Applications of Boundary Effect Model to Quasi-Brittle Fracture of Concrete and Rock

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Abstract

A recently-developed asymptotic boundary effect model for the quasi-brittle fracture of finite-sized fracture mechanics specimens is used to analyse the experimental results available in the literature. Three different experimental results are chosen in this study to cover various experiment designs, including geometrically similar specimens, specimens of identical size with different crack lengths and specimens of different configurations and geometries. It is shown that the size effect associated with quasi-brittle fracture, as reported in the literature, is in fact, due to the influence of specimen boundaries. The observed dependence of fracture behaviour on specimen size and crack length can be described by the same boundary effect model. The new asymptotic model is also compared with other size effect models dealing exclusively with geometrically similar specimens, and its distinct advantages over other models are discussed.

1. Introduction

Size-dependence of the fracture properties of quasi-brittle materials such as concrete, rock and ceramics has been a major barrier for the application of the classic linear elastic fracture mechanics (LEFM) for more than four decades (Kaplan 1961; Walsh 1972; Higgins & Bailey 1976; Bažant 1984; Bažant & Kazemi 1990; Bažant et al. 1991; Bažant & Yu 2004; Li & Liang 1986; Wittmann et al. 1990; Carpinteri 1994; Carpinteri et al. 1995, 1997; Mihashi & Nomura 1996; Karihaloo 1999; Karihaloo et al. 2003; van Vliet & van Mier 1999; Hu & Duan 2002; Duan & Hu 2002, 2004a,b; Duan et al. 2001, 2002, 2003a,b,c, 2004, 2005). The size effect implies that the fracture properties measured in a laboratory condition cannot be directly used for predicting the performance of real structures. This has undoubtedly been one of main drives behind the intensive studies on the size effect mechanisms and models. Currently, there are at least three size effect models that are often cited in the size effect literature, including the size effect law (SEL) by Bažant (1984), the multi-fractal scaling law (MFSL) by Carpinteri et al. (Carpinteri 1994; Carpinteri et al. 1995) and the size effect model for “large” 3-point-bending specimens by Karihaloo (1999). Even though derived through different approaches, all three models emphasize particularly the influence of physical specimen size and therefore, are normally applied to geometrically similar specimens. These models are not developed for the crack/ligament length dependence of the fracture properties measured from the specimens with identical sizes (Higgins & Bailey 1976; Wittmann et al. 1990; Shinohara et al. 1991; Hu & Duan 2002; Duan & Hu 2002; Duan et al. 2001, 2002, 2003a,b,c, 2004, 2005), because the specimen size does not vary in this case. The size effect models for characterising the fracture of un-notched specimens have also been proposed, which assume that the failure-controlling crack initiates in the boundary layer (Bažant & Li 1995; Carpinteri et al. 1997; van Vliet & van Mier 1999). Because the boundary layer and therefore, crack size is dependent on specimen size, the strength of the un-notched specimens is also size dependent.

To address the limitations associated with the size effect models, the present authors have recently shown that the effects of both specimen size and crack/ligament length on fracture properties actually result from the interaction between the crack-tip fracture process zone (FPZ) and specimen boundaries, and therefore, from the influence of specimen boundaries (Hu & Duan 2002; Duan & Hu 2002; Duan et al. 2001, 2002, 2003a,b,c, 2004, 2005). This has led to the development of the boundary effect concept and two boundary effect models, the asymptotic boundary effect model and the bilinear local fracture energy approach (Hu & Duan 2002; Duan & Hu 2002; Duan et al. 2001, 2002, 2003a,b,c, 2004, 2005).

The asymptotic model is based on the asymptotic solution developed by Hu for the strength behaviour of a large plate with a small edge crack (Hu 1998, 1999, 2002; Hu & Wittmann 2000). According to this asymptotic solution, the failure mode and strength value of the large plate depend on the ratio of the crack length to a reference crack length $a_\text{ref}$ (a material constant determined by the tensile strength $f_t$ and fracture toughness $K_{IC}$). The asymptotic solution has later been further developed to describe the fracture properties of specimens with limited sizes where the effects of both front and back boundaries exist (Duan & Hu 2002, 2004a,b; Duan

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The second approach is based on the local fracture energy concept (Hu 1995; Hu & Wittmann 1992) and uses a bilinear function to approximate the local fracture energy distribution along the ligament (Hu & Duan 2002; Duan et al. 2001, 2002, 2003a,b, 2004). In the bilinear function, a transition ligament \( a^* \) is introduced to separate the ligament into two regions, the inner region where the local fracture energy is a constant that is equal to the size-independent specific fracture energy \( G_I \), and the outer boundary region where the local fracture energy will decline to the surface energy at the specimen back boundary. The observed ligament-dependence of the fracture energy is due to the decrease in the local fracture energy in the boundary region, which results from the limitation of the specimen back boundary on the development of FPZ.

In the present paper, the asymptotic boundary effect model for finite-sized specimens is applied to a number of experimental results available in the literature and associated size-dependence is analysed. It will be shown that the asymptotic boundary effect model covers much broader applications, and geometrically similar specimens are only a special case for the applications of the present model. It will also be shown that the size effect in quasi-brittle fracture observed in a laboratory is in fact, due to the influence of specimen boundaries.

2. Asymptotic boundary effect model for Quasi-Brittle fracture of Finite-Sized specimens

The recently-developed asymptotic boundary effect model for finite-sized specimens is established on the equivalence between finite-sized specimens and a large plate with a small edge crack (Duan & Hu 2002, 2004a,b; Duan et al. 2005). This equivalence is achieved by introducing the equivalent crack length \( a^* \) that contains the influence of both the crack length \( a \) and specimen characteristic size \( W \), and can be expressed as a function of the ratio of the crack length to specimen characteristic size, \( D = \frac{a}{W} \). As a result, the fracture behaviour of finite-sized specimens is same as that of a large plate when \( a^* \) is used as crack length (Duan & Hu 2004a,b),

\[
\sigma_n = \frac{f_t}{\sqrt{1 + a^*/a^*_{ref}}} \quad (1a)
\]

\[
a^* = a \left[ \frac{A(\alpha) \cdot Y(\alpha)}{1.12} \right]^{\frac{1}{2}} \quad (1b)
\]

where as shown in Fig. 1, the “net-area based” nominal strength \( \sigma_n \) represents the failure stress at the crack tip (without considering stress concentration and singularity), and is related to the “gross-area based” nominal strength \( \sigma_N \) (corresponding to the maximum failure stress in the specimen without considering the presence of the crack) (Duan & Hu 2004a,b)

\[
\sigma_n = A(\alpha) \cdot \sigma_u
\]

In Eq. (1), \( f_t \) is the tensile strength, \( a^*_{ref} \) is the reference crack length and represents the intersection of the tensile strength and linear elastic fracture mechanics (LEFM) criteria (Hu 1998, 1999; Hu & Wittmann 2000)

\[
a^*_{ref} = \frac{1}{1.12^2} \left[ \frac{K_{IC}}{f_t} \right]^2 \quad (3)
\]

and \( Y(\alpha) \) is the geometry function used in the stress intensity factor expressions, i.e. \( K_{IC} = Y(\alpha) \cdot \sigma_N \sqrt{a^*} \).

For a large plate with a small edge crack \( a \), i.e. \( \alpha = 0 \), one has \( A(\alpha) = 1 \), \( Y(\alpha) = 1.12 \) and \( \sigma_n = \sigma_N \). As a result, Eq. (1) is reduced to
\[ \sigma_s = \sigma_N = \frac{f_i}{\sqrt{1 + a/a_s}} \] (4)

This shows that the large plate case is a special case of Eq. (1).

Examining further Eqs. (1) and (4), it can be found that the asymptotic solutions for both cases use \( f_i \) and \( a_s \) as scaling factors, and have two common asymptotic limits. First when \( a \) or \( a_s \ll a_s \), the specimen nominal strength \( \sigma_s \) is equal to \( f_i \), the first asymptotic limit. That is, the maximum tensile stress criterion \(( f_i)\) dominates the fracture of the large plate with a short crack \( a \) or \( a_s \). The second asymptotic limit is reached when \( a \) or \( a_e \gg a_s \) where the nominal strengths \( \sigma_s \) and \( \sigma_N \) can be obtained by combining Eqs. (1) to (3).

\[ \begin{align*}
\sigma_s &= \frac{f_i}{\sqrt{a_e/a_s}} = \frac{f_i}{\sqrt{1 + a/a_s} \left[ \frac{A(\alpha) \cdot Y(\alpha)}{1.12} \right]^{1/2} \frac{a}{a_s}} \\
&= \frac{K_{IC}}{A(\alpha) \cdot Y(\alpha) \cdot \sqrt{\pi a}} \\
\sigma_N &= \frac{K_{IC}}{Y(\alpha) \cdot \sqrt{\pi a}}
\end{align*} \] (5a, 5b)

It is found that the second asymptotic limit is, in fact, the LEFM \((K_{IC})\) criterion.

The quasi-brittle fracture behaviour of both finite-sized specimens and the large plate case as given in Eqs. (1) to (5) is schematically illustrated in Fig. 2. In the Log \( a \) – Log \( \sigma \) plot (Fig. 2a), the maximum tensile stress criterion \((f_i)\) is represented by a horizontal straight line of \( f_i \), and the LEFM criterion, Eq. (5) is given by a declining straight line with a slope of \(-1/2\). The intersection of these two linear functions represents the reference crack length \( a_s \). The asymptotic solution, Eq. (4) is represented by a thick curve that provides a smooth transition from the \( f_i\)-controlled failure to the LEFM criterion with increasing crack length \( a \). It is noted here that this crack length dependence of the large plate strength reflects the effects of the front boundary on the plate fracture. This is because the crack length \( a \) is equal to the distance of the crack tip to the front boundary, and the specimen size \( W \) is considered infinite and is not involved in changing the strength behaviour as shown in Eq. (4). That is, the changes in the strength \( \sigma_s \) with varying crack length \( a \) in the large plate is a result of boundary effects.

Because of the equivalence between finite-sized specimens and a large plate with an edge crack of \( ae \), the maximum tensile stress theory \((f_i)\), the LEFM criterion \((K_{IC})\) and the quasi-brittle transition curve (Eq. 1) for finite-sized specimens in the plot of Log \( \sigma \) versus Log \( a \), as shown in Fig. 2b, should be similar to those for the large plate in the Log \( a \) – Log \( \sigma \) plot. The key difference is that the distance of the crack tip in a finite-sized specimen to the specimen boundaries is limited (Duan & Hu 2002, 2004a,b; Duan et al. 2005). This is reflected in the limitation of specimen size on the maximum equivalent crack length \( a_e \) that can be achieved. For example, if the specimen size \( W \) is fixed and the \( \alpha\)-ratio increases from 0 to 1, the equivalent crack length \( a_e \) will grow from a very small value to the maximum \( a_e \) at \( \alpha \approx 0.2 \), and then turn back towards small \( a_e \). Meanwhile, the nominal strength \( \sigma_s \) also fluctuates from \( f_i \) to its minimum at \( \alpha \approx 0.2 \), and then increases back to the tensile strength \( f_i \) when \( \alpha \rightarrow 1 \). The changes in \( ae \) with the \( \alpha\)-ratio are illustrated in Fig. 2b by a dashed curve. The strength behaviour of a finite-sized specimen shown in Fig. 2b shows that the \( f_i\)-criterion dominates the strength of a finite-sized specimen when the crack tip is close to either its front boundary (short crack) or back boundary (short ligament). This again implies the boundary effect as a key in quasi-brittle fracture.

For the convenience of analysing experimental data from fracture mechanics tests, Eq. (1a) is rearranged to a linear form.
\[
\frac{1}{\sigma^*_n} = \frac{1}{f_t} + \frac{1}{f_*} \cdot \frac{a_*}{a_n}
\]

(6)

3. Boundary effect on geometrically similar specimens and size effect law

For geometrically similar specimens, the \( \alpha \)-ratio is kept constant, and the distances of crack tip to the front boundary (crack length \( a \)) and the back boundary (ligament \( W-\alpha \)) will increase with increasing specimen size \( W \). This leads to changes in the nominal strength \( \sigma_n \) with specimen size \( W \). This “size-dependent” strength of geometrically similar specimens has been studied extensively over the past two decades. The widely cited size effect law (SEL) by Bazant (1984) was proposed to deal with the “apparent” size effect phenomenon.

\[
\sigma_n = \frac{A \cdot f_*}{\sqrt{1 + W/W^*}}
\]

(7)

where the tensile strength \( f_* \) has to be determined in a separate test. The parameters \( A \cdot f_* \) and \( W^* \) are dependent on loading configuration and specimen geometry, and can be estimated from experiments. However, the parameters \( A \cdot f_* \) and \( W^* \) estimated from the experiments on the geometrically similar specimens with a given \( \alpha \)-ratio cannot be used to predict the strengths of those specimens with a different \( \alpha \)-ratio. For the analysis of experimental data, Eq. (7) is often given in a linear form.

\[
\frac{1}{\sigma_n} = \frac{1}{(A \cdot f_*)^2} + \frac{W^*}{(A \cdot f_*)^2} \cdot W
\]

(8)

The size-dependent nominal strength \( \sigma_n \) can also be described using Eq. (1). Let \( \alpha = \) constant in Eq. (1), the asymptotic solution for geometrically similar specimens is obtained.

\[
\sigma_* = \frac{f_*}{\sqrt{1 + W/W^*}}
\]

(9a)

\[
\sigma_n = A(\alpha) \cdot \sigma_* = \frac{A(\alpha) \cdot f_*}{\sqrt{1 + W/W^*}}
\]

(9b)

where \( W^* \) is a constant for a given loading configuration and specimen geometry, and is referred to as the transition specimen size (Duan & Hu 2002, 2004a,b; Duan et al. 2005).

\[
W^* = W \cdot a_*^2 = \frac{a_*^2}{\alpha \cdot \left[ A(\alpha) \cdot Y(\alpha) \right]^2}
\]

(10)

Eq. (9b) appears similar to SEL of Eq. (7). However, the explicit expressions for the parameters have been provided. This means that once the parameters \( f_* \) and \( W^* \) are determined from a set of geometrically similar specimens, all the strengths for other specimens with different \( \alpha \)-ratios can be predicted by using either Eq. (1) or (9). The advantages of Eq. (9) over Eq. (7) will be further discussed in the following sections.

4. Boundary effect formulae for fracture mechanics specimens

To use the new asymptotic model to analyse fracture mechanics tests on quasi-brittle materials, the detailed expressions of the equivalent crack length \( a_* \) and the nominal strength \( \sigma_n \) in Eq. (1) should be derived first. This involves the derivation of the functions \( A(\alpha) \) and \( Y(\alpha) \) for the specimens used. In the present paper, all the experimental results chosen for analysis were measured on the widely-used 3-p-b and CT specimens (Fig. 1). Therefore, only the \( a_* \) and \( \sigma_n \) expressions of these two geometries and associated \( A(\alpha) \) and \( Y(\alpha) \) functions are discussed in this section.

The function \( A(\alpha) \) of a given specimen configuration can be determined by using a simple mechanics analysis. For example, the \( A(\alpha) \) functions for widely-used 3-p-b and CT geometries given in Fig. 1 are given by

\[
A(\alpha) = \left(1 - \alpha \right)^2
\]

3-p-b (11a)

\[
A(\alpha) = \frac{(1 - \alpha)^2}{2(2 + \alpha)}
\]

CT (11b)

Geometry function \( Y(\alpha) \) for many fracture mechanics specimens can be found in many handbooks and standards (e.g. ASTM E399-90; Tada et al. 2000). For the convenience of further analysis, the \( Y(\alpha) \) functions for the standard 3-p-b (the ratio of span \( S \) over depth \( W \), \( S/W = 4 \)) and CT specimens are given.

\[
Y(\alpha) = \frac{1.99 - \alpha \cdot (1 - \alpha) \cdot (2.15 - 3.93 \cdot \alpha + 2.7 \cdot \alpha^2)}{\sqrt{\pi} \cdot (1 + 2 \cdot \alpha) \cdot (1 - \alpha)^{3/2}}
\]

3-p-b (12a)

\[
Y(\alpha) = \frac{(2 + \alpha) \cdot (0.886 + 4.64 \alpha - 13.32 \alpha^2 + 14.72 \alpha^3 - 5.6 \alpha^4)}{\sqrt{\pi \alpha} \cdot (1 - \alpha)^{3/2}}
\]

CT (12b)

The \( Y(\alpha) \) functions given in the above equations are all with a relative error less than 0.5%. \( Y(\alpha) \) for CT geometry given in Eq. (12b) is valid for an \( \alpha \)-ratio between 0.2 and 1, and \( Y(\alpha) \) expression for 3-p-b specimen in Eq. (12a) is valid for any \( \alpha \) value between 0 and 1. For non-standard 3-p-b specimens, modifications to Eq. (12a) are necessary, and can be done by linear interpolation of \( Y(\alpha) \) functions for \( S/W = 4 \) and 8 (Karihaloo & Nallathambi 1989).

Substituting Eqs (11) and (12) into Eq (1), the \( a_* \) expressions for the standard 3-p-b and CT specimens as
shown in Fig. 1, are obtained.

\[ a_e = \frac{(1 - \alpha)}{1.12^2 \pi} \left( \frac{1.99 - \alpha (1 - \alpha) (2.15 - 3.93 \alpha + 2.7 \alpha^2)}{(1 + 2 \alpha)} \right)^{\frac{1}{2}} \]

3-p-b (13a)

\[ a_e = \frac{1 - \alpha}{1.12^2 \cdot 4 \pi \alpha} \left( 0.886 + 4.64 \alpha - 13.32 \alpha^2 + 14.72 \alpha^3 - 5.6 \alpha^4 \right)^{\frac{1}{2}} \cdot a \]

CT (13b)

5. Applications of asymptotic boundary effect model to experiments

A number of experimental observations of the size effects on fracture properties of quasi-brittle materials have been reported. These experimental observations are conducted on both geometrically similar specimens and specimens of same size with different notch length. Also fracture mechanics tests on the same materials using different specimen geometries are reported. In the present study, we chose a few experimental results to show how the asymptotic solution work for the experimental results of geometrically similar specimens, the specimens of identical size with different crack length and different types of specimens.

5.1 Experimental results from geometrically similar specimens

Shown in Fig. 3 are the fracture strengths of an Indiana limestone reported by Bažant et al. (1991). The experiments were carried out using geometrically similar 3-p-b specimens with the span-to-depth ratio of 4 and the fixed \( \alpha \)-ratio of 0.4. Four beam depths of 102, 51, 25 and 13 mm were used for fracture testing. Substituting the geometry parameters and the measured peak loads into Eqs. (1), (2) and (11) to (13), both \( a_e \) and \( \sigma_f \) can be calculated, and are plotted in the plot of \( 1/\sigma_f^2 \) versus \( a_e \), as shown in Fig. 3. Shown in Fig. 3 is also a straight line obtained from the regressive analysis of the \( a_e - 1/\sigma_f^2 \) plot. From the interception and slope of this straight line, the tensile strength \( f_t \) and the reference crack \( a^* \) of the Indiana limestone are obtained as 8.71 MPa and 2.99 mm, respectively. Substituting \( f_t \) and \( a^* \) into Eq. (3), the fracture toughness \( K_{IC} \) is worked out as 0.94 MPa m\(^{1/2}\).

The tensile strength of 8.71 MPa is as high as two and a half times of the splitting strength of 3.45 MPa. This is attributed to the volume and configuration effects associated with statistical fracture behaviour of brittle materials. According to statistical fracture theory (Hunt & McCartney 1979), 3-p-b specimens have higher tensile strength \( f_t \) than splitting specimens because of high stress concentration in a small volume. In addition, the splitting specimens used for the Indiana limestone are very large comparing to the 3-p-b specimens used (Bažant et al. 1991). The \( f_t \) value of 8.71 MPa is also significantly higher than the values of 5.0 and 6.5 MPa given in the literature. This is considered more adequate because it is very close to the maximum \( \sigma_f \) of 8.38 MPa observed in the experiment. The fracture toughness \( K_{IC} \) of 0.94 MPa m\(^{1/2}\) is close to the values given in the literature (Bažant et al. 1991).

Substituting the estimated \( f_t \) and \( a^* \) into Eq. (1), the quasi-brittle fracture curve of the Indiana limestone are predicted, and are plotted in Fig. 4a to compare with those experimental results. The maximum tensile stress and LEFM criteria are also shown in Fig. 4a. It can be seen here that for even the largest beam used in this experiment, the equivalent crack length \( a_e \) is only about 2 times of \( a^* \), and it is much lower than 10\( a^* \) required for negligible boundary effect (Duan & Hu 2004a). This explains the observation of “all the data points lie in the transition zone between the LEFM criterion and the strength criterion” (Bažant et al. 1991).

The strength properties of the Indiana limestone were also analysed using SEL given in Eq. (7) or (8) (Bažant et al. 1991). Using Eq. (8), the geometry-dependent experimental parameters, \( A f_t \) and \( W' \) were calculated (Bažant et al. 1991). To decide the tensile strength \( f_t \) a splitting test was conducted, yielding \( f_t \) value of 3.45 MPa, which is much lower than the maximum \( \sigma_f \) of 8.38 MPa, observed in the experiments.

It is noted here that the geometry-dependent experimental parameters, \( A f_t \) and \( W' \) from SEL are valid only for the geometrically similar 3-p-b specimens of the Indiana limestone with the \( S/W' \) of 4 and \( \alpha \)-ratio of 0.4. In a contrast, \( f_t \) and \( a^* \) from the present boundary effect model can be used to predict the fracture behaviour of other geometries. As an example, Fig. 4b shows the predicted curves for the 3-p-b specimens with the \( \alpha \)-ratios of 0.1, 0.4 and 0.7.

From the above analysis, two important advantages of the present asymptotic boundary effect model over...
common size effect models can be observed. Firstly, the important fracture properties, the tensile strength $f_t$ and fracture toughness $KIC$ can be worked out directly from general fracture test. No extra test is needed to determine the tensile strength $f_t$. Secondly, once $f_t$ and $a^*_f$ values are determined from a single set of geometrically similar specimens, the strength behaviour for all other geometries can be predicted.

5.2 Experimental results from specimens of identical size with different notch lengths

The fracture tests on concrete by Shinohara et al. (1991) provide an example for the application of the present asymptotic boundary effect model to the non-geometrically similar specimens. Shinohara et al. (1991) tested the effects of water/cement ratio, aggregate size, loading rate and notch depth on the fracture properties of concrete using 3-p-b specimens of identical size where both beam width and depth are 100 mm, and span was kept at 300 mm. For the concrete with the maximum aggregate of 20 mm and the water/cement ratio of 0.55, four notch lengths of 10, 30, 50 and 70 mm were tested.

Similar to the analysis in the previous section, both the equivalent crack length $a_e$ and the nominal strength $\sigma_n$ or $\sigma_N$ can be calculated by substituting the correct geometry factors and peak loads into Eqs. (1), (2) and (11) to (13). The nominal strengths $\sigma_n$ and $\sigma_N$ and the equivalent crack length $a_e$ as well as notch length $a$ are plotted in Figs. 5 to 7. In the plot of $1/\sigma_n^2$ versus $a_e$, a linear function can be observed as shown in Fig. 5a. Using this linear function and Eqs. (3) and (6), the tensile strength $f_t$, the reference crack length $a^*_f$ and the fracture toughness $KIC$ are worked out as 1.84 MPa, 8.84 mm and 0.34 MPa $\tilde{m}^{1/2}$ for the 4 weeks old concrete,
and 1.80 MPa, 25.84 mm and 0.58 MPa \( \tilde{m}^{1/2} \) for the 1 year old concrete, respectively. Substituting the \( f_t \) and \( a^* \) into Eq. (1), \( \sigma \) values for various notch sizes are predicted, and are plotted in Figs. 6 and 7 to compare with those measured values. It is found that the predictions in Figs. 6 and 7 agree with the experiments reasonably well.

**Figure 5b** shows the same strength results in the plot of \( 1/\sigma_n^2 \) versus \( W \) as used in SEL, and clearly, no trend can be observed. Therefore, the SEL, Eq. (8) cannot be used to describe and predict the strength behaviour of non-geometrically similar specimens. The linear functions shown in **Fig. 5b** for \( \alpha \)-ratios of 0.3, 0.5 and 0.7 are predicted using the parameters in **Fig. 5a** and Eqs. (9) and (10). These straight lines are in very good agreement with those measured strengths of specimens with the same \( \alpha \)-ratio. This implies the significant effects of both specimen size (\( W \)) and geometry (\( \alpha \)-ratio) on the fracture properties. These combined specimen size and geometry influences have been observed by Higgins and Bailey (1976), and were attributed to the boundary effect in the previous study (Duan & Hu 2002, 2004a,b; Duan et al. 2005).

The analysis in this section shows another advantage of the present asymptotic boundary effect model over common size effect models. That is, the new asymptotic boundary effect model can be used to describe the strength behaviour of non-geometrically similar specimens, and using the parameters from these specimens, the fracture behaviour of geometrically similar specimens can be predicted.

### 5.3 Experimental results from different types of specimens

Fracture tests on a granite completed by Hashida and Takahashi (1985) were conducted using both 3-p-b and CT specimens. Because all the geometry parameters such as specimen thickness and \( \alpha \)-ratio as well as the span-to-depth ratio of 3-p-b specimens varied, these specimens are not geometrically similar. Using Eqs. (1) and (11) to (13) and the geometry parameters from (Hashida & Takahashi 1985), \( \sigma_n \) and \( a^* \) for both 3-p-b and CT specimens can be estimated. The \( 1/\sigma_n^2 - a^* \) plots of the granite are displayed in **Fig. 8** where it is found that all the 3-p-b and CT results follow the same linear relationship except the largest 3-p-b specimen. Therefore, linear regressive analysis using Eq. 6 is done without the largest 3-p-b specimen. The tensile strength \( f_t \), the reference crack length \( a^* \) and the fracture toughness \( K_{IC} \) for the granite are estimated as 13.59 MPa, 3.44 mm and 1.58 MPa\( \tilde{m}^{1/2} \), respectively.
Substitute the estimated $f_t$ and $a^*_f$ into Eq. 1, the fracture strength of the granite is predicted, and is plotted in Figs. 9 and 10 to compare with the experimental results. It can be found that the predictions agree well with the measured strength except the largest 3-p-b specimen. In the $\log V_n$–$\log a_e$ plot (Fig. 9a), the experimental results from both 3-p-b and CT specimens agree well with the unique asymptotic fracture curve. Figure 9b shows the reasonable agreement between the experimental results and predictions in the $\log V_N$–$\log W$ plot. The predicted strength curves in Fig. 9b are based on average $\alpha$-ratios for 3-p-b and CT specimens.

The experimental results of the granite have also been analysed by Bažant and Kazemi (1990) using the SEL. To satisfy the geometrical similarity required by the SEL, an average $\alpha$-ratio has to be used for either 3-p-b or CT specimens. Because the $\alpha$-ratios of the CT specimens vary in a narrow range of 0.537 to 0.553, only the results of these specimens were used to predict the strength behaviour. In a contrast, the results of both the 3-p-b and CT specimens contribute to the predictions in the present asymptotic boundary effect model. This reduces the bias from a single geometry.

6. Concluding remarks

The recently developed asymptotic boundary effect model (Duan & Hu 2002, 2004a,b; Duan et al. 2005) is used to analyse three different sets of experimental results from the literature. It is demonstrated that the size effect on the fracture of quasi-brittle structures often observed in a laboratory is, in fact, due to the influence of specimen boundaries, and both the crack size dependence of fracture properties of a large plate with a
small edge crack and the structural size dependence of fracture properties of geometrically similar specimens are special cases of the boundary effect.

Three types of experiments chosen for this study include experiments conducted on geometrically similar specimens, specimens of identical size with different crack lengths and specimens of different configurations and geometries. This aims at covering various specimen and/or crack size dependent experimental data available in the literature. Among these three types of fracture mechanics data chosen, SEL can only apply to size effect on the fracture of geometrically similar specimens. However, it can be noted here that for even the analysis of geometrically similar specimens, the new asymptotic boundary effect model has the distinct advantage of being able to predict the strength $f_p$, the fracture toughness $K_{IC}$ and the quasi-brittle fracture behaviour of specimens with different configurations, geometries and sizes directly from one single set of geometrically similar specimens.

Second type of data chosen for verifying the boundary effect model is the fracture properties measured on non-geometrically similar specimens. While SEL cannot be used, the predictions from the asymptotic boundary effect model agree well with those measured properties in both value and trend.

The results from different specimen geometries on the same material are also analysed using the new boundary effect model. It is interesting to see that the nominal strengths measured on both 3-p-b and CT specimens follow the same quasi-brittle fracture curve, and all results of 3-p-b and CT specimens contribute the prediction of the tensile strength and the fracture toughness.

Overall the asymptotic boundary effect model is more versatile and flexible comparing to common size effect models such as SEL. It can be used to characterise the quasi-brittle fracture behaviour of various types of specimens, geometrically similar or non-geometrically similar specimens, large or small specimens, single geometry or multi-geometry specimens. The two important fracture properties, the tensile strength and the fracture toughness can directly estimated from the simple fracture mechanics test with only maximum load recorded. No separate test is necessary to measure the tensile strength. When fracture mechanics tests are carried out using different types of specimens and/or several sets of geometrically similar specimens with different $\alpha$-ratios, all the data from these tests will contribute to the predictions of the fracture properties $f_p$ and $K_{IC}$ and the material quasi-brittle fracture behaviour. This reduces the bias from a single set of geometrically similar specimens.

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