Modeling the Influence of Pore Structure on the Acoustic Absorption of Enhanced Porosity Concrete

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Abstract

This paper describes a model to predict the acoustic absorption of Enhanced Porosity Concrete (EPC). The acoustic absorption coefficient was determined experimentally using an impedance tube, while an electro-acoustic analogy was implemented to develop the predictive model, considering the pore structure of EPC as a series of resistors and inductors. The physical features of the pore network were experimentally characterized using image analysis and a pore volume characterization technique. A parameter termed “structure factor” was introduced to account for the increased density of air that is not displaced by the acoustic wave pressure. The maximum acoustic absorption coefficient was found to decrease linearly with an increase in the structure factor. The development of this model and its correlation with physical measurements enable the prediction of acoustic absorption in EPC based on the geometric features of the pore structure. This model enabled a parametric study to be conducted to ascertain the effects of pore size, aperture size, porosity, and specimen thickness on acoustic absorption. An optimal pore to aperture diameter ratio was observed to exist, that maximizes acoustic absorption. The parametric study is believed to be able to aid in the design of EPC for acoustic absorption by better understanding the type of pore features that should be targeted for best performance.

1. Introduction

Recent studies have indicated that the use of a porous concrete layer (Enhanced Porosity Concrete – EPC) over a conventional concrete surface may provide an efficient noise-reducing pavement system (BRITE-EURAM 1994, BRRC 2000). Enhanced porosity concrete (EPC) is proportioned by gap grading the coarse aggregates and either limiting or eliminating the sand volume in the matrix, thereby developing a network of interconnected pores in the material. This interconnected network is believed to reduce tire-generated noise primarily through two mechanisms (Bernhard 2002). First, the generation of noise is altered by reducing the air pumping that occurs when air is drawn into and pumped out of the gaps between tread blocks in tires (by permitting this air to escape through the pavement). Second, the reflection and scattering of acoustic waves in the pores result in absorption and dissipation of sound through internal friction (Neithalath et al. 2004). Research has suggested that in order for such a layer to be effective it should have a porosity of 20-25% (BRRC 2000). However, little guidance is provided as to the exact characteristics that the pore structure should possess in order to achieve satisfactory performance.

In acoustics literature, it is common to consider porous materials as one of two different classes of materials: rigid-framed or elastic. In rigid-framed porous materials, the pore walls are non-deforming, and as a result the acoustic absorption occurs primarily due to the viscous losses and thermo-elastic damping as the sound propagates through a large number of air cavities in the material (Wang and Lu 1999, Voronina 1999). The sound propagation is therefore governed by the effective density and effective bulk modulus of the air in the pore spaces. In elastic porous materials, the pore walls are flexible membranes, and the displacement of the membrane results in acoustic losses (Voronina 1999). Generally, a density of 150 kg/m³ is considered to be the dividing line between rigid-framed (high density) and elastic (low density) acoustic materials. EPC has a typical density of approximately 2000 kg/m³, and therefore could be considered as a “rigid-framed” porous material.

Within the realm of rigid-framed porous materials, the acoustic characteristics are typically treated using either phenomenological or microstructural models (Francois and Michel 1993). Both these approaches describe the material in terms of its specific acoustic impedance (ratio of pressure to particle velocity) and wave number (ratio of angular frequency to velocity). The concepts of acoustic mass, stiffness, and damping are employed to arrive at non-dimensional expressions for characteristic impedance and wave number (Brennan and To 2001). The phenomenological models consider the porous medium to be a compressible fluid in which the dissipation of acoustic energy occurs, and the equa-
tions for analysis (Delany-Bazley laws) are established at a global scale. The microstructural models consider that the sound propagation occurs in straight pores, while the tortuosity of the pore network is accounted for with shape factors (Biot 1962, Johnson et al. 1987) to describe the dissipation of acoustic energy. One commonly used approach to account for the influence of material structure on acoustic wave propagation was formulated by Kirchhoff (Zwikker and Kosten 1949). This theory accounts for the viscous and thermal effects of sound propagation in cylindrical tubes, however it does not account for non-circular cross sections (Allard 1993). Several shape factors have been used to extend the circular pore geometries to more complicated shapes (Allard 1993, Stinson and Champoux 1992). This study employs a microstructural model to predict the acoustic absorption behavior of EPC.

When an acoustic wave impacts a pervious surface, a portion of the energy is absorbed, and another portion is reflected. Absorption occurs along the direction of wave propagation and the transmitted wave is essentially damped (Boutin et al. 1998). Biot’s theory is one of the frequently used theories to model acoustic wave propagation in a porous dissipative media like EPC. Based on this theory, empirical power law relationships between the fundamental acoustical properties of porous materials (i.e., the characteristic acoustic impedance and propagation constant, and the ratio of static flow resistivity to frequency) have been developed (Zwikker and Kosten 1949, Boutin et al. 1998). The drawback of this theory, however, is that the low frequency predictions are generally somewhat unrealistic (Lu et al. 2000).

While the foregoing discussion provides insight into some of the common models that are used to describe the acoustic behavior of porous materials, this work investigates the application of one modeling approach to predict and quantify the acoustic absorption in EPC. A modeling approach which was developed for highly porous metal matrix composites (Lu et al. 2000) was used for EPC in the current study. This paper uses a microstructural approach (rather than a phenomenological approach) to relate the acoustic absorption of EPC to the pore structure. Numerical predictions are compared with experimental observations to illustrate the ability of this model to capture how changes in pore structure parameters are reflected in acoustic absorption.

2. Research significance

This paper describes a portion of a larger investigation aimed at developing, optimizing, and characterizing EPC as a material for absorption of tire-pavement interaction noise as well as for improved drainage. This paper is intended to illustrate a procedure for characterizing the optimal pore structure characteristics of EPC (pore volume, pore size, and aperture size), and relating these properties to acoustic absorption. Experiments were performed to determine the influence of aggregate gradation on acoustic absorption, and to characterize the pore structure of EPC (Marolf et al. 2004, Neithalath et al. 2005). These experimental results are used in conjunction with the electro-acoustic model described in this paper to indicate the appropriateness of employing this type of model for EPC. Favorable comparison of the experiment and the theory suggests that this approach can be used to simulate a wide range of mixtures, thereby enabling the optimization of EPC for specific noise-related performance levels.

3. Experimental program

3.1 Specimen preparation

EPC mixtures were made using single sized aggregates - # 8 (2.36 - 4.75 mm), # 4 (4.75 – 9.5 mm), and 3/8" (9.5 – 12.5 mm), as well as the binary blends of these aggregate sizes (obtained by replacing 25, 50, and 75% by weight of the larger aggregates with smaller sized aggregates). The mixtures were cast in 150 x 150 x 700 mm beam molds, and cylindrical specimens having a diameter of 95 mm were cored from these beams for acoustic absorption measurements. Further details on the constituent materials, mixture proportions, and specimen preparation procedure can be found elsewhere (Marolf et al. 2004).

3.2 Determination of normal incidence acoustic absorption coefficient

The acoustic absorption of EPC was evaluated in accordance with ASTM E 1050-98, using a Brüel & Kjær™ impedance tube as shown in Fig.1. Three specimen lengths – 150 mm, 75 mm, and 37.5 mm were tested. The sample was placed inside a thin cylindrical Teflon sleeve to reduce the risk of scratching the inside of the impedance tube. The assembly, consisting of the cylinder and the Teflon sleeve, was placed against a rigid backing at one end of the impedance tube. The other end of the tube was equipped with a sound source. Microphones placed along the length of the tube were used to detect the sound wave pressure transmitted to the sample and the portion of the wave that is reflected. The pressure reflection coefficient (R) is defined as the ratio of the pressure of the reflected wave to that of the incoming wave, at a particular frequency. The pressure

![Fig.1 Impedance tube set up to measure normal incidence absorption coefficient.](image-url)
refraction coefficient can be expressed as:

\[ R = \frac{e^{jd_1} - e^{jd_2}}{e^{jd_1} p - e^{jd_2}} \]  

where \( d_1 \) and \( d_2 \) are the distances from the specimen to the closest and farthest active microphones respectively (as shown in Fig.1), \( j \) is an imaginary number \((\sqrt{-1})\), \( k \) is the wave number (ratio of angular frequency to the wave speed in the medium), and \( P \) is the ratio of acoustic pressures at the two active microphone locations.

Once the pressure reflection coefficients \( (R) \) are determined at each frequency, the absorption coefficient for each frequency can be calculated as shown in Eq.2.

\[ \alpha = 1 - |R|^2 \]  

The frequency range of interest was limited from 100 Hz to 1600 Hz in this study. A threshold of 100 Hz was established because at very low frequencies, the acoustic pressures were difficult to stabilize. The reasons for limiting the high frequency range to 1600 Hz were two-fold. First, the diameter of impedance tube has to be small to sustain a standing wave in the tube for higher frequencies (Allard 1993). Due to the aggregate sizes in the material, it is difficult to prepare samples of such small sizes. Second, the range of frequencies in which the tire-pavement noise is the most objectionable to the human ear is 800-1200 Hz.

### 3.3 Porosity and pore size

Porosity and pore size are the two of the most important pore structure features that can be used to describe the acoustic performance of any material using the microstructural model. Two methods were used to determine the porosity of EPC specimens – an image analysis method and a volumetric method. In the image analysis method, the EPC was impregnated with a low viscosity epoxy, and sectioned at different depths. The surface of each section was scanned, and an image analysis procedure was employed to differentiate the accessible and inaccessible porosities (Marolf et al. 2004). In the volumetric method, the sample was weighed in a dry condition, sealed with a latex membrane, filled with water and re-weighed. The weight change was then converted into volume (Neithalath et al. 2005). Both methods gave comparable results, as shown in Table 1 (in the Mixture IDs in Table 1, the number represents the weight percentage of the aggregate size that follows; for instance, 75-#4 – 25-#8 denotes an EPC mixture with 75% #4 aggregates and 25% #8 aggregates by weight). In addition to the pore volume, pore sizes and their distribution are also important features for describing the structure of the material. Though the system consists of varying pore sizes, it was assumed for the purpose of simplicity that a characteristic pore size \( D_p \) (median of all pore sizes greater than 1 mm) could represent the system adequately. Image analysis method was also used to estimate the characteristic pore sizes of the EPC system (Marolf et al. 2004). The values of \( D_p \) are also given in Table 1.

### 4. Numerical simulation of the acoustic properties of EPC

The following sections illustrate the use of a shape-specific model to describe the acoustic absorption behavior of EPC. This model was compared to physical measurements to illustrate the adequacy of the model. The theory is based on the acoustic impedance of the apertures and pores due to viscous effects. The actual pore structure of EPC is complex and consists of non-uniform pore sizes, which result in a variable cross section. For the sake of analysis, this cross section has been idealized into one consisting of single sized pores, and smaller apertures connecting these pores. In effect, the non-uniform pore structure of EPC is replaced by an equivalent uniform material or effective medium having the same total pore volume, and same characteristic pore size. This effective medium is characterized by characteristic acoustic impedance, which is a function of the idealized microstructural parameters. A correction factor for the pores that do not convey air directly (lateral cavities) has been included. The model was used to perform a parametric study to ascertain the influence of pore structure parameters on acoustic absorption. A

<table>
<thead>
<tr>
<th>Mixture ID</th>
<th>%3/8&quot; agg.</th>
<th>%#4 agg.</th>
<th>%#8 agg.</th>
<th>Porosity (image analysis)</th>
<th>Porosity (Volume method)</th>
<th></th>
<th>Optimal ( D_p/D_a )</th>
<th></th>
<th>Aperture dia. (( D_a ))</th>
<th></th>
<th>Aperture length (( L_a ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 - #8</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>0.214</td>
<td>0.207</td>
<td>2.17</td>
<td>1.95</td>
<td>4</td>
<td>0.54</td>
<td>2.09</td>
<td></td>
</tr>
<tr>
<td>75-#8-25-#4</td>
<td>0</td>
<td>25</td>
<td>75</td>
<td>0.305</td>
<td>0.208</td>
<td>2.69</td>
<td>2.42</td>
<td>4</td>
<td>0.67</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>50-#8-50-#4</td>
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<td>50</td>
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<td>0.247</td>
<td>2.48</td>
<td>2.23</td>
<td>4</td>
<td>0.62</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>25-#8-75-#4</td>
<td>0</td>
<td>75</td>
<td>25</td>
<td>0.276</td>
<td>0.225</td>
<td>3.14</td>
<td>2.83</td>
<td>4</td>
<td>0.79</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>100 - #4</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0.203</td>
<td>0.206</td>
<td>3.29</td>
<td>2.96</td>
<td>4</td>
<td>0.82</td>
<td>1.85</td>
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</tr>
<tr>
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<td>50</td>
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<td>0.190</td>
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<td>7</td>
<td>0.43</td>
<td>1.74</td>
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<tr>
<td>100-3/8</td>
<td>100</td>
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<td>0</td>
<td>0.237</td>
<td>0.193</td>
<td>4.77</td>
<td>4.29</td>
<td>8</td>
<td>0.60</td>
<td>1.78</td>
<td></td>
</tr>
<tr>
<td>50-#4-50-3/8</td>
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<td>0.303</td>
<td>0.264</td>
<td>3.29</td>
<td>2.96</td>
<td>5</td>
<td>0.66</td>
<td>0.83</td>
<td></td>
</tr>
</tbody>
</table>
clear relationship was observed between pore size and acoustic absorption. It is believed that these observations can be used to tailor the pore structure of EPC for specific applications.

4.1 Physical geometry of the simulated pore structure

A typical cross section of EPC is shown in Fig. 2(a), whereas an idealized system with only single sized openings is shown in Fig. 2(b). The acoustic waves are alternatively compressed and expanded through the idealized pore-aperture network, forcing them to lose a part of their energy. This follows the approach used by Lu et al. (2000) for cellular metals; however this model was modified for EPC to account for the substantial reduction in pore volume as a result of the thickness of the walls separating the pores. In addition, a structure factor was used to account for the lateral cavities (other than pores and apertures), as explained later.

The pore network is assumed to be a series of alternating cylinders of varying diameter. Each unit of the pore network consists of a pore and aperture as shown in Fig. 2(c). It can be argued that each of the units must exhibit the same fractional porosity as the total system. The porosity ($\phi$) can therefore be related to the length and diameter of the pore ($L_p$ and $D_p$ respectively), the length and diameter of the aperture ($L_a$ and $D_a$ respectively), and the wall thickness ($t$) as shown in Eq. 3.

$$\phi = \frac{(D_a^2L_a + D_p^2L_p)}{(L_a + L_p)(D_p + t)}$$  \hspace{1cm} (3)

Further, it is assumed that the apertures are arranged in a hexagonal array (to facilitate closest packing), and appropriate circular dimensions approximate these hexagons (Lu et al. 2000). In Fig. 2(d), the arrangement of the apertures in the Z-direction is shown. If the apertures are spaced at a distance $D$, then the hexagons enclosing them have a side of $D/2$ each. For a hexagonal prism with a side that has a length of $D/2$, and length $L_p$, the volume of the hexagonal prism ($V_{hex}=3\sqrt{3}D^2L_p/8$) is set equal to that of a sphere with a diameter of $D$ ($V_{sphere} = \pi D^3/6$). As a result, it can be seen that $L_p = 0.806D$. Similarly, the diameter of pore ($D_p$) can be related to the side of the hexagon ($D$) as $D_p = 0.909D$. From these, it could easily be deduced that the length of the pore $L_p \approx 0.90 \times D_p$.

4.2 Structure factor and its influence on acoustic absorption

EPC has been idealized in this work as a series of alternating cylinders as shown in Fig. 2(b). Though it is assumed that the sound waves pass through these sequential cylinders, it should be noted that there are potential lateral losses arising from the fact that the pore system, in reality, consists of lateral pores also in addition to the main pores which are considered to be a part of the ide-
alized pore network. An additional term, the structure factor \( k_s \), is defined as the ratio of total porosity (\( \phi \)) to the volume of the main pores (\( \phi_{\text{main pores}} \)), to account for this effect. Therefore, the pore volume that is effective in acoustic absorption is a ratio of the measured pore volume to the non-dimensional structure factor.

The physical significance of \( k_s \) is illustrated in Fig.3(a). When a pressure gradient (a standing wave in an impedance tube, for instance) is applied, the air in the main pores vibrate, but in lateral cavities, the air is largely at rest. As a result, the air in lateral cavities acts as if it were “heavier” than the air in the main pores. The structure factor \( k_s \) describes the fact that all the pores do not contribute equally to the loss mechanisms. It should be noted that structure factor \( k_s \) does not depend on the fluid inside the pores, but only on the geometry of the frame. The value of \( k_s \) is usually found to lie between 3 and 7, though it can technically have any value above 1 (Zwikker and Kosten 1949).

4.2.1 Determination of structure factor \( k_s \)
The pore network shown in Fig.2(b) was used to compute the structure factor (Allard 1993). Since determination of the volume of lateral cavities is an experimentally challenging process, the structure factor is typically calculated from the fluid velocities when accelerated by the pressure gradient, and pore geometry, as described below.

Using the linearized Euler equation to solve for the 1-D wave equation, the acoustic particle velocity in the X-direction, \( u_x \), is given by:

\[
\frac{\partial p}{\partial x} = \frac{\partial p_{\text{pore}}}{\partial x} L_p + \frac{\partial p_{\text{aperture}}}{\partial x} L_a \quad (4a)
\]

where \( k_s \) is the structure factor, \( \omega \) is the angular frequency, \( \rho_0 \) is the density of air and \( p \) is the acoustic pressure.

The acoustic particle velocity has two components, one corresponding to the velocity in the pore \( (u_{x,\text{pore}}) \), and another corresponding to the velocity in the aperture \( (u_{x,\text{aperture}}) \), which are related by the expression shown in Eq. 4b (Allard 1993):

\[
\frac{u_{x,\text{pore}}}{u_{x,\text{aperture}}} = \frac{D_p^2}{D_a^2} \quad (4b)
\]

The volume averaged velocity in the X-direction is given by:

\[
\langle u_x \rangle = \frac{L_p A_p}{L_p A_a + L_a A_p} \langle u_{x,\text{pore}} \rangle + \frac{L_a A_p}{L_a A_a + L_p A_p} \langle u_{x,\text{aperture}} \rangle \quad (4c)
\]

where \( A_p \) and \( A_a \) are the areas of a pore and aperture respectively.

The pressure in a pore-aperture combination (along the X-direction) is given as:

\[
\hat{p} = \hat{p}_{\text{pore}} L_p + L_a \quad \hat{p}_{\text{aperture}} \quad \hat{p}_{\text{aperture}} L_a \quad (4d)
\]

From these expressions, the structure factor can be obtained as:

\[
k_s = \frac{(L_p D_p^2 + L_a D_a^2)(L_p D_p^2 + L_a D_a^2)}{(L_p + L_a)^2 D_p^2 D_a^2} \quad (4e)
\]

where \( L_p \) and \( L_a \) are the lengths of the aperture and the pore and \( D_p \) and \( D_a \) are the diameters of the aperture and pore respectively.

The value of the diameter of the pore \( (D_p) \) can be obtained from image analysis as mentioned in Section 3.3. The length of pore \( (L_p) \) was shown to be equal to 0.90 \( D_p \) in Section 4.1. To solve for the unknowns in Eq. 3 \( (D_a \) and \( L_a) \), a value of pore diameter to aperture diameter ratio \( (D_p/D_a) \) is initially assumed so that \( D_a \) becomes known. As a first approximation, the value of \( D_p/D_a \) is assumed to be 4. From geometry, the wall thickness \( (t) \) then becomes equal to \( D_p/2 \). Eq. 3 then becomes:

\[
\phi = \frac{4(D_p^2 L_a + D_a^2 L_p)}{9(L_p + L_a) D_p^2} \quad (5)
\]

For cases where \( D_p/D_a \) is modified not to be equal to 4, the calculation is modified taking into consideration the new value of \( D_p/D_a \). The value of \( D_p/D_a \) is successively changed and corresponding aperture length \( (L_a) \) determined. A plot of \( D_p/D_a \) versus \( L_a \) is shown in Fig.3(b). The optimal \( D_p/D_a \) is chosen as the value at which \( L_a \) tends to asymptotically approach a constant value. The values of \( D_p, D_a, L_p \) and \( L_a \) for the mixtures investigated in this study, as calculated from the above procedure are shown in Table 1.

4.2.2 Acoustic absorption and structure factor
The maximum acoustic absorption \( (\alpha) \) for the EPC mixtures measured using the impedance tube is plotted against the calculated structure factor \( (k_s) \) in Fig.4. An increase in the structure factor results in a linear decrease in the maximum absorption coefficient. This occurs primarily because the displacement of air through the porous medium is only possible through the main pores of the material. When the volume of the main
pores decreases, the structure factor increases, resulting in a reduction in effective porosity (porosity effective in acoustic absorption), thereby reducing the maximum acoustic absorption.

4.3 Electro-acoustic analogy

The acoustic performance of the EPC system can be modeled using an electro-acoustic analogy (Zwikker and Kosten 1949, Lu et al. 2000). The electro-acoustic model consists of a series of resistors and inductors, as shown in Fig.5. In this model, the resistors represent the real component of the impedance of the apertures. The inductors in the model represent the imaginary component of the impedance of the apertures. The impedance of the air inside the pores (except the last pore against the rigid backing) is also modeled as a resistor, since the internal resistance of air in the pore is the product of the density of air and the speed of sound in air. The thermal effects are neglected in this model due to their minimal impact on the acoustic wave number of rigid framed porous materials (Brennan and To 2001).

The impedance of the air inside the pore adjacent to the rigid backing \( Z_D \) is a function of the pore diameter also, and can be calculated as follows:

\[
Z_D = -j \rho_0 c_0 \cot \left( \frac{\alpha D_p}{c_0} \right) \tag{6a}
\]

which can be simplified (since \( \frac{\alpha D_p}{c_0} \ll 1 \)) as:

\[
Z_D = -j \rho_0 c_0^2 / \alpha D_p \tag{6b}
\]

where \( \rho_0 \) is the density of air (1.2 kg/m\(^3\)), and \( c_0 \) is the speed of sound in air (343 m/s).

The impedance of the apertures \( Z_0 \) is a sum of the real and imaginary components \( Z_0 = R_0 + i M_0 \), where \( R_0 \) is the real component and \( M_0 \) is the imaginary component of impedance of the apertures.

One term that has to be considered in the calculation of impedance is the acoustic Reynolds number \( \beta \), which is a dimensionless parameter related to the quotient of the stresses caused separately by sound pressure and viscosity (Wang and Lu 1999).

\[
\beta = \frac{D_p^2 \rho_0}{\eta} \tag{7}
\]

where \( \eta \) is the dynamic viscosity of air. \( \beta<1 \) implies the low frequency or very low aperture radius limit whereas \( \beta>10 \) is the high frequency or large aperture radius limit. An intermediate situation \( (1<\beta<10) \) is required for the case of EPC for the frequency range considered (Maa 1987). The impedance of aperture then becomes:

\[
Z_a = \frac{32 n L_c}{D_p^2} \left( 1 + \frac{\beta^2}{32} \right) + i \omega \eta L_c \left( 1 + \frac{1}{\sqrt{\beta^2 + 4L_c}} \right) \tag{8}
\]

This case does not consider the end effects associated with the aperture. For the real component of impedance, an additional factor needs to be added to the length of the aperture (i.e., \( \frac{\beta D_p}{4L_c} \)). For the imaginary component,
the factor is $\frac{8D_p}{\pi L_p}$ (Stinson and Shaw 1985). With end

For a unit-cell case (i.e., one pore and aperture),
acoustic impedance of the system $Z_1$ is given by:

$$Z_1 = z_0 + Z_D = R + iM$$ (10a)

where $Z_0$ is the impedance of air inside the pores, as
given by Eq. 5b, and $z_0$ is the relative specific imped-
ance of apertures, given by:

$$z_0 = Z_0 \frac{D_p^2}{D_a^2}$$ (10b)

$R$ and $M$ are the real and imaginary components of the
impedance respectively (note that there is no subscript
“0” here, and these are different from $R_0$ and $M_0$).

For the case where there is more that one cell along
the length of the sample, the electro-acoustic analogy
must be applied to find the total impedance of the sample.
For a series of $n$ cells along the length of the sample, the
impedance at each cell depth is given by the relation-
ship:

$$Z_n = z_0 + \frac{1}{Z_D} + \frac{1}{Z_{n-1}}$$ (11)

where $Z_n$ is the impedance of the system with $n$ cells
along the length of the sample, $z_0$ is the relative imped-
ance of apertures, $Z_D$ is the impedance of air inside
pores, and $Z_{n-1}$ is the impedance of a system with ($n-1$)
cells. Using this value of impedance $Z_n$ (Eq.11), the real
and imaginary components can be used to calculate the
absorption coefficient of the sample ($\alpha$).

$$\alpha = \frac{4R}{\rho_0 c_0} \left(1 + \frac{R}{\rho_0 c_0}\right)^2 + \left(\frac{M}{\rho_0 c_0}\right)^2$$ (12)

where $\alpha$ is the absorption coefficient, $R$ is the real com-
ponent of $Z_n$, $M$ is the imaginary component of $Z_n$, $\rho_0$ is
the density of air, and $c_0$ is the speed of sound in air.

It has been described in Section 4.2 that the apparent
air density is higher by a factor $k_s$ since air displacement
is possible only through a fraction of the pore space. To
account for this effect, the air density $\rho_0$ in the model
has been modified by the multiplicative factor $k_s$.

4.4 Comparison with experimental measurements

The acoustic absorption spectra generated by the model
described in the previous section are compared with the
spectra obtained from impedance tube for selected mix-
tures, as shown in Fig.6. The input parameters for the
model are pore diameter ($D_p$), pore length ($L_p$), aperture
diameter ($D_a$), aperture length ($L_a$), dynamic viscosity of
air ($\eta$), density of air ($\rho_0$), and speed of sound in air ($c_0$).
The pore diameter ($D_p$) was obtained from image analy-
ysis, and the pore length $L_p$ was shown to be equal to
$0.90 D_p$. The length and diameter of the apertures were
obtained using the algorithm described in a previous
section. The density of air (1.2 kg/m$^3$), was modified by
multiplying with structure factor ($k_s$) (i.e., $\rho = \rho_0 \cdot k_s$).
Based on the pore length and aperture length, and the
length of the specimen, the number of cells (combination
of pore and aperture) along the length of the sample is
determined.

Fig.6(a) shows both the experimentally obtained and
predicted acoustic absorption spectra for EPC mixture
made with 100% #4 aggregate, for specimens having
lengths of 150 mm and 75 mm. It can be seen that there
is a good correlation between the measured and pre-
dicted values, and the first absorption peaks match well.
The frequencies of only up to 1000 Hz are shown in the
figure (though measurements are made up to 1600 Hz)
since the first peaks of both the thickness considered
appears within that frequency range, and this makes the
comparison easier.

Similarly, the absorption spectra for EPC mixture
with 100% 3/8” aggregate, for specimen lengths of 150
mm and 75 mm are shown in Fig.6(b), whereas Fig.6(c)
shows the spectra for a mixture with a blend of 75% #4
and 25% #8 aggregates. The predicted response reason-
ably matches the experimental observations, except for
the notable exception of the mixture with 100% 3/8”
aggregates. The possible reason for this could be the
fact that the pore sizes are very large in these mixtures,
and the model perhaps underestimates the reflection of
acoustic waves from these large sized pores.

5. Parametric study and optimization of
pore structure features

A parametric study was conducted to characterize the
influence of various microstructural features on acoustic
absorption. General agreement between the model pre-
dictions and the experimental measurements (Fig.6(a)
to (c)) indicates that this model may be used to deter-
mine the beneficial pore features of EPC in acoustic
absorption. The effect of pore size, pore aperture size,
porosity, and specimen length (thickness) on the acous-
tic absorption behavior of EPC is described in the fol-
lowing sections. It is believed that this information can
be used to select the optimal mixture proportions capable of providing the beneficial pore structure features with respect to acoustic absorption.

5.1 Effect of varying pore sizes
To model the influence of varying pore sizes on acoustic absorption of EPC, a sample thickness of 100 mm, and a porosity of 25% were assumed. Pore sizes ($D_p$) of 2, 3, 4, 5, and 10 mm were considered, maintaining the pore to aperture ratio ($D_p/D_a$) constant at 4. The pore length is fixed once the pore diameter is decided ($L_p = 0.9D_p$ from the model). The aperture length ($L_a$) was then calculated from Eq.5. The structure factor ($k_s$) could then be determined from Eq.4e. Based on the aperture length, pore length, and the thickness of EPC, the number of cells, $n$ (pore and aperture combination) could be determined.

Table 2 shows the values of all these parameters.

Figure 7 shows the plot of predicted normal incidence absorption coefficients as a function of frequency for the pore sizes selected for this study. An increase in pore size results in a reduction in the maximum absorption coefficient, though it does not substantially change the frequency at which the maximum absorption occurs. It has experimentally been shown that an increase in pore size reduces the maximum absorption coefficient (Marolf et al. 2004). This reduction in maximum acoustic absorption coefficient occurs because, for a system with large sized pores, the frictional losses are low and a considerable portion of the acoustic waves that enter the pore system gets reflected. It has also been shown that the pore sizes for the EPC systems could be related back to the aggregate sizes, and the proportions of each aggregate sizes in the mixture. This is a useful design tool.

Table 2 Pore structure features considered – effect of varying pore sizes.

<table>
<thead>
<tr>
<th>$D_p$, mm</th>
<th>$L_a(=0.9D_p)$, mm</th>
<th>$D_p/D_a$</th>
<th>$D_p$, mm</th>
<th>$L_p$, mm</th>
<th>No. of cells, n</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>1.8</td>
<td>4</td>
<td>0.50</td>
<td>1.58</td>
<td>20</td>
</tr>
<tr>
<td>3.0</td>
<td>2.7</td>
<td>4</td>
<td>0.75</td>
<td>2.36</td>
<td>13</td>
</tr>
<tr>
<td>4.0</td>
<td>3.6</td>
<td>4</td>
<td>1.00</td>
<td>3.15</td>
<td>10</td>
</tr>
<tr>
<td>5.0</td>
<td>4.5</td>
<td>4</td>
<td>1.25</td>
<td>3.94</td>
<td>8</td>
</tr>
<tr>
<td>10.0</td>
<td>9.0</td>
<td>4</td>
<td>2.50</td>
<td>7.88</td>
<td>4</td>
</tr>
</tbody>
</table>

Fig.6 (a) Measured and predicted acoustic absorption spectra for EPC with 100% #4 aggregates.

Fig.6 (b) Measured and predicted acoustic absorption spectra for EPC with 100% 3/8” aggregates.

Fig.6 (c) Measured and predicted acoustic absorption spectra for EPC with 75% #4 and 25% #8 aggregates.

Fig.7 Variation of acoustic absorption coefficient with frequency for varying pore size.
since the aggregate sizes and their proportions could be selected based on the pore sizes that would maximize acoustic absorption.

5.2 Effect of varying aperture sizes
In this case, the porosity was maintained at 25%, and a thickness of 100 mm was assumed. The pore size \((D_p)\) was maintained at 3 mm. Various \(D_p/D_a\) values (2,3,4, and 10) were chosen so that aperture sizes \((D_a)\) could be varied. The aperture length \((L_a)\), number of cells \((n)\), and the structure factor \((k_s)\) were determined. These values are listed in Table 3.

Figure 8(a) shows a plot of acoustic absorption as a function of frequency for varying pore to aperture size ratios chosen. For ease of interpretation, only the first peaks are plotted in this figure. It can be noticed that the maximum absorption coefficient increases with an increase in \(D_p/D_a\) values, until a \(D_p/D_a\) value of 5.0. For a \(D_p/D_a\) value of 10, the maximum absorption coefficient was found to decrease. The explanation for this observation is as follows: At very high values of \(D_p/D_a\) (smaller aperture sizes), the amount of air that can get into the pores is reduced, resulting in energy reflection at the surface, and consequently lower acoustic absorption. As \(D_p/D_a\) gets smaller (aperture sizes approach the pore sizes), there is less of a change in cross section that can lead to alternate compression and expansion of acoustic waves, resulting in a lower acoustic absorption. It can also be seen that the frequencies at peak absorption decrease with an increase in \(D_p/D_a\), i.e., the absorption is enhanced in the low-frequency range.

Another representation of this effect is shown in Fig. 8(b), where it can clearly be found that there is an optimal pore to aperture size ratio that maximizes acoustic absorption. The aperture length was kept constant at 5 mm, and \(D_p/D_a\) values of 3, 4, 5, 6, 9, 12, and 15 were chosen. Three pore sizes \((D_p)\) of 2, 3, and 5 mm were also selected. The maximum absorption coefficient \((\alpha)\) predicted by the model is plotted against the pore to aperture size ratios. In this parametric study, the pore sizes chosen are close to those obtained for EPC mixtures investigated (2 to 5 mm), and the optimal pore to aperture size ratios vary from 4 to 7, which was also found to be true experimentally (as shown in Table 1).

5.3 Effect of varying porosities
The influence of porosity on acoustic absorption coefficient was studied using a constant pore size \((D_p)\) of 3 mm, and pore to aperture size ratio \((D_p/D_a)\) of 4, with aperture length \((L_a)\) and number of cells \((n)\) calculated as described in previous sections. Five different porosities ranging from 15% to 35%, at an interval of 5% were considered. Table 4 gives the values of these parameters. Fig.9 shows the variation in absorption coefficient with frequency for different porosities.

Increasing porosities slightly increases the frequencies at maximum absorption. It can also be observed that the maximum absorption coefficient decreases with increase in porosity, which may seem counter-intuitive. But, Table 4 shows that with increase in porosity, for a constant pore size, the aperture length \((L_a)\) decreases, and this is illustrated in Fig.10.

It is in the apertures that the frictional losses occur,

Table 3 Pore structure features considered – effect of varying aperture sizes.

<table>
<thead>
<tr>
<th>(D_p), mm</th>
<th>(L_p = 0.9D_p), mm</th>
<th>(D_p/D_a)</th>
<th>(D_a), mm</th>
<th>(L_a), mm</th>
<th>No. of cells, (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>2.7</td>
<td>2</td>
<td>1.50</td>
<td>3.78</td>
<td>10</td>
</tr>
<tr>
<td>3.0</td>
<td>2.7</td>
<td>3</td>
<td>1.00</td>
<td>2.62</td>
<td>12</td>
</tr>
<tr>
<td>3.0</td>
<td>2.7</td>
<td>4</td>
<td>0.75</td>
<td>2.36</td>
<td>13</td>
</tr>
<tr>
<td>3.0</td>
<td>2.7</td>
<td>5</td>
<td>0.60</td>
<td>2.26</td>
<td>15</td>
</tr>
<tr>
<td>3.0</td>
<td>2.7</td>
<td>10</td>
<td>0.30</td>
<td>2.14</td>
<td>17</td>
</tr>
</tbody>
</table>
and hence reduced aperture length implies reduced absorption. If it is assumed that the aperture length remains constant, and the increase in porosity is achieved through increase in pore size, the same explanation as in Section 5.1 holds good. However, it should be noted that the porosity is influenced by the pore size, pore to aperture size ratio, and pore length, in addition to aperture length, and it is the synergetic effect of all these factors that dictate the effect of porosity in acoustic absorption.

### 5.4 Effect of varying specimen thickness

The influence of varying specimen thickness on acoustic absorption is illustrated by using a pore size ($D_p$) of 3 mm, pore to aperture size ratio ($D_p/D_a$) of 4, porosity of 25%, and an aperture length $L_a$ determined as 2.36 mm. The number of cells required for a specimen thickness of 150 mm is 20, for a thickness of 75 mm is 10, and for a thickness of 37.5 mm is 5. Figure 11 shows the variation in acoustic absorption with frequency for three different lengths. Increasing the specimen thickness reduces the frequencies at the maximum absorption, as well as the spacing between the peaks. The same effect is observed in Figs. 6(a) to (c) where the measured and predicted absorption coefficients are plotted for three different EPC mixtures.

Figure 11 shows the variation in acoustic absorption coefficient with change in thickness, predicted by the model.

The frequency at maximum absorption coefficient could be thought of as a function of wave speed in the medium and the specimen thickness. Because the speed of the wave in air is effectively a constant, the thickness of the sample must be changed, to shift the peaks. The peaks occur at frequencies that can be calculated according to the relationship (Zwikker and Kosten 1949):

$$f = \frac{1}{2\pi} \sqrt{\frac{C}{\rho}}$$

where $f$ is the frequency, $C$ is the speed of sound, and $\rho$ is the density of the medium.

### Table 4 Pore structure features considered – effect of varying porosities.

<table>
<thead>
<tr>
<th>Porosity</th>
<th>$D_p$, mm</th>
<th>$L_p (=0.9D_p)$, mm</th>
<th>$D_p/D_a$</th>
<th>$D_a$, mm</th>
<th>$L_a$, mm</th>
<th>No. of cells, n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>3.0</td>
<td>2.7</td>
<td>4</td>
<td>0.75</td>
<td>6.50</td>
<td>9</td>
</tr>
<tr>
<td>0.20</td>
<td>3.0</td>
<td>2.7</td>
<td>4</td>
<td>0.75</td>
<td>3.83</td>
<td>11</td>
</tr>
<tr>
<td>0.25</td>
<td>3.0</td>
<td>2.7</td>
<td>4</td>
<td>0.75</td>
<td>2.36</td>
<td>13</td>
</tr>
<tr>
<td>0.30</td>
<td>3.0</td>
<td>2.7</td>
<td>4</td>
<td>0.75</td>
<td>1.43</td>
<td>15</td>
</tr>
<tr>
<td>0.35</td>
<td>3.0</td>
<td>2.7</td>
<td>4</td>
<td>0.75</td>
<td>0.79</td>
<td>17</td>
</tr>
</tbody>
</table>

Fig. 9 Variation of acoustic absorption coefficient with change in porosity.

Fig. 10 Illustration of variation in porosity with change in aperture length ($L_a$) for a single cell.

Fig. 11 Variation of acoustic absorption coefficient with change in thickness, predicted by the model.

Fig. 12 Illustration of the product of frequency and thickness being a constant with respect to maximum absorption coefficient irrespective of specimen thickness.
The thickness of the sample.

odd integer number corresponding to the peak (1 for 1st peak, 3 for 2nd peak and so on), \( c \) is the effective speed of wave in the medium (343 m/s for air at 20°C), and \( l \) is the thickness of the sample.

Since \( c \) and \( B \) are constants, the product of frequency and thickness of the sample should also be a constant. This implies that when the product of frequency and thickness is plotted against the absorption coefficient, the peaks appear at the same frequency, irrespective of the specimen thickness. For illustrative purposes, this feature is shown in Fig.12. This plot shows that the product of frequency and the specimen thickness is a constant. This is advantageous in selecting the most acoustically efficient thickness of EPC overlays for pavements, based on the dominant frequency of sound to be attenuated.

For example, 800-1200 Hz is the frequency range in which more of the objectionable highway noise falls. Hence, using a frequency of 1000 Hz and speed of sound in air as 343 m/s, the thickness of overlay required (1st absorption peak, hence \( B=1 \)) can be calculated from Eq. 13 as 8.6 cm (3.4 inches).

6. Summary and conclusions

This paper describes the implementation of a model, originally proposed by Lu (2000) for cellular metals, to explain the acoustic absorption behavior of EPC. The model is modified and used to illustrate how the microstructural characteristics of EPC influence the acoustic absorption with the goal of better understanding how the pore structure can be improved for better acoustic performance.

The following conclusions are derived from this study.

- The idealized model for EPC consists of pores and apertures. This network was found to adequately describe the acoustic behavior of the system.
- An electro-acoustic analogy, considering a series of resistors and inductors (to represent the pores and apertures) has been employed to model the acoustic absorption of EPC.
- The concept of structure factor has been employed to correct for the density of air in the pores. An algorithm to determine the aperture length from the pore diameter and length was used, which can be used in the calculation of structure factor. The maximum absorption coefficient decreases linearly with an increase in structure factor.
- The experimentally measured and predicted acoustic absorption responses match well. A detailed parametric study has been conducted to isolate the effects of pore size, aperture size, porosity and the specimen thickness on acoustic absorption. An increase in pore size was found to reduce the maximum absorption coefficient while it was observed that there is an optimum pore to aperture size ratio that maximizes acoustic absorption. The acoustic absorption was enhanced in the low frequency range by increasing the pore to aperture size ratio.
- It was observed that increasing the porosity by keeping the aperture length constant results in a reduced maximum acoustic absorption. Increasing the thickness of the specimen results in maximum absorption peaks being shifted to lower frequencies, which can be described using a simple equation. This equation can be used to design EPC based on the frequencies that are required to be attenuated.

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References


