substantial period of time to reach a certain limit of crack widths. The stress relaxation during corrosive expansion depends on various factors such as the corrosion rate, cover depth, concrete micro-pore structure, strength, nature of corrosion products and its crystallization process. Thus, sufficiently accurate and versatile model for predicting corrosion induced damage is of importance for selecting effective maintenance strategies.

A number of empirical (Andrade et al. 1993; Oh et al. 2009) and analytical (Liu and Weyers 1998; Michel et al. 2014; Bazant 1979; Val et al. 2009) approaches have been proposed to model corrosion induced crack initiation and propagation in RC. The process of corrosion involves soluble species that can dissolve in the concrete pore solution and subsequently migrate or diffuse through the cement paste matrix away from the corroding steel (Sagoe-Crentsil and Glasser 1989). The pores near corroding steel bars act as storehouses, hence delaying stress build-up that leads to cracking of concrete cover (Liu and Weyers 1998; Pantazopoulou and Papoulia 2001; Alonso et al. 1998). The migration of corrosion products to micro-pores around reinforcing bars (Michel et al. 2014) and migration to cracks (Wong et al. 2010; Takaya et al. 2012) has been confirmed experimentally. Recent approaches for predicting time to initial cover cracking have incorporated ‘porous zone’ (Liu and Weyers 1998; Val et al. 2009) or ‘corrosion accommodating region’ (Michel et al. 2014) to account for the time needed for corrosion products to fill the porous cement paste around reinforcing bars. Toongoenthong and Maekawa (2005a, 2005b) and Val et al. (2009) explicitly modeled the penetration of corrosion products into cracks formed.

While most of these model approaches can provide

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consistent predictions to few experiments, this does not imply global accuracy to be used in practice for the following reasons. First, the analytical platforms currently being used are rooted in solid mechanics while we are dealing with a problem containing fluid in a porous medium. Second, scant attention is given to the nature of corrosion products under natural environments in contrast to the electric impressed corrosion in terms of its composition, expansion ratio and proportion of solid/liquid phases. This is because, the characteristics of corrosion products in naturally occurring process are substantially different in contrast from those produced in an electrically accelerated corrosion (Takaya et al. 2012).

Recently, the authors proposed a model for assessing corrosion impact and its mechanistic actions based upon multi-phase theory (Biot 1941, 1963; Maekawa and Fujiyama 2013; Gebreyouhannes et al. 2014), where the space averaged time and path dependent constitutive laws of cracked concrete (Maekawa et al. 2003) and anisotropy of both stiffness and permeability caused by cracking are taken into account. They have applied this approach for the assessment of bridge decks subjected to high-cycle loads coupled with the existence of moisture (Maekawa and Fujiyama 2013). At the same time, the past contributions on corrosion experiments highlight that corrosion products include both solid and liquid phases (Sagoe-Crentsil and Glasser 1989; Takaya et al. 2012). Apart from the corrosion products remaining on the interface of reinforcing bars, some portion of the corrosion gel is a liquid phase and is thought to be an agent to migrate inside both micro-pores and crack gaps, whereas concrete skeleton represents the solid phase with “varying” porosity or spaces. These facts indicate that, the concept of two-phase theory originated by Biot (1941, 1963) is logically suitable to assess the impact of corrosion to the service life of RC. Thus, the authors aim to use and expand this poro-mechanics for the assessment of bridge decks subjected to an electrically accelerated corrosion (Takaya et al. 2012).

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The proposed modeling is a part of the multi-scale thermo-hydro-integrated system (Maekawa et al. 2008), which may numerically trace cement-chemical events, mass transport (water, Cl-, CO2, O2, etc.), chemical reactions and their mass balance by using the multi-ionic approach from the onset and under different boundary conditions of environments. In this study, the authors opt to focus on the verification of the meso-scale mechanics part of the computational scheme targeting its structural impact after the initiation of corrosion. The schematic representation of the analytical framework is shown in Fig. 1.

2. Modeling of corrosion gel in concrete skeleton

The governing equation in this study is based on multi-phase theory (Biot 1941, 1963; Maekawa et al. 2008), which has been also applied to foundation consisting of particle assembly as soil skeleton and underground water as porous media. This scheme is extended to cracked concrete mechanics as a solid skeleton with water and corrosion gel as a porous media, which may exist in both micro-pores and crack gaps. Then, we have

\[ \rho = (1 - n) \rho_s + n \rho_f \]  \hspace{1cm} (1)

where, \( \rho \) is the total density of concrete with corrosion gel which could fill the whole pore volume, \( \rho_s \) is the density of concrete skeleton, \( \rho_f \) is the density of corrosion gel, and \( n \) is defined as the pore volume ratio. Strictly speaking, the total density of concrete with gels is not constant but variable since the pore volume changes when the crack opens/closes and pores are not fully saturated by gels. However, the variation of the total density can be assumed negligible because of much less weight of the gel compared to that of concrete. Then, Eq. (1) is directly used for estimating the inertial force with the total density in Eq. (4) of motion.

Here, the motion of corrosion gel is defined by the relative displacement from the concrete solid matrix. Then, the movement of corrosion-gel particle denoted by \( U_i \) is expressed by the displacement of concrete skeleton denoted by \( u_i \) and the mass-substantial relative displacement of corrosion gel phase denoted by \( w_i \) as,

\[ U_i = u_i + \frac{w_i}{n} \]  \hspace{1cm} (2)

The geotechnical engineering scheme based upon Biot’s original concept (1961, 1963) assumes isotropy of the porous media. This means that individual soil particles are surrounded by a liquid matrix. Similarly, for the structural concrete surrounded with corrosion gels, we have the total stresses (\( \sigma_{ij}^p \)) of soil-liquid system as a simple summation of the skeleton stress of un-cracked concrete and the isotropic corrosion gel’s pressure term as,

\[ \sigma_{ij}^p = \sigma_{ij}^s + \delta_{ij} p \]  \hspace{1cm} (3)

where, \( p \) is defined as pore pressure and \( \sigma_{ij}^s \) is the effective stress tensors defined on the solid skeleton. This isotropy assumption inside capillary pores of concrete may hold before cracking. This is applied to the corrosion gel inside capillary pores. In formulating the dynamic equilibrium equations of both the concrete skeleton and porous media in cracks, the interacting term between two phases is needed. Assuming that different kinematics of two phases may create mutual interacting forces, the governing equations can be formulated with the term of dragging forces rooted in the permeability (\( k \)) of corrosion gel phase in each direction-\( i \) through concrete pore and crack gaps as,

\[ \sigma_{ij}^p + \rho g_i = \rho \left( u_{i,mm} + p \right) w_{i,mm} \]  \hspace{1cm} (4)

\[ p_i = \rho_f (u_{i,mm} - g_i) + \rho_f w_{i,mm} / n + \left( \sqrt{k_i} / k_m \right) w_{i,m} \]  \hspace{1cm} (5)
where, the last term of Eq. (5) means the dragging action of pore gel and often referred as Darcy’s law of motion, and subscript \( m \) and \( mm \) indicate the first and the second degree differentiation with respect to time.

In the case of soil mechanics, the skeleton constitutive model is applied for calculating \( \sigma_{ij}^{\ast} \) as effective stress, and the constitutive model for soil particle assembly is used for simulation. The space averaged path and time dependent constitutive equation rooted in the multi-directional crack modeling (Maekawa and Fujiyama 2013; Maekawa et al. 2003) is available for RC. As for the mechanical modeling of liquefied phase of corrosion gels, the volumetric linear stiffness is assumed for compression from the atmospheric pressure (0.1 MPa) as,

\[
p = K_f \left( w_i + \epsilon_i - (1 - \beta) \cdot v_{cr} \right) < 0,
\]

\[
K_f = \left( \frac{1-n}{K_s} + \frac{n}{K_{cg}} \right)^{-1}
\]

where, \( K_s \) and \( K_{cg} \) are bulk stiffness of concrete matrix and corrosion gel, \( w_i + \epsilon_i \) represents the mean volumetric strain of gel phase inside concrete, \( v_{cr} \) is the net specific volume of the created gels inside a finite element (see Section 2.2), \( \beta \) is the ratio of the crystalline phase of gels. Then, the term \((1-\beta)\) means the liquid part. The stresses caused by the solid part of gels are calculated with respect to \( \sigma_{ij}^{\ast} \) as discussed later in Chapter 4.

Here, the shear stiffness of the liquid phase is assumed zero. Strictly speaking, the averaged porosity denoted by \( n \) is not constant but varies according to concrete micro-structure and crack opening and closure. But, as the range of strain for porous media denoted by \((w_i + \epsilon_i)\), which has to be considered in engineering practice, is at most one percent and negligible, the averaged porosity in governing equations is treated as the initial one of constant. Through this formula, the mechanical interaction of cracked concrete and gel media is consistently installed in the framework of continuum mechanics. Equation (6) was originally presented by Biot (1941, 1963) by assuming that the volumetric change of porous media is the simple summation of solid and pore gel due to Pascal’s Principle of ignoring the shear stiffness of porous media. The overall framework of combined solid skeleton and corrosion gel mechanics is summarized in Fig. 2.

### 2.1 Cracks and anisotropy

After cracking, the corrosion gel pressure acts anisotropically onto the concrete pore (capillary and crack gaps). The large amount of corrosion gel is located...
within a thin layer surrounded by a couple of crack planes, and the interaction with cracked concrete is rather anisotropic. In this study, the authors assume that the pore pressure inside crack gaps acts perpendicular to a pair of parallel crack planes as,

$$\sigma_{ij} = \sigma_{ij}^{*} + \delta_{ij} p$$  \hspace{1cm} (7)

where, $l_i$ is the unit directional vector normal to a crack plane (see Fig. 3). Here, the anisotropic action of the pore gel is assumed after complete formation of connected planes through the fracture process zones.

### 2.2 Corrosion gel permeation to capillary pores

After the initiation of corrosion, some of the liquefied corrosion products permeates into the surrounding pores of concrete. Experimentally, Michel et al. (2014) using the x-ray attenuation and Wong et al. (2010) using the image analysis observed the migration of corrosion gel to the pores of surrounding concrete. In this study, the migration of un-crystalized gel to the capillary pores of concrete is considered by Washburn’s equation (8) (1921) as,

$$V_{eff} = \frac{V_p}{1 + \left(\frac{\sigma_{ij} + \sigma_{kk}}{a}\right)}$$  \hspace{1cm} (8)

where, $V_{eff}$ is the amount of corrosion gel permeated to the capillary pores under the isotropic pressure denoted by $p$, $V_p$ is the volume ratio of the cement paste matrix.
containing micro-pores, $\phi_p$ is the total capillary porosity on $V_{cp}$, $B_{cp}$ is the distribution parameter corresponding to the capillary porosity and taken as $3.34 \times 10^{-7} / \text{m}$ (Maekawa et al. 2008), $\sigma_f$ and $\theta$ are the surface tension and the contact angle of the liquefied pore media (Washburn 1921) and $\sigma_f \cos \theta$ is assumed to be 0.1 (Maekawa et al. 2008) in this study.

The isotropic pressure denoted by $p$, which is induced by the increased volume of liquid corrosion gel and is computed by Eq. (6), acts as the main force driving the liquid phase through the concrete micro-structure against the surface tension. The referential radius denoted by $r$ in Eq. (8) means the limit size of the micro-pore which may contain the pore media. For normal concrete mixture, 60% or more of concrete volume is occupied by aggregates and the rest of the binder volume ratio is about 40% or less. Then, for simplicity, a typical value of 0.4 is applied for $V_{cp}$ which is on the higher side and is taken intentionally to reflect the increase in local porosity due to calcium depletion caused by corrosion products, as reported by Wong et al. (2010). This explicit consideration of the interaction between concrete micro-structure and corrosion products enables us to trace the simultaneous build of stress and migration of corrosion gel to concrete pores. The value of $v_{cr}$ in Eq. (6) is calculated by subtracting $V_{cpg}$ from the gross volume of created gels.

2.3 Corrosion gel permeability through concrete cracks

Cracks are the source of stress-induced anisotropy of migration for pore media though the skeleton solid. Then, the permeability of cracked concrete is to be a function of post-cracking deformation of finite elements. In particular, the averaged water permeability was studied with single (Wang et al. 1997) and multiple cracks' specimens (Lepech and Li 2005) in which the permeability of un-cracked concrete is reported about $10^{-10}$ to $10^{-9}$ cm/sec of isotropy, but can be greatly magnified up to $10^{-3}$ cm/sec or more as shown in Fig. 3. It shows the experimentally observed sensitivity of the gap of a single crack on the macroscopic permeation of mass transport. To estimate the potential values for the uncracked corrosion gel permeability, the sensitivity analysis was carried out by varying its value from $10^{-15}$ to $10^{-9}$.

The results are shown in Fig. 4. For the purpose of sensitivity analyses, the experimental investigation by Oh et al. (2009) on crack initiation was taken as a benchmark with a cover depth of 4cm, where the behavior is more governed by the un-cracked one. The detail of the experimental set-up used is indicated in the following Section 5.1 and shown in Fig. 6. It can be seen that the value for the uncracked corrosion gel permeability lies between $10^{-10}$ to $10^{-9}$ cm/sec. The estimated value is in the order of water permeability of concrete. Thus in this study, the corrosion gel permeability of un-cracked concrete $k^*$ is simply taken as $5 \times 10^{-10}$ cm/s in consideration of the sensitivity to the computed results, the reproducibility of experiments on which the data identification is made as,

$$k_i = k^* \left(1 + \frac{e_{ij}^+ e_{jk}^+}{a}\right)^4$$

(9)

where, $(i, j, k)$ means the axis of orthogonal Cartesian coordinate system defined on 3D extent of analysis domain and the tensorial expression in Eq. (9) follows the Einstein summation convention.

In considering the fact that the average strain of concrete normal to the crack plane is proportional to the crack width, the authors assume the poly-nominal function in terms of the average strain normal to crack planes as Eq. (9). The value of $k_i$ in Eq. (5) and Eq. (9) is the mean permeability of cracked concrete along $i$-axis, $k^*$ is the intrinsic permeability of un-cracked concrete, and $e_{ij}^+ + e_{jk}^+$ is the mean transverse in-plane strain. The orthogonal coordinate system $(i, j, k)$ is defined along and normal to the crack plane. In this study, the value of "$a$" is taken as 1.0.

Equation (9) is formulated based on the experimental results obtained (Wang et al. 1997) and the comparison of the curve with the experimental result is shown in Fig. 3.

![Fig. 4 Sensitivity of corrosion gel permeability and FE model used for sensitivity analysis.](image)
Given the fact that flow is dependent on the crack surface roughness and the mix proportion of concrete, the formulation in Eq. (9) is rather tentative and further research is needed to upgrade its reliability. In the subsequent chapters, the modeling formulated here is used for the simulation of RC members with corroding reinforcing bars.

### 3. Characteristics of corrosion products

#### - Implication to modeling -

The pressure caused by the increased volume of corrosion products is responsible for generating stresses on the surrounding concrete. Differing values for the volumetric expansion ratio of corrosion products has been reported, ranging from 1.2 to 6.9 (Oh et al. 2009; Takaya et al. 2012; Mehta and Monteiro 1997). The amount of volume increase varies according to the test method (accelerated versus natural), presence or absence of chloride ions, corrosion rate, supply of oxygen, PH and other environmental factors.

Majority of proposed numerical models uses a thermal analogy to mimic the increased volume of corrosion products, with less giving due consideration of characteristics of corrosion products and associated expansion ratio. Basically, they assume a solid phase for the corrosion products even though some relaxation is considered to account for the migration of corrosion gel to micro-pores. At the same time, the existing models rely on calibration with experimental data obtained from electrically accelerated corrosion tests. Owing to these observations, it is important to initially understand the nature of the corrosion products and its dependency on the test environment and the rate of application prior to the development of any numerical model. Based on the available published experimental data, this chapter attempts to discuss the characteristics of the corrosion products in natural and the cathodic processes in terms of composition, expansion ratio and the proportion of solid/liquid phases. A key focus is given the importance for reasonable estimates of the various parameters related to the composition and properties of corrosion products based on the available published experimental data.

### 3.1 Electrically accelerated vs. natural corrosion

Recent valuable contribution by Takaya et al. (2012) indicates that the nature of corrosion products in naturally occurring process is substantially different in contrast to those produced in an electrically accelerated corrosion. Williamson and Clark (2002) reported that variation of corrosion rate changes the morphology of the corrosion products. At high corrosion rates (approximately 2000μA/cm²), they observed the corrosion products to behave more like a dark thick liquid type.

Moreover, a wide range of mass loss values (10 to 150 mg/cm²) that correspond to a visible crack have been reported. Interestingly, values obtained from electrically accelerated corrosion tests indicate lower values (10 to 50 mg/cm²) whereas, values obtained from natural corrosion process indicate higher values (50 to 150 mg/cm²). These differences suggest that for the same mass loss, the impact of corrosion might not be the same depending on the process of occurrence. Takaya et al. (2012) reported that the amount of rust remaining on the interface between steel bars and concrete is smaller for the case of high current corrosion tests. They also observed noticeable flow of rust through the cracks in the high current corrosion test. These results indicate that corrosion products induced with a slow process have more chance to crystallize around the reinforcing bars before they are being transported to crack gaps, implying that the extent of crystalline portion of a corroding reinforcing bar in concrete is rate dependent. To address this point, the authors propose a corrosion rate dependent apportionment of crystalline and liquid phases of corrosion products.

Here, it should be noted that the corrosion cracking under natural environments is generally judged by visual observations, but on the other hand, strain gages are often used to judge the occurrence of corrosion cracking in laboratory accelerated tests. Then in this paper, the authors carefully do not mix up the data of laboratory-based experiments and field testing data for the verification of the numerical modeling.

### 3.2 Volumetric expansion ratio of corrosion products

The amount of volume increase in corroding reinforcing bar varies according to the types of oxidation products. Table 1 presents values of the volumetric expansion ratio reported by Oh et al. (2009), Takaya et al. (2012), Mehta and Monteiro (1997) and Suda et al. (1993). This summary indicates that the product of Calcium Iron Oxide (III) Chloride is observed only in the electrically induced corrosion with the presence of chloride ions, and its

<table>
<thead>
<tr>
<th>Corrosion products</th>
<th>Expansion ratio Mehta/Oh</th>
<th>Corrosion products</th>
<th>Expansion ratio Suda*</th>
<th>Expansion ratio Takaya**</th>
</tr>
</thead>
<tbody>
<tr>
<td>FeO</td>
<td>1.20/1.70</td>
<td>Goethite</td>
<td>α-FeOOH</td>
<td>2.9-3.8</td>
</tr>
<tr>
<td>Fe(OH)₂</td>
<td>3.75/3.60</td>
<td>Akaganite</td>
<td>β-FeOOH</td>
<td>--</td>
</tr>
<tr>
<td>Fe₂O₃</td>
<td>2.20/2.10</td>
<td>Lepidocrocite</td>
<td>γ-FeOOH</td>
<td>3.0</td>
</tr>
<tr>
<td>Fe₂O₄</td>
<td>2.00/2.00</td>
<td>Magnetite</td>
<td>Fe₂O₄</td>
<td>2.1</td>
</tr>
<tr>
<td>Fe(OH)₃</td>
<td>4.20/4.00</td>
<td>Calcium Iron Oxide (III) chloride</td>
<td>CaFeO₂Cl</td>
<td>--</td>
</tr>
<tr>
<td>Fe(OH)₃·3H₂O</td>
<td>6.40/6.15</td>
<td>Iron chloride Hydroxide</td>
<td>Fe₅Cl₆(OH)₁₂</td>
<td>--</td>
</tr>
</tbody>
</table>

Note: * results obtained from natural corrosion process ** results obtained from electrically induced corrosion
volumetric expansion rate is 6.9 (Takaya et al. 2012). This product not only has a higher volumetric expansion ratio but also is the agent which has much to do with the kinematics of corrosion gel. In the range of 10 - 15 mM/l chloride, iron solubility at PH ~12 increases abruptly from near zero to 170 to 180 mM/l (Sagoe-Crentsil and Glasser 1989). This chloro-complex, where iron and chloride ions are bound together, is observed to migrate away from the corroding steel bar and subsequently precipitate in oxygen-rich areas, releasing the chlorides to transport more ferrous ions from the corrosion steel (Sagoe-Crentsil and Glasser 1989).

These facts highlight that the approaches based solely on a single volumetric expansion ratio may not be realistic to simulate corrosion induced damage of structural concrete for all conditions. These are because the degree of volumetric expansion may reach up to 5 times even more for the electrically accelerated cases in the presence of chloride ion. Here, the authors use different expansion ratio depending on the method adopted to induce corrosion and depending on the presence or absence of chloride ions.

3.3 Crystalline and liquid phase of corrosion products

The process of corrosion involves soluble species that can dissolve in the concrete pore solution and subsequently migrate or diffuse through the cement paste matrix away from the corroding steel (Sagoe-Crentsil and Glasser 1989). Owing to this fact, it is important to delineate the proportions of the crystalline and liquid parts of the corrosion products in modeling corrosion induced crack initiation and propagation. Strictly speaking, it is not easy to clearly delineate the boundary between the two due to the fact that the products eventually change depending on the micro-climate of the surrounding concrete.

Corrosive product was regarded as liquid based on the investigation reported by Molina et al. (1993). At high corrosion rates (approximately 2000μA/cm²), Williamson and Clark (2002) observed the corrosion products to behave more like a dark thick liquid type. From a study of corrosion products formed in bars embedded in concrete exposed to marine environment, crystalline magnetite, goethite and lepidocrocite were found to make up only 30% of the rust formed within concrete (Suda et al. 1993). Similarly, Takaya et al. (2012) observed crystallized rust around the reinforcing bar nearly 25% of the total rust formed, for a corrosion rate of 1680μA/cm² and 80% for a corrosion rate of 168μA/cm². These past contributions reveal one key fact that the morphology of the corrosion products is rate dependent. Grounded on the experimentally established facts, the authors propose a corrosion rate dependent proportioning of rust into crystallized and liquid parts.

4. Model properties of corrosion gel

Based on the discussions in previous sections, properties of corrosion products consist of both liquid and crystalline parts and both play an important role in the simulation of corrosion-induced damage of concrete. Before the occurrence of cracking, the role of both the liquid and crystalline phases is substantial. However, after cracking, the impact of the liquid portion drops owing to the increased permeability induced by cracking (see Fig. 5).

Here, two cases are separately treated. The first is, corrosion induced in a chloride rich water saturated environment, and the second case is corrosion induced in a chloride free water saturated environment.

Two models are proposed for the delineation of the crystalline and liquid phases of corrosion products and the associated volumetric expansion ratio. For the former case, where chloride rich environment prevails the experimental results reported by Takaya et al. (2012) are used. Since the presence of the chloro-complex is evident where up to 6.9 times expansion is expected, the total equivalent volumetric expansion ratio is assumed to be 5.0. Similar result is reported (Takaya et al. 2012). Furthermore, in their experiment, nearly 20% of the rust remained around the reinforcing bar for a corrosion rate of 1680μA/cm² and about 80% for a corrosion rate of 168μA/cm².

In the chloride-free environment, the expected corrosion products are goethite with a volumetric expansion of 2.9, magnetite (=2.1) and lepidocrocite (=3.1). The total

![Fig. 5 Schematics of crystalized and liquid phases of corrosion products in chloride-rich environment.](image-url)
The equivalent expansion ratio is then expected to be between 2.5 and 3.0. In this study, a value of 3.0 is assumed for the volumetric expansion ratio. Here, the liquid kinematics of the crystalline phase of the total product of gels, which is described by Eq. (6) as \((1-\beta)\), is solved in the multi-phase scheme as shown in Fig. 2.

The solid phase of the total produced gel (denoted by \(\beta\)) firmly stays with corroded mother steel bars. Then, the created stress rooted in the solid phase of produced gels is computed by conventional methods to assume that the steel bars equivalently expand uniformly (Toongoenthong and Maekawa 2005a, 2005b), and this stress is involved in the skeleton stress in Eq. (3). According to the experiments discussed in this study, the value of \(\beta\) is thought to be 0.75 or less and will be further discussed in more detail again in Chapter 6. In the next chapter, the interaction of the corrosion products with the concrete solid skeleton will be discussed.

### 5. Experimental verification – behavioral viewpoint and magnitude –

#### 5.1 Electrically induced corrosion in a chloride rich water saturated environment

With the aim of determining the critical corrosion amount to cause cracking of RC structures, Oh et al. (2009) carried out accelerated corrosion experiment on RC specimens submerged in chloride solution. In their study, the major test variables include the cover thickness and the compressive strength of concrete (see Table 2). The evolutions of strain on the surface of concrete cover were measured for each of the specimens. The region of corrosion was limited to the central portion of the steel bar as shown in Fig. 6. The presence of the non-corroding region of the reinforcing bar provides more control on the evolution of surface strain, where the impact of each incremental corrosion product can be traced. In this regard, the experiments are critical to verify the reliability of the entire proposed model which incorporates the apportionment of solid and liquid phases, permeability model before and after cracking and the macroscopic expansion rate of the corrosion products.

The experiments were simulated by using the proposed multi-scale poro-mechanical approach and the FE model as indicated in Fig. 7a. Finer time step in the range of 10 to 15 minutes is used to capture smooth occurrence of physical events such as cracking and corrosion gel migration. For the same reason, finer mesh size in the range of 2mm-4mm is used around the reinforcing bar. Here the volumetric expansion of rust is taken as 5.0 with 0.24±0.01 crystalline ratio (Chapter 4). The computed results for the evolution of the surface strain with cover depths of 2cm, 3cm, 4cm and 5cm are shown in Fig. 7b in contrast to the experimentally obtained results. Initially, the corrosion gel gradually permeates to the capillary pores as a result of the isotropic pressure created in the vicinity of the reinforcing bar. This is followed by subsequent build-up and relaxation of stresses due to penetration of the corrosion gel to crack spaces. The phenomenon can be observed both in the experiment and analysis, leading to consistent results with fair agreement.

Once the surface crack occurs, the space-averaged strain on each finite element starts to be localized in 3D extent. As shown in Fig. 7a, the strain profile on the upper surface is not uniform but a little greater strain zone painted by dark blue remains around the highly localized zone illustrated in yellow. It means that the several cracking with different orientations has taken place. Afterwards, the deforming field was converged to the highly localized deformation by cracking of the vertical direction. This transient process fairly matches the reality of the experiment and the past experiences as well.

The further penetration of corrosion gel into crack gaps is provoked. Owing to the increased permeability of gel into the wider cracks, migration of liquefied corrosion gel becomes easy. On the surface of the specimen,
there is no restriction of the corrosion gel phase to be discharged, but the injection of gel from outside is not allowed according to the experimental conditions.

**Figure 8** shows the effect of mix proportion on the progress of surface strain for concrete with a cover depth of 4cm. Three different mixes with water to cement ratio of 0.55, 0.45 and 0.35 were considered as reported (Oh et al. 2009). The time elapsed after steel de-passivation...
until the occurrence of surface cracking is elongated for the mix with 0.35 of the water to cement ratio. The phenomenon is fairly reproduced computationally and the results are consistent with the experimental measurements. While the trends are similar both in the experiment and analysis, the effect seems more pronounced in the experiment.

5.2 Rate-dependency of electrically induced corrosion

In order to monitor the amount of corrosion needed to induce cracking and its subsequent propagation, Andrade et al. (1993) conducted artificial corrosion experiments by an impressed current with different corrosion rates. Single bar was placed inside the specimen at the center axis. Corrosion rates of 10 μA/cm² and 100 μA/cm² were investigated for cover depths of 2cm and 3cm. The average tensile strength of the concrete used is 3.55MPa. The experiments were simulated by using the proposed poro-mechanics based analysis and the results are presented in Fig. 9, accompanied by the experimental results which were processed on the coordinate of the crack width and the corrosion ratio. In all cases, the corrosion penetration depth required for crack initiation is similar to the experiments. This indicates that the assumed volumetric expansion ratio (5.0 in this case) is reasonable. Furthermore, the crack propagation recorded in the experiment can be reproduced when the crystalline ratio of the corrosion products denoted by β in Eq. (6) is 0.40 for the specimens with corrosion rate of 100 μA/cm² and 0.58 for the specimen with corrosion rate of 10 μA/cm². The computed result indicates that the crystalline portion of the corrosion products becomes lower as we increase the corrosion rate. Similarly, Williamson and Clark (2002) reported that variation of corrosion rate changes the morphology of the corrosion products. Noteworthy, the results here highlight that engineering judgement based on corrosion experiments with impressed current at high rates underestimate the propagation of crack width as indicated in Fig. 9d.

Recent investigation by Takaya et.al (2012) also discussed the effect of rate dependency and the results are indicated in Fig. 10. In the experiment, they employed the electric corrosion test at theoretical rates of 168 μA/cm² and 1680 μA/cm² with a cover depth of 2cm. The average efficiency of the impressed currents were approximately 50% for the corrosion rate of 168 μA/cm² and 80% for the corrosion rate 1680 μA/cm², which makes the actual rates correspond to equivalent values of 84 μA/cm² and 1344 μA/cm², respectively.

The experiments were simulated by applying the proposed poro-mechanical approach and the computed

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**Fig. 9 Crack initiation - propagation with the progress of corrosion; computed results and experiment.**
results are indicated in Figs. 10b, 10c and 10d. This total volumetric expansion ratio is assumed to be 5.0 as discussed in Section 3.2. The crack propagation recorded in the experiment can be reproduced if the crystalline ratio of the corrosion products is 0.40 for the specimens with the corrosion rate of 1344 μA/cm² and 0.48 for the specimen with the corrosion rate of 84 μA/cm². The results further strengthen the fact that the morphology of the corrosion products is rate dependent and the crystalline portion of the total corrosion products becomes lower with the increase of corrosion rate. This time-dependency will be further discussed in Chapter 6.

5.3 Electrically induced corrosion with no chloride: Broken Symmetry of Corrosion Profile

Michel et al. (2014) carried out experimental investigation on mortar specimens with varying water-to-cement ratios subjected to current-induced corrosion at rates of 10, 50 and 100 μA/cm². The experimental set-up is indicated in Fig. 11, and no chloride was used for inducing corrosion. In this case, formation of the chloro-complex products of corrosion is not expected thus leading to the reduced expansion ratio. Detailed experimental results are presented for the specimen with 0.5 of W/C and subjected to 100 μA/cm². The specimen has dimensions of length × width × height (23mm × 100mm × 100mm). With the same input, the experiments were simulated by using the proposed model and the FE mesh is indicated in Fig. 12. The mesh discretization is in perfect symmetry on both horizontal and vertical axes. The node at the center of the bar circle is fixed with respect to the translational...
displacement to avoid the rigid body motion so that the boundary conditions also satisfy the perfect symmetry.

Interestingly in the experiment, Michel et al. (2014) measured the corrosion-induced deformations between the steel and mortar at various clock positions by digital image correlation as shown in Figs. 13a and 13b. The digital image correlation results clearly show that corrosion-induced deformations, which are thought to be the increased thickness of the interface between steel surface and mortar. They are non-uniform at the various measurement locations along the steel circumference and the maximum deformation of approximately 15μm was measured after 9.2 days. At some points, minus deformations can be seen and no explanation from the microscopic viewpoint was made. Anyhow, it can be said that the symmetric corrosion profile is broken despite of

![Fig. 12 FE model used for simulation.](image1)

![b) Processed X-ray image: non-symmetric, non-uniform broken field of interface deformation (Michel et al. 2014).](image2)

![c) Simulated gel permeation prohibited and point-symmetry of deformation around a bar - shown in solid red ring -](image3)

![d) Simulated gel permeation allowed and the broken symmetry of deformation around a bar - shown in broken red ring -](image4)

![e) Coupled broken symmetry of interface deformation and corrosion](image5)

Fig. 13 Broken symmetry of deformational profiles on steel-mortar interface: experiment vs. analysis.
the four plane-symmetric geometry of the specimen.

In order to mechanically investigate this broken symmetry, the migration of corrosion gel was first prohibited computationally by setting the zero permeation of gel into crack gaps. Thus, the produced gel is numerically confined on the surface of the bar. The computationally obtained results are indicated in Fig. 13c, where the maximum interface deformation computed is about 24μm in terms of relative displacement of adjacent nodes, and the space-averaged principal tensile strain uniformly develops like a ring around the bar. This result is greater as compared to the experimentally observed one. Because of no degree-of-freedom on gel migration, the broken symmetry barely takes place within the possible solutions. This is some sort of “mathematical locking”.

For further investigation, the original model was applied with non-zero gel permeability in motion (see Fig. 3). The computed results are shown in Fig. 13d. The non-uniform local deformation along the surface can be seen as the reality does, and some nodes of FEM are indicating nearly a third of the maximum deformations as the tendency observed in the experiment. Besides the maximum interface deformation is 12μm, which is quite consistent with the deformation measured in the experiment. The broken symmetry at the very beginning of corrosion is successfully reproduced by formulating the migration of the corrosion gel.

At this point, it is emphasized again that consideration of broken symmetry of corrosion profile is crucial as for the sound mathematical scheme of corrosion simulation. In other words, the gel migration coupled with concrete mechanics is equivalent to “unlocking” from ill conditions of solution. This is similar to thin plates and slender columns’ buckling under compression whose solution can be captured by introducing the geometrical nonlinearity. Otherwise, no buckling can be simulated in structural mechanics as well.

Here, it must be noted that searching of the numerical solution to satisfy the governing equations was conducted under multi-core parallel processing of computation (In this study, 16-CPU's were utilized). The multi-core process definitely introduces a very small perturbation of digit to the target solution. Then, it is not necessary to artificially introduce any fluctuation in geometry and/or material properties in space for the trigger of broken symmetry, but it is naturally reproduced like a nature. When we cycle the same parallel computation for 10 or 20 times, a rather uniform corrosion profile tends to appear as an equilibrium solution but unstable one. In nature, perfect uniformity of material property and geometry never exists in the first place, and as a matter of fact, natural perturbation is the trigger of broken symmetry, no way to be back to the symmetry of corrosion.

It should be emphasized again that the broken symmetry of “corrosive deformation” is made at the beginning when a uniform “corrosive reaction” develops around the surface of bars in computation. This broken symmetry of interface deformation becomes a trigger of the subsequent non-uniform corrosive reaction caused by the non-uniform supply of oxygen and chloride ion associated with the broken field of deformation. Then, to the end, the broken symmetry of “corrosive reaction” (Tran et al. 2011; Thybo et al. 2013) rises as a result of coupled mechanics and mass transport (see Fig. 1).

Figure 13e shows the interface deformation corresponding to the broken symmetry of corrosive reaction (almost one-half of the bar surface). The non-uniformity of interface deformation gets enlarged than the one shown in Fig. 13d, where the corrosion product is forced to be uniform on the surface of the bar. As the mother steel is non-uniformly decayed, minus deformation is also somewhat made, and it is similar to the experimental measurements as shown in Fig. 13a.

In all cases of sensitivity analyses, the corrosion symmetry is hardly broken when the magnitude of steel corrosion is so small at the beginning of the test. This is computationally in sound condition. However, the experimental results indicate the substantial asymmetry before 5 days (see Fig. 13a). This initially induced asymmetry may attribute to the non-symmetric location of devices like titanian meshes as well as the variational properties of materials in space. At this moment, some unknown issues remain and further investigation is needed. The mechanically and chemically induced broken symmetry is to be further investigated too as a highly coupled problem of interest. Qiao et al. (2015) further investigated the non-uniform corrosion of reinforcing bar section at the greater magnitude in consideration of the electro-field as well. This is of importance to understand corrosion processes under more natural environments.

The experimental verification in this chapter is conducted based upon the laboratory-based acceleration tests. As the consistent criterion of corrosion cracking and rather simple states for corrosion can be maintained as stated in Section 3.1, the mechanistic discussion became possible by comparing the experimental results and those of analyses. However, realistic corrosion processes under ambient conditions will be more complex owing to the coupling of different chemical and physical events with higher variation. The experimental verification for coupled problems will be made in near future by referring to the corrosion processes which develop under real environmental states. In other words, this study may serve as the opening stage to more realistic phases of corrosion with complex coupling.

6. Crack propagation dependency on corrosion rate

The commonly used accelerated corrosion tests by using the impressed current allow for shorter time to assess the impacts of corrosion. While the approach is time-effective, interpretation of the results for natural environmental states should be taken with attention. In the literature, apparently different results of interest have
been reported. The one reported by Andrade et al. (1993) and Wong et al. (2010) indicates that the faster rate of crack propagation is caused by the slower speed of corrosion (see Section 5.2). The different trend was also reported as shown in Fig. 14a (El-Maaddawy and Soudki 2007).

It should be noted that this rate oriented investigation was based upon the cases of a bar positioned at a corner with the equal cover on the orthogonal directions. Besides, the measurement of the crack width was taken at a single corner point. On such a case of bar arrangement at the corner, the variation in the measured crack width is generally scattered (see Fig. 14b) unlike the case of experiments where the bar is arranged at the center of the specimens. The authors are not sure how to quantitatively explain this interesting phenomenon of different tendency, but for further discussion, the experimental stability must be thought as one of factors of consideration. Then, for experimental verification in terms of the corrosion magnitude and the crack width, the authors selected the cases where the crack extension pattern mechanically gets reproducible with less scatter in Chapter 5. The computational results from this study exhibit consistency with the experimental observations as discussed in Section 5.2. The rate dependency discussed above is of importance in the interpretation of the accelerated tests for those of natural conditions. Here, the direct use of the results obtained from the high corrosion rate would be carefully made for the corrosion occurring in natural environments. Otherwise, it may lead to underestimation of the crack width.

According to the governing formulae, the rate dependency as discussed above is closely associated with the modeling of the crystalline ratio. Then, for preparation for the future discussion, the inversely obtained crystalline ratio, which is denoted by $\beta$ and used in Chapter 5, is again focused on in terms of the corrosion rate as shown in Fig. 15, where the dot points derive all from the experiments in Chapter 5 (Andrade et al. 1993; Oh et al. 2009; Wong et al. 2010). The less crystalline solid of corrosion gel can be seen under the higher rate of corrosion. One of the possible explanations is that the crystallization or precipitation of ionized gel to solids takes some time. As a matter of fact, any phase-change more or less deviates from the idealized quasi-equilibrium process of thermodynamics. This point will be further discussed in future study.

Tentatively, Eq. (10) is proposed to roughly determine the ratio ($\beta$) of corrosion remaining around the surface of the reinforcing bar. The equation is formulated as the upper bound and a guide for the structural assessment of corroding RC so that the safer decision will be made in engineering viewpoint at this moment.

$$\beta = 0.75 - 0.1 \log_{10}(i_{\text{cor}}) \leq 0.75$$

(10)

where, $\beta$ is always greater than zero and represents the ratio of crystalized rust around the reinforcing bar, $i_{\text{cor}}$.

7. Conclusions

The initiation and subsequent propagation of corrosion-induced cracks are simulated based upon poro-mechanics
where a coupled migration of liquefied gel and cracking fracture are coherently treated. The overall mechanistic behavior is found to be dependent on the crystalline-liquid phase proportion of total corrosion products, kinematics of corrosion-gel to concrete micro-pores and crack gaps, corrosion application rate, concrete mix proportion, and the thermo-physical characteristics of the corrosion medium. The behavioral simulation indicates that the poro-mechanical approach can rationally be used for assessing the impact of corrosion, which may reproduce so-called broken symmetry of corrosion profiles and instability caused by perturbation in nature. The followings are the summary of conclusions.

(1) The poro-mechanics approach is implemented for the prediction of corrosion-induced crack initiation and propagation by explicitly considering pressure dependent migration of corrosion gel to micro-pores and crack gaps. This scheme enables us to reproduce realistic non-uniform corrosion fields even though the symmetric geometry of analysis domains is targeted.

(2) Corrosion rate dependency on the propagation of corrosion-induced cracks is addressed and its implication to engineering judgment in the maintenance of RC structures is tentatively proposed.

(3) By using the computationally obtained macro-behaviors of corroded RC beams, the average volumetric expansion ratio of corrosion products in chloride-rich water-saturated environments is found to be approximately 5 times, with the crystalline portion ranging from about 20% to 60%.

(4) The corrosion rate dependent liquid-crystalline proportioning of corrosion products in chloride-rich water-saturated environment is proposed by using inverse analyses from the meso and macro-scale experiments.

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