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Composite Strut and Tie Model for Reinforced Concrete Deep Beams

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Abstract

Monotonic shear loading tests were conducted on three half-scaled reinforced concrete deep beams with shear span-to-depth ratios of 0.5 to 0.75. The obtained test results were investigated in detail based on the experimental measurements and finite element analysis. From these investigations, a new macro model for deep beams was established. This model is composed of two crooked main struts formed between both beam end sections and branched-off sub struts. The compressive force introduced to main struts balances the flexural compression and the external shear force. The bond stress of the longitudinal reinforcement and the tensile force of the stirrup produce the diagonal compression in the sub strut. Theoretically predicted shear strengths of tested deep beams showed good agreement with experimentally observed shear strengths, where the effective strength of concrete was assumed to be 75% of the cylinder strength.

1. Introduction

Deep beams are widely used as perimeter beams of reinforced concrete (RC) frames for functional and architectural reasons. In multi-story wall structures, deep beams are also used for coupling walls constructed side by side. Because the beams have small span-to-depth ratios, brittle failure in shear may be observed under seismic lateral loads for some of them. Examples of such failure due to earthquakes have been reported in the literature (Kim et al. 2001; Paulay 1981; Paulay and Binney 1974). To avoid such failure, a sufficient amount of transverse reinforcement should be provided to induce ductile flexural failure prior to shear failure. An alternative method is the arrangement of diagonal reinforcement in a beam, which was proposed by Park and Paulay (1975).

When a deep beam is subjected to combined bending and shear with anti-symmetric moment distribution, the truss mechanism is hardly developed due to the overlap of tension shift regions of longitudinal reinforcement. That is, the main resisting mechanism is developed by a single strut formed between both end sections of the member. Thus the final shear failure may occur due to crushing of diagonally compressed concrete struts. Based on this basic mechanism, several shear design equations for deep beams have been proposed (ACI 2008; AIJ 1990, 1999; CEB-FIP 1995; Nielsen 1984). However the role of the transverse reinforcement is not necessarily clear.

This paper presents a new macro model to predict the shear strength of deep beams with anti-symmetric moment distribution. Shear loading tests and a nonlinear finite element (FE) analysis for deep beams were conducted to derive the mechanical shear resisting model. The proposed model is composed of two crooked main struts and branched off sub struts, where the contribution of the transverse reinforcement and the bond action of the longitudinal reinforcement are taken into account.

2. Experimental program

Three specimens with various shear span-to-depth ratios and amounts of transverse reinforcement, as indicated in Table 1, were cast and tested to verify the accuracy of the proposed model. The details of typical specimen S-0.75-50 are shown in Fig. 1. Each specimen has a central test region with a 200×300 mm sectional dimension and loading stubs at both ends of the specimen. The clear spans of S-0.5-50, S-0.75-50 and S-0.75-75 were 300, 450 and 450 mm, respectively.

The specimens were designed to fail in shear before yielding of the flexural reinforcement. All the specimens contained four deformed bars with a nominal diameter of 16 mm and a yield strength of 795 MPa as top and bottom reinforcements. Deformed and welded closed stirrups were used in this experiment. The transverse steel bars had the yield strength of 305 MPa and the nominal diameter of 6 mm. The spacing of stirrups of S-0.5-50, S-0.75-50 and S-0.75-75 were 300, 450 and 450 mm, respectively. Normal weight concrete was used in the construction of the test specimens. The average compressive strength of concrete at the time of the beam test was 30.6 MPa.

Figure 2 illustrates the loading equipment and the test setup of the specimens. This loading method was developed by Jirsa et al. (1978), where the rotation of the top stub of the specimen is restricted by two vertical jacks and results in the anti-symmetric moment distribution of the central beam test region. In this system, the
vertical movement of the top stub is free, that is, no additional vertical load is applied to the specimen subjected to lateral shear force by two parallel horizontal jacks. During the test, the rotation angle (drift) of the specimen was measured by two electronic displacement transducers and recorded by a computer controlled data acquisition system. Wire-type strain gauges were attached on the surface of the steel bars to measure the strain of longitudinal and transverse reinforcements. Readings from the strain gauges were used to estimate the tensile and bond stresses of the reinforcing bars.

Table 1 Details of test specimens.

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Concrete ($f'_c$, MPa)</th>
<th>Shear reinforcement</th>
<th>Longitudinal reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Pitch (mm)</td>
<td>$\sigma_{wy}$ (MPa)</td>
</tr>
<tr>
<td>S-0.5-50</td>
<td>30.6</td>
<td>50</td>
<td>305</td>
</tr>
<tr>
<td>S-0.75-50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-0.75-75</td>
<td></td>
<td>75</td>
<td></td>
</tr>
</tbody>
</table>

Note: S-0.5-50 means S (specimen), 0.5 (shear span-to-depth ratio), and 50 (spacing of shear reinforcement, mm)

Fig. 1 Details of specimen (S-0.75-50).

Fig. 2 Test setup of specimen.
3. Experimental results

3.1 Load-displacement response and crack pattern

All test specimens failed in shear due to crushing of diagonally compressed concrete. Table 2 indicates the experimentally observed maximum strength of each specimen. From the comparison between S-0.75-50 and S-0.75-75, the contribution of the shear reinforcement seems to be small.

The shear strengths predicted by ACI 318-08 Code (ACI 2008) and AIJ (Architectural Institute of Japan) Code (AIJ 1990, 1999) are also indicated in Table 2. It can be seen from the table that all the code equations give very conservative shear strengths for test deep beams. More reliable design equations for the shear strength of deep beams with anti-symmetric moment distribution should be established. With respect to the contribution of the shear reinforcement, Fig. 3 shows the experimentally observed shear force versus drift angle responses of test specimens. While all specimens showed almost the same responses up to peak loads, the ductility of the specimen increased as the amount of transverse reinforcement grew higher in the post-peak behavior.

Crack propagation of each specimen for three loading stages is indicated in Fig. 4. After initial development of a diagonal crack, the crack width increased with increases in shear force up to the maximum load. Finally, each specimen reached the maximum capacity showing the crushing of diagonally compressed concrete near the beam end sections.

3.2 Strain of reinforcement

Figure 5 shows the variation of the strain distribution of reinforcing bars. The x-axis of the figure represents the location of strain gauges along a beam length. In the figure, the thick solid line corresponds to the strain at the maximum capacity. After diagonal cracking, stirrup strain increased rapidly and most stirrups yielded at the maximum capacity. With respect to the strain of the longitudinal reinforcement, tension shift was observed after diagonal cracking, and then the strain was maintained in tension. This means that the beam action disappeared over the entire beam length. It can be also seen from Fig. 5 that the longitudinal reinforcement did not yield in any of the loading stages as intended in the design of specimens.

The average bond stress along a beam length was computed from the experimentally obtained strain of the longitudinal reinforcing bars. The computed average bond stresses are indicated in Fig. 6 with the drift angle. The bond stresses were very small and ranged from 1.5 to 2.0 MPa though no splitting bond failure of the longitudinal reinforcement was observed. This may be due to the specific mechanical behavior of deep beams.

4. Analytical investigation

To investigate the shear resisting mechanism of deep beams, a nonlinear FE analysis was carried out for specimens S-0.5-50 and S-0.75-50, with shear span-to-depth ratios of 0.5 and 0.75, respectively. The FE analysis was performed using a two-dimensional FE analysis program, VecTor2, developed by Vecchio and his research group at the University of Toronto in Canada. The analytical model used in the VecTor2 was the Modified Compression Field Theory (MCFT) (Vecchio and Collins 1986) and the constitutive models that were used (Kent and Park 1971; Kupfer et al. 1969; Popovics 1970) are listed in Table 3. The details of each constitutive model and their implementation into the VecTor2

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Concrete ( f'_c ) (MPa)</th>
<th>( \rho_w \sigma_{wy} ) (MPa)</th>
<th>( V_{AIJ} ) (kN)</th>
<th>( V_{ACI} ) (kN)</th>
<th>( V_{exp} ) (kN)</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-0.5-50</td>
<td>30.6</td>
<td>1.95</td>
<td>190</td>
<td>149</td>
<td>297</td>
<td>Shear failure (crushing)</td>
</tr>
<tr>
<td>S-0.75-50</td>
<td></td>
<td>1.30</td>
<td>181</td>
<td>149</td>
<td>290</td>
<td></td>
</tr>
<tr>
<td>S-0.75-75</td>
<td></td>
<td></td>
<td>174</td>
<td>149</td>
<td>274</td>
<td></td>
</tr>
</tbody>
</table>

\( V_{AIJ}, V_{ACI} \): shear force calculated by AIJ and ACI Code, respectively; \( V_{exp} \): Experimentally obtained shear force.
Fig. 4 Crack propagation.

Fig. 5 Strain of reinforcements along length of specimen.
A comparison of the analytical and experimental results for the shear force versus displacement response of test specimens is shown in Fig. 7. In S-0.5-50, the maximum load was reached at step 5 in the computational process, and in S-0.75-50, at step 12. These predicted maximum shear strengths were almost the same as the experimentally observed ones. After the peak load, however, the predicted curve showed a somewhat premature reduction of load carrying capacity compared to the experimental one.

Figure 8 illustrates the crack patterns of S-0.5-50 and S-0.75-50 at loading steps 8 and 19, respectively. It can be seen from this figure that the theoretically predicted crack propagation agreed well with experimental observation (see Fig. 4). Based on the comparison results, the applied FE analysis method in this paper can be used for predicting the internal stress flow of specimens.

Figure 9 shows the magnitude and direction of principal tensile and compressive stresses at three loading stages, i.e. pre-peak, peak and post-peak loads. From the figure for the pre-peak stage, it can be seen that one main compressive strut developed on a beam diagonal. At peak and post-peak loads, however, this diagonal strut seems to separate into two main struts as the width of the diagonal crack increases. These two main struts are located at both sides of a beam diagonal. During this process, the truss action produced by the stirrup tension and the diagonal compression of the concrete existed outside of the main strut. However, this truss action was small and most of the shear force could be resisted by the diagonal main strut.
The above-mentioned phenomena can be confirmed by the shear stress and principal compressive stress distributions along a section height, as shown in Fig. 10. At the critical section of the beam, Section A-A’, the location of the peak stress shifted from the section edge to the center of the section as the progress of loading steps. This peak shift was observed in both beams and may be due to the local crushing of the concrete at the extreme compression fiber. A similar phenomenon was observed at Section B-B’ of the S-0.5-50 beam. The other specific behavior was the separation of the diagonal main strut at and after peak load. At Section C-C’ of S-0.5-50, and at Sections B-B’ and C-C’ of S-0.75-50, the stress distributions had two peaks at and after peak load. This suggests that the main diagonal strut is going to separate into two struts due to the change of internal mechanism. Final failure of the beam occurred due to crushing of the diagonally compressed concrete, and average principal compressive stress at peak load was from 0.7 to 0.8 times the compressive strength of concrete.

5. Curved dual strut model (CDS-M)

When a deep beam with a shear span-to-depth ratio of one or less is subjected to bending moment and shear, uniform compression stress field (in other word, truss mechanism) is hardly developed due to the tension shift of the flexural reinforcement. This means that the main resisting mechanism for shear should be a diagonal strut developed between both beam ends. This has been pointed out experimentally and theoretically in past research works. Therefore design equations for deep beams have been developed based on the strut and tie model. However, the bond action of longitudinal reinforcement and the force of the transverse reinforcing bar cannot be properly evaluated using the strut and tie model, because the model is able to control the shear strength of deep beams only by the compressive strength of diagonal strut.

In this chapter, a new shear resisting model incorporating dual curved struts is proposed, where the bond action of longitudinal reinforcement and the force of the stirrup are taken into account. Based on the numerical
Experimental observations and the theoretical analysis of the concrete near the pivot of a fan is not clear and hardly modeled because of the changing width of each fan bone. In this study, the shear resisting mechanism is modeled using the lower bound of the theory of plasticity. The feature of this modeling is that two curved diagonal main struts are introduced as the main shear resisting mechanism in conjunction with supplemental sub struts. This curved dual strut model was devised from the experimental observations and the theoretical analysis using the FE method.

The new model is indicated in Fig. 11 for two beams with different shear span-to-depth ratios. All of the main

5.1 Modeling and basic assumptions

When a deep beam is subjected to a reverse symmetric moment under constant shear (e.g. a coupling beam), the shear resisting mechanism is generally expressed by a diagonal direct strut and supplemental fan shaped mechanism (AIJ 1999). However, when the theory of plasticity is applied for the modeling, the stress condition of the concrete near the pivot of a fan is not clear and hardly modeled because of the changing width of each fan bone.

In this study, the shear resisting mechanism is modeled using the lower bound of the theory of plasticity. The feature of this modeling is that two curved diagonal main struts are introduced as the main shear resisting mechanism in conjunction with supplemental sub struts. This curved dual strut model was devised from the experimental observations and the theoretical analysis using the FE method.

The new model is indicated in Fig. 11 for two beams with different shear span-to-depth ratios. All of the main

![Fig. 10 Distribution of shear and principal stresses.](image)
and sub struts are subjected to an axial compressive stress, $\sigma_e$. Two main struts are changing their angle along a beam length due to the external compression given by the sub struts. At an intersection point between the main and sub struts, a triangular hydro pressure stress field is assumed. The axial compressive forces of sub struts balance the horizontal bond forces of the longitudinal reinforcement and the vertical tension forces of the stirrup at the intersection points of the sub strut and the longitudinal reinforcement. The basic assumptions used in the proposed model are as follow:

(a) At the maximum shear resistance, all of the shear reinforcement is to be yielded. This comes from the experimentally observed strain distributions of stirrups indicated in Fig. 5.

(b) Based on the experimental results described in Section 3.2, the distribution of the bond stress of longitudinal reinforcement is assumed to have a triangular shape as recognized from Fig. 11.

(c) The resultant vector of the force of the sub-strut located in the C region in tension indicates point T. Equation (1) shows the vector equation, and the angles of the other two sub-struts can be computed from Eq. (2) based on the assumptions of (a) and (b) above.

\[ \tan \theta_1 = \frac{2d}{a} \]  
\[ \tan \theta_2 = \frac{1}{3} \tan \theta_1, \quad \tan \theta_3 = \frac{1}{5} \tan \theta_2 \]  

where $d$ is the distance between the centers of the strut and the bottom re-bar; $a$ is a third of the clear span of a beam; and $\theta$ is the angle between the sub-strut and the longitudinal axis.

(d) As shown in Fig. 11, the rim of crooked main strut A in the A region is bent by the influence of the sub-struts, and thus it is assumed that the Crooked Main Strut starts from the center of the main re-bar.

(e) The width of crooked main strut A is set below one tenth of the effective depth of the member. This imitation is based on the result of the FE analysis for the reason that the region traversing the diagonal shear crack becomes quite large if the width of strut A is too large, as can be seen in Fig. 11.

**5.2 Computation of basic values for analysis**

First of all, the angles of the sub-struts are determined based on the experimental and analytical results. Figure 11 shows that the shear resistance mechanism is governed by a geometric condition. Based on the assumption that the stress of each strut is the same, the shear force can be obtained by computing the width of the strut that satisfies the geometric constraint.

Figure 11 can be drawn in more detail as Fig. 12(a). The force per unit length and the coordinate of strut B can be given by Eqs. (9) and (10), respectively.

\[ C_{str} = \sigma_e \cdot b \cdot s_1 \]  

where $\sigma_e$ is the stress of concrete on the strut; $b$ is the width of the cross section; $x$ is the width of strut A; $\beta$ is the angle between the strut A and the longitudinal axis; $x$ and $y$ are the x- and y-axes in the x-y coordinate system, respectively.

Similarly, the force of strut E and its coordinate can be obtained as

\[ C_{ste} = \sigma_e \cdot b \cdot s_1 \]  
\[ y = -(\tan \beta) x + d \]  

where $\sigma_e$ is the stress of concrete on the strut; $b$ is the width of the cross section; $x$ is the width of strut A; $\beta$ is the angle between the strut A and the longitudinal axis; and $L$ is the clear span of the member.

The force per unit length and the coordinate of strut B can be given by Eqs. (15) and (16) based on Eqs. (4) and (6).

\[ x = \left( \frac{d + (a/2) \tan \theta_1}{\tan \beta - \tan \theta_1} \right) = \alpha \]  
\[ y = -(\tan \theta_1) \cdot x - (a/2) \tan \theta_1 \]  

The B region of Fig. 11 is illustrated in detail in Fig. 12(b). The force per unit length and the coordinate of strut B can be given by Eqs. (9) and (10), respectively.

\[ C_{str} = \sigma_e \cdot b \cdot s_2 \]  
\[ y = -(\tan \phi_1) x + \alpha (\tan \phi_2 + \tan \theta_1) - (a/2) \tan \theta_1 \]  

where $s_2$ is the width of strut F and $\phi_1$ is the angle between the strut B and the longitudinal axis.

\[ x_2 = \left[ \left( x \cdot \sin \beta + s_2 \cdot \sin \theta_1 \right)^2 + \left( x \cdot \cos \beta + s_2 \cdot \cos \theta_1 \right)^2 \right]^{1/2} \]  

The coordinates for the nodal point II of struts A and E can be given by the following Eqs. (7) and (8) based on Eqs. (4) and (6).

\[ x = \left( \frac{d + (a/2) \tan \theta_1}{\tan \beta - \tan \theta_1} \right) = \alpha \]  
\[ y = -(\tan \theta_1) \cdot x - (a/2) \tan \theta_1 \]  

where $\sigma_e$ is the stress of concrete on the strut; $b$ is the width of the cross section; $x$ is the width of strut A; $\beta$ is the angle between the strut A and the longitudinal axis; and $L$ is the clear span of the member.

The coordinates for the nodal point III of struts B and F can be given by the following Eqs. (15) and (16) based on Eqs. (10) and (14).

\[ y = \left( \frac{(a/2) \tan \theta_1 - (3a/2) \tan \theta_2 - \alpha (\tan \phi_1 + \tan \theta_1)}{(\tan \theta_2 - \tan \phi_1)} \right) = \gamma \]  

where $\sigma_e$ is the stress of concrete on the strut; $b$ is the width of the cross section; $x$ is the width of strut A; $\beta$ is the angle between the strut A and the longitudinal axis; and $L$ is the clear span of the member.
Fig. 11 Crooked dual strut model.
\[ y = -(\tan \theta_2) \cdot \gamma - (3a / 2) \tan \theta_2 \]  

(16)

**Figure 12(c)** depicts the C region of Fig. 11 in detail. The force per unit length and the coordinate of strut C can be given by Eqs. (17) and (18), respectively.

\[ C_{SRC} = \sigma_c \cdot b \cdot x_3 \]  

(17)

\[ y = -(\tan \theta_2) \cdot x + \gamma (\tan \phi_2 - \tan \theta_2) - (3a / 2) \tan \theta_2 \]  

(18)

\[ \tan \phi_2 = \left( \frac{x_4 \cdot \sin \beta + s_3 \cdot \sin \theta_1 + s_2 \cdot \sin \theta_2}{x_4 \cdot \cos \beta + s_3 \cdot \cos \theta_1 + s_2 \cdot \cos \theta_2} \right) \]  

(19)

where \( x_4 \) is the width of strut C, and \( \phi_2 \) is the angle between the strut C and the longitudinal axis.

Sub-strut G can be expressed as the following equations.

\[ C_{STG} = \sigma_c \cdot b \cdot s_\theta \]  

(21)

\[ y = -(\tan \theta_3) \cdot x - (5a / 2) \tan \theta_3 \]  

(22)

where \( s_\theta \) is the width of strut G and \( \theta_3 \) is the angle between the strut G and the longitudinal axis.

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**Diagram**

**Figure 12** Details of crooked dual strut model.

(a) A region  
(b) B region  
(c) C region  
(d) D region
The coordinates for the nodal point IV of struts C and G can be given by the following Eqs. (23) and (24) based on Eqs. (18) and (22).

\[
x = \left[ \frac{y(\tan \phi_1 - \tan \theta_j) + (5a/2) \tan \theta_j - (3a/2) \tan \theta_j}{\tan \phi_1 - \tan \theta_j} \right] = \lambda
\]

\[
y = -(\tan \theta_j) \cdot \lambda - (5a/2) \tan \theta_j
\]

The force per unit length and the coordinate of strut D can be given by Eqs. (25) and (26), respectively.

\[
C_{STD} = \sigma_c \cdot b \cdot x_d
\]

\[
y = -(\tan \phi_d) \cdot x + \lambda(\tan \phi_d - \tan \theta_j) - (5a/2) \tan \theta_j
\]

\[
\tan \phi_d = \left( \frac{x_1 \cdot \sin \beta + s_1 \cdot \sin \theta_j + x_2 \cdot \sin \theta_j + s_2 \cdot \sin \theta_j}{x_1 \cdot \cos \beta + s_1 \cdot \cos \theta_j + x_2 \cdot \cos \theta_j + s_2 \cdot \cos \theta_j} \right)
\]

\[
x_d = \left[ (x_1 \sin \beta + s_1 \sin \theta_j + x_2 \sin \theta_j + s_2 \sin \theta_j)^2 + (x_1 \cos \beta + s_1 \cos \theta_j + x_2 \cos \theta_j + s_2 \cos \theta_j)^2 \right]^{1/2}
\]

where \( x_d \) is the width of strut D and \( \phi_d \) is the angle between the strut D and the longitudinal axis.

The coordinates for the point V of strut D in Fig. 12(d) can be given by the following equations.

\[
x = -(3a)
\]

\[
y = j_i - d - \eta
\]

\[
\eta = \left[ x_1 \cdot \cos \beta + (x_1) / (2 \sin \phi_d) + x_1 \cdot \sin \beta \cdot \tan \phi_1 - x_1 / (2 \cos \beta) \right]
\]

where \( j_i \) is the distance between the centers of the main re-bars.

Substituting Eqs. (29) and (30) for Eq. (26), Eq. (31) is obtained. Then, the value of the concrete stress in the strut is used to solve the equation to determine the width of each strut.

\[
j_i \cdot d - \eta + (5a/2) \tan \theta_j - 3a \cdot \tan \phi_1 - \lambda(\tan \phi_1 - \tan \theta_j) = 0
\]

In addition, the width \( c \) of strut H in the longitudinal axis can be expressed as in the following equation.

\[
c = x_4 \cdot \sin \beta + x_4 \cdot \cos \phi_4
\]

Thus, the desired shear force of the member is given by the following equation.

\[
V' = \sigma_c \cdot b \cdot c = \sigma_c \cdot b(x_4 \cdot \sin \beta + x_4 \cdot \cos \phi_4)
\]

As has been shown so far, the shear force can be computed by solving a geometric problem. The flow of the analysis is shown in Fig. 13. It involves several processes, namely (1) determining the angles \( \theta_1 \), \( \theta_2 \), and \( \theta_3 \) of the three sub-struts that are formed by the sum of the resultant forces of the bond stress of the main re-bar and the tensile force of shear reinforcement; (2) assuming the compressive stress (\( \sigma_c \)) of the strut and the angle \( \beta \), which is formed by main strut A and the longitudinal axis; and (3) calculating the width of the crooked main strut, which satisfies the geometric constraints illustrated in Fig. 11.

6. Analysis results and discussion

Experiments and FE analysis with the shear-span ratio as the variable of interest were carried out to propose a new shear resistance model for the crooked main- and sub-struts and an analytical method to predict the shear force, as illustrated in Fig. 11(a) and Fig. 11(b) for the shear-span ratios of 0.5 and 0.75, respectively. From the figures, it can be seen that the main strut A inclines (is crooked) more as the shear-span ratio gets shorter, and the main strut exhibits the tendency to bend a little more due to the sub strut under the influence of the bond stress of the main re-bar, which gets larger as the shear-span ratio becomes greater.

Additionally, the characteristic mechanism of the proposed model in this study is that the shear force is
transmitted through a by-passing route to avoid the diagonal shear cracks, which connect both ends of the beam, as the width of the diagonal cracks increases after the occurrence of the diagonal shear crack. This allows the rather free use of the effective factor of concrete compressive strength inside the strut.

Shown in Fig. 14 obtained from analytical simulations is the relationship between the shear-span ratio and the effective strength of concrete inside the strut. There is good agreement between the model prediction and the test results of the two cases. Additionally, the crooked main-strut and sub-strut mechanism, based on the experimental and FE analytical results, are most consistent with each other when the effective compressive strength of concrete is set to 0.75 times the cylinder strength.

Additionally, when the effective factor of concrete compressive strength of Fig. 14 is set to 0.75, the crooked main-strut and sub-strut mechanism works only for the case of the member with shear-span ratio below 0.75. When the geometric behavior for the member with shear-span ratio below 0.75 is determined by the analysis results so far discussed, the shear resistance of the member can be predicted with much improved accuracy by the analytical computing method proposed and explained in this study. However, to increase the precision of the macro model proposed in this study, it is necessary to determine the bond behavior of the main re-bars in more detail. Thus, additional studies in this area are expected in near future.

7. Conclusions

An experimental investigation was carried out for the RC beams with a main variable of the shear-span ratio. The test specimen was subjected to monotonic antisymmetric moment. An analytical method to compute the shear force of the deep beams and a new macro model for the shear resistance mechanism were proposed based on the results of the experiments and numerical analysis. From the experimental and analytical investigations, the following conclusions can be deduced:

1. FE analysis was carried out on the test specimens with shear-span ratios of 0.5 and 0.75 to delineate the stress flow and the failure process, and the numerical results showed good correlation with the experimental results.
2. An analytical method for computing the shear force and a new macro model, which considered the bond action of the main re-bar and used the geometric property of the crooked dual strut model, was proposed.
3. The analytical results revealed that the proposed shear resistance action was observed at a shear-span ratio of less than 0.75. In addition, when the effective factor of concrete compressive strength was 0.75, the results of the analysis and experiment were the most consistent with each other.

Notations

The following symbols are used in this paper:

- $a$: a third of the clear span of member
- $b$: width of beam section
- $c$: length of strut H in the longitudinal axis
- $C_{ST}$: force per unit length of strut
- $d$: distance between the centers of the strut and bottom re-bar
- $f'_{c}$: cylinder compressive strength of concrete
- $j_{t}$: distance between centers of main re-bars
- $L$: clear span of member
- $s_{i}$: width of sub-strut
- $V$: shear force of member
- $W_{cr}$: crack width
- $x$, $y$: $x$- and $y$-axes in the $x$-$y$ coordinate system, respectively
- $x_{i}$: width of main axes
- $\beta$: angle between main strut A and longitudinal axis
- $\phi$: angle between main strut B-D and longitudinal axis
- $\theta$: angle between sub-strut and longitudinal axis
- $\rho_{s}$: shear reinforcement ratio

Fig. 14 Shear force versus effectiveness factor of concrete relationships.

(a) S-0.5-50

(b) S-0.75-50
σc = stress of concrete on the strut
σwy = yield strength of shear reinforcement

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