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PC Tendon Damage Detection Based on Change of Phase Space Topology

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Abstract

PC tendon is a vital component for strength of prestressed concrete structure. However, there is no warning sign when it is degraded or damaged, because of its invisibility from outside. Vibration-based method is one of the technique for damage detection due to simplicity. However, the extensively used method such as modal-based analysis is still insensitive to such damages. Recently, phase space-based analysis, a novel method for damage detection, was adopted in Civil engineering and a new index called Change of Phase Space Topology (CPST) has been developed. It is effective in identifying the damage existence regardless a known of damage location. Therefore, this paper adopted such technique to investigate damages of PC tendon. A pretension concrete girder with artificial PC tendon damages was used in experiment and it was excited by two vibration tests; free vibration and impact hammer test. The CPST values and the results from modal-based analysis were investigated and compared. The index shows increasing trend with increasing of damage levels. Moreover, it is more sensitive to the PC tendon damages than the results from modal-based analysis. This implies that CPST might be an effective index to detect the damages of PC tendon.

1. Introduction

Nowadays, prestressed concrete is widely used in construction, especially for large structures such as bridges and buildings due to its advantages on having PC tendons that improve structural strength, resulting in a small cross-section and lightweight structure. However, in general, all structures must deteriorate depended on their usage, types of materials, and surrounding environment. Prestressed concrete bridge is one of the structures that inevitably face with such problems because of its location and usage condition. Deterioration of concrete or overweight vehicles can lead to several cracks

on a bridge structure. Moreover, incompletely filled of grout material also causes voids around PC tendons. Water, Oxygen, and other chemicals can get into the voids from sheath joints, vicinity of anchor, and much easier when the ducts crack or corrode. This causes corrosion and diminishes the structural strength of the bridge (Duke 2002). Such reduction can lead to failure of the bridge and causes a great loss. Hence, to avoid such problems, early detection of the damage of PC tendons is necessary.

In literature, several methods have been proposed to identify the damages of PC tendons such as acoustic emission technique (Yuyama *et al.* 2007; Ramadan *et al.* 2008; Shiotani *et al.* 2013), magnetic inspection (Scheel and Hillemeier 2003), and electromechanical impedance method (Kim *et al.* 2010; Jiang *et al.* 2017). However, they still have some difficulties and limitations in real-life application for long term monitoring. For instance, acoustic emission (AE) technique has been successful in detection and evaluation of tendon failures in prestressed concrete structure. However, because of ambient noise, it requires sensitive sensors, reliable amplifiers and sophisticate data processing technique to detect and localize the signal (Grosse and Ohtsu 2008). Magnetic signal used in magnetic inspection method seems to be disturbed by other steel elements. Durability of piezo-impedance transducer (PZT) used in the electromechanical impedance method is limited to some periods and not suitable in high temperature (Yang *et al.* 2008). Many attempts have been made to overcome such challenges such as the suggestion of new tendon monitoring technique with the use of alternative filler material (Abdullah *et al.* 2014, 2015, 2016).

Vibration-based method is an alternative technique which is simple, cheaper and widely used (Casas and

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Aparicio 1994; Salawu 1997; Doebling *et al.* 1998; Wang *et al.* 2016; Moughty and Casas 2017). Only sensors attach on structural surface without any surface preparation. The basic idea is that the changes of measured modal parameters (frequencies, mode shapes and modal damping) are related to the changes of structural properties (such as mass and stiffness). Several researches apply such techniques to study the dynamic behaviors of prestressed concrete bridge (Kato and Shimada 1986; Döhler *et al.* 2014) and also propose new damage identification methods (Wahab and Roeck 1999; Cavell and Waldron 2001; Huth *et al.* 2005). Similarly, dynamic behaviors of deteriorated PC tendon are also necessary to study. Many researches were interested in the change of structural frequencies due to the effect of prestress force magnitude. Both analytical studies (Saiidi *et al.* 1994; Miyamoto *et al.* 2000; Hamed and Frostig 2006) and experimental studies (Saiidi *et al.* 1994; Hop 1991; Noble *et al.* 2015, 2016; Wang 2017) were investigated. Results revealed that the prestress force magnitude effect on the natural frequency. However, in the most of experiments, the prestress force magnitude was controlled by loading jack and load cell (Hamed and Frostig 2006; Hop 1991; Noble *et al.* 2015, 2016; Wang 2017). Therefore, the force was equally changed throughout a specimen. In reality, the damage of pre-tension tendon and bonded tendon are very local and the changes are not uniform. Such effects result in insensitive change of natural frequency.

Phase space analysis based on vibration data is an alternative method. The concept of phase space is that the method transforms measured data of time domain into spatial domain. A small change of one parameter will affect the entire system. Recently, it is extensively used in other engineering field such as mechanical engineering (Todd *et al.* 2001; Nichols *et al.* 2003) and proved that it is sensitive to damage. For the field of Civil engineering, the phase space analysis has been adopted and developed. The damage index called *Change of Phase Space Topology* (CPST) (Nie *et al.* 2013) was proposed and used to detect damages of continuous reinforce concrete slab. They found that the CPST index was very effective in identifying the existence of damages because its values increased with damage levels regardless of damage location. Several researches used the CPST as an indicator for damage detection (Paul *et al.* 2017; Nie *et al.* 2017).

This paper conducts experiment on a prestressed concrete girder with local damages. The damages are simulated by artificial cut of PC tendons. Measured acceleration data is used to analyze based on the change of phase space topology in order to detect the damages. In Section 2, method of phase space reconstruction and its damage index are explained. Experiment on the damages of PC tendons and vibration tests are described in Section 3. In Section 4, the testing results on modal based method and phase space based method are inves-

tigated and compared. Finally, the conclusions are given in Section 5.

2. Damage detection method based on phase space topology

2.1 Phase space reconstruction

Phase space analysis is a novel approach for damage identification. The method alters measurement data of a time series into a spatial domain, where all possible state of a system can be represented. Every parameter of the system is plotted as an axis of a multidimensional space. Such plot is called phase space topology and a change of one parameter will affect the entire system.

The reconstruction of phase space is based on delay coordinate and embedding dimension. The corresponding phase space $\mathbf{X}(n)$ of the measurement in time series of N data points can be reconstructed as (Takens 1981)

$$\mathbf{X}(n) = [x(n), x(n+T), \dots, x(n+(d-1)T)], \quad (1)$$

where T is the delay time and d is the embedding dimension and each dimension can be represented as

$$\begin{aligned} x(n) &= [x(1), x(2), \dots, x(N-(d-1)T)] \\ x(n+T) &= [x(1+T), x(2+T), \dots, x(N-(d-2)T)] \\ &\vdots \\ x(n+(d-1)T) &= [x(1+(d-1)T), x(2+(d-1)T), \dots, x(N)] \end{aligned} \quad (2)$$

The delay time T must be large enough so that each dimension does not contain redundant information. Generally, it can be determined from the first minimum of Average Mutual Information (AMI) (Jiang *et al.* 2010). Moreover, the embedding dimension d must be large enough to unfold the system and it is normally calculated from the minimum value of False Nearest Neighbors (FNN) approach (Rhodes and Morari 1997).

2.2 Change of Phase Space Topology (CPST) as a damage index

Change of Phase Space Topology (CPST) index was proposed to detect damages of reinforce concrete slab (Nie *et al.* 2013). The index measures how much difference of a damage state from a predicted damage state. The predicted damage state is calculated based on reference topology. Suppose a reconstruction phase space of a damage state, constructed as Eq. (1), is defined as $\mathbf{Y}(n)$. To construct the predicted damage state, a fiducial point at time index r , $\mathbf{Y}(r)$ on the damage state is selected and mapped on to the reference state $\mathbf{X}(n)$. The nearest p neighbors of this point based on the reference state are selected as

$$\mathbf{X}(p_j) : \min \|\mathbf{X}(p_j) - \mathbf{Y}(r)\|, \quad j = 1, \dots, p \quad (3)$$

where p denotes the nearest neighbors to the fiducial point, $\mathbf{Y}(r)$ and the $\|\cdot\|$ operator computes the Euclidean norm. Noted that each two nearest neighbor points

$\mathbf{X}(p_a)$ and $\mathbf{X}(p_b)$, $a \leq p, b \leq p$, should not be in the same trajectory. Therefore, the time difference of each two points should be greater than the delay time T . Then, the future value at the next s -time step, $\mathbf{Y}(r+s)$ of the damage state can be predicted as

$$\hat{\mathbf{Y}}(r+s) = \frac{1}{p} \sum_{j=1}^p \mathbf{X}(p_j+s), \quad j=1, \dots, p \quad (4)$$

where the predicted value $\hat{\mathbf{Y}}$ is the average of the evolved values for the neighborhoods for the next time step s . The difference of the predicted value $\hat{\mathbf{Y}}$ with the real future value of the damage state $\mathbf{Y}$ can be calculated as

$$CPST_r = \frac{1}{p} \|\hat{\mathbf{Y}}(r+s) - \mathbf{Y}(r+s)\|, \quad r=1, \dots, N_f \quad (5)$$

where N_f is total number of fiducial points. Since the reconstructed phase space has $N-(d-1)T$ points in total, together with s -time step, this calculation will be also done for $N_f = N-(d-1)T-s$ times. Then its estimate value is used as the damage index, named *Change of Phase Space Topology (CPST)* defined as

$$CPST = \frac{1}{N_f} \sum_{r=1}^{N_f} CPST_r. \quad (6)$$

The schematic view of CPST is illustrated in Fig. 1. In order to be able to compare with other indices. The CPST will be normalized by the reference state as following

$$\Delta CPST = \frac{|CPST^i - CPST^{ref}|}{CPST^{ref}}, \quad (7)$$

where $CPST^i$ is the CPST value of event i^{th} and $CPST^{ref}$ is the CPST value of the reference state.

3. Experimental program

3.2 Specimen and experimental setup

The specimen of this experiment is a pretension concrete girder taken from an existing PC bridge in Kyushu area where it was removed due to a new construction. However, the bridge was still in a good condition. The schematic of PC girder on a loading machine is shown in Fig. 2. The PC girder is simply support with a span length of 8.5 m and I-shape cross-section with a depth of 425 mm. Nine of seven-wire strands of nominal diameter 10.8 mm are used as the PC tendons where two and seven of them were assigned in the upper and lower part, respectively. Four point bending test was performed on the PC girder with two levels of loading, dead load and live load calculated from FEM analysis where live load represented 90% of elastic limit and the

Fig. 1 Schematic view of CPST calculation.

Fig. 2 Schematic of the PC girder on a loading machine, accelerometer locations, and hitting locations for impact hammer. (a) side view (b) top view (unit: mm)

Table 1 Name of each event.

Event	E0	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10
Condition	Healthy	Concrete removed (all sections)	A1 cut	A2 cut	C5 cut	C4 cut	B1 cut	B5 cut	B2 cut	B4* cut	Rupture

*The PC tendon at B4 was partially cut.

dead load was 50% of live load. To measure vibrations, three accelerometers were attached at the bottom surface of PC girder; CH01 and CH03 were at 2.5 m away from the supports while CH02 was at the middle of span. Type of accelerometer is KIONIX (KXR94-2050) with a sensitivity of 660 mV/g measuring only vertical vibrations. Sampling rate was set as 1 kHz.

3.2 PC tendons breakage sequence

Damages were simulated by cutting the PC tendons at different sections along the PC girder. Section A and C were located at the middle of support and loading, while section B was at the center of span. For convenience, it was assumed that the concrete parts covering on the damaged PC tendons simultaneously deteriorated with the PC tendons. Therefore, small size of concrete parts, approximately $120 \times 60 \times 100$ mm, at all cutting sections were removed before conducting the experiment. The removed parts were shown in Fig. 3 by dash line. Four out of five PC tendons at the lowest part of section were cut. Name of each PC tendon was defined by its section (A, B, or C) followed by the order of PC tendons (1 to 5). For example, A1 means cutting the first

PC tendon at section A. Cutting events of each section were performed at both levels of loading as shown by the blue arrows in Fig. 4. Noted that the last event of PC tendon cutting was partially cut. Only 3 strands of the tendon B4 were cut and the PC girder was loaded until it ruptured. Therefore, the remaining strands of tendon B4 would be ruptured by loading. Name of each damage state are shown in Table 1.

3.3 Vibration testing

Vibrations of the PC girder at each state of damages were excited by its free vibration and impact hammer. The free vibration response was obtained by hanging and releasing a mass of 50 kg at the middle span of PC girder. It was conducted at initial state and after each cutting event when the PC girder was unloaded to 0 kN and the distributing girder was detached from the specimen. The free vibration test is shown by the yellow arrows in Fig. 4. Due to a large excitation force of the free vibration test, it was expected to give a clear decay curve for analysis and can be used as reference. On the other hand, even a low excitation force of an impact hammer, the impact hammer test is also necessary be-

Fig. 3 PC tendons are named according to section locations and the order of PC tendon. Area in the dashed line are the removed parts of concrete. (unit: mm)

Fig. 4 Sequence of loading, PC tendon cutting, free vibration test and impact hammer test.

cause it is more practical method for structural inspection.

Impact hammer (PCB 063C03) was used in this experiment. The impact hammer has a mass of 0.16 kg, length of 21.6 cm, head and tip diameters of 1.57 and 0.63 cm, respectively. The measurement range is ± 2224 N pk. Two types of cap were used in this experiment; soft cap and hard cap. The soft cap is a rubber insert in aluminum used for lower frequency response while the hard cap is made from stainless steel used for higher frequency response. The impact hammer test was con-

ducted at all states of damages when the PC girder was unloaded to 0 kN. In general, impacting at different locations gives different responses. For example, hitting at one point, magnitudes of some frequency mode may be clearly observed while the others may not. To capture most modes of vibration, several hitting locations (in total 19 points) were set along the PC girder as shown in **Fig. 2b**. Then, the impact hammer hit vertically on the defined location.

(a) Sections of PC tendon cutting and accelerometer locations

(b) PC tendon cutting

(c) Rupture of PC girder

(d) Rupture at the center of PC girder (Side view)

(e) Rupture at the center of PC girder (Bottom view)

Fig. 5 Pictures taken during experiment.

4. Results and discussion

Pictures taken during the experiment are shown in **Fig. 5**. The measured time series data obtained from both vibration tests as shown in **Fig. 6** were analyzed and discussed in this section. Due to a large excitation force in free vibration test, the measured time series data takes approximately 5 seconds to decay to zero. Hence, acceleration data of 6.5 seconds (6500 data points) was used in analysis. On the other hand, due to a small excitation force on impact hammer test, the measure acceleration data decay to zero rapidly, less than 1 seconds. Hence, the acceleration data of 1024 data points was used in analysis.

4.1 Modal properties

Modal-based analysis is considered in this section. Fast Fourier Transform and Half Power Method are firstly adopted to determine natural frequency and damping ratio, respectively. However, in order to ensure the analysis results, the more accuracy method, called Eigensystem Realization Algorithm (ERA) (Juang and Pappa 1985), is also utilized with the Hankel matrix size of 150×150 and 400×400 and the minimum system order of 100 and 400 for the measured data from free

vibration test and impact hammer test, respectively. Noted that analysis results presented here are the average values of the three accelerometers. And, for the impact hammer test in this paper, data from hitting location pt.10 and pt.17 are chosen to investigate due to the clearness of peaks. With these methods, the first three modes of natural frequency and damping ratio were extracted as illustrated in **Fig. 7** with the values shown in **Table 2** (only for ERA).

Effects of the concrete part removal can be seen by comparison the event E0 and E1 in **Table 2**. It is found that the second mode of natural frequency is the most influence with the decrease of 1.90%.

Moreover, during the PC tendon cut, results demonstrate that the first and the third mode of natural frequency gradually decrease where their percentage changes (comparing with event E1, after removed concrete part) are not greater than -2.5%. And interestingly, the first mode of natural frequency at the event E9 (partially cut of B4) from impact hammer test recovers and nearly equal to those of the initial event. However, when the PC girder ruptures, the natural frequency of the first and the third mode dramatically reduce with -9.27% and -6.67%, respectively. On the other hand, the

Fig. 6 Example of measured time series data. The data measured from CH02 at event E1. For the impact hammer test, the hitting location is pt.10.

Table 2 ERA results from vibration test.

Event	Real values						% difference from E1					
	Natural frequency [Hz.]			Damping Ratio			Natural frequency			Damping Ratio		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
E0	-	-	-	-	-	-	-	-	-	-	-	-
	(13.01)	(48.72)	(100.14)				(+1.26)	(+1.90)	(+0.17)			
E1	12.68	47.64	99.85	0.0074	0.0228	0.0133	0.00	0.00	0.00	0.00	0.00	0.00
	(12.84)	(47.81)	(99.97)				(0.00)	(0.00)	(0.00)			
E2	12.62	47.70	99.04	0.0081	0.0329	0.0156	-0.49	0.13	-0.81	9.56	44.02	17.23
	(12.82)	(46.85)	(99.37)				(-0.22)	(-2.02)	(-0.60)			
E3	12.60	47.26	98.67	0.0087	0.0274	0.0147	-0.65	-0.80	-1.18	18.53	19.90	9.85
	(12.75)	(46.75)	(99.19)				(-0.71)	(-2.21)	(-0.78)			
E4	12.58	47.50	98.63	0.0069	0.0314	0.0144	-0.78	-0.29	-1.22	-5.70	37.50	7.76
	(12.72)	(48.08)	(99.01)				(-0.98)	(+0.57)	(-0.96)			
E5	12.57	48.25	98.80	0.0071	0.0182	0.0147	-0.88	1.28	-1.06	-3.75	-20.35	10.24
	(12.78)	(48.45)	(98.96)				(-0.48)	(+1.33)	(-1.01)			
E6	12.53	48.06	98.42	0.0070	0.0152	0.0123	-1.20	0.88	-1.43	-5.33	-33.46	-7.52
	(12.65)	(48.08)	(98.43)				(-1.53)	(+0.57)	(-1.54)			
E7	12.49	47.95	98.01	0.0076	0.0127	0.0149	-1.51	0.65	-1.84	2.98	-44.52	11.31
	(12.69)	(47.99)	(97.49)				(-1.20)	(+0.37)	(-2.48)			
E8	12.50	47.74	97.70	0.0083	0.0137	0.0145	-1.46	0.21	-2.15	12.52	-40.10	8.59
	(12.61)	(47.98)	(98.18)				(-1.84)	(+0.36)	(-1.78)			
E9	-	-	-	-	-	-	-	-	-	-	-	-
	(12.98)	(48.20)	(98.24)				(+1.04)	(+0.81)	(-1.73)			
E10	11.55	47.48	93.18	0.0113	0.0171	0.0154	-8.95	-0.33	-6.68	53.69	-25.36	15.48
	(11.65)	(47.74)	(93.30)				(-9.27)	(-0.16)	(-6.67)			

*Results from free vibration test are shown outside parenthesis and those from impact hammer test are inside the parenthesis.

Fig. 7 Modal properties from vibration test.

second mode of natural frequency shows a different trend. It gradually decreases from the event E1-E4. Then, from the event E5 where the PC tendon at Section A and C are equally cut, the natural frequency recovers

nearly to the initial condition and does not much change until the PC girder ruptures. This implies that damages at the middle span do not effect on the second mode of vibration even the rupture of PC girder.

Table 3 Vibration test results by FFT and Half Power Method.

Event	Real values						% difference from E1					
	Natural frequency [Hz.]			Damping Ratio			Natural frequency			Damping Ratio		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
E0	-	-	-	-	-	-	-	-	-	-	-	-
E1	(12.94)	(48.49)	(100.30)				(+0.63)	(+1.47)	(+0.02)			
	12.69	47.49	99.85	0.0081	0.0150	0.0062	0.00	0.00	0.00	0.00	0.00	0.00
E2	(12.86)	(47.79)	(100.28)				(0.00)	(0.00)	(0.00)			
	12.63	47.30	98.92	0.0081	0.0145	0.0081	-0.48	0.39	-0.94	-0.08	-3.77	30.34
E3	(12.79)	(47.45)	(99.06)				(-0.55)	(-0.70)	(-1.22)			
	12.63	47.79	98.49	0.0079	0.0400	0.0067	-0.48	-0.64	-1.37	-2.43	166.24	8.73
E4	(12.63)	(46.84)	(98.86)				(-1.74)	(-1.98)	(-1.42)			
	12.57	48.31	98.71	0.0081	0.0196	0.0041	-0.96	-1.74	-1.14	-0.19	30.12	-33.47
E5	(12.70)	(46.23)	(98.80)				(-1.27)	(-3.26)	(-1.48)			
	12.57	48.13	98.63	0.0078	0.0170	0.0065	-0.96	1.35	-1.22	-3.50	12.81	5.70
E6	(12.71)	(48.31)	(98.53)				(-1.19)	(+1.09)	(-1.74)			
	12.57	47.82	98.12	0.0083	0.0135	0.0048	-0.96	0.71	-1.73	3.06	-10.17	-22.88
E7	(12.57)	(48.00)	(98.37)				(-2.22)	(+0.45)	(-1.91)			
	12.51	47.91	98.04	0.0080	0.0134	0.0074	-1.44	0.90	-1.81	-0.58	-10.79	19.02
E8	(12.60)	(47.94)	(98.31)				(-1.98)	(+0.32)	(-1.97)			
	12.51	47.91	98.02	0.0085	0.0128	0.0030	-1.44	0.90	-1.83	5.01	-14.90	52.14
E9	(12.56)	(47.91)	(98.02)				(-2.29)	(+0.26)	(-2.25)			
	-	-	-	-	-	-	-	-	-	-	-	-
E10	(12.99)	(48.19)	(98.02)				(+1.03)	(+0.83)	(-2.25)			
	11.60	47.64	92.83	0.0092	0.0122	0.0065	-8.65	-0.32	-7.03	13.63	-18.95	5.58
	(11.67)	(47.52)	(93.04)				(-9.26)	(-0.57)	(-7.22)			

*Results from free vibration test are shown outside parenthesis and those from impact hammer test are inside the parenthesis.

Fig. 8 Example of (a) delay time and (b) embedding dimension. The data measured from CH02 at event E1 (free vibration test).

Damping ratio of the first mode shows decreasing trend until the event E5 (after cut tendon C4). After that it shows increasing trend where the location at the middle span starts damaging. Conversely, the second mode of damping ratio shows a large oscillation before the event E5 and little change in later. There is no trend in the third mode of damping ratio.

4.2 Change of Phase Space Topology (CPST)

In order to remove the effect of different excitation forces, the measured time series data are normalized as

below

$$\hat{x} = \frac{x - \bar{x}}{\sigma}, \quad (8)$$

where $\bar{x}$ and σ are the mean and standard deviation of the measured time series data, respectively.

(1) Results from free vibration test

First, phase space topology was reconstructed by parameters shown in Fig. 8. The first minimum of Average

Mutual Information and the lowest value of False Nearest Neighbors were selected as the appropriate delay time $T = 20$ and embedding dimension $d = 20$, respectively. Phase space topology at each event is plotted in Fig. 9 as example. Noted that pairs of two dimensions were chosen only for demonstration in this figure but in the calculation of CPST all dimensions were used. It can be noticed that phase space topology of event E10 (Rupture) is obviously different from others.

CPST value in this case was calculated from 10 nearest neighbor points with 5 s -time step. The event E1 (after removal of concrete part) was selected as the reference state. The normalized CPST value from the three accelerometers are shown in Fig. 10. It is observed that the normalized CPST value gradually increase during each cutting event and suddenly increase when the PC

girder ruptures. Moreover, it is also noticed that the CPST at the middle span shows higher value than the others.

(2) Results from impact hammer test

For the measured time series data from impact hammer test, the delay time was firstly determined as $T = 3$ for both data from soft cap impact hammer and hard cap impact hammer. However, with this value, the percentages of False Nearest Neighbors are quite high. For example, at 20 dimensions, the percentages of FNN are 13% and 14% for the soft cap hammer and the hard cap hammer, respectively. This is because the number of data for reconstruction is too small. Therefore, the measured time series data was then resampled to 6144 data points with the same shape as its original data. The

Fig. 9 Example of phase space topology.

Fig. 10 Normalized CPST from free vibration test where E1 is reference event.

delay time $T = 18$ was used and the percentages of FNN as in **Fig. 11a** of the soft cap and hard cap hammer decrease to 3% and 2%, respectively.

Because there is some error in the measured time series data at event E1 of the hard cap impact hammer, this data was excluded from the analysis and for the consistency with the results of soft cap impact hammer the event E2 was selected as the reference state instead. CPST value for this case was calculated from 10 nearest neighbor points with 5 s-time step. The normalized CPST of the original data and the resample data are compared in **Fig. 11b**. Those of resample data give a bit smaller value, however, all cases show the same trend.

Next, to compare with the free vibration test, normalized CPST of all hitting location were calculated and their average values are shown in **Fig. 12**. Both normalized CPST from free vibration test and impact hammer test show the same trend. They oscillate during the PC tendon cut showing peak at the event E4 and E7. However, the values have increasing trend as the damage occurs which imply that the CPST values calculated from the both vibration tests can detect PC tendon damages. To obtain a clear result, it is suggested to use a large excitation force.

4.3 Comparison with modal properties

In this part, the normalized CPST is compared with the change of modal properties (first mode of natural frequency and damping ratio obtained from ERA). The modal properties were normalized with the reference event E2 as Eq. (9). The stability, reliability, and sensitivity of the normalized CPST comparing with indices from modal properties are discussed.

$$\Delta f^i = \frac{|f^i - f^{E2}|}{f^{E2}}, \Delta \xi^i = \frac{|\xi^i - \xi^{E2}|}{\xi^{E2}}, \quad (9)$$

In view of stability, unlike the natural frequency, the value of normalized CPST depends on measurement location as shown in **Fig. 10**. At different channels, the values of normalized CPST are different. Moreover, it also depends on the types of vibration tests (free vibration and impact hammer tests) and excitation force (hard cap and soft cap) as shown in **Fig. 12**. To have a stability, the normalized CPST should be calculated and compared within the same configuration. As a result, the normalized CPST of any configuration shows the same trend.

As the PC girder was gradually damaged by increasing the number of PC tendon cuts, the values of reliable index should also increase with the damages. As in **Fig. 13a**, the normalized CPST and the normalized 1st frequency reflect such damages while the normalized 1st damping ratio does not. This implies that the reliability of the normalized CPST is the same as the normalized 1st frequency and more reliable than the normalized 1st damping ratio.

Moreover, as shown in **Fig. 13b** where all indices were plotted together, it is obvious that the normalized CPST is more sensitive to the damages of PC tendons than indices from modal properties. This implies that CPST might be an effective index to detect the damages of PC tendon.

5. Conclusion

This paper presented an alternative method to detect the

Fig. 11 Comparison between original data and resample data.

Fig. 12 Normalized CPST where E2 is the reference event.

Fig. 13 Comparison of all normalized indices.

damage of PC tendon based on a change of phase space topology. Tendon damages were simulated by artificial cut in experiment. Two types of vibration test; free vibration and impact hammer were performed on the PC girder after each cutting event. Measured time series data in vertical direction were used for analysis. Due to the damages of PC tendon, the change of modal proper-

ties and the change of CPST were investigated and compared.

Modal-based analysis results revealed that different modes of vibration showed different trends due to the damages. The first and the third mode of natural frequency showed similar trend, slightly decreased due to the damages of PC tendon and significantly decreased

due to the rupture of PC girder. Different trends were found in the second mode of natural frequency. It decreased if the damage location was not at the middle span. However, their percentage of change were insignificant. For the damping ratio, although their percentage of change were quite large, they oscillated and had no trend.

For the phase space-based results, the change of CPST of the both vibration tests showed increasing trend with the increase of PC tendon damage levels and were much more sensitive to PC tendon damages than the results from modal-based analysis which implied that the CPST might be an effective index to detect the damages of PC tendon.

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