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Micro-mechanical Analysis of Fiber Reinforced Cementitious Composites

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Abstract

This paper discusses the mechanism appearing during fiber debonding in fiber reinforced cementitious composites with special emphasis on Engineered Cementitious Composites (ECC). The investigation is performed on the micro scale by use of a Finite Element Model. The model is 3 dimensional and the Fictitious Crack Model (FCM) and a mixed mode interface model are implemented. It is shown that the cohesive law for a unidirectional fiber reinforced cementitious composite can be found through superposition of the cohesive law for mortar and the fiber bridging curve. A comparison between the numerical and an analytical model for fiber pull-out is performed.

1. Introduction

Fiber bridging, i.e. fibers bridging a propagating and opening matrix crack, is a fundamental mechanism governing the nonlinear behavior of fiber reinforced cementitious composites. The stresses carried across the crack by the bridging fibers are often described with an (average) cohesive law. When a cohesive law is applied for the matrix crack as well, an average cohesive law emerges describing the crack in the composite.

Li et al. (1993) suggested using a superposition scheme for the fiber bridging cohesive stresses and the matrix cohesive law (the Fictitious Crack Model, FCM, Hillerborg et al., 1976) in order to derive an average cohesive law for fiber reinforced concrete. Engineered Cementitious Composite (ECC), (see e.g. Li, 2003) is a high performance, discontinuous fiber reinforced cementitious composite, which - in contrast to conventional fiber reinforced concrete - is characterized by its ability to undergo strain-hardening in tension. Strain-hardening is achieved through multiple cracking of the material. For strain-hardening to occur in an ECC material it is required, that the criteria for multiple cracking are satisfied. These criteria require that (1) the maximum fiber bridging stress is higher than the stress at which cracking is initiated and that (2) the cracks propagate in a steady state manner in an infinitely large specimen (see Li et al. 1992). Both criteria can be expressed in terms of the average cohesive law for the composite. Thus, in order to engineer ECC materials it is essential to be able to predict the average cohesive law for the composite. The cohesive law for the mortar can be found e.g. from a wedge splitting test and an inverse analysis (see e.g. Østergaard 2003). A closed form solution for the total response of the fibers has been derived (see e.g. Lin et al. 1999). The approach in the derivation is first to derive an analytical solution for the fiber debonding and pull-out case and then integrate this solution over the crack surface for random orientation and position of the fibers. Having arrived at the cohesive laws for the matrix and the fibers respectively, the remaining question now is: can the cohesive law for the ECC material be found through a simple superposition of the two fundamental laws or will there be effects that make superposition invalid? Or in other words, is the debonding and subsequent pull-out case representative of the fiber bridging, which takes place in the composite?

One effect that might be able to cause superposition to be invalid is described by Cook and Gordon (1964). In their paper they describe how a tensile stress field is formed in front of a crack tip and how this tensile stress field can cause debonding of a weak interface in front of the crack. If significant fiber-matrix debonding takes place before the crack tip reaches the fiber, the fiber debonding case with an initially perfect fiber-matrix interface would not be representative of the fiber bridging case and a direct superposition would not be valid during crack initiation. In the present paper this effect will be denoted the Cook/Gordon effect. Another effect possibly invalidating superposition could be matrix spalling (Leung and Li 1992, Leung and Chi 1995) taking place at the crack surface during inclined fiber debonding and pull-out. However, here matrix spalling is assumed to be a phenomenon primarily associated with fiber pull-out and thus not associated with the initial matrix crack propagation and fiber-matrix debonding. In the present investigation only initial matrix crack propagation and the associated fiber-matrix debonding is investigated thus matrix spalling is not included and only a fiber oriented perpendicular to the matrix crack

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surface is modeled.

Though the present approach has many similarities with other investigations of the fiber-matrix debonding and pull-out problem (e.g. Stang et al. 1990, Naaman et al. 1991, Leung and Li 1991 and others) and matrix crack formation in fiber composites (e.g. Aveston, Cooper and Kelly 1971, Budiansky, Hutchinson and Evans 1986), emphasis is here placed on the interaction between fiber-matrix debonding and matrix crack formation. In the present approach matrix cracking is described using cohesive crack modeling originating from the FCM. Though the ECC matrix is often considered brittle, Dick-Nielsen et al. (2004) found the cohesive fracture mechanical approach to be more suitable than Linear Elastic Fracture Mechanics (LEFM) for modeling of crack propagation in paste and mortar. The cohesive crack model assumes the presence of a process zone and implicitly links this process zone to various micro-cracking and bridging mechanisms in the matrix.

The analysis presented is based on the concept of a Representative Volume Element (RVE). An RVE can traditionally be thought of either as an element containing a sufficiently number of microstructural inhomogeneities in order for it to be considered macroscopically homogeneous. Alternatively, it can be thought of as a (small) repetitive element from which a true RVE can be constructed. In both cases the element is subjected to boundary conditions which would introduce a homogeneous stress and strain state in the element, if the element was homogeneous, thus allowing for simple interpretation of the behavior of the inhomogeneous element in terms of average properties such as stiffness. Here the latter approach will be taken.

In the present paper the validity of the superposition scheme of the fundamental cohesive laws in order to obtain the average cohesive law for a typical ECC composite is investigated. To investigate whether superposition of the cohesive laws for the mortar and fiber bridging is valid, a Finite Element Method (FEM) model is set up. The three basic cases: debonding of a straight fiber perpendicular to the crack face, crack propagation in pure mortar and crack propagation in an RVE with mortar and fiber are analyzed and a parameter study is performed. Finally, a comparison between the numerical and an analytical model for fiber debonding and pull-out is performed.

2. Model description

2.1 Mesh

To investigate crack propagation in an RVE, a FEM model is used. Symmetry is assumed and the RVE is only representing one side of the crack (see Fig. 1). The aim of the investigation is to determine whether superposition of the cohesive laws for the fiber and the mortar can be applied in order to find the cohesive law for the RVE. Particularly, it will be investigated how the initial fiber-matrix debonding takes place in the fiber pull-out case and in the model involving matrix cracking, respectively. Therefore only the first part of the fiber-matrix debonding is relevant and the height of the RVE is chosen so the debonding of the fiber will not reach the end of the model for relevant matrix crack openings. To control where the crack propagation starts a straight notch is introduced in the crack plane.

For modeling of the fiber, 15 node, wedge shaped, solid elements are used and for the mortar 20 node, cubic shaped, solid elements are used. The elements are based on quadratic interpolation and Gauss integration. Between the mortar and the fiber and in the symmetry crack plane 8+8 nodes interface elements are used. The model contains 3984 elements. The computation time was in the order of 14 hours on a computer with a 2.4 GHz Intel Xeon processor and 3 Gb RAM.

In the model the fiber is assumed to have a diameter of 40 µm and the length of the sides, b in the (x,y)-plane are 251 µm (symmetry plane). This gives a fiber volume percent of 2, which is typical for ECC. The height, $h_z$ of the RVE is 4 mm. The length of the PVA fibers are typically 12 mm, but because the simulations end before the debonding of the fiber is completed, only 4 mm is modeled.

The model containing the fiber as well as the mortar crack (in the symmetry crack plane) is referred to as the total model. When the mortar crack is replaced with a free surface, the model can be used to simulate fiber debonding and pull-out. For convenience this model is referred to as the fiber pull-out model. Replacing the fiber by mortar and letting the mortar crack interface cover the entire bottom surface the model can simulate crack propagation in pure mortar.

2.2 Constraints

The displacements perpendicular to the end planes in the (x,y)-plane are constrained. This causes the load to be applied as a displacement load. The nodes in each side of the RVE in the (z, x)- and (z, y)-plane are tied together in the direction perpendicular to the plane. This causes the edges in the (x, z)- and (y, z)-planes to remain plane and result in a stress state throughout the RVE in average equal to plane stress.
2.3 Mixed mode cohesive crack model

The fiber-matrix interface crack as well as the mortar crack is modeled using a mixed mode cohesive model where the normal stresses, $\sigma$, and the shear stress, $\tau$, depend on both the displacement in the tangential $\delta_t$ and the normal $\delta_n$ direction (see Walter et al. 2005).

$$\sigma = \sigma(\delta_t, \delta_n)$$

$$\tau = \tau(\delta_t, \delta_n)$$

In addition to the usual softening relations between normal deformation and normal stress and shear deformation and shear stress, softening of the cohesive normal stress law takes place due to a shear deformation and the same apply for the shear stress law and deformations in the normal direction. The model is incremental and has been implemented into the commercial finite element program DIANA.

2.4 Deriving the average cohesive law from the RVE

The aim is to derive an average cohesive law $(\sigma_w, w)$ for the RVE. Here $\sigma_w$ is the applied stress and $w$ is the average crack opening. The average crack opening, $w$, can be found by taking the total elongation, $\delta$, of the RVE and subtract the elastic elongation (see Fig. 2) where the total elongation is found from the FEM calculation.

The average crack opening, $w$, can be found from the relation below, where the elastic strain, $\varepsilon$, can be found from the applied stress, $\sigma$, and the composite stiffness. Using these equations all effects from non-uniform crack opening and initial debonding will be included in the average crack opening.

$$\delta = \varepsilon \cdot L + w/2$$

$$\delta = \sigma \left( E_f \cdot \xi_f + \left(1 - \xi_f\right) \cdot E_m \right) \cdot L + w/2$$

$$w/2 = \delta - \sigma \left( E_f \cdot \xi_f + \left(1 - \xi_f\right) \cdot E_m \right) \cdot L$$

Here $E_f$ and $E_m$ are the plane stress elastic moduli for the fiber and the mortar respectively, $\xi_f$ is the fiber volume concentration and $\sigma$ is the average stress applied by the prescribed displacement. This formulation enables the determination of an average crack opening without integrating over the actual crack opening profile.

Equation (4) is applied in the interpretation of the fiber pull-out model as well as the pure mortar model. This makes comparison between the obtained $(\sigma_w, w)$-relations from the different models consistent.

2.5 Material parameters

In the next section the results from the performed investigations are presented. First the results for one set of material parameters are presented and afterwards a parameter study is performed. Typical data from a PVA fiber ECC material are considered.

To perform the FEM calculation two sets of cohesive laws are needed for each calculation, one for the mortar crack and one for the interface between mortar and the fiber. The cohesive law for the mortar is based on experimental investigations on a typical ECC matrix (obtained by Shuxin Wang, University of Michigan) where a bi-linear cohesive law is determined from wedge splitting tests and inverse analysis. The cohesive law for the interface between the mortar and the fiber is difficult to measure. Therefore values determined by Shao, Li and Shah (1993) are taken as a basis for the mode II cohesive law, while the mode I cohesive law is estimated. To get a better understanding of the influence of these cohesive laws a parameter study is performed.

The mode I cohesive law for the mortar is assumed to be bi-linear as shown in Fig. 3. The following parameters were determined: the tensile strength $f_t = 2.83$ MPa, the stress-separation constants $a_1 = -156$ mm$^{-1}$, $a_2 = -9.74$ mm$^{-1}$ and $b_2 = 0.241$, the fracture energy $G_F = 14.1$ N/m, the elastic modulus $E = 31.0$ GPa and the Poisson’s ratio $\nu = 0.2$. The cohesive law for the shear stresses (mode II) in the mortar is not significant, since the crack will mainly propagate in a mode I. This is con-
firmed by calculations performed during the present investigation. The cohesive mode II law for the mortar is chosen as a linear relation: \( \tau_{\text{max}} = 3.0 \text{ MPa} \) and the slope of the stress-separation curve \( a_1 = -50 \text{ mm}^{-1} \).

The Young’s modulus for the PVA fibers is \( E_f = 42.8 \) GPa and the Poisson’s ratio is chosen to \( v = 0.2 \). This Poisson’s ratio is chosen in order to isolate the Cook/Gordon effect from any influence of a possible Poisson’s effect. In the parameter study the effect of the Poisson’s ratio will be investigated separately. The cohesive mode II law for the fiber-matrix interface is also bi-linear and has the following values: The shear strength \( \tau_{\text{max}} = 3 \text{ MPa} \), constants \( a_1 = -222 \text{ mm}^{-1}, a_2 = -19.6 \text{ mm}^{-1} \) and \( b_2 = 0.392 \). These values are based on measurements of Shao et al. (1993), where a shear strength of 3 MPa was the largest measured. The mode I cohesive law for the fiber-matrix interface is estimated to vary linearly with a tensile strength of 0.5 MPa and a constant \( a_1 \) of -1000 mm\(^{-1}\). Because of the difficulty in measuring the fiber-matrix cohesive law a parameter study will be performed.

3. Results

The approach now is to determine average cohesive laws in terms of stress-separation curves for the pure mortar model, the fiber pull-out model and total model. When these three curves are determined, the interrelationship between the curves is investigated and discussed.

3.1 Stress-separation curves

In Fig. 4 average cohesive laws for the mortar model, the fiber pull-out model and total model are shown. In addition to these curves a curve made from superposition of the pure mortar curve and the fiber pull-out curve is shown. It appears from the figure that the average cohesive law from the total model can be found from a direct superposition of the pure mortar cohesive law and the fiber pull-out curve.

In the next sections some of the mechanisms that appear during the crack propagation and fiber debonding will be discussed. This will give a better understanding of why superposition is valid for the present choice of material parameters.

3.2 Crack propagation in symmetry plane

In Fig. 5 the crack front in the mortar interface is plotted for different loading stages. For an applied stress of 2.32 MPa only the elements close to the notch are open. The crack front is the point where the crack initiation takes place and the normal stress, \( \sigma \) is equal to the tensile strength, \( f_t \).

In the figure it is shown that the crack does not propagate directly across the fiber with a straight crack front. When the crack front gets close to the fiber the crack starts to propagate around the fiber leaving a section behind the fiber closed. As the load is increased, the

\[ \text{Fig. 4 Average cohesive laws.} \]

\[ \text{Fig. 5 Crack front for different loading stages (Load in MPa).} \]
3.4 Stresses along the fiber-matrix interface in the total model

In sections 3.4 and 3.5 the stresses along the fiber-matrix interface at the front of the fiber (where the matrix crack first meets the fiber) will be discussed for the total model and the fiber pull-out model respectively. The fiber front is marked with a thick line in Fig. 6. Fig. 7 (a) shows the applied stress for the total model, Fig. 7 (b) the shear stresses on the fiber-matrix interface and Fig. 7 (c) the normal stresses on the fiber-matrix interface. In Figs. 7 (b) and (c) the curves refer to the values for the different load levels indicated in Fig. 7 (a).

The maximum normal and shear stress in the cohesive laws for the mortar/fiber interface are 0.5 MPa and 3.0 MPa, respectively. In the Fig. 7 it is seen that neither of the maximum stresses are reached due to the mixed mode material model applied to the interface. From the figures it is seen that debonding of the fiber in the total model is initiated by a combination of normal and shear stresses. But this mixed mode debonding process is not a phenomenon that is strictly related to the initiation phase of the debonding process. Early in the debonding process a characteristic shear and normal stress profile are formed and these profiles then move up along the fiber. Further it should be noted that the Poisson’s ratio in the fiber and the mortar in this calculation are identical. A separate investigation of the effect of different Poisson’s ratio in the fiber and matrix is performed later.

3.5 Stresses along the fiber-matrix interface in the fiber pull-out model

In the previous section the stresses along the fiber front was discussed for the total model. In this section a similar investigation is performed for the fiber pull-out model. Fig. 8 (a) shows the applied stress for the fiber pull-out model, Fig. 8 (b) the shear stresses on the fiber-matrix interface and Fig. 8 (c) the normal stresses on the fiber-matrix interface. In Fig. 8 (b) and (c) the curves refer to the values for the different load levels indicated in Fig. 8 (a).

The results from the fiber pull-out model are very similar to the results obtained from the total model. Again the debonding is initiated due to a mixed mode stress combination. The tensile stress field in front of the debonding zone is not present in the analytical fiber pull-out model, which is the foundation for the derivation of the fiber bridging relation by Lin et al. (1999). The influence of leaving out the tensile stress field in

![Fig. 6 (a) Cut in model.](image)

![Fig. 6 (b) Debonding versus crack propagation.](image)

![Fig. 7 (a) Applied load, (b) the shear stresses on the fiber-matrix interface and (c) the normal stresses on the fiber-matrix interface.](image)
the derivation of the analytical fiber pull-out model will be discussed in section 3.9. As stated in the previous section these tensile stresses are present even without a difference in the Poisson's ratio in the fiber and the matrix.

The similarity of the stress profiles for the total model and the fiber pull-out model, together with the weak effect of the Cook/Gordon mechanism and the fact that the mortar crack propagates through the RVE without initiating significant debonding, explains why superposition between the stress-separation curves for the mortar and fiber is valid.

### 3.6 Comparison of the Cook/Gordon approach and the cohesive approach

In order to investigate the effect of the applied fracture mechanical approach on the significance of the Cook/Gordon effect, the stress field ahead of the crack model applied by Cook and Gordon (1964) is compared with the stress field ahead of the cohesive crack applied in the present approach.

Cook and Gordon analyzed the stress state around a crack shaped as a flat ellipsoidal hole in an infinite sheet loaded in uni-axial tension. The ellipse has the semi-axes $a$ and $b$ resulting in a crack tip radius $r (r = b^2/a)$. They assumed that the crack tip radius is of molecular dimensions and that it remains constant during crack propagation in a brittle medium. In Fig. 9 the stress state in front of the crack tip is plotted according to Cook and Gordon (1964) for the ratio $a/b = 100$. In the figure $\sigma_{xx}$ are the stresses parallel with the crack plane and $\sigma_y$ are perpendicular to the crack plane. Stresses have been normalized with respect to the maximum normal stress $\sigma_{yy}$, which is identified as the tensile strength of the material.

The magnitude of the crack tip radius is assumed by Cook and Gordon to be 0.2 nm. This means that the peak in the normal stresses parallel to the crack plane $\sigma_x$ will appear very close to the crack tip. The ratio between the two peak stresses is found to be just below 1/5. To relate these results to the cohesive approach a sheet with the width 0.6 m and the height 0.5 m is modeled using FEM. The sheet is loaded in uni-axial tension and contains a slit like stress free crack with the length 4 mm ($2a_0$). The dimensions are chosen such that the sheet can be regarded as infinite compared to the initial crack. The model contains an interface in which the crack can propagate. The model consist of $20 \times 412$ (height x width) quadrilateral, 8 nodes plane stress elements. The element size increase with a factor 1.025 from the crack tip towards the edge along the width, and with a factor 3 from the crack towards the ends along the height. The material used is identical to the mortar described in section 2.5 ($f_c = 2.83$ MPa). In Fig. 10 (a) matching values of the far field stress and the half crack length, $a$ are plotted.

In Fig. 10 (b) the ratio between the two normal stresses at the crack tip in the cohesive approach is plotted for different crack lengths, $a$. The ratio found in the cohesive approach is of the same magnitude as in the approach adopted by Cook and Gordon. For crack lengths in the micro scale the ratio between the normal stresses found in the cohesive model are significant higher than predicted by Cook and Gordon. The Cook/Gordon effect is therefore more pronounced in the cohesive approach.
In sections 3.1-3.5, the cracking and debonding process for one set of material parameters have been investigated. In order to evaluate the sensitivity of the conclusions on the choice of material parameters, a parameter study is carried out. The parameters being varied are the parameters characterizing the mixed mode cohesive law for the mortar/fiber interface as shown in Fig. 11.

For the normal stresses the tensile strength undergoes the values 0.125 MPa, 0.25 MPa and 0.5 MPa. For the shear stresses the slope of the first branch is varied. This is done by letting the shear strength undergo the values 1.5 MPa, 2 MPa and 3 MPa. Calculations for all combinations of normal and shear strength have been performed. The results obtained in this study are very similar to the ones obtained in the previous sections. The new results will therefore not be illustrated here.

An interesting combination was a low tensile strength (0.125 MPa) and a high shear strength (3 MPa). This could cause debonding in the total model to be initiated due to the Cook/Gordon effect while debonding in the fiber pull-out model would be initiated mainly due to shear stresses. The analysis showed that the low tensile strength did in fact lower the shear stress needed to initiate debonding in the total model. But because the same was valid in the fiber pull-out model, superposition between the cohesive laws for the mortar and fiber pull-out was still valid.

### 3.8 Parameter study of the Poisson's ratio

In the previous sections the fiber and mortar had the same Poisson's ratio. This was chosen in order to isolate the Cook/Gordon effect on the fiber debonding in the total model. In this section the fiber is given a more realistic Poisson’s ratio of 0.35 in order to investigate the effect of the Poisson’s ratio. For the present investigation four calculations are performed. The normal strength in the mode I cohesive law is varied from 0.125 MPa to 0.5 MPa. Finally, a pure mode II calculation is performed. In all calculations the cohesive mode II law with a shear strength of 3 MPa is used (see section 3.7).
fiber pull-out curves for the two Poisson’s ratios end up being very similar, with the curve for a Poisson’s ratio of 0.2 slightly higher than the curve corresponding to a Poisson’s ratio of 0.35.

In Fig. 13 fiber pull-out curves are plotted to illustrate the effect of the Poisson’s ratio for different mode I cohesive laws on the interface.

In the extreme case where the mode I law is so strong that debonding will occur in pure mode II, the Poisson’s effect can be neglected. When comparing Fig. 13 (a) and (b) it is observed that a mode I law with a tensile strength of 0.5 MPa results in a fiber pull-out curve close to the one obtained in the pure mode II calculation. This explains why the Poisson’s ratio has so little influence when applying a mode I law with a tensile strength of 0.5 MPa. For a mode I cohesive law with a tensile strength of 0.125 MPa the difference in Poisson’s ratio between the mortar and the fiber is seen to have a significant influence of results. This is because the increase in normal stresses becomes significant compared with the tensile strength. The conclusion is that if the interface between the fiber and the mortar has a weak mode I cohesive law, it is important to include the effects introduced by the Poisson’s ratio. In this investigation all fibers have been pulled out normal to the crack plane.

Another relevant investigation is the Poisson’s effect on pull-out of fibers inclined to the crack plane. When pulling out a fiber inclined to the crack plane, the Poisson’s effect might be insignificant compared to the large contact pressure between fiber and crack face or possible spalling of the matrix.

3.9 Comparison of the analytical and numerical model for fiber pull-out

In this section a comparison is made between the present numerical model for fiber pull-out and the corresponding analytical model by Lin et al. (1999). Three assumptions are made in order to derive the analytical model: (1) an aspect ratio larger than 100. (This is fulfilled for most fibers and this is also valid for the present numerical model). (2) The slip between matrix and fiber during debonding is negligible hence the shear stresses are constant, \( \tau \). (This is not valid in the numerical model when applying a cohesive law. As illustrated in Fig. 8 a shear stress profile is formed after debonding is initiated and as the debonding propagates the shear stress profile travels with the debonding). The Poisson’s effect is negligible. As shown in the previous section the Poisson’s effect can be neglected, but only in the case where the mode I cohesive strength between

![Fig. 12](image-url) (a) Applied load, (b) shear stresses on the fiber-matrix interface and (c) normal stresses on the fiber-matrix interface. In Figs. (b) and (c) the curves refer to the values for the different load levels indicated in Fig. (a).

![Fig. 13](image-url) Fiber pull-out curves as functions of fiber Poisson’s ratio for different mode I cohesive laws on the interface: (a) Pure mode II, (b) tensile strength of 0.5 MPa and (c) tensile strength of 0.125 MPa.
fiber and mortar is strong. Apart from these three assumptions the analytical model does not take into consideration the tension stress field in front of the debonded zone, hence debonding in the analytical model is pure mode II. The relation between the pull-out force, $P$, and the relative displacement, $\delta$, between the fiber and the matrix in the analytical model is given by the expression below:

$$P = \sqrt{\frac{\pi^2 \tau_\text{f} d_\text{f}^2 (1 + \eta)}{2 \delta} + \frac{\pi^2 G_\text{f} d_\text{f}^2}{2}}$$

(5)

Here $E_f$ and $d_f$ are Young’s modulus and diameter of the fiber respectively, and $\eta = V_m E_m / V_f E_f$ where $V_m$ and $V_f$ is the volume fraction of mortar and fiber respectively. The chemical bond strength, $G_f$, is related to the fracture energy, $G$, of the mode II cohesive law, however in the analytical model, $G_f$ is assumed to be dissipated in a point. Because of the difficulties in deciding which constant frictional bond strength, $\tau_\text{f}$, correspond to the applied cohesive law, a direct comparison between the numerical model and the analytical model is not performed. Instead an investigation is performed with the numerical model on the effect of the mode I cohesive law, which is left out in the analytical model. This is done by comparing fiber pull-out curves obtained from applying a fixed mode II law with a shear strength of 3 MPa together with a variation of the cohesive mode I law. The cohesive laws used are the ones shown in section 3.7. In Fig. 14 the fiber pull-out curves are plotted for a Poisson’s ratio in the fiber of 0.35.

In the figure it is seen that the pull-out load decreases for a decreasing tensile strength in the mode I cohesive law. If the analytical model is calibrated with a fiber pull-out test, the influence of leaving out the mode I part is probably not so significant, because the constants will be affected by the mixed mode conditions. If on the other hand the fiber pull-out is modeled based on independently obtained mode II constants then a significant error can be made by leaving out the mode I part, particularly if the mode I properties are relatively weak.

### 4. Concluding remarks

In the present investigation it has been examined whether superposition of the average cohesive law for the fiber pull-out and the cohesive law for mortar was valid in order to find the average cohesive law for the total model. Special attention was given to the tensile stress field in front of the crack tip, possibly leading to the so-called Cook/Gordon effect, while the investigations were limited to the case of straight fibers arranged perpendicular to the crack surface. The present investigation showed that in general superposition is valid. Furthermore it was found that a low mode I strength in the mortar/fiber interface did lower the shear stress needed to initiate debonding in the total model. However, because the same applied in the fiber pull-out model superposition of the cohesive laws was still valid.

The present investigation further showed that the mortar crack propagation and the subsequent fiber debonding and pull-out essentially are two separate mechanisms in the sense that the mortar crack propagated through the RVE without initiating any significant debonding; the mortar crack grows past the fiber and then gradually back towards the fiber before significant debonding begins.

The influence of a difference in the Poisson’s ratio between mortar and fiber was examined. It was found that the Poisson’s ratio did not have any influence on the problem and the debonding would propagate in an almost pure mode II, when the strength of the mode I interface between fiber and mortar was sufficient high. If on the other hand the mode I strength was low, then the increase in normal stresses due to the Poisson’s effect did lower significantly the force needed to pull-out the fiber due to the mixed mode condition.

Finally, a comparison between an analytical and numerical model for fiber pull-out was conducted. In the analytical model the mixed mode stress condition is not taken into consideration. The conclusion is that as long as the analytical model is calibrated with fiber pull-out tests then the influence of leaving out the mode I part is not significant, because the material constants will be affected by the mixed mode conditions. If the mode I strength is low and the analytical model is not calibrated with tests then leaving out the mode I part can lead to overestimation of the load carrying capacity.

### References

