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Multi-Directional Fixed Crack Model Extended to Masonry Structures

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Abstract

The mechanical nonlinearity of a masonry structure depends on debonding and slip of masonry joints and the fracture of blocks like bricks and concrete. The present study proposes a behavioral simulation to idealize masonry joints by allocating three orthogonal planes, which are governed by the existing multi-directional crack model, and to represent the damage of blocks by using another three crack planes. The shear stiffness of the joint changes due to the disintegration of the infilling mortar caused by the shear slip of the joint, as well as the confining pressure dependence of the shear strength. Shear response analysis of a masonry structure considering this disintegration was carried out, and the applicability of the analysis model was validated. The following three types of structures were selected for validation: 1) masonry structures in which both joints and masonry blocks are damaged, 2) structures in which masonry blocks are primarily damaged but deformation is not concentrated in the joints, and 3) structures in which mortar joints are exclusively damaged but the masonry blocks are exempt of damage. It was confirmed that the proposed analysis model can analyze the damage mode of masonry structures. It evaluates yield strength as well and deformability is estimated on the safe side of an engineering viewpoint.

1. Introduction

Masonry structures have a long history and are still effectively used today, but numerous instances of earthquake damage to such structures have been reported (Bruneau 1995; Hisada and Shibayama 2004; Javed *et al.* 2006; Mukai *et al.* 2016; Coburn and Spence 2002). Aiming for more sophisticated seismic capacity evaluation, methods for evaluating the seismic performance of existing masonry structures have been developed. One such method is to represent the mortar joint and the masonry block by separate structural elements, each of which is given a constitutive model (Pandey and Meguro 2004; Lourenco and Rots 1997; Hashimoto *et al.* 2014, 2017). In addition, a method to obtain the response of a masonry structure has been proposed by describing the space-averaged behaviors of masonry components including the mortar joint in a constitutive model (Fig. 1). The former can analyze the structural response and the behavior of the constituent materials in detail. The latter can analyze large-scale and complexly shaped masonry structures rationally and with a small number of degrees of freedom by dividing them into a small number of finite

elements.

Lourenco *et al.* (2007) have proposed a non-linear model of masonry structures in which mortar joint deformation is prominent. Maier *et al.* (1991) have proposed an isotropic damage model for each of the blocks and mortar joints that make up the masonry structure. Facconi *et al.* (2014) have formulated the local stress-strain relationship of mortar joint and designed a constitutive model considering anisotropy.

These models focus on the nonlinearity of mortar joint. The constituent blocks are represented by elastic bodies or nonlinear materials that allow for cracking in one direction. To the best of the authors' knowledge, there are no analysis models that can handle the state where cracks intersect in multiple directions within masonry blocks in having interaction with multi-directional masonry joints. When the strength of the mortar joint exceeds that of the masonry block, or in a masonry structure where the periphery is reinforced with RC columns, fracture damage to masonry blocks is often inevitable. In view of the above, the objective of this study is to present a structural model that may take into account both the complex fracture of the structural blocks and the local deformation of joints.

For the behavioral analysis of RC structures under reversed cyclic loading, a constitutive model to consider non-orthogonal cracking in up to 6-directions has been developed (Maekawa and Fukuura 2014). Then, the authors propose a method in which the possible crack axis is fixed in the direction of the joint planes to represent the response of the joints (3-directions), and a multi-directional crack model (another 3-directions) is further provided for the evaluation of damage that develops in the masonry blocks, as in the conventional model. The accuracy and applicability of the multi-directional crack

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model extended to masonry structures is validated experimentally.

2. Extended multi-directional crack model

2.1 Allocation of crack coordinates to masonry joints

An RC in-plane constitutive model to deal with interacting two-way cracking has been formulated based on the active crack method (Okamura and Maekawa 1991). Later, it was extended to three-dimensional stress fields (Fukuura and Maekawa 1998; Hauke and Maekawa 1999). Currently, a constitutive model that can consider non-orthogonal cracking in 6 directions is used (Maekawa and Fukuura 2014). An outline is shown in Fig. 2. For 3-directional quasi-orthogonal crack planes, a non-orthogonal coordinate system is applied. Further, a 3D space averaged constitutive law for a total of six directions of crack groups has been formulated by adding

a new crack coordinate system.

The authors opted to allocate one of the above two quasi-orthogonal crack coordinate systems to mortar joint planes orthogonal to each other. The remaining quasi-orthogonal coordinate system was allocated to the behavioral analysis of the constituent blocks (Fig. 2). The opening of the joint was defined if the stress normal to the mortar joint plane exceeds the tensile strength of the joint (βf_t in Fig. 2). A tensile strain-softening model is applied to subsequent deformations (σ'_1 in Fig. 2). It goes without saying that this crack coordinate system is spatially fixed parallel to the joints.

When the maximum principal stress in the masonry block exceeds the tensile strength, a second crack coordinate system is mobilized. This makes it possible to handle kinetics in which multiple cracks occur not only in mortar joint but also in masonry blocks under complex load histories. The crack criterion and the shear transfer characteristics of cracks can be set separately for the

Fig. 1 Behavioral modeling of masonry structures in terms of referential volume.

Fig. 2 Extension of non-orthogonal multi-directional crack modeling.

mortar joint and block.

The slip and the opening modes are considered for space-averaging in which a set of block joints within a finite element forms a continuous plane. On the other hand, in the space-averaging process where masonry joints may intersect discontinuously, the bricks or blocks interlock. In this case, only cracks in the opening mode are considered. This makes it possible to handle various ways such as British and Flemish bond brick works.

2.2 Disintegrated cementitious composites on masonry joints

For a finite region with multi-directional intersecting cracks, the active crack plane that dominates the nonlinearity is selected (Maekawa *et al.* 2003). The material constitutive model of compression, tension, and shear as shown in Fig. 3 is applied to this plane. The stresses carried by cracked concrete can be obtained as a result.

For the shear stress-shear strain relationship along the crack plane, a shear transfer model based on the contact density function was applied (Li *et al.* 1989). This model represents the aggregate interlock resulting from the restricted rotation of cemented aggregates (Fig. 3). The shape of the crack plane is determined by the contact density function with respect to the contact angle. In other words, the differences in crack plane's shape and shear transfer characteristics between concrete and mortar can be reflected in the analysis. This was also applied to the mechanics of masonry blocks.

The concrete-to-sand transition model (Yamanoi and Maekawa 2020), in which the mortar transitions to a sand particle assembly with shear slip, was used in this study. Originally formulated in a three-dimensional stress field,

the vertical displacement of the joint is negligibly small compared to the shear slip displacement. Therefore, the existing concrete-to-sand transition model was degenerated to a two-dimensional joint plane (joint) for use in this study.

$$\tau_{cr} = (1 - \alpha)\tau_c + \alpha\tau_s \quad (1)$$

where, τ_{cr} is total shear transfer stress on the crack surface, τ_c is shear transfer stress calculated based on the contact density model, τ_s is shear transfer stress based on the frictional law and α is mixture ratio of concrete particle assembly.

The authors decided to use the state function denoted by " α " in Eqs. (1) and (2) to represent the process by which the consolidation of mortar joint by cement disintegrates as shear slip progresses and transitions to a sand particles aggregation model. Shear stress component τ_s of the sand particle assembly is represented by the Mohr-Coulomb law based on the cohesive strength, which is equal to the tensile strength of concrete, and the internal friction angle as,

$$\alpha = 1.0 - \exp\left(-A(\varepsilon_{eq} - B)\right), \varepsilon_{eq} = \sqrt{\frac{1}{2}(\varepsilon_{xy}^2 + \varepsilon_{xz}^2)} \quad (2)$$

where, A and B are constants, ε_{eq} is equivalent shear strain on the active crack surface, and ε_{xy} and ε_{xz} are shear strain on the active crack surface (x-direction coincides with the one normal to the joint plane). Here, the coefficient B , representing the start point of the transition, is set to 1.0×10^{-4} so that no transition occurs in the elastic region. The coefficient A , which governs the slope of the ε_{eq} to α relation, was inversely estimated from the experimental results.

Fig. 3 Concrete-to-sand transition model in view of shear transfer.

3. Validation – Coupling of joints and blocks damage

3.1 Experimental validation with shear walls

Ganz and Thurlimann (1984) reported a loading experiment on masonry shear walls. In the W1 specimen as shown in **Fig. 4(a)**, there are openings and slips in the joints. Furthermore, cracks were introduced in the masonry blocks. As two types of nonlinearities are provided, this experiment was considered appropriate for the verification of the extended multi-directional crack model proposed in this study. The bricks are arranged in a staggered pattern and this stacking pattern does not provide effective interlock between masonry blocks against horizontal displacement of joints.

On the other hand, interlock between blocks that works against the displacement of joints in the vertical direction is exhibited. The method of entering interlocks between blocks into the calculation (Section 2) can also be confirmed by this verification.

The specimen consists of hollow clay bricks stacked in 10 layers. The bricks are bonded to each other with 10 mm of mortar. Each brick measures 300 mm × 190 mm × 150 mm. The finite elements placed in the wall were 300 mm × 200 mm × 150 mm, almost the same dimensions as the brick blocks. Then, one element may contain at most one joint. As shown in **Fig. 4(b)**, the in-plane structural system was discretized with 3D enhanced strain elements, which may describe the induced out-of-plane deformation and the continuous in-plane crack propagation across the element boundaries made by 8 nodes (Simo *et al.* 1993). An RC constitutive model was applied to the loading beams and specimens (Maekawa *et al.* 2003). Horizontal displacement was applied to the top plate under a vertical load of 415 kN. The material properties used in the analysis are listed in **Table 1**.

Experimental values were used for the strength of the masonry bricks (Ganz and Thurlimann 1982). It is known that the strength of the mortar joint in a structure generally differs from the strength of the standardized

Table 1 Material properties for analysis of Ganz's wall.

Material property	RC element (brick)	RC element (RC beam)
Young's modulus (MPa)	5460	22800
Compressive strength (MPa)	7.61	29.0
Tensile strength of concrete (MPa)	0.67	2.30
Tensile strength of mortar (MPa)	0.01	-
Frictional coefficient	0.40	-

specimen due to water absorption into the bricks during curing, early age drying, and the quality of construction. In the analysis, sensitivity analysis was performed by setting the tested strength to multiple levels while referring to the specimen strength by the standard testing procedure. Here, the aforementioned concrete-to-sand transition model was applied to the shear transfer characteristics of the cracks in the mortar joint. From sensitivity analysis and in reference to the commonly known internal friction angle of sands, the assumed friction coefficient was set to 0.4 for the case when the solidification caused by cement paste disappears.

Figure 5 shows a comparison of the experimental and the analysis results obtained by applying the proposed model. In this analysis, the exact position of the joint is not specified within the finite element, but space averaged continuous strain distribution within the element is addressed. The shear strain distribution and the load-displacement relations were reasonably well reproduced by the analysis. Both joint openings/slips and damage to the brick observed in the experiment. It was confirmed that this shear wall experiment is an appropriate target for validating the capabilities of this analysis that can consider both mortar joint and brick cracks.

3.2 Sensitivity analysis in terms of joint disintegration

Furthermore, we carried out validation through sensitivity analyses by comparing the results when the shear transfer characteristics of the mortar joint would not

(a) Test specimens (W1)

(b) FEM mesh

Fig. 4 Experimental setup and finite element discretization of Ganz's wall (Ganz and Thurlimann 1984).

change ($\alpha = 0.0$), and an extreme case of the mortar joint being disintegrated from the beginning ($\alpha = 1.0$). **Figure 6** shows the respective deformation diagrams when each of the models is applied. In the former case, bending damage is predominant in the analysis. This differs from the experimental results in which the shear damage mode is predominant. According to the blue line in **Fig. 7**, the analysis gives a larger load capacity than the experiment. On the other hand, in the latter case, the strain distribution is consistent with the experimental results as shown in **Fig. 6(b)**. However, the load bearing capacity is lower than the experimental value (orange line in **Fig. 7**). In both of these extreme cases, it is difficult to explain the overall structural response. This transition model that combines both characteristics is judged to be effective as a shear transfer model for mortar joint.

4. Validation – Uncoupled single damages of joints or blocks

The proposed model is unique to consider the damages that appear in both joints and masonry blocks and their inter-dependence. Then in Section 3, the proposed model was validated using a structural experiment in which both the joints and the masonry blocks were cracked. To determine the extent of applicability, it is necessary to examine the extreme case where the nonlinearity of the masonry blocks is so great that the joints do not move (Case A). Similarly, the case where the nonlinear behavior of the joints dominates the overall structural responses and the damage does not extend to the blocks is also to be verified (Case B). In nonlinear problems where the model cannot be verified and validated by theoretical solutions, it is essential to check the reliability of the

Fig. 5 Behavioral simulation of masonry wall for experimental validation (Ganz and Thurlimann 1984).

Fig. 6 Shear strain distribution and deformational modes in accordance with the mechanism of masonry joints (the deformation magnification is 10 in both figures).

model for both extreme cases as well and use it in actual practice. In this section, these two cases will be discussed.

4.1 Experiment: Case A

Bourzam *et al.* (2008) subjected a brick wall surrounded by RC columns and beams to cyclic loading. Assuming an ordinary window pier, the aspect ratio [h/l in Fig. (8a)] of the specimen is set to 1.5. The nominal dimensions of the bricks that make up the specimen are 210 mm $\times$ 100 mm $\times$ 60 mm. The bricks are bonded to each other with 10 mm of mortar. The compressive strength of the brick is 30 MPa, whereas the strength of the mortar

joint is 27 MPa. Since the strength of the bricks and the mortar joint are nearly the same, there is almost no opening of the joints and the cracks present a failure mode that penetrates the masonry blocks.

The specimens were subjected to cyclic horizontal loading with average vertical stress of 0.4 MPa, assuming a load acting on the first floor of a four-story building. The levelness of the top loading beam is maintained by the loading frame installed on the side of the specimen (Fig. 8). The upper and lower loading slabs were sufficiently reinforced and no damage was observed. The analysis was carried out on the assumption that the loading slabs are elastic bodies.

In the validation of modeling, the measured material properties were set (Table 2). However, as mentioned previously, the tensile strength of mortar joint is known to be much lower than its material strength obtained by the standard testing procedure. Then, the strength value obtained from the empirical equation of Nakahama *et al.* (2009) was used in the analysis.

4.2 Experiment: Case B

In masonry structures constructed in the early 20th century, the strength of the mortar joint was considerably lower than that of the brick. The experiments of Abrams and Shah (1992) were selected for validation as satisfying this condition. As shown in Fig. 9(a), the specimens were nearly unrestrained brick walls. Three types of specimens with different aspect ratios were prepared

Fig. 7 Comparison of load-displacement relations in view of the kinetics of joints.

(a-1) Test specimens

(a-2) Loading system

(b) FEM mesh

Fig. 8 Case A (Strong mortar joint): Experimental setup and FE mesh discretization (Bourzam *et al.* 2008).

Table 2 Material properties for Case A and Case B.

Material property value	Experiment Case A			Experiment Case B	
	RC element (brick)	RC element (RC pillar)	Elastic element	RC element (brick)	Elastic element
Young's modulus (MPa)	8240	18300	210000	4710	210000
Compressive strength (MPa)	30.0	20.0	-	24.0	-
Tensile strength of concrete (MPa)	2.20	1.70	-	1.91	-
Tensile strength of mortar (MPa)	0.27	-	-	0.06	-
Frictional coefficient	0.50	-	-	0.50	-

(W1, W2 and W3). The nominal dimensions of the bricks used were 198 mm × 89 mm × 56 mm and the mortar joints were 9.5 mm thick. **Table 2** lists the physical property input in the analysis. The strength of the bricks in the experiment was 24.0 MPa, and the compressive strength of the mortar was 6.3 MPa. The friction coefficient of mortar joint is reported to be 0.50. The tensile strength of the mortar joint in the structure was set to 1/100 of the standard specimen strength.

A constant vertical load was applied to each specimen by applying the tension force of the PC steel bar to the

top loading beam, and horizontal displacement was applied to the center of the top beam as shown in **Fig. 9(b)**.

4.3 Comparison of analysis and experiment in Case A

Figure 10 shows the comparison of the load-displacement relations of experiment and analysis. In the experiment, at the load of about 70 kN in a positive side, the stiffness is greatly reduced. Then, when the horizontal displacement reaches about 2.5 mm, the load comes up to the maximum followed by gradual softening. At the point of

Fig. 9 Case B (Weak mortar joint): Experimental setup and FE mesh discretization (Abrams and Shah 1992).

Fig. 10 Case A (Strong mortar joint): Computed load-displacement relations (Bourzam *et al.* 2008).

Table 3 Summary of experimental validation results of capacity.

Case	Ganz	Case A (Strong mortar joints)		Case B (Weak mortar joints)					
				W1		W2		W3	
		Positive	Negative	Positive	Negative	Positive	Negative	Positive	Negative
P_E	256	76	-86	409	-436	191	-200	89	-89
P_A	232	57	-62	348	-355	143	-150	64	-64
P_A/P_E	0.9	0.8	0.7	0.9	0.8	0.7	0.7	0.7	0.7
G_E	-	476		5,930		2,790		1,085	
G_A	-	459		4,400		1,790		599	
G_A/G_E	-	1.0		0.7		0.6		0.6	

P_E , P_A : Maximum load in experiment and analysis (kN)

G_E , G_A : Energy absorbed in the final loop in experiment and analysis (J)

maximum horizontal displacement, the specimen retains the strength about half of its capacity. Looking at the hysteresis characteristics, the unloading path is origin-directed up to the maximum load. After the maximum load was reached, the residual displacement was accumulated, and the maximum residual displacement at load removal was about 4 mm. Comparing the loading histories of the negative and positive sides, the load bearing capacity and residual strength of the negative side are a little larger, while its residual displacement is smaller.

The initial stiffness and maximum load can be reproduced by analysis. On the other hand, the residual strength and residual displacement were evaluated on the higher side. In the analysis, the difference between the positive and negative loading histories is small. In Fig. 11, the ultimate state of the specimen is compared with the analysis value of the corresponding shear strain distribution. In Fig. 10, red circle indicates the point where the deformation is captured. The diagonal cracks in the center of the specimen and the dominant damage at the base are well reproduced.

Owing to the relative strength of the mortar joint, many cracks were observed to penetrate the bricks in the experiment. The proposed model is capable of evaluating the load bearing capacity on the safe side, and the damage mode is confirmed correctly.

4.4 Comparison of analysis and experiment in Case B

Figures 12 and 13 show comparisons between the experiment and the analysis. Due to the low strength of the mortar joint, the diagonal cracks propagated along the joints while avoiding the masonry blocks. In the analysis too, the diagonal cracks did not propagate linearly, and the main strain was distributed stepwise along the joints. Diagonal cracks were found in experiments on W1 and W2 with a large aspect ratio (Fig. 13), but in W3 with a small aspect ratio, cracks were concentrated along the joints at the base.

Although the load bearing capacity and residual strength were a little smaller than the experimental fact, the cyclic hysteresis indicated by the S-shaped load displacement diagram is reproduced. This nonlinear but quasi-elastic mode of displacement attributes to the kinetics of rigid body rotation of the entire masonry wall.

Table 3 gives a summary of the recorded experimental and analytical load bearing capacity values. In each case, the proposed model estimates the load bearing capacity on the safe side. Focusing on the historical characteristics, the energy absorption capacity in the analysis is also calculated somewhat on the small side. Therefore, from the viewpoint of practical design, the analysis for both monotonic and reversed cyclic loading gives an evaluation on the safe side.

Fig. 11 Case A (Strong mortar joint): Computed failure mode and shear strain distribution (Bourzam *et al.* 2008).

Fig. 12 Case B (Weak mortar joint): Computed load-displacement relations (Abrams and Shah 1992).

Fig. 13 Case B (Weak mortar joint): Computed failure mode and shear strain distribution (Abrams and Shah 1992).

5. Conclusions

The constitutive law based on the existing non-orthogonal multi-directional crack model can take into account the nonlinearity presented by a group of cracks intersecting in up to six directions in a finite region. In this study, a nonlinear response analysis is presented in which a fixed crack plane with three directions is placed in the nonlinear response of joints of a masonry structure, and the remaining three directions are assigned to represent the damage to unreinforced masonry blocks. Furthermore, a transition model that takes into account the degradation of mortar solids due to shear deformation was applied to a low-strength mortar joint. Three cases were selected to validate the analysis model: a masonry wall structure with cracks in both the joints and the blocks, a structure where the cracks exclusively penetrate the blocks, and a structure where the crack damage is concentrated in the joints. The findings of this study are summarized as below.

- (1) The proposed model is capable of reproducing the damage mode of a masonry structure subjected to shear, and it evaluates the load bearing capacity.
- (2) Structural analysis also confirmed that the shear transfer characteristics of low-strength mortar joint combine the shear transfer characteristics of concrete and those of sand.
- (3) To improve the accuracy of the analysis, it is necessary to take into account the characteristic of loss of consolidation of sand particles in mortar joint due to shear slip.

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