Scientific paper

Local Stress Field Approach for Shear Failure Assessment of Reinforced Concrete Members

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Received 14 December 2009, accepted 24 May 2010

Abstract

Despite extensive research on behavior of reinforced concrete in shear, there is still considerable disagreement among researchers in proposing and using a rational way of calculating the shear strength of reinforced concrete members. Due to lack of a universally accepted model for shear, shear design provisions still generally consist of empirical relationships that differ from code-to-code. In this study, a simple rational formula for calculating the shear strength of reinforced concrete elements is proposed by conducting a systematic parametric study on the response of RC elements in shear. This is done by using a computational model which simulates the post cracking behavior of RC membrane element on the basis of local stress transferring mechanism between adjacent cracks and microscopic stress transfer across cracks. Through a comparison with extensive available experimental tests conducted on RC elements, it is shown that the proposed rational relation can appropriately predict the shear strength of reinforced concrete elements under different in-plane stress conditions. Then the proposed relation is extended to the shear design of RC members and again it is shown that the predicted shear strengths are in good agreement with the experimental test results under different combination of axial, bending and shear forces.

1. Introduction

Although extensive research has been conducted on the behavior of reinforced concrete elements, there is still considerable disagreement among researchers in proposing and using a rational way of calculating the shear strength of reinforced concrete for different loading conditions and material properties. This is because of different influencing phenomena and parameters which extensively control the overall response of reinforced concrete members in shear and make it complex to be predicted. Most of the current design procedures for reinforced concrete elements in shear are based on the truss analogy yet, which was developed by Ritter and Moersch.

Many years ago Ritter proposed a 45 degree truss model for computing the shear strength of reinforced concrete members and became the basis of many design codes around the world (Rahal and Collins 1999). Ritter assumed that after cracking of concrete, the behavior of a reinforced concrete member is likely to that of a truss with a top longitudinal concrete chord, a bottom longitudinal steel chord, vertical steel ties, and diagonal concrete struts inclined at 45 degrees and further it was assumed that the diagonally cracked concrete cannot resist tension and the shear force is resisted by transverse steel,

commonly referred to as the steel contribution (V_s).

Since then, extensive research has been conducted with the aim of improving the design methods for shear and behavior of concrete members. These studies resulted in several modifications to improve the Ritter's model like the addition of a semi-empirical concrete contribution term (V_c) to the steel one (V_s). Also the researches have shown that the angle of inclination of the compressive strut is not always 45 degree. Along with the experimental researches, analytical models have been developed to account for this problem. The development of the compression field theory (CFT), for torsion (Mitchell and Collins 1974), and then for shear (Collins 1978), made the calculation of the angle of inclination of the compressive strut possible but still ignored tensile stresses in the concrete. This model uses the strain conditions in the web to determine the inclination of the diagonal compressive stresses. The development of this method was a significant step toward the shear design of RC members and has appeared in CSA A23.3 (1984) Standard.

After that, extensive experimental research by Vecchio and Collins (1982) resulted in modified compression field theory (MCFT) with improvements to the CFT method, in which the tensile stresses in concrete was considered in the computations. The main assumptions made in MCFT method are as follows: 1-The principle strain directions are coincident with the principle stress direction; 2-Reinforcing bars are uniformly distributed over the element. Also, the average stresses and strains are used in the computations. This analytical method can accurately predict the behavior and shear strength of RC members in most cases and some modifications are made in the constitutive models in recent

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years to improve its disadvantages. The MCFT method shows good results in comparison to the experimental results in both predicting the behavior and shear strength in most cases. Because of the complexity of this method for design procedures, it has been simplified to be more applicable (Bentz *et al.* 2006), but it resulted in a decrease in the accuracy of predictions.

In this study the shear behavior of RC panels is studied based on a computational method proposed by Soltani *et al.* (2003) for formulating the post cracking response of reinforced concrete membrane elements in terms of stresses at the crack surfaces and taking into account all characteristics of stress transferring in RC elements. Based on this computational method, a parametric study is conducted on the response of RC elements under different conditions of material properties, reinforcement ratio and loadings which resulted in two simple and precise relations for computing the shear strength of RC elements reinforced with deformed bars and welded wire mesh.

The exactness of the proposed relation in predicting the shear strength of the RC elements is shown by comparing the results with extensive available experimental results. Then the proposed relation is extended for design of RC members under the combination of axial force, bending moment and shear and it is shown that the predicted shear strengths are appropriately accurate and also applicable to any special conditions such as large member size (size effect) or high strength concrete.

2. Local stress field approach

The computational method used in this research, is a method that attempts to formulate the post cracking response of reinforced concrete membrane elements in terms of stresses at the crack face, taking into account all characteristics of stress transferring in RC elements (Fig. 1). This so called "Local Stress Field Approach" was proposed by Soltani *et al.* (2003, 2005). The local stress-strain characteristics of concrete and reinforcing bars embedded in concrete, used in this work, are based on the extensive research activities that have been undertaken at the University of Tokyo during the past decades to investigate the fundamental mechanism of local stress transfer in RC domain (Maekawa *et al.* 2003). Using this method, the spatial average behavior of both concrete and reinforcing bars during the crack propagation and stabilizing of cracks can be investigated.

This method comprises different stress transfer mechanisms and phenomena such as local stress transfer along reinforcing bars, shear and dilatancy stresses transfer through aggregate interlock on the crack planes, bridging stress transfer across rough crack, dowel action and kinking of the reinforcing bars at the crack location, stress transfer of concrete parallel to the cracks, softened compression of concrete due to coexisting transverse tensile deformation and compressive damage localization between cracks (Fig. 1). So on the basis of

this local analysis approach the influences of reinforcement ratio and arrangement, crack inclination and material properties on the post cracking response of concrete and reinforcement can be investigated numerically (Soltani *et al.* 2005).

2.1 Local stress analysis of RC membrane elements

Figure 2(a) shows the state of stresses of a reinforced concrete element subjected to general in-plane stresses. The RC element is orthogonally reinforced with longitudinal and transverse bars placed in x and y direction. The principle directions of applied stresses are also shown in Fig. 2(b). The crack direction is initially considered normal to principle tensile stress at an angle θ with respect to x direction. After cracking the local stress and strain states in RC element change. At a distinct cracked section, the local force normal to crack is carried by the reinforcing bars (f_{crx}, f_{cry}) and bridging stress of the cracked concrete (σ_{br}). The local forces across the crack is carried by aggregate interlock (τ_{agg}) and dowel action of reinforcing bars (f_{dx}, f_{dy}). Due to interlocking of aggregates at the crack plane, the compression stress due to dilatancy (σ_d), is applied normal to the crack surface (Fig. 3(a)). The local force of reinforcing bars at the crack surface is partially transferred to the concrete between adjacent cracks through bond stress and anchorage effect of longitudinal and transverse bars, while the bridging stress, aggregate interlock stress, and stresses due to dowel action at crack location are directly applied to the fracturing planes.

Using the equilibrium equations, the applied stress components can be expressed based on the local stresses of cracked concrete and those of reinforcing bars. The local stresses, shown in Fig. 3, are statically equivalent with the applied stresses so:

$$\sigma_x = (\sigma_d + \sigma_{br}) \sin^2 \theta + \sigma_2 \cos^2 \theta - 2\tau_{agg} \sin \theta \cos \theta + \rho_x f_{crx} + \rho_y f_{dy} \quad (1)$$

$$\sigma_y = (\sigma_d + \sigma_{br}) \cos^2 \theta + \sigma_2 \sin^2 \theta + 2\tau_{agg} \sin \theta \cos \theta + \rho_y f_{cry} + \rho_x f_{dx} \quad (2)$$

$$\tau_{xy} = (\sigma_d + \sigma_{br} - \sigma_2) \sin \theta \cos \theta - \tau_{agg} (\cos^2 \theta - \sin^2 \theta) \quad (2)$$

where σ_x, σ_y are space-average normal stress acting on the whole domain of RC element in x and y direction respectively, σ_2 is the principle compressive stress (Fig. 2), ρ_x, ρ_y are the reinforcement ratios in x and y directions respectively, and τ_{xy} is the applied shear stress in x - y direction.

By satisfying the equilibrium and compatibility conditions at a crack plane as well as un-cracked concrete, the local stress and crack deformation can be obtained. Prior to cracking, concrete is modeled as an elastoplastic and fracture continuum by using biaxial EPF model (Okamura and Maekawa 1991). The first crack is introduced in the direction of principle tensile stress

Fig. 1 Local stress transfer mechanism in RC membrane elements.

when its value reaches the cracking stress. After cracking, the implicit iterative solution is conducted in terms of local deformation of crack i.e average opening (ω) and sliding of crack planes (δ) and also axial strain in local “2” direction (ε_2) as primary un-known parameters.

The computational scheme and general description of the method has been explained in (Soltani *et al.* 2003, 2005) and can be summarized as follows.

(a) The axial and shear deformations of steel bars are obtained by considering the local deformation in

the vicinity of cracks. Using the governing equations of stress transfer along the steel bar, the profile of strains and stresses along the reinforcing bar and its local stress at the crack surface can be computed for any given steel strains by using the appropriate bond model. In this analysis the bond-slip-strain model proposed by Shima *et al.* (1987), which is applicable for both elastic and inelastic ranges, is adopted. Also the shear force carried by reinforcing bars, dowel action, is explicitly calcu-

Fig. 2 RC element under in-plane stresses: (a) applied stresses; (b) principle stresses.

Fig. 3 Stress state of cracked concrete and reinforcing bars: (a) local stresses; (b) average stresses.

lated by considering the local curvature developed in reinforcing bar in the vicinity of crack.

- (b) The local stress conditions at crack planes are computed by the amount of local deformation of crack planes using the appropriate material models (Soltani *et al.* 2005). The average tensile stress of concrete between adjacent cracks (tension stiffening) is obtained from the local steel stresses transferred to the concrete body. The possibility of a newly formed crack is checked with the maximum local concrete stress in un-cracked concrete region. When the maximum tensile stress in concrete reaches cracking stress, a new crack is introduced in the RC domain and computation is carried out with the updated crack spacing.
- (c) The average strain of RC element in global coordinate is obtained by transformation of average concrete strains on the local coordinate along a crack. The concrete and steel stresses computed in the

above mentioned steps should be in equilibrium with the applied stresses defined on the global coordinates (Eqs. 1-3). Then, for any given applied stresses, the unknown average strains can be determined in an iterative way.

A comprehensive verification of this model with experimental work has been conducted for RC elements with deformed bars or welded wire mesh in (Soltani *et al.* 2003, 2004). Here, the accuracy of this analytical method in prediction of response of some reinforced concrete panels subjected to in-plane forces is re-examined. The first group of selected panels, made of deformed bars, have been tested at the University of Houston by Pang and Hsu (1995) (panels A2,A3), and the second group are the panels, made of welded wire mesh, which have been tested at the University of Toronto by Vecchio and Collins (1982) (panels PV3,PV11). The specimens' properties and loading conditions are listed in **Table 1**. The comparison between the experimental and analytical results is shown in **Fig. 4** which confirms the reliability of the numerical method.

2.2 Steel and concrete contribution to overall response of RC elements

The grasp of individual contributions of differing stress transfer mechanisms and their influence to the structural response is the key for development of a rational model for failure assessment of RC elements. Herein, the discussion is directed to the individual influence of these mechanisms on the behavior and failure capacity of RC membrane elements.

As in the proposed method, all the mobilized stress components in the RC domain can be tracked in terms of local stress-strain characteristics and local response of the components, the average stress-strain relationship of reinforcing bars and cracked concrete can be obtained. Also, the contribution of each component i.e. the reinforcing bars and concrete to the overall behavior of the RC elements can be investigated which is the basis of the current study in formulizing and computing the shear strength of reinforced concrete members.

As an example the individual contribution of cracked concrete and steel bars to the overall response of RC panel PV11 was computed in terms of both average stress and local stress at the crack plane and shown in **Fig. 5**.

Also, the contribution of each component to the shear response of the panel can be computed, writing the equilibrium equation in y direction, which helps us to investigate the effect of transverse reinforcement and cracked concrete in the overall response and derivation of the design relationships. For the case of RC element reinforced with orthogonal reinforcing bars in direction of applied stresses, the equilibrium equation of local stress state at the crack location in y direction can be expressed as (**Fig. 3**):

$$\tau_{xy} = (\sigma_d + \sigma_{br}) \cot \theta + \tau_{agg.} + \rho_y f_{cry} + \rho_x f_{dx} - \sigma_y \cot \theta \quad (4)$$

Table 1 Specimen details and loading condition.

Panel	Concrete		Reinforcing Bars								Loading $\tau_{xy}:\sigma_x:\sigma_y$
	f'_c (MPa)	$\varepsilon_c \times 10^{-3}$	ρ_x (%)	f_{yx} (MPa)	d_{bx} (mm)	L_x (mm)	ρ_y (%)	f_{yy} (MPa)	d_{by} (mm)	L_y (mm)	
PV3	26.6	-2.3	0.483	662	3.3	50	0.483	662	3.3	50	1:00:00
PV11	15.6	-2.6	1.79	235	6.35	50	1.31	235	5.44	50	1:00:00
A2	41.2	-2.1	1.193	463	15	189	1.193	463	15	189	1:00:00
A3	41.6	-1.94	1.789	447	20	189	1.789	447	20	189	1:00:00

Fig. 4 Comparison with experimental results (A2,A3) by Pang and Hsu (1995) and (PV3,PV11) by Vecchio and Collins (1982).

where, the right hand side of this equation shows the effects of applied axial stress in y direction and also the contribution of concrete (in tension and shear) and reinforcing bars (axial and dowel) to the shear capacity. The interesting point in this type of formulation is the elimination of the effect of other component of applied forces, (σ_x), on the shear failure capacity. The effect of this term is indirectly considered in the crack inclination. Also in the average form, this equilibrium is derived as:

$$\tau_{xy} = \sigma_1 \cot \theta + \tau_{12} + \rho_y \bar{f}_y - \sigma_y \cot \theta \quad (5)$$

Figure 6 shows the analytical result for RC Panel PV11, in terms of average stress between cracks (**Fig. 6a**) and the stresses at the crack faces (**Fig. 6b**). The large participation of concrete (normal and across cracks) in the shear capacity of this element is obvious. This concrete contribution is the term, which is entirely

neglected in the original truss model, where the shear capacity is considered totally equal to the yielding capacity of transverse reinforcing bars. This term is roughly considered in the present design codes and mostly independent of the reinforcement ratio, reinforcement and crack inclination and the size of structures.

This figure obviously shows that even after yielding of transverse reinforcing bars, considerably large stresses can be transferred through combined contribution of longitudinal reinforcement and cracked concrete.

Up to yielding of transverse reinforcing bars, almost the shear stress across cracks is zero. This has substantial meaning; both rotating crack approach (crack stress free method like MCFT) and fixed crack model tend to produce the same results as the cases of fairly isotropic RC elements. After yielding of the transverse reinforcing bars the shear stress across cracks increases to bal-

Fig. 5 Contribution of different mechanisms to the overall response of panel PV11.

Fig. 6 Contribution of different mechanisms to the overall response of panel PV11 (equilibrium equations in y direction).

ance the difference of stress at crack location as well as between cracks. However after yielding of reinforcing bars in both directions, the shear stress almost remains constant and shear slip and crack opening rapidly increase up to failure of the element.

3. Derivation of the relations

It is well known that the cracked reinforced concrete transmits loads in a relatively complex manner involving opening and closing the cracks, slipping of the crack surfaces, new cracks formation, shear transfer at the crack surface, and significant variation of the steel stresses due to bond effect. A rational design method should consider all these parameters and their effects on the shear strength of RC members. The local stress field approach can consider all the influencing parameters on the shear behavior and strength of RC members and can simulate the shear failure response accurately.

Here it is assumed that the shear capacity of concrete is a function of tensile strength (f_t), maximum aggregate

size (a_g), crack width (ω) and spacing (S_θ), and crack slippage (δ):

$$v_c = f_1(f_t, f'_c) \times f_2(a_g) \times f_3(\omega, \delta) \times f_4(S_\theta) \quad (6)$$

Based on the researches conducted by several authors, including (Walraven 1981), the shear capacity of RC elements increase with increasing the maximum aggregate size. But in high strength concrete, as the cracks pass through the aggregates, the aggregate interlock is not effective in shear transmitting ability of concrete. Crack width and slippage are two other important factors affecting the shear capacity of RC elements. Increasing in crack width and slippage results in decrease in shear capacity of elements (Li *et al.* 1998; Bujadham and Maekawa 1992). To account for this effect, a parameter called "Crack Deformation Parameter (S_c)" is defined as the product of the local longitudinal strain and the crack spacing. Longitudinal strain in the member is a representative of loading conditions and average crack widths along the element. Therefore longitudinal

strain together with the crack spacing can explicitly consider the effects of crack width and slipping in the shear strength of reinforced concrete elements. This results in modification of Eq. 6 to:

$$v_c = f_1(f_t, f'_c) \times f_2(a_g) \times f_3(S_c) \quad (7)$$

Considering Eq. 7, in order to develop a rational and simple relation for shear failure assessment of RC members, a systematic parametric study is conducted on RC elements under different conditions of loading and material properties and the effect of these parameters on the contribution of cracked concrete to the shear strength of RC elements is studied. The variation of shear capacity of concrete (v_c) by increasing the crack deformation parameter is shown in **Fig. 7**. The effect of maximum aggregate size and tensile strength of concrete is also shown in **Fig. 8**. This study resulted in Eq. (8) for deformed bars:

$$v_c = 0.012 f_c^{0.67} \sqrt{a_g} S_c^{-0.65} \leq f_t \quad (8)$$

and in Eq.(9) for welded wire mesh:

$$v_c = 0.0083 f_c^{0.67} \sqrt{a_g} S_c^{-0.65} \leq f_t \quad (9)$$

Fig. 7 Shear capacity of RC members that failed before yielding of longitudinal bars.

Fig. 8 Influence of tensile strength and aggregate size on shear capacity.

where f_c is the concrete compressive strength (MPa), a_g is maximum aggregate size (mm), and S_c is the crack deformation parameter (mm). As the shear capacity of RC members is defined in terms of compressive strength of concrete in most design codes, the tensile strength of concrete in Eq. 7 is substituted with its equivalent value in terms of compressive strength, $f_t = 0.194 f_c^{0.67}$ (Maekawa *et al.* 2003), in Eq. 8 and Eq. 9. The specialty of these equations is that they can be simply used for any material properties, reinforcement ratio and distribution and also any size of elements.

The experimental tests performed on RC columns (Preistley *et al.* 1992) showed that shear failure may occur before or after the formation of flexural hinges, which limits the ductility of the flexural behavior of the member. As plastic hinge rotations increase, the widening of cracks reduces the shear capacity because of the reduction of aggregate interlock shear transfer. To investigate the applicability of the proposed relation (Eq. 8) for the RC elements in which the shear failure occurs after yielding of the longitudinal bars, the response of some RC elements with this condition is drawn together with the curves of the RC elements in which the shear failure occurs prior to yielding of longitudinal bars in **Fig. 9**. The peak value of each curve can be explained as the shear strength of the element. It can be seen that the proposed relation (Eq. 8) is also correct for these elements. Accuracy of the proposed relation to predict the shear capacity of RC members that failed after longitudinal reinforcements yielding shows that the proposed method is capable of predicting the governing behavior and ductility of RC members.

In high strength and lightweight concrete, the cracks pass through the aggregate, making the crack surfaces smooth, and so the aggregate size will not influence the crack roughness. To apply the Eq. 8 and Eq. 9 to members made of these types of concrete, the effective aggregate size should be used. It is proposed to use the actual value of maximum aggregate size for concrete elements with compressive strength of lower than 60 MPa, and to use a value of 4 mm for lightweight con-

Fig. 9 Shear capacity of RC members that failed after yielding of longitudinal bars.

crete elements as well as high strength concrete elements with compressive strengths of more than 70 MPa. For concrete elements with compressive strengths of between these values, linear interpolation is adopted.

As it is discussed before, the crack deformation parameter, S_c , is calculated as the product of the local axial strain and crack spacing:

$$S_c = \varepsilon_x \times S_\theta \quad (10)$$

$$S_\theta = \frac{1}{(\sin \theta / S_x) + (\cos \theta / S_y)} \quad (11)$$

where ε_x is longitudinal strain, S_θ is average crack spacing normal to cracks and S_x and S_y are crack spacing in x and y directions, respectively.

The longitudinal strain (ε_x) in the RC elements can be calculated as follows:

$$\varepsilon_x = \frac{N_u + V_u \cot \theta}{E_s A_{sx}} \quad (12)$$

where N_u and V_u are axial and shear forces acting on RC element respectively. E_s is the steel modulus of elasticity and A_{sx} is area of reinforcing bars in the longitudinal (x) direction. This relation has a good accuracy in predicting the longitudinal strain of RC elements which will be verified in the next section.

Salem (1998) proposed a relation for calculating the crack spacing in RC elements. The simplified form of this equation is adopted here:

$$L_e = 1.76 \times \rho^{-0.5} f_y^{0.1} f_c'^{0.4} d (1 + f_t) \quad (13)$$

where, L_e is the crack spacing in mm, ρ is the reinforcement ratio in the direction under investigation, f_y is the reinforcing bar yield stress in MPa, d is the bar diameter in mm, f_c' is the concrete compressive strength in MPa and f_t is the concrete tensile strength in MPa.

Having calculated the longitudinal strain and crack spacing, the crack deformation parameter can be com-

Fig 10 State of stresses in cracked RC element.

puted and then the shear capacity of RC element can be achieved by using Eq. 8 or Eq. 9.

The transverse steel contribution (v_s) can be computed by the conventional relation as follows:

$$v_s = \rho_v f_{yv} \cot \theta \quad (14)$$

where ρ_v and f_{yv} are the reinforcement ratio and steel yield stress in transverse direction, respectively. Depending upon the concrete and steel properties and the reinforcement ratio, the compressive failure of the concrete may occur before yielding of the transverse reinforcing bars and so the contribution of transverse steel will be less than the value obtained by Eq. 14. Using this equation in such conditions leads to overestimating the contribution of steel bars to the shear capacity of reinforced concrete members. To consider this effect, the equilibrium equation between stresses acting on an RC element can be considered (Fig. 10):

$$v = v_c + v_s = (f_2 + \sigma_1) \sin \theta \cos \theta \quad (15)$$

where θ is the crack direction (Fig. 3), f_2 is absolute value of concrete compressive stress and σ_1 is concrete tensile stress. At the ultimate condition, the concrete stress, f_2 , will be equal to the compressive strength of the element, f_{2max} , and the contribution of σ_1 , in Eq. 15 becomes small in comparison to the value of f_2 , and can be conservatively neglected. Thus,

$$v = v_c + v_s = f_{2max} \sin \theta \cos \theta \quad (16)$$

So the maximum local stress of transverse reinforcing bars is:

$$v_s = f_{2max} \sin \theta \cos \theta - v_c \quad (17)$$

The tensile strain in the crack plane weakens the ability of concrete to withstand compression in the direction perpendicular to the crack surface. So the value of f_{2max} will be less than the concrete compressive strength because of tensile strains in the crack surface. Based on the experimental tests on reinforced concrete elements, Vecchio and Collins (1986) proposed the following relation to compute the maximum strength of cracked concrete (f_{2max}):

Fig. 11 Concrete compressive strength reduction factors.

$$f_{2\max} = \frac{f'_c}{0.8 + 170\varepsilon_1} \quad (18)$$

Computing the value of ε_1 can be done based on the following equation:

$$\varepsilon_1 = \varepsilon_x + (\varepsilon_x + \varepsilon_2) \cot^2 \theta \quad (19)$$

replacing ε_1 in Eq. 18 by Eq. 19, it can be expressed as:

$$f_{2\max} = \frac{2 \sin^2 \theta}{(1.6 \sin^2 \theta + 340 \varepsilon_2 \cos^2 \theta) + 340 \varepsilon_x} f'_c \quad (20)$$

As the value of $1.6 \sin^2 \theta + 340 \varepsilon_2 \cos^2 \theta$ is approximately equal to 1, this equation can be expressed as follows:

$$f_{2\max} = \frac{2 \sin^2 \theta}{1 + 340 \varepsilon_x} f'_c \quad (21)$$

The reduction factors computed with Eq. 21 in comparison to the values obtained with Eq. 18 is shown in Fig. 11 for 42 RC panels. It can be seen that the reduction factors obtained with the proposed relation are almost in a good correspondence with the values obtained with Eq. 21 and also it highly simplifies the design process.

4. Strength predictions for RC elements

Among different available experimental results, 42 RC elements (Bentz *et al.* 2006) with different properties under pure shear or shear combined with axial stress loading are selected to investigate the accuracy of the shear strengths predicted with the proposed method. The properties of the elements and the shear strengths predicted with the MCFT method (Collins *et al.* 1996), ACI 318-05 method, and the proposed method are shown in Table 2 to Table 5.

The ACI code expression for the capacity of the

Table 2 Zhang and Hsu (1998) experimental results.

Zhang & Hsu, $G_{\max}=13 \text{ mm}$													Calculations		Vexp./Vcomp.		
Panel	f'_c (MPa)	ρ_x (%)	d_{bx} (mm)	f_{yx} (MPa)	L_x (mm)	ρ_y (%)	d_{by} (mm)	f_{yy} (MPa)	L_y (mm)	f_x/v	S_θ (mm)	ε_x (x1000)	S_c (mm)	Vexp. / f'_c	Full MCFT	ACI	Prop. Method
VA1	95.1	1.19	11.3	445	94	0	11.3	445	94	0	192	2.51	0.48	0.068	1.04	0.75	1.00
VA2	98.2	2.39	16	409	94	0	16	409	94	0	208	2.16	0.45	0.103	1.03	1.02	0.92
VA3	94.6	3.59	19.5	455	94	0	19.5	455	94	0	231	2.3	0.53	0.163	0.94	1.59	0.90
VA4	103.1	5.24	25.2	470	94	0	25.2	470	94	0	200	2.33	0.46	0.22	1	2.21	0.89
VB1	98.2	2.39	16	409	94	0	11.3	409	94	0	152	1.16	0.18	0.08	1.01	0.91	1.07
VB2	97.6	3.59	19.5	455	94	0	11.3	455	94	0	158	0.47	0.07	0.097	0.95	1.1	1.09
VB3	102.3	5.98	25.2	445	94	0	11.3	445	94	0	159	0.31	0.05	0.099	0.9	1.17	1.04
VB4	96.9	1.79	19.5	455	188	0	11.3	455	188	0	190	0.62	0.12	0.052	0.97	0.85	1.01
Ave.															0.98	1.20	0.99

Table 3 Vecchio *et al.* (1994) and Vecchio and Chan (1990) experimental results.

Vecchio <i>et al.</i> , $G_{\max}=10 \text{ mm}$													Calculations		Vexp./Vcomp.		
Panel	f'_c (MPa)	ρ_x (%)	d_{bx} (mm)	f_{yx} (MPa)	L_x (mm)	ρ_y (%)	d_{by} (mm)	f_{yy} (MPa)	L_y (mm)	f_x/v	S_θ (mm)	ε_x (x1000)	S_c (mm)	Vexp. / f'_c	Full MCFT	ACI	Prop. Method
PA1	49.9	1.65	8	522	45	0.82	5.72	522	45	98	1.71	0.17	0	0.126	0.94	0.95	0.93
PA2	43	1.65	8	522	45	0.82	5.72	522	45	94.2	1.45	0.14	0	0.145	0.94	0.96	0.95
PHS1	72.2	3.23	8	606	44	0	5.72	521	44	138	0.4	0.06	0	0.037	1.07	0.97	0.82
PHS2	66.1	3.23	8	606	44	0.41	5.72	521	44	103	0.47	0.05	0	0.093	1.13	1.27	1.18
PHS3	58.4	3.23	8	606	44	0.82	5.72	521	44	93.6	0.66	0.06	0	0.14	0.99	1.2	1.15
PHS8	55.9	3.23	8	606	44	1.24	5.72	521	44	92.1	1.22	0.11	0	0.193	1.02	1.45	1.19
PC1	25.1	1.65	5.72	500	44.5	0	5.72	500	89	88.6	1.54	0.14	0	0.197	0.84	0.99	0.88
Ave.															0.99	1.11	1.01

Table 4 Pang and Hsu (1995) experimental results.

<i>Pang & Hsu, $G_{max}=19\text{ mm}$</i>										Calculations					Vexp./Vcomp.		
Panel	f'_c (MPa)	ρ_x (%)	d_{bx} (mm)	f_{yx} (MPa)	L_x (mm)	ρ_y (%)	d_{by} (mm)	f_{yy} (MPa)	L_y (mm)	S_θ (mm)	ϵ_x (x1000)	S_c (mm)	f_x /v	Vexp. /f _c	Full MCFT	ACI	Prop. Method
A2	41.3	1.19	16	463	189	1.19	16	463	189	215	2.75	0.59	0	0.136	1.01	0.87	0.90
A3	41.6	1.79	19.5	447	189	1.79	19.5	447	189	232	2.55	0.59	0	0.19	0.98	1.23	0.91
A4	42.5	2.98	25.2	470	189	2.98	2.5	470	189	224	2.28	0.51	0	0.28	0.97	1.82	0.92
B1	45.2	1.19	16	463	189	0.6	1.13	445	189	223	1.66	0.37	0	0.092	1.01	0.87	1.10
B2	44.1	1.79	19.5	447	189	1.2	1.6	463	189	222	1.95	0.43	0	0.146	0.96	0.97	0.96
B3	44.9	1.79	19.5	447	189	0.6	1.13	445	189	222	1.20	0.27	0	0.102	0.94	0.96	1.11
B4	44.8	2.99	25.2	470	189	0.6	1.13	445	189	223	0.80	0.18	0	0.119	0.92	1.12	1.05
B5	42.8	2.98	25.2	470	189	1.2	1.6	463	189	226	1.23	0.28	0	0.177	0.89	1.16	1.08
B6	42.8	2.98	25.2	470	189	1.79	1.95	447	189	222	1.68	0.37	0	0.23	0.95	1.53	1.03
														Ave.	0.96	1.17	1.01

Table 5 Vecchio and Collins (1982) experimental results.

<i>Vecchio & Collins, G_{max} =6 mm</i>									Calculations				Vexp./Vcomp.		
Panel	f'_c (MPa)	ρ_x (%)	f_{yx} (MPa)	L_x (mm)	ρ_y (%)	f_{yy} (MPa)	L_y (mm)	f_x /v	S_θ (mm)	ϵ_x (x1000)	S_c (mm)	Vexp. /f _c	Full MCFT	ACI	Prop. Method
PV1	34.5	1.79	483	51	1.68	483	54	0	36.1	2.47	0.089	0.23	0.93	1.37	0.88
PV2	23.5	0.18	428	51	0.26	428	50	0	37.1	3.64	0.135	0.049	1.41	0.48	0.96
PV3	26.6	0.48	662	51	0.48	662	50	0	53	3.72	0.197	0.115	0.95	0.63	0.82
PV4	26.6	1.03	242	51	1.06	242	50	0	60.1	1.55	0.093	0.109	1.12	0.68	0.86
PV5	28.3	0.74	621	100	0.74	621	100	0	72.1	3.32	0.239	0.15	0.91	0.8	0.84
PV6	29.8	1.79	266	51	1.78	266	51	0	36.1	1.61	0.058	0.153	0.95	0.84	0.79
PV10	14.5	1.79	276	51	1	276	51	0	36.1	1.01	0.037	0.27	1.06	1.05	1.08
PV11	15.6	1.79	235	51	1.31	235	51	0	84.8	1.00	0.085	0.23	0.98	0.9	0.91
PV12	16	1.79	469	51	0.26	469	51	0	106	0.54	0.057	0.196	1.09	1.24	1.31
PV16	21.7	0.74	255	51	0.74	255	51	0	36.1	1.82	0.065	0.099	1.12	0.62	0.82
PV18	19.5	1.79	431	51	0.3	431	51	0	36.1	0.73	0.026	0.156	1.08	1.1	1.15
PV19	19	1.79	458	51	0.46	458	51	0	36.1	0.92	0.033	0.21	0.95	1.1	1.23
PV20	19.6	1.79	460	51	0.57	460	51	0	36.1	1.03	0.037	0.22	0.93	1.04	1.18
PV21	19.5	1.79	458	51	0.86	458	51	0	36.1	1.34	0.048	0.26	0.91	1.14	1.06
PV22	19.6	1.79	458	51	1.4	458	51	0	36.1	1.70	0.061	0.31	0.98	1.38	0.95
PV26	21.3	1.79	456	51	1.02	456	51	0	36.1	1.53	0.055	0.25	0.88	1.18	0.97
PV27	20.5	1.79	442	51	1.79	442	51	0	36.1	1.76	0.063	0.31	0.96	1.41	0.96
PV30	19.1	1.79	437	51	1.09	437	51	0	36.1	1.54	0.055	0.27	0.88	1.18	0.93
												Ave.	1.01	1.01	0.98

members under shear and flexural actions is as follows:

$$V_c = v_c b_w d \quad (22)$$

$$v_c = 0.17 \sqrt{f'_c} \quad (23)$$

In using this expression for elements made from high-strength concrete, the term $\sqrt{f'_c}$ is limited to a value of 8.8 MPa.

The average ratio of the predicted strengths to the experimental ones for these 42 elements with the ACI 318-05 method is 1.10, with the full MCFT method is 0.99, and with the proposed method is 1.00. As it is shown, the ACI 318-05 method predicts the strengths in some cases so un-conservative. The MCFT method in its full format, predicts the strengths accurately, but it is hard to use for design processes. Because of this problem, Bentz *et al.* (2006) proposed a simplified form of the

MCFT that is more simple to apply for design, but the results are less accurate in predicting the shear strengths (the average of ratio of shear strengths predicted for selected elements to the experimental results is equal to 1.11). It is also shown that the proposed method is simple and gives accurate results in comparison to experimental results.

To control the accuracy of Eq. 12 for calculating the longitudinal strain in RC elements, the obtained longitudinal strain is compared with the longitudinal strain correspondent to the shear strength of some RC elements in Table 6.

5. Application to design of RC members

The proposed method is also applicable for design of RC members. For computing the crack spacing of reinforced concrete members, the CEB-FIP relation can be

Table 6 Computed longitudinal strains for some RC elements.

Panel	Expected Longitudinal Strain	Computed Longitudinal Strain
PV3	0.0024	0.00383
PV4	0.0013	0.00167
PV11	0.0009	0.001
PV27	0.0015	0.00149
A2	0.00261	0.00273
A3	0.0029	0.00254
B1	0.00164	0.00161
B2	0.00194	0.00191

Fig. 12 Computation of longitudinal strain in the member.

used, which describes the average crack spacing as a function of the bar diameter, reinforcement ratio, concrete cover and spacing between bars:

$$L_{av} = 2\left(c + \frac{S_o}{10}\right) + k_1 k_2 \frac{d_b}{\rho} \quad (24)$$

where c is the clear cover, S_o is the spacing between center of bars, k_1 is coefficient equal to 0.4 for deformed bars and 0.8 for smooth bars, k_2 is coefficient to account for strain gradient which equals to 0.25 for pure tension and 0.125 for bending, d_b is bar diameter and ρ is effective reinforcement ratio which can be calculated according to the given instructions in the next section.

The longitudinal strain (ϵ_x) in RC members is given by:

$$\epsilon_x = \frac{0.5N_u + \alpha M_u / d + 0.5V_u \cot \theta}{E_s A_{sx}} \quad (25)$$

where E_s is the steel modulus of elasticity, A_{sx} is the steel area in the flexural tension zone, d is the effective depth of the section, N_u is the factored applied axial load (positive if tensile), M_u is the factored bending moment (positive), α is longitudinal strain modifier factor that accounts for the average crack width, V_u is

the factored shear force (positive) and θ is the crack direction (Fig. 12) and can be calculated as follows:

$$\tan 2\theta = \frac{-2V}{P} \quad (26)$$

Simplifying this equation results in:

$$\begin{cases} \theta = 0.5 \arctan\left(\frac{-2V}{P}\right) & \text{if } \frac{-2V}{P} < 0 \\ \theta = \frac{\pi}{2} + 0.5 \arctan\left(\frac{-2V}{P}\right) & \text{if } \frac{-2V}{P} \geq 0 \end{cases} \quad (27)$$

where V is the applied shear force and P is the applied axial force (positive in compression). The conventional form of the Eq. 25 calculates the longitudinal strain in the maximum width of the flexural cracks. But as in the design procedures the average crack spacing is used to calculate the shear capacity of the element, the longitudinal strain should also be calculated on the basis of the average crack width. The parameter α is used to account for this problem. Based on the conducted parameter study, 0.8 is adopted here for this parameter.

It should be noted that Eq. 12 can be used just before yielding of longitudinal reinforcing bars. Although the general model proposed in Eq. 8 and 9 can also be used for the shear failure assessment of RC elements after yielding of main reinforcing bars as shown in Fig. 9, but for such a case the axial steel strain should be computed based on the member ductility demand which has not been discussed in this paper. Therefore, the design procedure for calculating the shear capacity of RC members can be summarized as follows:

- 1- Calculating the crack spacing in longitudinal and transverse directions of the member according to Eq. 24 and the instructions given in the next section.
- 2- Calculating the angle of the cracks according to Eq. 26 and 27.
- 3- Calculating the average crack spacing normal to cracks using Eq. 11.
- 4- Calculating the longitudinal strain in the member according to Eq. 25.
- 5- Calculating the Crack Deformation Parameter, S_c , according to Eq. 10.
- 6- Calculating the contribution of concrete to the shear capacity of the member, V_c , according to Eq. 8 or Eq. 9 depending on the type of the reinforcing bars and using Eq. 22.
- 7- Calculating the total shear capacity of the member by adding the concrete and steel shear contributions.

6. Size effect

The failure of large RC elements during earthquakes resulted in investigations (Bazant and Kim 1984; Walraven and Lehwalter 1994; Kani 1967; Taylor 1972; Kennedy 1967; Shioya *et al.* 1989), in which it was indicated that when the size of the RC element increases, the shear strength decreases. So using the conventional design process for large RC members leads to overesti-

mating their strength which may be up to 50% or more.

In 1967, Kani (1967) showed the importance of the size effect and demonstrated that as the depth of the beam increases, the shear stress at failure decreases. Shioya *et al.* (1989) reaffirmed this fact with an extensive experimental program conducted in Japan and made a convincing evidence for this phenomenon. The result of this work is shown in **Fig. 13**, which can be stated that by increasing the member depth or decreasing the maximum aggregate size, the shear stress at failure decreases. Since then, several experimental and analytical researches have been conducted to study the size effect which resulted in this general agreement that the main reason of this phenomenon is the larger crack widths in the larger members.

Until 1980, this so called “size effect” phenomenon was not considered in the design processes. After that, according to the experimental and analytical results, some codes tried to consider this important effect by empirical formulations. Okamura and Hiagai (1980) proposed a design equation for shear strength upon investigation of published test results on shear strength of reinforced concrete beams without web reinforcement. Bazant and Kim (1984) believed that the most important consequence of wider cracks is the reduced residual tensile stresses and proposed a size reduction factor based on nonlinear fracture mechanics to account for this effect. The MCFT method accounts for this effect by computing a parameter which is a function of crack spacing. This method is used in some recent codes such as AASHTO-LRFD (1998), but still most of the design codes such as ACI-318 (2005) do not consider size effect in the design methods, and as the structures are becoming larger in size, a rational design method should be able to consider this important effect appropriately.

In the proposed design method, the size effect is taken into consideration by computing the crack spacing while considering the effective size of RC zone (An *et al.* 1996). It is well known that the concrete behavior near the reinforcing bars is different from far from. This difference is taken into account by considering two different zones in RC members, one for the concrete near the steel bars, and showing tension-stiffening behavior (called as RC zone), and another one for the concrete outside the RC zone showing strain-softening behavior (called as PL zone). So, the cracking behavior and consequently the shear strength of RC zone and PL zone are different. The size of RC zone is related to the bond effect, which is related to the diameter and amount of the steel bar and is a very important factor for size effect investigation. As the diameter of the steel bar will not always increase proportionally with the scale of the member, the size of RC zone will not increase consequently. The size of RC zone is related to the bond effect of reinforcing bars and should be decided by the bond characteristics. There are many experimental and analytical researches conducted by several authors to study the bond strength, the area of effective RC zone

and bond slip relation of bars. An example of the definition of RC zone is shown in **Fig. 14** (Collins and Mitchell 1991), which is used in the proposed method to calculate the effective area of RC zone. Computing the effective area of RC zone, the effective ratio of reinforcement can be calculated and then the crack spacing within this zone is calculated. Just a few primary cracks within RC zone grow to the plain concrete zone, so the spacing between critical unstable cracks is larger than the crack spacing within RC zone. In order to consider this effect, herein it is assumed that the crack spacing is equal to crack spacing within the RC zone multiplied by the ratio of whole member cross sectional area to the area of RC zone. If the member has no transverse reinforcement, the crack spacing in y direction should be assumed equal to the member depth. By using this method the crack spacing and consequently the member capacity will be calculated with considering the bond influencing zone of reinforcing bars.

The precision of the proposed method in predicting the shear strength of large beams is shown in the following section. It is shown in **Table 7** that even in large beams the predicted shear strength is accurate in comparison with the experimental results.

Fig. 13 Influence of member depth and maximum aggregate size on shear stress at failure, tests by Shioya *et al.* (1989).

Fig. 14 Definition of RC zone (Collins and Mitchell 1991).

7. Strength predictions for RC members

To study the accuracy of the relations in different conditions like size effect, high strength concrete and bar distribution over the element, the shear capacity of beams (Collins and Kuchma 1999) and columns (Kirschner and Collins 1986) were calculated with the proposed method and the predicted shear strengths are compared with the experiment results in **Table 7** and **Table 8**.

For the case of beams, 17 specimens (Collins and Kuchma 1999) were selected. All the specimens were under a point load at the mid-span, resulting in a shear-span-to-depth ratio of about three. The beams have different depths, reinforcement ratio, and distributions, and all of them failed in shear prior to yielding of the longitudinal reinforcement. In 5 specimens high strength concrete was used. As it is shown in **Table 7** the average of the predicted shear strengths with the proposed method is almost 1 (Std. Dev. of 0.095), while the ACI 318-99 method predicts the shear strengths with the average of 0.971, and the MCFT with the average of 1.204 but with lower standard deviation in comparison to ACI 318-99 method.

For the columns, 24 specimens (Kirschner and Collins 1986) were selected. The parameters that change in specimens are the ratio of compression to shear force, N/V , the concrete compressive strength, the specimen width, and the amount of shear reinforcement. As it is shown in **Table 8** the proposed method can predict the shear strengths precisely with the average of 1.04 (Std. Dev. of 0.18), while the average of shear strength predicted by the detailed ACI 318-99 method is 1.03 (Std.

Dev. of 0.4), and is 1.31 (Std. Dev. of 0.25) for the MCFT method.

It is shown in **Table 7** that even in large beams; the predicted shear strength is accurate in comparison with the experimental results.

Fig. 15 Details of Collins beams. (Collins and Kuchma 1999)

Table 7 Collins and Kuchma (1999) beams verification.

Properties & geometry									S_{θ} mm	ε_x (x1000)	S_c mm	Comp. Shear (kN)	Exp. Shear (kN)	Vexp./ Vcomp.	Vexp./ V _{ACI}	Vexp./ V _{MCFT}
Beam	h (mm)	b (mm)	d (mm)	f'_c (MPa)	A_s (mm ²)	ρ_x (%)	ρ_y (%)	f_{yy} (MPa)								
BN100	1000	300	925	37.2	2100	0.76	0	0	60.1	0.85	0.051	186	192	0.968	0.68	1.14
BN50	500	300	450	37.2	1100	0.81	0	0	30	1.08	0.032	124	132	0.941	0.96	1.33
BN25	250	300	225	37.2	600	0.89	0	0	14.8	1.49	0.022	78	73	1.073	1.06	1.25
BN12	120	300	110	37.2	300	0.91	0	0	7.5	1.84	0.014	51	40	1.271	1.19	1.24
BH100	1000	300	925	98.8	2100	0.76	0	0	60.1	0.86	0.052	191	193	0.992	0.5	1.08
BH50	500	300	450	98.8	1100	0.81	0	0	30	1.11	0.032	126	132	0.951	0.7	1.2
BH25	250	300	225	98.8	600	0.89	0	0	14.8	1.51	0.022	80	85	0.945	0.91	1.28
BND100	1000	300	925	37.2	2900	1.05	0	0	22.8	1.05	0.024	300	258	1.162	0.91	1
BND50	500	300	450	37.2	1500	1.11	0	0	16.5	1.17	0.019	169	163	1.035	1.19	1.13
BND25	250	300	225	37.2	884	1.31	0	0	9.7	1.37	0.013	110	112	0.982	1.63	1.36
BHD100	1000	300	925	98.8	2900	1.05	0	0	22.8	1.07	0.024	311	278	1.118	0.72	0.94
BHD50	500	300	450	98.8	1500	1.11	0	0	16.1	1.18	0.019	177	193	0.916	1.03	1.15
BHD25	250	300	225	98.8	884	1.31	0	0	9.8	1.49	0.013	113	111	1.016	1.19	1.37
B100	1000	300	925	36	2800	1.01	0	0	60.1	0.7	0.042	205	225	0.913	0.81	1.22
B100D	1000	300	925	36	3300	1.19	0	0	22.5	1.01	0.023	325	320	1.015	1.15	1.21
BM100	1000	300	925	47	2100	0.76	0.079	508	50	1.16	0.057	311	342	0.909	0.8	1.39
BM100D	1000	300	925	47	2900	1.05	0.079	508	21	1.28	0.027	447	461	0.969	1.08	1.18
													Ave.	1.010	0.971	1.204
													St. dev.	0.095	0.259	0.122

Table 8 Kircshner and Collins (1986) columns verification.

Specimen	Properties & geometry									Comp. Shear (kN)	Exp. Shear (kN)	Vexp./Vcomp.	Vexp./V _{ACI}	Vexp./V _{MCFT}
	h (mm)	b (mm)	d (mm)	f'_c (MPa)	ρ_x (%)	ρ_y (%)	f_{yy} (MPa)	f_{ty} (MPa)	N/V					
PC1	375	489	345	62.2	2.62	0.153	520	684	0	429	437	1.02	1.22	1.07
PC2	375	489	345	62.2	2.62	0.153	520	684	4	844	863	1.02	0.69	1.38
PC3	375	375	345	62.8	2.62	0.131	520	684	4	696	845	1.21	0.73	1.68
PC4	365	375	335	83.1	2.63	0.142	509	785	0	342	401	1.17	1.47	1.26
PC5	365	375	335	86.9	2.63	0.142	509	785	8	749	679	0.91	0.79	0.91
PC6	365	375	335	86.9	2.63	0.142	509	785	4	749	668	0.89	0.61	1.37
PC7	365	375	335	39.9	2.63	0.142	509	785	0	436	387	0.89	1.66	1.44
PC8	365	375	335	42.4	2.63	0.142	509	785	4	471	497	1.05	0.65	1.24
PC9	365	375	335	42.9	2.63	0.142	509	785	8	474	516	1.09	0.95	1.07
PC10	365	625	335	42.9	2.63	0.142	509	785	4	946	726	0.77	0.58	1.08
PC11	365	625	335	44.7	2.63	0.142	509	785	8	946	754	0.80	0.84	0.92
PC12	365	375	335	60	2.63	0.142	509	785	0	389	490	1.26	1.87	1.66
PC13	365	375	335	60	2.63	0.142	509	785	4	642	680	1.06	0.72	1.52
PC14	365	375	335	64.5	2.63	0.142	509	785	8	660	686	1.04	0.98	1.13
PC15	365	375	335	56.6	2.63	0.142	509	785	20	362	358	0.99	1.02	1.02
PC16	365	365	335	56.6	2.63	0.667	435	630	4	1112	945	0.85	1.05	1.05
PC17	365	365	335	56.6	2.63	0.667	435	630	8	1112	707	0.64	1.07	1.07
PC18	365	625	335	29.5	2.63	0.085	509	785	4	655	832	1.27	0.93	1.72
PC19	365	625	335	29.5	2.63	0.085	509	785	8	655	751	1.15	1.1	1.32
PC20	365	625	335	47.3	2.63	0			6	534	715	1.34	0.64	1.19
PC21	365	625	335	47.3	2.63	0			4	534	767	1.44	0.69	1.52
PC22	365	375	335	53.2	2.63	0.142	509	785	0	361	443	1.23	1.75	1.55
PC23	365	375	335	53.2	2.63	0.142	509	785	3	613	603	0.98	0.8	1.63
PC24	365	375	335	53.2	2.63	0.142	509	785	1.5	526	528	1.00	1.92	1.67
											Ave.	1.04	1.03	1.31
											Std. Dev.	0.18	0.40	0.25

Fig. 16 Application of loads and measurement of deformation.

Here, the experiments (Shioya *et al.* 1989) are used to investigate more the ability of the proposed method to predict the shear strength of large RC members with different depths or maximum aggregate size. As it is

shown in Fig. 17 the shear strength predicted by the proposed formula is in good agreement with the experimental results, which shows the accuracy of the method of considering the size effect. Also the calculation procedure is shown in Table 9.

8. Conclusions

Despite extensive research on the behavior of reinforced concrete in shear, there is still considerable disagreement among researchers in proposing and using a rational way of calculating the shear strength of reinforced concrete. Most of the design codes have adopted semi-empirical methods with several different expressions for calculating the shear strength under different loading conditions and different types of members. Thus, the shear strength predicted by different current design codes for a particular RC member can vary by factors of more than 2.

In this paper, on the basis of complex computation of local stresses in the crack surface of RC elements a rational simple formula and computation procedure are proposed for shear design and failure assessment of reinforced concrete members. This proposed method is in good agreement with the experimental results and can

Table 9 Shioya *et al.* (1989) beams shear capacity calculation.

Properties & geometry										S_x (mm)	S_y (mm)	θ (mm)	S_θ (mm)	ε_x (mm)	S_c (mm)	V_c (MPa)	Comp. shear (kN)
Spec.	h (mm)	b (mm)	d (mm)	$f'c$ (MPa)	f_t (MPa)	ρ_x (%)	ρ_y (%)	f_{yy} (MPa)	N/V								
3	655	300	600	21.1	1.81	0.4	0	440	0	480	655	45	391.7	0.00152	0.596	0.418	75
5	1200	500	1000	21.9	2.23	0.4	0	370	0	800	1200	45	678.8	0.00092	0.624	0.408	204
6	2100	1000	2000	28.5	2.73	0.4	0	370	0	1600	2100	45	1284.2	0.00162	2.080	0.350	700
7	3140	1500	3000	24.3	2.19	0.4	0	360	0	2400	3140	45	1923.6	0.00123	2.366	0.290	1305

Fig. 17 Size effect verification, tests by Shioya *et al.* (1989).

be used for different conditions of material properties and loading condition with only one equation. It is shown that the proposed relations can predict the strength of RC members when having high strength concrete, welded wire mesh, different bar distribution and size effect with a good accuracy.

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