Analysis of FRP-Strengthened RC Members with Varied Sheet Bond Stress-Slip Models

HuneBum Ko¹ and Yuichi Sato²

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Abstract

Finite element analysis was conducted for ten RC specimens, which were shear-strengthened with fiber-reinforced polymer (FRP) sheets. Four of the ten specimens were wrapped with unbonded sheets to observe influence of the bond to the strengthening effectiveness. The tests and analyses demonstrated that the unbonded sheets are not effective to increase the shear strength of a member without steel stirrups while the flexural ductility of a member can be improved by confinement with unbonded sheets. Parametric calculations were attempted with varied interfacial fracture energy G_f and maximum bond stress τ_y between the sheet and concrete. The calculations indicated that the G_f and the τ_y have limited influence on the shear strength while the local stresses and crack widths can be controlled to a certain extent.

1. Introduction

Fiber-reinforced polymer (FRP) sheets transversely jacketed around RC members enhance shear strength and flexural ductility (JCI 1998 and JCI 2003). In parallel to researches on structural performance of members, local bond behavior between the sheet and concrete has been investigated with great efforts since the bond is considered as an important factor for the strengthening effectiveness. The JCI Technical Committee on Retrofitting Technology for Concrete Structures issued a state of the art report in 2003 (JCI 2003). About half of this report discusses the bond behavior between the strengthening material and the concrete.

Flexural strengthening is another major usage of FRP sheets, where the sheets are longitudinally applied along members. Flexural strengthening requires tension development of the sheet, and importance of the bond is widely recognized. On the other hand, no unified agreement has been made on contribution of the bond to shear strengthening. For this reason, a working group of the Technical Committee conducted analyses of benchmark specimens and discussed on the bond characteristics and strengthening effectiveness.

The benchmark specimens consisted of five beams and ten columns strengthened with carbon or aramid fiber sheets (Asakura 1994, Ishizaki 1997 and Fukuyama 1998). Six of the specimens were wrapped with-

out providing bond to the concrete. The analyses were conducted by three different kinds of method: (1) a method based on Modified Compression Field Theory (Vecchio 1986), (2) a model, which consists of two rigid bodies separated by a diagonal shear crack, and (3) finite element method. Based on this analytical research a parametric study has been conducted, varying fracture energy G_f of the bonding interface. The report finally pointed out the following remarks:

- (a) Increase of shear crack width causes significant increase of FRP sheet strain when the sheet is bonded.
- (b) Relatively small sheet strain was observed when the sheet is wrapped without bonding.
- (c) Shear strength increment depends on shear-reinforcing steel bar content, and sheet stiffness.
- (d) The bond characteristics have limited influence on the ductility of strengthened member.

This paper selects ten specimens from the benchmark specimens and presents FE analysis results to make detailed discussions on the influence of the sheet bond. The analysis adopts an improved model for post-peak response of concrete. The calculations were conducted with varied bond stress-slip relations. Relationships between the bond, shear strength, ductility, sheet strains, and crack widths are discussed based on the analysis.

2. Analyzed specimens

Table 1 and **Fig. 1** summarize the configurations of the selected ten specimens, five RC beams made by Ishizaki et al. (Ishizaki 1997) and five RC columns by Fukuyama et al. (Fukuyama 1998). Four of these specimens possessed bond between the sheet and concrete while the other four were wrapped with unbonded sheets. Two control specimens without sheet wrapping were also scheduled. Ishizaki reported that unbonded sheets re-

¹Assistant Professor, Department of Architecture, Inha Technical College, Incheon, Korea (Visiting researcher in Kyoto Univ., Kyoto, Japan).

E-mail: hbko@inhac.ac.kr

²Research Associate, Department of Urban and Environmental Engineering, Kyoto University, Kyoto, Japan.

E-mail: satou@archi.kyoto-u.ac.jp

sulted in an inferior shear capacity increment for a beam with no shear-reinforcing steel bars, while slight difference was observed in effectiveness of ductility enhancement between bonded and unbonded sheets in Fukuyama's study.

The FE program used for the analysis is VecTor2 developed by Vecchio (Vecchio 2000). **Fig. 1** illustrates mesh dividing of models for the specimens. The models adopt four-node plane-stress rectangular elements to represent concrete with smeared reinforcement (=stirrup or hoop). Longitudinal steel bars and FRP sheet are modeled by two-node truss elements, and bond of steel and FRP by four-node joint elements (contact elements). Degree of freedom of the model was 1,387 for Ishizaki's specimens and 2,292 for Fukuyama's. Total number of elements was 868 for Ishizaki's and 1498 for Fukuyama's (**Table 2**).

3. Bond stress-slip relationship

Figure 2 shows typical bond stress-slip relations between FRP sheet and concrete obtained from tests by the authors, based on the JCI's specification (JCI 1998). The analyses in this paper adopted a bilinear assumption, which is defined by maximum bond stress τ_y and two characteristic slips S_y and S_u . In a cyclic loading condition, linear unloading and reloading paths, which intersect the origin, were assumed. The area enveloped by the curve is defined as interfacial fracture energy G_f . Previous researches have indicated that the G_f varied from 0.3 N/mm up to 1.5 N/mm (JCI 1998 and JCI 2003). Sato proposed maximum bond stress τ_y as a function of G_f by Eq.(1) (Sato 2003a).

$$\tau_y = 6.6 \sqrt{G_f} \quad (1)$$

Table 1 Configurations of analyzed specimens.

Specimen	f'_c MPa	M /VD	Longitudinal reinforcement	Lateral reinforcement	ρ_F %	t_F mm	E_F GPa	f_{Fu}^{*1} MPa	Bond ^{*2}	V_{exp} kN
No.1	32.6	1.9	$\rho_l = 1.82\%$ $f_{ly} = 350$ MPa $d_{bl} = 38$ mm	$\rho_w = 0.0\%$ $f_{lw} = --$ $d_{bw} = --$	0.0	--	--	--	--	181
No.2					0.036	0.11	244	2900	B	285
No.3					0.054	0.17	80	2400	B	236
No.4					0.036	0.11	244	2900	U	184
No.5					0.054	0.17	80	2400	U	177
S-RC-0	33.9	1.5	$\rho_l = 0.56\%$ $f_{ly} = 792$ MPa $d_{bl} = 13$ mm	$\rho_w = 0.13\%$ $f_{lw} = 360$ MPa $d_{bw} = 6$ mm	0.0	--	--	--	--	236
S-RC-B	29.6				0.074	0.11	236	4490	B	288
S-RC-N	35.0				0.074	0.11	236	4490	U	270
S-M12-B	34.8				0.074	0.11	236	4490	B	338
S-M12-BP	35.3				0.074	0.11	236	4490	U	340

*1: f_{Fu} = rupture stress of sheet. *2: B = bonded, U = unbonded.

(a) Ishizaki (1997)

(b) Fukuyama (1998)

Fig.1 Geometry and mesh dividing of specimens.

Table 2 Analysis models.

Literature	Ishizaki	Fukuyama
Degree of freedom	1,387	2,292
Number of elements	Rectangular	420
	Truss (steel)	30
	Truss (FRP)	224
	Contact (steel)	28
	Contact (FRP)	192
Total	868	1,498
Total steps	40	568
Prescribed disp. increment	0.5mm	1.0 mm

Fig.2 Bond stress-slip relationship between FRP sheet and concrete.

Characteristic bond slips S_y and S_u are given by Eqs.(2) and (3).

$$S_y = 0.057 \sqrt{G_f} \quad (2)$$

$$S_u = 2 G_f / \tau_y \quad (3)$$

The units of τ_y , G_f , S_y and S_u are MPa and N/mm, mm and mm.

4. Crack spacing model

Sato and Vecchio made a model equation to estimate crack spacing of RC with FRP sheet wrapping based on calculations of equilibrium and compatibility conditions between reinforcement and concrete (Sato 2003a). Equations (4) to (6) give crack spacing s_r based on this model.

$$s_r = \frac{f'_t}{2 \sum_{i=1}^m \frac{\rho_{e,i} \tau_{b0,i} \cos \theta_{s,i}}{d_{b,i}} + \frac{1}{220} \sum_{j=1}^n \frac{\rho_{F,j} c_{3,j} \cos^2 \theta_{F,j}}{t_{F,j}}} \quad (s_r \leq 220 \text{ mm}) \quad (4)$$

$$s_r = \frac{f'_t - \sum_{j=1}^m \frac{\rho_{F,j} c_{3,j} \cos^2 \theta_{F,j}}{t_{F,j}}}{2 \sum_{i=1}^n \frac{\rho_{e,i} \tau_{b0,i} \cos \theta_{s,i}}{d_{b,i}}} \quad (s_r > 220 \text{ mm}) \quad (5)$$

where

$$c_3 = (15.8 + 1.34 \sqrt{t_F E_F}) \sqrt{G_f} \quad (\text{N/mm}) \quad (6)$$

f'_t = tensile strength of concrete (MPa);
 ρ_e = reinforcement ratio of steel bar;
 τ_{b0} = average bond stress along steel bar (MPa) = $2f'_t$;
 θ_s = angle between steel bar and direction of principal tensile concrete stress;
 d_b = steel bar diameter (mm);
 ρ_F = reinforcement ratio of FRP sheet;
 θ_F = angle between FRP sheet and direction of principal tensile concrete stress;
 t_F = thickness of FRP sheet (mm);
 E_F = elastic modulus of FRP sheet (MPa); and
 G_f = area enveloped by bond stress-slip curve (N/mm).

5. Strength and post-peak response of confined concrete

Confinement of the concrete by transverse stresses enhances the strength and ductility of the concrete, transforming the failure in compression from brittle to ductile. Although the finite element algorithm used in this paper was formulated for the plane stress state, the pro-

gram accounts for out-of-plane stresses due to confinement of lateral expansion by out-of-plane reinforcement. The triaxial stress state is then utilized in computing the strength enhancement effects due to confinement (Vecchio 1992).

Figure 3 presents compressive stress-strain relations of concrete obtained from cyclic tests of five column specimens (Sato 2003b and Sato 2003c). The relations represent stress-strain hysteretic response of plastic hinge region, which were heavily confined with steels and FRP sheets. The specimens were 400 mm by 400 mm in cross-section and 1,200 mm in clear span ($M/VD = 1.5$). The flexural reinforcement ratio ρ_l was 0.90 % for Specimen K1 to K4, and 1.08 % for H6C. Applied axial force ratio was $N/(bDf'_c) = 0.15$.

The concrete stresses and strains were derived from sixty-five displacement transducers and approximately one hundred strain gages applied on steels and FRP sheets. The confinement resulted in significant enhancement of ductility. In Specimen K2, a large tensile strain was observed under compression because of dilatation of concrete.

In the model of Park, Priestly and Gill (Park 1982), the residual compressive stress f_{pr} is given as $0.2 (1 + \rho_w f_{wy}) f'_c$. The residual stresses observed in the five specimens, however, were considerably larger than those given by $0.2 (1 + \rho_w f_{wy}) f'_c$. Hence, the authors attempt to modify the Park's formulation based on curve fitting with the experimental results:

$$f_{c2} = -\beta_1 |f'_c| \{1 + Z_m (\varepsilon_{c2} - \beta_1 \varepsilon_0)\} < f_{pr} \quad \text{for } \varepsilon_{c2} < \varepsilon_p < 0 \quad (7)$$

where

f_{c2} = principal compressive concrete stress;
 ε_{c2} = principal concrete strain;

$$Z_m = \frac{0.5}{\frac{3 + 0.29 |f'_c|}{145 |f'_c| - 1000} \cdot \left(\frac{\beta_1 \varepsilon_0}{-0.002} \right) + 0.75 \rho_w \sqrt{\frac{b_e}{s_w}}} \quad (8)$$

$$f_{pr} = - (4800 \rho_w)^{0.8} (f_{wy}/360)^{0.55} - 4.1 \rho_F E_F \varepsilon_{Fe} < -0.3 |f'_c| \quad (9)$$

b_e = effective width of member (mm);
 E_F = elastic modulus of FRP sheet (MPa);
 s_w = spacing of hoop or tie (mm);
 f_{wy} = yield stress of hoop or tie (MPa);
 β_1 = strength enhancement factor of confined concrete;
 ε_0 = strain at peak stress of unconfined concrete;
 ε_{Fe} = effective confining strain of FRP sheet;
 ρ_F = reinforcement ratio of FRP sheet; and
 ρ_w = reinforcement ratio of stirrup, hoop or tie.

Fig.3 Compressive stress-strain relationships of concrete in plastic hinge.

Equation (9) gives the residual compressive stress f_{pr} of concrete. Several previous researches, including Park's work, have defined the residual compressive stress as linear function of $\rho_w f_{wy}$ term. This assumption, however, overestimates ductility of columns confined with ultra high strength steel bars.

For this reason, the first term of Eq.(9) defines the contribution of steel confinement with through the variables of ρ_w and f_{wy} . The second term gives the contribution of the FRP sheet. The coefficient 4.1 is adopted in accordance with work of Richart et al. (Richart 1928). Previous tests by the authors showed that the FRP sheets wrapped around hinge regions of shear-flexural members rarely ruptured although those wrapped around columns under uniaxial compression always did (Sato 2003b). Hence, the confining action of FRP sheet for shear-flexural members is given by effective strain ε_{Fe} . Figure 4 shows FRP sheet strain at maximum shear force of shear critical beams and columns with respect to stiffness of the sheet $\rho_F E_F$ (Sato 1999). The data shown in Fig. 4 represents *in-plane* strains whereas the ε_{Fe} in Eq.(9) is *out-of-plane* strain. The *out-of-plane* strain, however, is usually lower than the *in-plane* strain when the member is subjected to an *in-plane* shear-flexural load. In other words, the strains in Fig. 4 can be viewed as an upper limit of the *out-of-plane* strains. For this reason, the effective sheet strain in *out-of-plane* direction is given by Equation (10):

$$\varepsilon_{Fe} = -0.001 - 0.007 \ln(\rho_F E_F / 1000) \quad (10)$$

A curve given by Eq.(10) is drawn in Fig. 4. Note that Eq.(10) is only available for in-plane shear-flexural condition, but not for uniaxial compression nor biaxial shear-flexural condition.

The strength enhancement factor β_1 for the direction of the largest compressive stress, f_{c3} , is determined by the equation below according to researches of Richart et al. and Kupfer et al. (Kupfer 1969).

$$\beta_1 = \left[1 + 0.92 \left(\frac{f_{cn}}{f'_c} \right) - 0.76 \left(\frac{f_{cn}}{f'_c} \right)^2 \right] + 4.1 \left(\frac{f_{cl}}{f'_c} \right) \quad (f_{c3} < f_{c2} < f_{c1} < 0) \quad (11)$$

where

$$f_{cn} = -(f_{c2} - f_{c1}) > 0 \quad (12)$$

Fig.4 Effective strain of FRP sheet (Sato 1999).

$$f_{c1} = -f_{c1} < 0 \quad (13)$$

Contribution of the FRP sheet to the enhancement factor β_1 is neglected because the confining force provided by the sheet at peak stress of the concrete is negligibly small. Equations (7) to (13) were incorporated with a hysteretic response formulation developed by Palermo (Palermo 2003).

6. Crack width limit

Vecchio pointed out that formation of a dominant shear crack of a lightly reinforced RC member is localized in a narrow band (Vecchio 2000). In finite element analysis, it is reasonable to discount the ability to sustain compressive stresses in elements located along the dominant crack. Vecchio proposed to reduce the principal compressive stress when the computed crack width exceeds 2 mm.

The authors modified this model in three aspects. Firstly, the crack width limit is usually needless in RC elements with heavy reinforcement contents. Typical instance is a plastic hinge with heavy confinement, which sustains under large compression regardless of a number of widely opened cracks contained. A threshold is therefore introduced to decide whether a finite element needs the crack width limit check. The analyses conducted in this paper checks the width only in elements, which satisfy a condition given by Eq.(14):

$$\Sigma(\rho_{si} \cos \theta_{si} / d_{bi}) < 0.00075 \quad (14)$$

where

- ρ_{si} = reinforcement ratio of i -th component of steel bar;
- θ_{si} = angle between orientation of i -th bar component and direction of principal tensile concrete stress; and
- d_{bi} = bar diameter of i -th component.

Secondly, it is found that wrapping a RC member with carbon fiber sheets provides considerably improved strength and ductility. On the other hand, those with low-modulus sheets (e.g. aramid or glass fibers) resulted in inferior strengthening effectiveness. Elastic modulus of carbon fiber ranges from 200 GPa up to 400 GPa while those of aramid fiber and glass fiber are lower than 100 GPa. It is therefore proposed to omit the crack width limit for specimens wrapped by FRP sheets with an elastic modulus of 100 GPa or higher.

Thirdly, the critical crack width was reduced by 1mm based on extended analytical studies of lightly reinforced RC elements.

7. Variation of bond characteristics

Five cases of calculation were performed for the speci-

Table 3 Bond stress-slip relation model.

Case	1	2	3	4	5
G_f N/mm	0.3	0.9	0.9	0.9	1.5
τ_y MPa	3.61	1.565	6.26	25.05	8.08
S_y mm	0.031	0.054	0.054	0.054	0.070
S_u mm	0.166	1.15	0.287	0.072	0.371

Table 4 Constitutive models and analysis conditions.

Compression Base Curve:	Parabola
Compression Post-Peak:	Eqs.(6)-(10)
Compression Softening:	Vecchio 1992
Tension Stiffening:	Sato 2003a
Crack spacing:	Eqs.(4)-(6)
Tension Softening:	Linear ($G_f = 0.075$ N/mm)
Confinement Strength:	Eqs.(11)-(13)
Concrete Dilatation:	Kupfer 1969
Cracking Criterion:	Mohr-Coulomb
Crack Width Check:	Eq.(14)
Steel Bond	Fujii 1981
Concrete Hysteresis:	Palermo 2003
Steel Hysteresis:	Seckin 1981
Rebar Dowel Action:	He 2001
Slip Distortion:	Walraven 1981

mens bonded with sheets to observe influence of the bond stress-slip curves (Table 3). Firstly, fracture energy G_f was varied by 0.3 N/mm, 0.9 N/mm, and 1.5 N/mm (Case 1, 3, and 5). In these cases, the characteristic bond stress τ_y and the slip S_y were determined by Eqs.(1) and (2). Secondly, the τ_y was varied by 0.25 times the value given by Eq.(1) (Case 2) and four times (Case 4), maintaining the G_f constant at 0.9 N/mm. Table 3 summarizes the variation of G_f , τ_y , and S_y . Table 4 presents other material models adopted in the analysis.

8. Analysis results

Table 5 shows analysis results of Ishizaki's specimens. The bond characteristics (G_f and τ_y) were varied for Specimens No.2 and No.3. The calculated shear strengths of Specimens No.2 and No.3 were slightly varied by the difference of G_f and τ_y . The calculated average sheet strains ranged from 1.36×10^{-3} up to 2.24×10^{-3} at maximum load for specimens with carbon fiber (No.2 and No.4) while 0.49×10^{-3} up to 0.94×10^{-3} for specimens with aramid fiber (No.3 and No.5). Figure 5 compares shear force-deflection relations between test and analysis. “(G_f)” indicates comparison between Cases 1, 3, and 5 while “(τ_y)” applies Cases 2, 3, and 4.

Figure 6 presents crack patterns, FRP strains, and bond stresses of Specimen No.2 at maximum shear load. Difference was observed in the analysis results between the five cases. The analysis of Fukuyama's specimens was conducted with total 568 calculation steps, which follows hysteretic rule adopted in the test. Thickness of Specimens S-M12-B and S-M12-BP were assumed to be 340 mm (300 mm for other specimens) in order to

Table 5 Analysis results (Ishizaki 1997).

Specimen	No.1	No.2					No.3					No.4	No.5
Case	--	1	2	3	4	5	1	2	3	4	5	--	--
G_f (N/mm)	--	0.3	0.9	0.9	0.9	1.5	0.3	0.9	0.9	0.9	1.5	--	--
τ_v (MPa)	--	3.61	1.57	6.26	25.1	8.08	3.61	1.57	6.26	25.1	8.08	--	--
Experimental shear strength V_{exp} (kN)	181	285					236					184	166
Analyzed shear strength V_{cal} (kN)	171	250	243	241	247	241	216	211	216	222	216	168	177
V_{cal} / V_{exp}	0.95	0.88	0.85	0.85	0.87	0.85	0.92	0.89	0.91	0.94	0.92	0.91	1.06
Analyzed sheet strain at V_{max} ($\times 10^{-3}$)	---	2.10	2.24	1.44	1.68	1.36	0.49	0.70	0.86	0.69	0.68	1.89	0.94

Fig.5 Shear force-deflection relationships of Ishizaki's specimens.

Fig.6 Analyzed crack patterns, sheet strains and bond stresses of Specimen No.2 at maximum shear force.

represent 20 mm-thick finishing mortars under FRP sheets. **Table 6** summarizes the analysis results. The shear force reached maximum at a drift angle around 1.5 % and then began to decrease because of compressive fracture of concrete (**Fig. 7**).

Five cases of calculations for Specimens S-RC-B and S-M12-B also resulted in slight variation in ductility as well as shear strength. The calculated average FRP strain (at drift angle of 1.5 %) ranged from 3.12×10^{-3} to 4.36×10^{-3} . Increasing G_f of Specimens S-RC-B and S-M12-B reduced the sheet strains. **Figure 8** presents crack patterns, FRP strains, and bond stresses of Specimen S-M12-B at drift angle of 1.5 %.

9. Sheet strains in beams with no stirrup

Figure 9 compares between analysis and test results of

the sheet strains of Ishizaki's specimens with respect to shear force. The presented strains are average of four strains measured along dominant shear crack. The sheet strains firstly remain under 0.0005 and begin to increase after shear cracking at an approximately 150 kN shear force. The sheet strain of Specimen No.2, where carbon fiber sheets were bonded, increased up to 0.005 while that of Specimen No.3 with aramid fiber remained under 0.001 until the shear force reached the maximum. Influence of the bond characteristics on the sheet strain is not significant in both Specimens No.2 and No.3 at pre-peak stage. On the other hand, comparison between specimens with bonded and unbonded sheets shows remarkable difference at post-cracked stage. The shear forces increased after shear cracking in specimens with bonded sheets (No.2 and No.3) while the shear force of those with unbonded sheets remained on lower levels.

Table 6 Analysis results (Fukuyama 1998).

Specimen	S-RC-0	S-RC-B					S-RC-N	S-M12-B					S-M12-BP
Case	--	1	2	3	4	5	--	1	2	3	4	5	--
G_f (N/mm)	--	0.3	0.9	0.9	0.9	1.5	--	0.3	0.9	0.9	0.9	1.5	--
τ_y (MPa)	--	3.61	1.57	6.26	25.1	8.08	--	3.61	1.57	6.26	25.1	8.08	--
Experimental shear strength V_{exp} (kN)	236	288					270	338					340
Analyzed shear strength V_{cal} (kN)	200	287	271	267	283	280	307	340	332	333	339	336	328
V_{cal} / V_{exp}	0.85	1.00	0.94	0.93	0.98	0.97	1.14	1.01	0.98	0.99	1.00	0.99	0.96
Analyzed sheet strain at $R = 1.5\%$ ($\times 10^{-3}$)	---	4.29	3.64	3.70	3.67	3.25	3.66	4.36	3.84	3.76	3.82	3.35	3.12

Fig.7 Shear force-drift angle relationships of Fukuyama's specimens.

Fig.8 Analyzed crack patterns, sheet strains and bond stresses of Specimen S-M12-B at drift angle of 1.5 %.

Fig.9 Shear force-sheet strain relationships of Ishizaki's specimens.

The above observation indicates importance of the bonding of FRP sheets. Nevertheless, the strengthening effectiveness is sufficiently achieved even with very low interfacial fracture energy G_f of 0.3 N/mm.

10. Sheet strains in ductile columns

Figure 10 compares analysis and test results of the sheet strains of Specimen S-M12-B with respect to drift angle.

The presented strain is the one measured at mid-depth at mid-span. In the test, the strains reached to 0.008 at a drift angle of 1.5 %, where maximum shear force of the column was observed. Then the strain continued to increase with a smaller gradient and reached around 0.01 at a drift angle of 5 %. The analysis results generally agreed with the test although discrepancy was found at drift angle of 5 %.

Fig.10 Sheet strain-drift angle relationships of Specimen S-M12-B.

11. Crack width

Ishizaki et al. measured crack width of the dominant shear crack in Specimens No.2, 3, 4, and 5. **Figure 11** compares between analysis and test results of crack widths at a location 400 mm distanced from the loading point. The crack width of Specimen No.2 remained under 0.2 mm while larger widths were observed after shear cracking in other specimens. As mentioned in the previous section, a lightly shear-reinforced member collapses when the dominant shear crack opens by a certain width.

Test result of Specimen No.2 indicated that the bonded carbon fiber sheets were effective to limit the crack width opening, whereas aramid fiber sheets or unbonded sheets were less effective. In the analyses, the

crack width of Specimen No.2 at maximum shear force was reduced when the fracture energy G_f of 0.3 N/mm (Case 1) was adopted. The bond stress with a large τ_y (Case 4) also reduced the width. However, the reduction in the crack width did not result in remarkable increase of the shear strength of Specimen No.2. These observations indicate that strengthening of a member without shear reinforcing steel requires usage of high-stiffness sheet and bonding of the sheet.

12. Influences of interfacial fracture energy and maximum bond stress

Figure 12 summarizes influences of the fracture energy G_f and the maximum bond stress τ_y on the calculated shear strength, average sheet stress ($\rho_F f_F$), and average crack width. The maximum bond stress τ_y in **Fig. 12** is normalized by dividing by $6.6\sqrt{G_f}$, where $G_f = 0.9$ N/mm. Four lines indicate results of Specimens No.2, No.3, S-RC-B and S-M12-B. As the fracture energy G_f increases, the sheet stress $\rho_F f_F$ decreases while the crack width increases, although Specimen No.3 presents exceptional tendencies. Note that the crack widths of Specimens No.2 and No.3 represent widths of the dominant shear cracks, while those of Specimens S-RC-B and S-M12-B average widths of all cracks in the column. On the other hand, influence of the maximum bond stress τ_y on the $\rho_F f_F$ and the crack width is insignificant. Furthermore, variations of the G_f and the τ_y resulted in slight changes of the shear strengths.

Fig.11 Shear force-crack width relationships of Ishizaki's specimens.

Fig.12 Influences of bond characteristics on calculated shear strength, sheet stress, and crack width.

These results indicate that improvement of the bond characteristics cannot increase the shear strength of retrofitted member although the sheet stress and the crack width can be controlled by the G_f and the τ_y .

13. Conclusions

A concrete member with no shear-reinforcing steel needs bonding of the FRP sheets to achieve sufficient strengthening effectiveness, although flexural ductility can be improved by confinement with unbonded sheets. The interfacial fracture energy G_f and the maximum bond stress τ_y influences to response of a strengthened RC member to a certain extent. The analysis results indicated that distributions of cracks, sheet strains, and bond stresses changed corresponding to the variation of G_f and τ_y .

The authors state that improvement of the bond capacity is probably worthwhile for strengthening of a low-reinforced member with high-stiffness FRP sheet (JCI 2003). However, the calculations with improved analytical models indicated that the G_f and the τ_y are not influential factor for the strengthening effectiveness. Even low interfacial fracture energy G_f of 0.3 N/mm between the sheet and concrete provides sufficient enhancement in shear strength as long as the sheets are wrapped around a RC member.

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