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Investigation of Influence of Section Pre-crack on Shear Strength and Shear Resistance Mechanism of RC Beams by Experiment and 3-D RBSM Analysis

Li Fu1*, Hikaru Nakamura2, Yoshihito Yamamoto3 and Taito Miura4

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Abstract
In order to investigate the influence of crack through section, which is often generated under cyclic loading, on the shear behavior and strength of RC beam, an experimental method was attempted to introduce a 0.5 mm and 1.0 mm-wide pre-crack, respectively, in beam cross section. The two pre-cracked beams were tested and the shear behaviors were compared with a non-cracked one. As the experimental result, it was found that the section pre-cracks led to a significant degradation of shear strength. Secondly, the different shear behaviors between pre-cracked and non-cracked beams were simulated by 3-D RBSM, and the developments of shear resistances were decomposed into the shear contributions provided by beam action and arch action. Based on the decomposition result, it became clear that the arch actions governed the grades of shear strengths and the section pre-cracks played a primary role in inducing extra crack behaviors and obstructing the longitudinal compressive stress transfer in concrete.

1. Introduction
When a reinforced concrete (short for RC) member is transversely one direction loaded, flexural cracks would be generated from the tension side and the un-cracked compression zone can bear great resistance. However, when a RC member is affected by reversed cyclic load, due to the effect of the negative load, flexural cracks would also initiate at the former un-cracked compression zone and connect with the former flexural cracks due to positive load, namely, the flexural cracks caused by the positive and the negative loads can cut through the member’s cross section and are likely to reduce the resistance in the compression zone. In this paper, the cracks that cut through member’s cross section is defined as the “section crack”. The formation of section crack is ordinarily regarded as a typical crack behavior for cyclic loaded RC member, and is likely to damage shear performance with a reduction of shear strength, since many studies mentioned that the RC members designed in flexure sometimes failed in shear after flexural yielding when subjected to cyclic loading (Ohta 1979; Asheim and Jack 1992; Wong et al. 1993; Priestley et al. 1994). In addition to that, it is usual to observe section cracks due to drying shrinkage, thermal stress at the early stage of service life of concrete structure. Therefore, it is required to investigate the influence of section crack on shear behavior and strength.

To date, however, only a few researchers have studied the influence of section crack on shear. Pimanmas and Maekawa (2001a) for first time experimentally penetrated multiple section pre-cracks in RC beams (the shear span to effective depth ratio is 2.4) designed in shear by four point reversed flexural loading; thereafter, they found that the section pre-cracks enhanced the shear strength compared with the non-cracked beam by three point loading, because the pre-cracks obstructed the continuous propagation of diagonal crack which was further demonstrated in their finite element analyses (Pimanmas and Maekawa 2001b, 2001c). As the characteristics, the section pre-cracks by four point reversed loading dispersely located in the entire shear span, and the crack widths varied in a large range from 0.02 to 5.0 mm and could not be accurately controlled. Quite differently, however, the section cracks due to cyclic loading generally initiate and propagate mainly in the zone where high moment affects, and usually present a similar width in a certain loading stage. Thus, it is desirable to develop an experimental method to introduce a section pre-crack with an expected width only in the cross section where a section crack is likely to form under cyclic loading, and investigate its damage to shear performance.

On the other side, it was revealed by Park and Paulay (1975) that the shear resistance mechanism of a RC member can be decomposed into the two contributions provided by beam action and arch action. And we have demonstrated the applicability of the decomposition approach, decomposing the development of shear resistance of a RC column, analyzed by three dimensional rigid-body-spring-method (3-D RBSM), into the contributions of beam and arch, utilizing the local stress

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result (Fu et al. 2016a). This approach is very useful to understand the mechanism of shear resistance of the beams having varying structural factors such as shear span to effective depth ratio, tension reinforcement ratio and shear reinforcement ratio considering decomposed beam action and arch action definitely.

In this study, firstly an experimental approach was attempted to introduce a section pre-crack into RC beam. In detail, one non-cracked RC beam for behavioral comparison and two beams with 0.5 and 1.0 mm-wide pre-crack, respectively, were three point loaded, and the influence of pre-crack on shear strength was clarified. Secondly, a numerical approach by employing 3-D RBSM was put forward to simulate the different shear behaviors of the test beams with and without pre-cracks. Finally, the shear resistances by numerical analysis of the three beams were decomposed into the contributions by beam action and arch action, utilizing local stress result, with an intention to understand the mechanism of shear strength degradation due to section pre-crack.

2. Experimental program

2.1 Design of specimens

One non-cracked RC short beam (No. A) and two beams with a single section pre-crack of 0.5 and 1.0 mm in width, respectively (No. B and No. C), were tested. The three beams with same dimensions were designed in shear failure. As shown in Table 1 and Fig. 1, the beams are 1,600 mm long and have a cross section of 300 mm × 150 mm. The shear span to effective depth ratio is 2.35, which ordinarily leads to shear compression failure. Two tension reinforcing bars of type D29 were arranged with a concrete cover thickness of 45 mm, i.e. the tension reinforcement ratio is 3.36%, and the elastic modulus and yield strength of them are 182 GPa and 358 MPa, respectively. In order to facilitate the observation of shear behavior, the stirrups of type D6 were arranged at one side (left span in Fig. 1) to fail the beams at the other span, where the shear behavior was focused on and video recorded. By standard cylinder compression test, it was known that the concrete compressive strength of No. A, C and No. B are 34.1 MPa and 29.2 MPa, respectively.

According to the studies by Niwa et al. (1986, 1983), the shear cracking load $V_c$ and the shear strength under shear compression failure $V_u$ for the three beams were predicted (see Table 1) by the following equations, without considering the influence of pre-crack. And the current JSCE standard specification (2012) were on the basis of these equations.

$$V_c = 0.20 f_c^ {1/3} \rho_t \left( \frac{d}{1000} \right)^{1/2} \left( 0.75 + \frac{1.4}{a/d} \right) b_w d$$  \hspace{1cm} (1)

$$V_u = 0.24 f_c^ {1/3} \left( 1 + \sqrt[100]{\rho_t} \right) \left( 1 + 3.33r/d \right) b_w d$$  \hspace{1cm} (2)

Where, $f_c$ is the concrete compressive strength; $b_w$ is the width of beam; $a$ is the length of shear span; $d$ is the effective depth of beam; $\rho_t$ is the tension reinforcement ratio; $r$ is the width of loading plate along beam axis.

Herein, as an improving factor for shear strength, the width of loading plate was neglected because the shear load was applied through a steel roller in our experiment (see Fig. 1). Thus, the predicted shear strengths were probably a low limit.

2.2 Method for Introducing Section Pre-crack

In beams No. B and C, a 0.5 mm and 1.0 mm single pre-crack was introduced, respectively, into the target section which was the effective depth (255 mm) far away from the loading point, since it was considered that a section crack due to cyclic loading ordinarily forms.

---

**Table 1 Material property of RC beam.**

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>$a/d$</th>
<th>$\rho_t$ (%)</th>
<th>$f_c$ (MPa)</th>
<th>$E_c$ (GPa)</th>
<th>Width of pre-crack $w$ (mm)</th>
<th>Design shear cracking load $V_c$ (kN)</th>
<th>Design shear strength $V_u$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.35</td>
<td>3.36</td>
<td>34.1</td>
<td>125</td>
<td>168</td>
<td>125</td>
<td>168</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>29.2</td>
<td>0.5</td>
<td>119</td>
<td>151</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td>34.1</td>
<td>1.0</td>
<td>125</td>
<td>168</td>
<td></td>
</tr>
</tbody>
</table>

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![Fig. 1 Details of RC beams.](image-url)
In order to introduce section pre-crack with a desirable width, the soft corrugated cardboard, i.e., the inlayers of common carton box, were set at the target section inside the steel framework before concrete casting (see Fig. 2a), considering that the corrugated shape of the cardboard can generate rough crack surface. The wave amplitude and the wave length of the corrugated shape roughly were 4 mm and 8 mm, respectively. That is, the section pre-crack was modeled by the cardboard as artificial crack. The merit of this method was that crack position and crack width can be controlled. The desirable pre-crack widths were achieved by superimposing cardboard (0.25 mm-thick for one piece), two pieces for 0.5 mm and four pieces for 1.0 mm. Moreover, the cotton thread was utilized to fix the cardboard in framework, in order to prevent the movement of cardboard in longitudinal direction caused by concrete flow during casting and vibrating as far as possible. The image of the introduced pre-crack in the beam No. C is illustrated in Fig. 2b.

In the process of shear loading, not only the shear load and the vertical displacements at loading and support points were measured but also the opening and closure behaviors of section pre-crack were recorded by PI-shape displacement transducers, which were set along the pre-crack at compression (PI-a) and tension (PI-b) sides, respectively (see Figs. 1 and 2b).

3. Experimental result

3.1 Load-displacement relations and failure mode

The load-displacement relations by experiment are displayed by solid curves in Fig. 3. In addition, the shear cracking loads \( V'_{cr} \) and the shear strengths \( V'_{ur} \) are listed in Table 2. Overall, it was notable that the initial stiffness of the pre-cracked beams were significantly decreased compared with the non-cracked one, attributed to the closure behavior of pre-crack. And the decreasing rate was higher in the wider pre-crack beam (No. C). In terms of the shear cracking load, the non-cracked beam (A3 in Fig. 3, 136 kN) was slightly greater than the previous prediction (125 kN for No. A in Table 1). In contrast, the shear load of the pre-cracked beam No. B was nearly in same grade (B4 in Fig. 3, 142 kN) and that of the pre-cracked beam No. C was slightly reduced (C4 in Fig. 3, 108 kN), which implied that the section pre-cracks had minor influence on the critical shear cracking load. With regard to the shear strength result, the non-cracked beam No. A presented a higher grade (A5 in Fig. 3, 234 kN) than the prediction (168 kN for No. A in Table 1) as previously explained, and it was worthy of note that the strengths of the pre-cracked beams obviously declined, 38% decrease for the beam No. B (144 kN) and 41% for the beam No. C (139 kN). Therefore, it became clear that the pre-cracks in the section which was effective depth far away from loading point could significantly reduce the shear strength of short beam, and this was incompatible to the conclusions in the study by Pimanmas and Maekawa (2001a) where the multiple section pre-cracks enhanced the shear strength of RC short beams. Although it was difficult to make clear the difference between their test result and ours, the effect of crack position, crack number and the shear strength of the non-cracked beam are supposed to be the possible reasons. For example, it was discovered in their experimental result that the shear strength of the non-cracked beam was nearly 50% lower than the calculation according to the equations for short members proposed by Niwa et al. (1983).

The failure modes of the three beams are illustrated in Fig. 4. It was observed that at the ultimate stage the shear compression failure occurred nearby the loading plate.

![Fig. 2 Introduction of section pre-crack (beam No. C).](image)

![Fig. 3 Load-displacement relations.](image)

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Width of pre-crack ( w ) (mm)</th>
<th>Test shear cracking load ( V'_{cr} ) (kN)</th>
<th>Numerical shear cracking load ( V'_{cr} ) (kN)</th>
<th>Test shear strength ( V'_{ur} ) (kN)</th>
<th>Numerical shear strength ( V'_{ur} ) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>136</td>
<td>121</td>
<td>234</td>
<td>227</td>
</tr>
<tr>
<td>B</td>
<td>0.5</td>
<td>142</td>
<td>116</td>
<td>144</td>
<td>149</td>
</tr>
<tr>
<td>C</td>
<td>1.0</td>
<td>108</td>
<td>111</td>
<td>139</td>
<td>137</td>
</tr>
</tbody>
</table>

Table 2 Result of shear load.
along with a severe spalling of concrete cover in the three beams. However, as the deformation characteristics of the pre-cracked beams, No. B and C, a second diagonal crack above the critical one and a horizontal crack along the tension reinforcing bars were noted (see Fig. 4b-c).

The process of crack propagation will be more detailed discussed in the following section to explain the influence of pre-crack on shear behavior.

3.2 Opening and closure behaviors of pre-crack
The opening and closure behaviors of the pre-cracks recorded by PI-shape displacement transducers (the measurement points are shown in Fig. 1) are displayed in Fig. 5 by solid curves, combined with the load-displacement relations. Herein, the average measurement by PI-a or PI-b on the front and back surfaces are presented and the negative width in the diagram standards for a closure procedure.

Fig. 5a shows the result for the beam No. B. In terms of the closure behavior by PI-a at the compression side, it was understood that the pre-crack closed at a constant speed since the start of loading until the point D1 when the closure speed began gradually decrease. The closure width at the point D1, which was 0.41 mm, was identified as the width of pre-crack and it approximately reached our target, i.e. 0.5 mm. Afterward, the pre-crack completely ceased to close at the point D2 when the closure width approached 0.5 mm, and simultaneously the critical shear crack was observed. After the point D2, the shear behavior governed the structural performance until the ultimate stage. In terms of the opening behavior by PI-b at the tension side, it was seen that the pre-crack opened at a constant speed until the occurrence of the critical shear crack at the point E2, and afterward, ceased to open since the shear behavior became dominant.

Based on the above opening and closure behaviors of pre-crack, it was proved that the experimental method for introducing pre-crack was practical and effective. In particular, the target pre-crack width could be achieved, which means that a parametric study on the influence of pre-crack width and location on shear becomes assessable.

4. Numerical analysis for test beams
4.1 Numerical method and model
In chapter 3, by experiment, it waslessoned that the pre-cracks with a width of around 0.5 and 1.0 mm can significantly reduce the shear strength of a RC short
beam. But it was desirable to understand the mechanical role of pre-crack on shear performance, i.e. mechanism of shear strength degradation. Therefore, a numerical analysis employing 3-D RBSM was performed to simulate the shear behaviors of the test beams.

Above all, the numerical method is outlined. 3-D RBSM is a discrete method and has been proved to be effective to accurately simulate the behaviors of reinforced concrete structures including bending and shear (Yamamoto et al. 2008; Gedik et al. 2011; Yamamoto et al. 2014; Fu et al. 2016a). In 3-D RBSM, concrete is modeled as an assemblage of rigid polyhedrons interconnected by springs at their boundary surfaces (see Fig. 6). Since the crack propagation is affected by mesh design, a random geometry of rigid polyhedrons is generated by Voronoi diagram, which can reduce mesh bias on the initiation of potential crack.

Regarding the reinforcing bar model, it is created by a series of regular beam elements (Fig. 7), which can simulate bending effect. In this model, the beam elements are freely arranged in the member, without consideration of mesh design of concrete elements. Moreover, beam elements are attached to concrete elements by zero-size link elements, which can provide a load-transfer mechanism between concrete polyhedron and beam element. The parameters and constitutive models applied in 3-D RBSM have been proposed in the work by Yamamoto et al. (2008, 2014).

The RBSM models of the test beams, the average element size of which were 15 mm, are shown in Fig. 8, and the same material properties of concrete and reinforcing bar listed in Table 1 were utilized.

4.2 Numerical method for introducing pre-crack in beam

For pre-cracked beams, before loading analysis, pre-cracks were introduced by a way applying loading on link elements for longitudinal reinforcing bar, which was first reported by Fu et al. (2016b) (see Fig. 8).

In the procedure of pre-crack introduction by 3-D RBSM, as the first step, the four link elements in longitudinal reinforcing bars located in the same section that 150 mm far away from loading point were fixed, and meanwhile another four link elements located in the same section 350 mm far away from loading point were horizontally tensioned by displacement control (see step 1 in Fig. 8). Consequently, due to the high deformation ability of the reinforcing bars, a pre-crack would be generated approximately in the middle plane between the previous two sections where the link elements located in, that is, a pre-crack at the same position as that of the test beams was introduced (see step 2 in Fig. 8). With regard to the width of pre-crack, it was determined by the displacement of the tensioned link elements.

Then in the second step, once a pre-crack with desirable width was introduced, the previously tensioned and fixed link elements were to be unloaded and become completely free in movement. The pre-crack, however, would remain there due to the residual deformation of the longitudinal reinforcing bars (see step 2 in Fig. 8).

By above two steps, a desirable pre-crack with a width of 0.5 or 1.0 mm could be introduced. And after that, in the third step, shear load analysis would be performed by
displacement control to investigate shear behavior and strength. Before shear loading, the effect of compressive reinforcing bars, which were just for introducing pre-crack, were removed, because we did not arrange any compressive reinforcing bars in actual experiment (see step 3 in Fig. 8).

It should be emphasized that the shape of the pre-crack surface, in numerical analysis, would change with the rearrangement of concrete element, since the concrete elements were randomly created; but it was confirmed that as long as the pre-crack width was determined, the macro deformation behavior would be consistent, regardless of the change of the distribution and the shape of concrete element.

Moreover, we also took into account of the investigation of opening and closure behaviors of pre-cracks in numerical analysis, which were achieved by the relative horizontal displacements of link elements at the compression and tension sides (see step 3 in Fig. 8).

4.3 Result of numerical analysis

4.3.1 Load-displacement relation and behavior of pre-crack

In this section, the result of numerical analysis is briefly discussed by comparing with the previous experimental result.

The global load-displacement relations by numerical analysis are plotted in previous Fig. 3 by dotted lines and the obtained shear cracking loads and shear strengths are listed in previous Table 2. It was evident that the numerical result overall captured the global behaviors of the three beams including the reduction of initial stiffness of pre-cracked beam. Furthermore, similar to experimental result, the shear cracking load was not remarkably affected by the pre-cracks (see a3, b4, c4 in Fig. 3 and Table 2), nevertheless the shear strength was considerably reduced in the pre-cracked beams (see a5, b5, c5 in Fig. 3 and Table 2).

The opening and closure behaviors of pre-cracks for the beams No. B and C obtained from the relative displacements of link elements are displayed in previous Fig. 5 by dotted lines. In a brief summary, it was apparent that for the two beams, both the closure and opening behaviors including the widths of pre-cracks could be roughly captured by 3-D RBSM. And from the closure widths at the stages of d1 and e1, where the closure speeds began decrease, the widths of pre-cracks for the beams No. B and C, in analysis, were identified as 0.40 mm and 1.24 mm, respectively.

Based on the above comparison of results by experiment and numerical analysis, it was proved that 3-D RBSM could simulate the global behaviors of the test beams with a high accuracy. Meanwhile, the good agreement of the experimental and numerical analysis in turn is a good evidence of the effectiveness of the experimental method for introducing a section pre-crack and the fact of shear strength reduction due to section pre-crack.

4.3.2 Crack propagation

In order to understand the influence of section pre-crack on shear behavior, the crack propagations of the three beams based on results by experiment and numerical analysis are focused on in this section.

Figures 9 to 11 display the crack patterns at the selected critical stages observed in experiment and numerical analysis for the three test beams, and the critical stages are marked in Fig. 3. In the result of numerical

![Fig. 9 Crack propagation of beam No. A.](image)
analysis, the scale of crack width varies from 0.01 to 0.5 mm while the deformation is magnified 15 times.

As the experimental result of the non-cracked beam No. A (see Fig. 9), it was observed that the visible flexural cracks first formed from the bottom surface under the loading point (A1) when the load was increased to 69 kN. Subsequently, the preceding flexural cracks propagated upward to the loading point and simultaneously new flexural cracks initiated in the shear spans, when the load was increased to 130 kN (A2). Afterward, the critical shear crack appeared for the first time as a result of the propagation of the preceding flexural crack in shear span when the load reached 136 kN (A3), and then the shear behavior became dominant, that is, the critical shear crack propagated rapidly to the loading point and the support point with an increase of crack width (A4).

Fig. 10 Crack propagation of beam No. B.

![Crack propagation of beam No. B](image)

Fig. 11 Crack propagation of beam No. C.

![Crack propagation of beam No. C](image)
Ultimately, as mentioned in chapter 3, the critical shear crack dramatically developed and resulted in the compression failure of concrete near loading point, when the load reached the peak, i.e. 234 kN (A5).

On the other hand, in numerical analysis, the similar behaviors such as flexural cracking (a1-a2) and shear cracking (a3-a4) were confirmed at the corresponding load stages. Moreover, the failure mode, i.e. compression failure due to shear at loading point, was well simulated (a5).

In terms of the pre-cracked beam No. B (see Fig. 10), in the experiment, a minor horizontal crack first initiated from the section of pre-crack and propagated toward the support point along the tension reinforcing bars (B1), instead of the first formation of flexural cracks that was observed in the non-cracked beam No. A (see A1 in Fig. 9). Thereafter, two flexural cracks initiated from the bottom surface under the loading plate when the load was increased to 39 kN (B2). After that, a flat diagonal crack at compression side originated from the section of pre-crack. Herein, it was worth noting that the combination of the flat diagonal crack, the previous horizontal crack along tension reinforcing bars and the vertical pre-crack formed a Z-shape crack (see Fig. 10). Finally, the critical shear crack was generated across the section of pre-crack when the load was increased to 142 kN (B4) and then rapidly developed, leading to the compression failure of concrete near loading plate (B5). With respect to the result by numerical analysis, evidently the first occurrence of Z-shape crack (b1-b5) and the ultimate shear compression failure mode were successfully achieved.

Because of the high similarity of the crack propagation between the beams No. B and C, the crack behavior of the pre-cracked beam No. C would be briefly described in Fig. 11. In the experiment, with the increase in load, besides flexural cracks, a horizontal crack along tension reinforcing bars (C1) and a flat diagonal crack from pre-crack at compression side (C3) first successively initiated resulting in a Z-shape crack (C3). Subsequently, the critical shear crack across the pre-crack was generated (C4) and dramatically developed triggering a compression failure of concrete near loading point (C5). For the numerical analysis, a similar crack behavior to that in the experiment has been confirmed (see c1-c5 in Fig. 11).

By contrasting the results between non-cracked and pre-cracked beams, it became clear that although the section pre-crack caused extra behavior, Z-shape crack, the critical shear cracks in the pre-cracked beams could form with a nearly same inclination as that in the non-cracked beam. And these observations were opposite to the shear-influence of the multiple pre-cracks, uniformly distributed in shear span, studied by Pimanmas and Maekawa (2001a), where the continuous development of critical shear crack was obstructed by the pre-cracks. In addition, it was also notable that the deformation of concrete near loading point at the failure stage became more intense in the pre-cracked beams (see a5 in Fig. 9 and c5 in Fig. 11).

5. Mechanism of shear strength degradation caused by section pre-crack

In this chapter, the developments of shear loads by numerical analysis shown in Fig. 3 are decomposed into the contributions of shear resistance mechanisms, i.e. beam action and arch action, with an intention to reveal the reason of shear strength degradation caused by section pre-crack. And the influence of pre-crack on local stress distribution in concrete is further clarified.

5.1 Decomposition of shear resistance into beam action and arch action

It is acknowledged that the force equilibrium in the cross section of a RC beam is ordinarily expressed by Eq. 3 (see Fig. 12).

\[ M = (T_c + C_c) \cdot \frac{j_c}{2} + C_r \cdot j_{cr} + T_r \cdot j_{rc} \]  \hspace{1cm} (3)

Where, \( M \) is the bending moment acting on a cross section of RC beam, \( T_c \) and \( C_c \) are the tensile and compressive resistances sustained by longitudinal reinforcing bars; \( C_r \) is the compressive resultant in concrete; \( T_r \) is the tensile resultant in concrete; \( j_c \) is the arm length between the centroids of tensile and compressive longitudinal reinforcing bars; \( j_{cr} \) is the arm length between the centroid of compressive resultant in concrete and the beam axis; \( j_{rc} \) is the arm length between the centroid of tensile resultant in concrete and the beam axis.

Based on the previous work (Park and Paulay 1975; Fu et al. 2016b; Iwamoto et al. 2017), by differentiating Eq. 3 in the small segment \( dx \) between two adjacent cross sections, an equation for calculation of shear resistance, Eq. 4, can be derived. Eq. 4 expresses the relationship between bending moment and the change rate of longitudinal forces in the concrete and reinforcing bars (Fig. 13a), and it can be divided into the two components of beam action and arch action expressed by Eq. 5 and Eq. 6 corresponding to Fig.13b and Fig. 13c, which show the components of axial stress contributing to each action.

\[ V = \frac{dM}{dx} = V_s + V_a \]  \hspace{1cm} (4)

\( V_s \) and \( V_a \) are the shear component and the arch component, respectively.
The shear resistance $V_s$ is defined as beam action, which is caused by the change rates of tensile resistance $dT_c$ and compressive resistance $dC_c$ in longitudinal reinforcing bars, and the change rates of compressive and tensile resultants in concrete $dC_c$, $dT_c$ between two adjacent cross sections (Fig. 13b). The shear resistance $V_a$ is defined as the arch action, which is caused by the change rate of centroids of compressive and tensile resultants in concrete $djC_c$, $djT_c$ along beam axis (Fig. 13c).

For example, the section stress states in concrete along beam axis are conceptually shown in Fig. 14. It became clear that if the stress states and the centroids of compressive resultants are determined in two adjacent cross sections spacing $dx$, the arch action provided by the second item ($C_c djC_c/dx$) in Eq. 6 in a segment can be computed.

By the same way, the variables required for determination of beam action and arch action were able to be obtained from the local stress states in concrete and reinforcing bar elements in 3-D RBSM, although they were not available by experiment. Herein, the zones at the ends of shear span within 50 mm from the loading and support points were removed for shear resistance calculation, because high stress concentration was found there (Iwamoto et al. 2017). Moreover, after several tests for accuracy, it was clarified that if the unit $dx$ was less than 100 mm, the average combination shear of beam action and arch action for all units $dx$ could well agree with the numerical load by 3-D RBSM and the ratio of beam action to arch action almost did not change. In order to ensure the calculation accuracy, a dimension of 50 mm for unit $dx$ was finally utilized to compute beam action and arch action.

The average beam action and arch action, at each load, for all units $dx$ located in the target shear span were computed for the three beams, and the results are displayed in Fig. 15a-c, combined with the numerical load-displacement relations that mentioned in Fig. 3. Although the distributions of decomposed arch action and beam action are slightly varied along beam axis (Iwamoto et al. 2017), the utilization of averaged values is convenient to discuss the macro behavior. It was apparent that the combination effect of beam and arch was in agreement with the shear resistance by 3-D RBSM for the three beams, that is, the decomposition method was applicable and effective in the case of 3-D RBSM. For the non-cracked beam No. A (see Fig. 15a), it was seen that the beam action first provided shear resistance since the start of loading, nevertheless the arch action was nearly not generated. A rapid shift in the resistant effect of beam and arch, however, was noted at the initiation of shear crack, that is, the beam action declined steeply to around 40 kN while the arch action rose sharply and became dominant to the development of shear resistance. It should be kept in mind that the arch action developed greater than the maximum effect of beam and governed the shear strength while the beam action almost remained a constant at a low grade after shear cracking.

For the pre-cracked beams (see Fig. 15b-c), similar developments of beam action and arch action to those of the non-cracked one, particularly the shift in the role of beam and arch, were confirmed. For understanding the primary role of arch action, the beam actions and arch actions corresponding to shear strengths were picked out and compared in Fig. 15d. It was concluded that the arch actions governed the shear strengths for the three beams, and the significant degradation of shear strength due to pre-crack was attributed to the decrease of arch action.
even though the minor beam action was well maintained.

### 5.2 Primary role of pre-crack on shear resistance mechanism

Moreover, the specific resistant components in arch actions were investigated. The components for maximum arch actions by concrete compression related effect (short for compression effect, item \(C_c \cdot djC_c/dx\) in Eq. 6) and concrete tension related effect (short for tension effect, item \(T_c \cdot djT_c/dx\) in Eq. 6) are illustrated in Fig. 16 in the form of distribution along beam axis that obtained in each unit \(dx\). Herein, the lower limit and upper limit of the horizontal coordination in Figs. 16-18 represent the location of loading point and support point, respectively. As a consequence, it was seen that arch action actually was nearly completely provided by the compression effect, whereas the tension effect was minor and could be neglected. For the compression effect of non-cracked beam (Fig. 16a), it was noted that the arch action was in a relatively higher grade nearby loading plate and was gradually decreased toward the support point. In contrast to that of the non-cracked beam, the compression effect in pre-cracked beam was overall in a lower grade, particularly in the zone between loading point and section of pre-crack, directly leading to the overall degradation of arch action.

For a deeper interpretation, the relevant factors for calculation of component by compression effect, i.e. the resultant compression \(C_c\) in concrete and the change rate of its centroid along longitudinal direction \(djC_c/dx\) were represented in Figs. 17 and 18, respectively. With respect to resultant compression \(C_c\), it was observed that its distribution considerably declined in pre-cracked beams, and the wider the pre-crack was the higher the decline rate became. The different grade of resultant compression \(C_c\) can be intuitively understood by the different longitudinal stress states in target shear span shown in Fig. 16 (hereby the lower and upper limits in stress legends stand for the tensile and compression strengths of concrete), that is, the area of high compression zone nearby loading plate at compression side of the pre-cracked beams were evidently smaller and the range of inclined

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![Fig. 16](image1.png)

**Fig. 16 Longitudinal distributions of decomposed arch action and stress states in central cross section at maximum load.**

![Fig. 17](image2.png)

**Fig. 17 Longitudinal distribution of resultant compression in concrete \(C_c\).**

![Fig. 18](image3.png)

**Fig. 18 Longitudinal distribution of centroid change rate of resultant compression in concrete.**
arch-shape stress flow were obviously narrower compared with those of the non-cracked beam.

On the other side, because of the similar orientation of critical shear crack in the three beams, a roughly same shape of distribution of centroid change rate was confirmed, that is, the centroid change rates of the three beams were in close grade in the entire span and were gradually decreased from loading point to support (see Fig. 18). Thus, it became clear and should be emphasized that the overall decrease of resultant compression $C_r$ was dominant to the degradation of arch action in pre-cracked beam.

The primary role of section pre-crack is more detailed explained in Fig. 19, where the different longitudinal stress states between non-cracked beam No. A and pre-cracked beam No. C in selected three sections are compared. The locations of selected cross sections ①-③ are shown in Fig. 19a combined with the crack patterns at maximum loads. As the Fig. 19b-c for contrast of results between beam No. A and No. C in cross sections ①-③, respectively, the diagram on left side illustrates the longitudinal stress intensities obtained from concrete elements in a cross section while the graphs in middle and right sides display the corresponding crack patterns and longitudinal stress contours in the same cross section for beam No. A and No. C.

As a result, for cross section ①, it was seen that the compressive stress in beam No. C was overall in a lower grade than the non-cracked beam one (see diagram in Fig. 19b). Moreover, compared with those in beam No. A, it was observed that an extra crack (i.e. Z-shape crack mentioned in previous discussion) formed above the critical shear crack in the pre-cracked beam No. C (see crack patterns of Fig. 19b), and the extra crack was likely...
to lead to the dramatic reduction of stress at its location (see stress contour in No. C of Fig. 19b). For cross section ② where the pre-crack generated, the reduction of stress in pre-cracked beam No. C became more obvious, namely, the majority of stresses nearly declined close to zero (see diagram in Fig. 19c), which was attributed to the much severer crack damage, i.e. larger number of cracks (see crack patterns in Fig. 19c). This was further confirmed by the remarkable loss of stress in the crack zone (see stress contour in No. C of Fig. 19c). Therefore, it was concluded that the section pre-crack in this study played a primary role in inducing extra crack behaviors and obstructing the transfer of longitudinal compressive stress, leading to the overall decline of resultant compression in concrete C, and the degradation of arch action. With respect to the result for cross section ③, an entire reduction in low rate of stress in pre-cracked beam No. C was seen as well (see diagram in Fig. 19d). Different from the mechanism in cross sections ①-②, however, the reduction of stress in section ③ was likely to be a subsequent process caused by the obstruction of stress transfer in cross section ②, because a similar crack pattern was clarified in the two beams.

6. Conclusions

This study investigated the influence of section crack on the shear strength of RC short beam by experiment and 3-D RBSM analysis. Finally the study work was briefly summarized and the following conclusions could be drawn.

1) As the summary of the study work, an experimental method by utilizing cardboard for introducing a pre-crack with a desirable width in a target cross section of RC short beam was attempted, and its effectiveness and applicability were proved by the measurement of opening and closure behaviors of the pre-cracks in the RC beams under shear loading. For the numerical work, by employing 3-D RBSM, the shear behaviors including load-displacement relation, crack propagation and pre-crack opening/closure of the test beams were successfully simulated. Particularly, the unique deformation behavior of Z-cracking in pre-cracked beams was well reproduced.

2) Shear loading was applied on a non-cracked beam and two beams with a single 0.5 mm and 1.0 mm pre-crack, respectively, and it was found that the section pre-cracks could lead to a significant degradation of shear strength compared with the non-cracked beam one. The numerical result also supported the finding of the shear strength degradation due to the initiation of section pre-crack.

3) By utilizing the local stress states obtained from 3-D RBSM, the development of shear resistances for the three beams were decomposed into the contributions by beam action and arch action. And it was lessened that the shear strengths of the three beams were governed by the contributions of arch action, and the degradation of the shear strengths in pre-cracked beams was attributed to the decrease of arch action.

4) Based on the decomposition result of resistant components for calculation of arch action and the comparison of longitudinal compressive stresses in critical cross sections between non-cracked and pre-cracked beams, it was concluded that the section pre-crack in this study played a primary role in inducing extra crack behaviors and obstructing the transfer of compressive stress in longitudinal direction after shear cracking, which was the essential reason why the resultant compression in concrete C, and arch action degraded. However, it should be noted that the above conclusions for the influence of section pre-crack on shear is only applicable to the RC members where the tensile reinforcement ratio was relatively high (around 3.36%) and the shear span to effective depth ratio was relatively small (around 2.35), and a further parametric study is needed to investigate the effect of various structural factors, such as tensile reinforcement ratio and shear span to effective depth ratio, on the shear failure mechanism.

References


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