Numerical Simulation on Shear Capacity and Post-Peak Ductility of Reinforced High-Strength Concrete Coupled with Autogenous Shrinkage

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Abstract

The shear capacity and post-peak ductility of reinforced high-strength concrete (HSC) beams, which are greatly affected by both autogenous shrinkage and a notable reduction in shear transfer along HSC crack planes, are simulated using nonlinear finite element (FE) analysis. The volumetric change caused by autogenous shrinkage is incorporated into the analysis by introducing an effective shrinkage strain related to the initial stress that develops in the reinforcement. The computed capacity, loading/unloading stiffness, crack pattern, and mode of failure replicate data obtained from systematic experiments. Approximately 50% of the plain concrete early-age shrinkage is observed to be consistent with self-induced stresses in structural concrete. The impact of autogenous shrinkage is further emphasized in assessing shear performance together with reduced crack shear transfer. Shrinkage changes the stress transfer path and may also alter the failure mode. The multi-directional fixed crack approach is verified as a reliable structural concrete model in the case of high autogenous shrinkage as well.

1. Introduction

The invention of super plasticizers and other mineral additives has enabled the use of high-strength concrete (HSC) in industrial structures, where it allows for more efficient use of space and enhances structural performance. HSC is characterized by a low water-to-binder ratio and is particularly vulnerable to self-desiccation during the hydration process at early ages. Currently, it is well recognized that HSC undergoes significant autogenous shrinkage as well.

Formerly, the impact of autogenous shrinkage on the structural performance of reinforced concrete (RC) was regarded as insignificant (Ahmad et al. 1986; Elzanaty et al. 1986; Fujita et al. 2002; Suzuki et al. 2003; Zink 2000; and others), because the self-equilibrated stress is thought to rapidly fade away due to greater creep and relaxation of young concrete. However, recent great findings have indicated the importance of considering this early-age shrinkage in determining the shear capacity of reinforced HSC members. As a matter of fact, higher risk of early-age cracking due to self-induced stresses resulting from autogenous shrinkage has been reported (Pailiere et al. 1989; Tazawa and Miyazawa 1992; Schrage et al. 1992; Tazawa et al. 1994; Igarashi et al. 2000; Maruyama et al. 2006; and others), but these investigations did not indicate that members had reduced ultimate capacity. Sato and Kawakane (2008) experimentally showed that early diagonal cracking in high-shrinkage reinforced HSC beams resulted in a 5% to 18% reduced capacity, depending on the effective depth, compared to that of low-shrinkage HSC beams using an expansive chemical agent. This important contribution is notable for addressing the vital importance of the material development process to the understanding of real structural mechanics. In this paper, the aim is to address this key qualitative knowledge from the perspective of path-dependent mechanistic modeling.

A number of empirical formulae have been proposed for the prediction of the diagonal cracking capacity of HSC members (Zink 2000; Fujita et al. 2002; Suzuki et al. 2003), but these incorporate no explicit consideration of autogenous shrinkage. To address the effects of autogenous shrinkage, Kawakane and Sato (2008) proposed a new design equation in which the concept of an equivalent reinforcement ratio was introduced. Although this model is not explicitly dependent on the magnitude of early-age deformation, the approach is doubtlessly useful in practice.

An alternative is the multi-scale chemo-physical thermodynamic approach (Maekawa et al. 2003), which is able to treat cement-chemical events as well as mechanistic processes without the need for any equivalent factors. Analysis can be carried out from the onset (post-casting) up to cement hydration in space. The microstructure, solid strength, elastic modulus, internal relative humidity, shrinkage, stress/strain, and cracking can be computed for specified ambient and loading conditions. Figure 1 provides an overview of the multi-scale scheme (Maekawa et al. 2003) and shows the load-displacement relation for an HSC beam (Kawakane and Sato 2008) as an example. Here, the initial stress of both reinforcement and concrete is simulated by solving for moisture states in the micro-pores and the associated driving force of early-age deformation (Asamoto et al. 2002; Suzuki et al. 1992; Fujita et al. 2003; Zink et al. 2003; and others), but these incorporate no explicit consideration of autogenous shrinkage, Kawakane and Sato (2008) proposed a new design equation in which the concept of an equivalent reinforcement ratio was introduced. Although this model is not explicitly dependent on the magnitude of early-age deformation, the approach is doubtlessly useful in practice.

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The macro-stress fields are computed by multi-directional crack modeling.

Here, it must be noted that the accuracy of this system depends on many models at different scales with complex interlinking. Even if the computational predictions obtained prove consistent with a small number of real cases, this does not imply global accuracy. For this reason, the authors first focus on verifying the sub-systems that together cover computations of the macro-scale behavior from meso-scale activities (see Fig. 1), leading up to representation of the effective shrinkage strain associated with initial stress just before shear loading. The other major sub-system, in which the meso-scale scalar is computed from nano-micro scale physical chemistry, will be examined in an accompanying study in future.

Path dependent constitutive models (Collins and Vecchio 1986; Maekawa et al. 2003) have been successfully applied to the behavioral simulation of HSC members where autogenous shrinkage has no substantial impact. Currently, multi-directional fixed crack modeling is being used not only for design but also in the maintenance phase of existing infrastructure. From this base, the authors discuss the impact of autogenous shrinkage as well as the reduced shear transfer along HSC crack planes. The focus is on the shear strength of various HSC beams with greatly varying autogenous shrinkage under the same crack-shear transfer character which is thought to be common for HSC, and the main aim of the work is a clear verification of the sub-system illustrated in Fig. 1.

2. Experiment on reinforced HSC beams subjected to autogenous shrinkage

This section outlines the experimental investigation carried out by Sato and Kawakane (2008), based on which the applicability of the FE framework with integrated path- and time-dependent RC constitutive models is examined. Table 1 indicates the dimensions and detailing of the specimens. None of the beams have web reinforcement so as to ensure that they fail in shear mode. In each series of experiments, beams with low and high autogenous shrinkage (LAS and HAS, respectively) were examined. For each RC beam, a corresponding plain concrete prism was made and the free autogenous shrinkage was measured. Material properties and main reinforcement strains in the RC beams before the shear loading are summarized in Table 2. In all cases, the shrinkage strain induced in the reinforcement is smaller than the free concrete shrinkage because of confinement by bonding. Overall, a wide range of parameters were covered in the experiment, including effective depth (234-1000mm), shear-span(702-3000mm), reinforcement ratio (1.5-3.39%), yield strength of steel (351-1208MPa), early age shrinkage (-690µm +216µm). Detailed information on the experimental

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**Fig. 1 Overview of the multi-scale computational scheme (Maekawa et al. 2003) and the current research target.**
yield strength of concrete. The beams described above are simulated, in beams 3.1 Behavior of shear critical reinforced HSC program is available in the original paper (Sato and Kawakane 2008).

Table 1 Details of experiment by Sato and Kawakane (2008).

<table>
<thead>
<tr>
<th>Series</th>
<th>Designation of Specimen</th>
<th>b/h/l (mm)</th>
<th>c (mm)</th>
<th>a (mm)</th>
<th>d (mm)</th>
<th>( \rho_s ) (%)</th>
<th>( \rho_c ) (%)</th>
<th>( f_t (MPa)/E_t (GPa) )</th>
</tr>
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<tbody>
<tr>
<td>I</td>
<td>I-HAS25-A,B</td>
<td>150/300/2300</td>
<td>200</td>
<td>750</td>
<td>250</td>
<td>50</td>
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<td>0.38 (2D10)</td>
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Table 2 Mechanical properties of concrete reported by Sato and Kawakane (2008).

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<tr>
<th>Designation of Specimen</th>
<th>( f_c ) (MPa)</th>
<th>( f_t ) (MPa)</th>
<th>( E_c ) (GPa)</th>
<th>( G_f ) (N/mm)</th>
<th>( \varepsilon_{sh,con} ) ((\times 10^{-6}))</th>
<th>( \varepsilon_{sh,steel} ) ((\times 10^{-5}))</th>
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<td>126</td>
<td>7.4</td>
<td>50.7</td>
<td>0.209</td>
<td>216</td>
</tr>
</tbody>
</table>

\( f_c \): cylinder compressive strength at the age of loading; \( f_t \): splitting tensile strength at the age of loading; \( E_c \): elastic modulus of concrete at the age of loading; \( \varepsilon_{sh,con} \): free autogenous shrinkage of concrete (positive for expansion, negative for shrinkage); \( \varepsilon_{sh,steel} \): steel bar strain induced by autoshrinkage; \( G_f \): fracture energy at the age of loading (*value assumed by Sato and Kawakane 2008*).

3. Two-dimensional nonlinear finite element simulation

3.1 Behavior of shear critical reinforced HSC beams

The beams described above are simulated, in two-dimensional space, using the multi-scale fixed four-way crack model in which the strain-path and time-dependent concrete constitutive models are integrated. This analytical platform has previously been used for the evaluation of shear critical reinforced normal strength concrete (NSC) and understanding its structural behavior (e.g. Maekawa et al. 1998; An et al. 1998). Provided that the properties of high-strength materials can be formulated as appropriate constitutive models with reasonable
accuracy, the range of applicability of the platform can be extended based on existing models (Okamura and Maekawa 1991). Using the same analytical framework, Tsuchiya et al. (2002) numerically evaluated the shear performance of reinforced HSC beams with reasonable accuracy. Their paper pointed out the phenomenon whereby the shear failure of reinforced HSC members cannot be addressed solely by changing the compressive strength and presented a number of reasons for this. First, crack surfaces in HSC are characterized by a smoother fracture plane than those in NSC, because cracks tend to form through the aggregate particles rather than around them. This can be addressed by reducing the cracked shear transfer resistance of HSC as compared to that of NSC (Bujadham and Maekawa 1992). Second, HSC is characterized by a sudden stress release after cracking, which can be addressed by the tension softening effect. Third, the compression response of HSC is characterized by relatively stiff and brittle response in contrast to that of the NSC. They studied concrete with a compressive strength of 60-70 MPa without silica fume, and used the factorized tensile strength to consider the initial tension caused by autogenous shrinkage.

3.2 Computational constitutive models of HSC

This section describes the material constitutive models for HSC in tension, shear, and compression that are employed in simulating the reinforced HSC beams. Cracks are expressed by means of a four-way fixed cracking approach based on the active crack method, in which mutual crack-to-crack interaction is considered (Maekawa et al. 2003). A brief outline of the core constitutive models is presented in the subsequent sub-sections.

3.2.1 Concrete tension constitutive model

The original tension model by Okamura and Maekawa (1991) which was later enhanced in consideration of high-cycle time-dependent fracturing (Maekawa et al. 2006) is adopted without modification for the high strength plain concrete domain. In the model, the envelope for the post-cracking response is expressed by Eq. (1).

\[
\sigma_c(\varepsilon_c) = f_t \left( \frac{\varepsilon_{tu}}{\varepsilon_c} \right)^c \quad \text{for} \quad \varepsilon_c > \varepsilon_{tu} \\
\varepsilon_c = \varepsilon_{tu} - \varepsilon_{eff\_free}
\]

where \( \sigma_c \) is the transferred tensile stress, \( \varepsilon_{tu} \) is the crack strain defined equal to \( 2f_t/E_o \), \( E_o \) is the initial stiffness of concrete, \( \varepsilon_c \) is the stress associated strain of concrete after cracking, \( \varepsilon_{to} \) is the total strain of concrete, \( \varepsilon_{eff\_free} \) is the effective stress-free strain of concrete, \( f_t \) is the uniaxial tensile strength, and \( c \) is a tension softening parameter, which describes the softening stress across the fracture process zone of the concrete. In the multi-directional fixed crack formulation scheme (Maekawa et al. 2003), the value of \( c \) depends on the concrete fracture energy in tension and the sizes of the finite elements (Bazant and Oh 1983). The softening parameter for plain concrete is determined based on the fracture energy balance, which is inversely related to the element size (An et al. 1997) in accordance with Eq. (2) and as illustrated in Fig. 2.

\[
\frac{G_f}{l} = \int_{\varepsilon_{tu}}^{\varepsilon_{te}} \sigma_c(\varepsilon) d\varepsilon + 1/4f_t\varepsilon_{tu}
\]

where \( G_f \) is fracture energy (N/mm), \( l \) is element size (mm), \( \varepsilon_{tu} \) is cracking strain, and \( \varepsilon_{te} \) is ultimate tensile strain.

In consideration of the fracturing process after cracking, the transient tension model proposed by Soltani and An (2001) is adopted for the cracked concrete volume in an RC element. The model exhibits the sudden release of tension that occurs just after crack localization where a low tensile strain field develops. Its applicability to NSC has been validated by experiment (Soltani 2002; Soltani et al. 2003). Even after fracturing advances in plain concrete, tensile stress is spatially transferred over the cracked concrete domain through the bond mechanism of deformed steel and concrete. This post-cracking space-averaged stress-strain relation is generally defined as tension-stiffness. The whole process from brittle tensile softening to stabilized tension transfer through the bond mechanism is given in Eq. (3) and schematic representation is shown in Fig. 3.

\[
\frac{G_f}{l} = \int_{\varepsilon_{tu}}^{\varepsilon_{te}} \sigma_c(\varepsilon) d\varepsilon + 1/4f_t\varepsilon_{tu}
\]
applied. Shrinkage appears in the reduced apparent cracking stress effect of the self-equilibrated stress induced due to shrinkage on the tensile response of an RC member. The shows the typical effect of pre-loading shifted tensile stress-strain model for structural concrete.

3.2.2 Shear model

It is well known that cracks in HSC are characterized by smoother fracture planes. Bujadham and Maekawa (1991) investigated the roughness profile of the crack interface in HSC and proposed a universal shear transfer model based on the concept of contact density (Li et al. 1989). To simulate the reinforced HSC beams under consideration here, this universal shear transfer model is applied to the simulation. The model is based on the integration of local contact forces along a finite crack surface. The transferred shear and confining stresses can be expressed by Eq. [4] as,

\[
T = \int_{-\pi/2}^{\pi/2} (N_N(\omega, \delta, \theta) \sin \theta + T_N(\omega, \delta, \theta) \cos \theta) d\theta \\
\sigma = \int_{-\pi/2}^{\pi/2} (N_T(\omega, \delta, \theta) \cos \theta + T_T(\omega, \delta, \theta) \sin \theta) d\theta
\]

(4)

where \(N_N\) and \(T_T\) are the local forces at each contact point along the contact surface in the normal and tangential directions, respectively, and are expressed as Eq. [4a].

\[
N_N(\omega, \delta, \theta) = K(\omega) \sigma_{con}(\theta) A \Omega(\theta) d\theta \\
T_N(\omega, \delta, \theta) = K(\omega) \tau_{con}(\theta) A \Omega(\theta) d\theta
\]

(4a)

where \(A \Omega(\theta)\) represents the area of crack surface per unit projected area and \(\Omega(\theta)\) is the density function for defining the geometry of the crack surface (and expressed for HSC as Eq. [4b]).

\[
\Omega(\theta) = \frac{5}{6} \exp \left(-21 \left(\frac{\theta}{\pi}\right)^2\right)
\]

(4b)

where \(K(\omega)\) represents the proportion of effective contact area and is expressed by the crack width as Eq. [4c].

\[
K(\omega) = 1 - \exp \left(1 - \frac{0.5 G_{\max}}{\omega}\right) \geq 0 \text{ and } \omega \neq 0
\]

(4c)

Since cracks in HSC pass through the aggregate particles, an apparent value of 6mm is assumed for \(G_{\max}\). This is based on an experimental investigation of the crack roughness profile of HSC by Bujadham and Maekawa (1991). The local normal stress, \(\sigma_{con}\), and the local tangential stress, \(\tau_{con}\), are governed by the local deformation of the contact points and are dependent on the kinematics of the crack interface. The effects of local anisotropic fracturing and plasticity on the contact points are also rooted in the model and the schematic representation of each is given in Fig. 5. The applicability of the model has been verified by extensive experimental results both under single and mixed mode crack kinematics (Bujadham et al. 1992). This highly nonlinear model has been used for RC joints in which discrete crack kinematics prevails. Recently, the applicability of this model has been extended into the smeared crack approach, with an input of crack spacing.

3.2.3 Compression model

As the main focus of the current study is on the shear failure of reinforced HSC beams prior to yielding of the main reinforcement and concrete compression softening, the time-dependent elasto-plastic and fracture compression model developed for NSC (El-Kashif and Maekawa 2004) is simply adopted without any modification. Generally, the behavior of HSC concrete under uniaxial compression is characterized by stiffer and more brittle response as compared to NSC. A schematic representa-
tion of the elasto-plastic and fracturing model (El-Kashif and Maekawa 2004) is shown in Fig. 6.

3.3 Finite element modeling
Two-dimensional nonlinear finite element analyses were carried out for the experimental series identified above. The analyses were performed using the multi-directional fixed crack FE framework. A typical finite element model representing one of the beams is shown in Fig. 7. To capture the post-peak localized response, full two-dimensional models were used for all beams. Rectangular meshes with eight nodes were used. All steel bars were modeled as embedded smeared reinforcement inside the elements. Concrete elements without steel bars were modeled as plain concrete elements characterized by unyielding behavior. For each element size, the softening parameter was determined based on the experimentally measured fracture energy. Concrete with steel reinforcement was modeled as RC elements characterized by tension stiffening. The stiffening parameter is set inversely estimated from the beam test results to give consistent first flexural cracking load.

3.3.1 Material properties and effective autogenous shrinkage
The experimentally obtained values for the material properties of concrete and steel reinforcement, as presented in Section 2, were used, except in the case of concrete tensile strength. Concrete tensile strength plays a major role in defining the failure load of RC beams when the failure mode is diagonal tension cracking. Figure 8 is a plot of the experimentally measured splitting tensile strength values versus the compressive strength values. There is a large variation in measured tensile strength values even for the same compressive strength values. The splitting tensile strength is known to result in estimates of concrete tensile strength that are near the upper bounds. Thus, direct use of splitting tensile strength values may not be appropriate for FE analysis. Here, the tensile strength values for FE analysis are inversely estimated from the beam test results to give consistent first flexural cracking load.

The determination of tensile strength by inverse analysis from the cracking load is not straightforward for the following reasons. First, a specimen is initially under self-equilibrated stresses generated by autogenous shrinkage. Second, the measured autogenous shrinkage values described in Section 2 are free shrinkage values and only a portion of this free shrinkage will be responsible for self-induced stress in an RC beam. A significant proportion of autogenous shrinkage develops at an early age before concrete maturity is achieved. Thus, the free autogenous shrinkage is not fully resolved into stress due to early age creep, relaxation, micro-cracking, and other

Fig. 5 Schematic representation of the universal shear transfer model (Bujadham and Maekawa 1991).
effects. Under such restrained conditions, concrete stress is determined by the balance between concrete deformation and the development of elastic modulus and creep behavior (Ito et al. 2004). This situation can be explicitly tackled by mutually linking a material development model based on the chemo-physical thermodynamics of concrete with the full three-dimensional structural mechanics framework (Maekawa et al. 2003). Here, the concept of effective autogenous shrinkage that is associated with stress is introduced as the authors’ focus is on verification of the multi-directional fixed crack approach to concrete modeling with high autogenous shrinkage. Thus, in modeling the RC beams, it remains important to compute the effective autogenous shrinkage that is associated with stress prior to the estimation of the tensile strength.

In their experimental study, Sato and Kawakane reported the shrinkage-induced strain values of the main reinforcement in the RC beams prior to shear loading. This information is used to estimate the effective autogenous shrinkage. The input free strain that induces the same level of steel strain as recorded in the experiments with a tolerance of ±10µ is defined as the effective autogenous shrinkage. Consequently, the computed values of effective autogenous shrinkage are initially input as a uniform free strain, similar to that used for thermal expansion, into the RC beam model before shear loading is applied. As autogenous shrinkage is an intrinsic characteristic of the cement paste, it can be assumed to have a uniform spatial distribution throughout the mass of the concrete. Thus, unlike the effects of drying shrinkage, the effects of autogenous shrinkage can be dealt in the form of a smeared uniform strain over the analysis domain in FE analysis. Then a displacement-controlled shear load is applied at a rate of 0.2mm/minute and the input concrete tensile strength that gives the same first flexural cracking load as that of the experimental result is taken as the corresponding tensile strength. Accordingly, for each beam, effective autogenous shrinkage and tensile strength values are determined and are presented in Table 3.

### 3.3.2 Evaluation of tensile strength obtained by inverse analysis

The tensile strength results obtained by inverse analysis are compared against the measured values, as well as against proposed empirical formulae, in Fig. 8. The inverse analysis tensile strength results are 94±4%, 90±4%, and 90±2.5% of those obtained by the formulae proposed by Zain (Eq. 5a) Iravani (Eq. 5c), and the extrapolated American Concrete Institute (ACI) formula (Eq. 5b). Taking a fracture mechanics point of view, Rocco et al. (2001) proposed a value around 0.92 for the ratio of tensile strength to splitting tensile strength in the case of cylindrical standard specimens tested according to ASTM C496. Similarly, the CEB-90 Model Code gives the uniaxial tensile strength as 90% of the splitting tensile strength. These other relations provide fair support for the results obtained by inverse structural analysis.

\[
f_u = 0.54 \sqrt{f'_{c}} (W / B)^{0.07} \quad \text{Zain et al. (2002)}
\]  

(5a)
It can be seen that nearly 50-60% of total free shrinkage is attributable to the reinforced concrete domain. The analytical observation is responsible for generating self-equilibrated stresses in young concrete. However, the current study indicates that around 50% of autogenous shrinkage is effective in generating self-induced stresses when the tensile reinforcement ratio is 1.55%. This value decreases to around 40-45% for tensile reinforcement ratios of 2.7% and 3.39%, as in beams I-HAS23 and II-HAS25. In the past, the impact of autogenous shrinkage on RC structural performance was regarded as being insignificant, because the self-equilibrated stress was thought to rapidly fade away due to greater creep and relaxation of young concrete. However, the current study indicates that around 50% of autogenous shrinkage is effective in generating self-equilibrated stress in RC members, implying that there is in fact a significant impact. In the case of HSC, though most of the autogenous shrinkage occurs at an early age, the development of elastic modulus is also rapid. Measurements of the development of elastic modulus in HSC (137 MPa) have been reported by Ma-ruyama et al. (2006), showing that more than 50% of the maximum modulus is developed within one day. Thus, the interaction between early-age shrinkage, development of elastic modulus, and creep behavior is important in determining the self-equilibrated stress arising in RC members. The authors want to strongly emphasize this point and hope to promote increased awareness of the importance of accurate measurement of early age (within one day) autogenous shrinkage in assessing the structural performance of reinforced HSC members.

### 3.3.3 Evaluation of effective autogenous shrinkage

As explained in the Section 3.3.1, the effective autogenous shrinkage is the portion of total free shrinkage that is responsible for generating self-equilibrated stresses in the reinforced concrete domain. The analytically obtained values are tabulated in Table 3 for the HAS beams. It can be seen that nearly 50-60% of total free shrinkage is effective in generating self-induced stresses when the tensile reinforcement ratio is 1.55%. This value decreases to around 40-45% for tensile reinforcement ratios of 2.7% and 3.39%, as in beams I-HAS23 and II-HAS25. In the past, the impact of autogenous shrinkage on RC structural performance was regarded as being insignificant, because the self-equilibrated stress was thought to rapidly fade away due to greater creep and relaxation of young concrete. However, the current study indicates that around 50% of autogenous shrinkage is effective in generating self-equilibrated stress in RC members, implying that there is in fact a significant impact. In the case of HSC, though most of the autogenous shrinkage occurs at an early age, the development of elastic modulus is also rapid. Measurements of the development of elastic modulus in HSC (137 MPa) have been reported by Ma-ruyama et al. (2006), showing that more than 50% of the maximum modulus is developed within one day. Thus, the interaction between early-age shrinkage, development of elastic modulus, and creep behavior is important in determining the self-equilibrated stress arising in RC members. The authors want to strongly emphasize this point and hope to promote increased awareness of the importance of accurate measurement of early age (within one day) autogenous shrinkage in assessing the structural performance of reinforced HSC members.

### 3.3.4 Loading

Using the model developed in the previous sections, the RC beams are subjected to induced displacement loading. Prior to loading, the effective autogenous shrinkage strain is input. During this process, the elastic modulus of the loading plates and support bearings, modeled as

![Fig. 8 Comparison of tensile strength obtained by inverse analysis against other proposed relations.](image)

\[ f_{sp} = 0.59 \sqrt{f_{c}} \; ; \; \text{for} \; f_{c} \leq 83 \text{MPa} \quad \text{ACI} - 363 R92 \]  
(5b)

\[ f_{u} = f_{sp} = 0.57 \sqrt{f_{c}} \; ; \; \text{for} \; f_{c} \leq 100 \text{MPa} \quad \text{Iravani (1996)} \]  
(5c)

**Table 3 Effective autogenous shrinkage and inversely obtained tensile strength values.**

<table>
<thead>
<tr>
<th>Series</th>
<th>Designation of Specimen</th>
<th>( f_{c} ) (MPa)</th>
<th>( f_{t} ) (MPa)</th>
<th>( \varepsilon_{sh,con} \times 10^{-6} )</th>
<th>( \varepsilon_{sh,con,effective} \times 10^{-6} )</th>
<th>( \varepsilon_{sh,con}/\varepsilon_{sh,con,effective} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>I-HAS25-A,B</td>
<td>115</td>
<td>5.8</td>
<td>-330/-360</td>
<td>120/-150</td>
<td>0.54-0.56</td>
</tr>
<tr>
<td>I</td>
<td>I-LAS25-A,B</td>
<td>101</td>
<td>5.3</td>
<td>-66/-39</td>
<td>246/-275</td>
<td>0.43-0.40</td>
</tr>
<tr>
<td>I</td>
<td>I-HAS23-A,B</td>
<td>115</td>
<td>5.8</td>
<td>-225/-225</td>
<td>120/-150</td>
<td>0.56</td>
</tr>
<tr>
<td>I</td>
<td>I-LAS23-A,B</td>
<td>101</td>
<td>5.3</td>
<td>-124/-124</td>
<td>246/-275</td>
<td>0.62</td>
</tr>
<tr>
<td>II</td>
<td>II-HAS25</td>
<td>127</td>
<td>5.8</td>
<td>-192</td>
<td>357/-30</td>
<td>0.44</td>
</tr>
<tr>
<td>II</td>
<td>II-LAS25</td>
<td>113</td>
<td>5.5</td>
<td>-23</td>
<td>357/-30</td>
<td>0.56</td>
</tr>
<tr>
<td>III</td>
<td>III-HAS50-A</td>
<td>124</td>
<td>6.0</td>
<td>-234</td>
<td>488/-275</td>
<td>0.56</td>
</tr>
<tr>
<td>III</td>
<td>III-HAS50-B</td>
<td>121</td>
<td>6.0</td>
<td>-247</td>
<td>415/-290</td>
<td>0.70</td>
</tr>
<tr>
<td>III</td>
<td>III-HAS50-C</td>
<td>128</td>
<td>6.1</td>
<td>-238</td>
<td>450/-280</td>
<td>0.62</td>
</tr>
<tr>
<td>III</td>
<td>III-LAS50-A</td>
<td>120</td>
<td>5.7</td>
<td>-40</td>
<td>5/-60</td>
<td>0.60</td>
</tr>
<tr>
<td>III</td>
<td>III-LAS50-B</td>
<td>121</td>
<td>6.0</td>
<td>-6</td>
<td>27/-10</td>
<td>0.60</td>
</tr>
<tr>
<td>III</td>
<td>III-LAS50-C</td>
<td>117</td>
<td>6.0</td>
<td>-22</td>
<td>11/-30</td>
<td>0.60</td>
</tr>
<tr>
<td>IV</td>
<td>IV-HAS50-A,B</td>
<td>115</td>
<td>5.5</td>
<td>-225/-228</td>
<td>495/-275</td>
<td>0.56</td>
</tr>
<tr>
<td>IV</td>
<td>IV-LAS50-A,B</td>
<td>109</td>
<td>5.5</td>
<td>-1/-6</td>
<td>35/-10</td>
<td>0.56</td>
</tr>
<tr>
<td>V</td>
<td>V-HAS25-A,B</td>
<td>121</td>
<td>5.9</td>
<td>-227/-242</td>
<td>539/-250/-270</td>
<td>0.46-0.50</td>
</tr>
<tr>
<td>V</td>
<td>V-LAS25-A,B</td>
<td>117</td>
<td>5.8</td>
<td>-59/-50</td>
<td>17/-68/-60</td>
<td>0.60</td>
</tr>
<tr>
<td>V</td>
<td>V-HAS50-A,B</td>
<td>117</td>
<td>5.7</td>
<td>-310/-310</td>
<td>-608/-365/-365</td>
<td>0.59</td>
</tr>
<tr>
<td>V</td>
<td>V-LAS50-A,B</td>
<td>124</td>
<td>5.8</td>
<td>-69/-77</td>
<td>30/-85/-100</td>
<td>0.59</td>
</tr>
<tr>
<td>V</td>
<td>V-HAS100-A,B</td>
<td>123</td>
<td>6.0</td>
<td>-284/-294</td>
<td>-597/-350</td>
<td>0.59</td>
</tr>
<tr>
<td>V</td>
<td>V-LAS100-A,B</td>
<td>126</td>
<td>6.0</td>
<td>-33/-37</td>
<td>216/-50</td>
<td>0.59</td>
</tr>
</tbody>
</table>

\( f_{c} \): cylinder compressive strength at the age of loading; \( f_{t} \): splitting tensile strength at the age of loading; \( \varepsilon_{sh,con} \): steel bar strain induced by autogenous shrinkage; \( \varepsilon_{sh,con,effective} \): effective autogenous shrinkage of concrete; \( \varepsilon_{sh,con}/\varepsilon_{sh,con,effective} \): effective autogenous shrinkage of concrete.
elastic elements, is intentionally made low: almost one-tenth that of the concrete. This is to reduce their effect in restraining concrete deformation. Later, the actual elastic modulus is input for these elements with the path-dependent parameters reset as null. Consequently, two-point loading is applied at a rate of 0.2mm/minute until failure. This analysis scheme is applied to all beams and the results are presented and discussed in Section 4.

4. Comparison with experimental results

Figure 9 shows comparison of the experimental and analytical results for I-HAS25A, B and I-LAS25A,B beams. All the simulated beams failed in diagonal tension, as observed in the experiments. The principal strain field plots for each beam just after diagonal cracking are indicated in the Figs. 9c, d, e, and h. Large strain concentrations can be observed at the localized diagonal cracks and the position of these localized cracks is closer to the loading plates in the LAS beams as compared to the HAS beams. However, the tensile strength of the HAS-A,B beams is higher by 9% as compared to that of the LAS-A,B beams, while the load corresponding to the formation of the diagonal shear crack is almost the same for both. The mid-span deflection just before the forma-

![Fig. 9 Comparison of experimental and analytical results for I-25 beams.](image-url)
tion of the diagonal crack for the HAS beams is nearly 13% larger than for the LAS beams, signifying a reduction in average stiffness due to the pre-induced stresses caused by autogenous shrinkage. Notably, the unloading and re-loading stiffness and the post peak response obtained computationally are consistent with the experiments. Overall, the behavior of the beams in this series is replicated quite well.

The series I-23 beams are similar to the series I-25 beams except for the high reinforcement ratio used (3.39% for the I-23 beams and 1.53% for the I-25 beams) and the width of the beams. Unlike the I-25 beams, which failed in diagonal tension, the I-23 beams failed in shear compression. This difference in mode of failure, reduction in stiffness, and reduction in capacity are fairly well simulated by the analysis, with minor overestimation of ultimate capacity for the high autogenous shrinkage beams (I-HAS23). Comparison of the load versus mid-span deflection curves for experiment and analysis is given in Fig. 10. The principal strain field plots during initiation and localization of the diagonal crack are also shown in Figs. 10c and d.

Series II beams have also high tensile reinforcement ratio, 2.7%. In this series, the early age shrinkage resulted in a remarkable decrease in load-carrying capacity. This large difference in capacity is manifested in the different modes of shear failure observed in the beams. The LAS beam failed in diagonal shear compression while the HAS beam failed in diagonal shear tension. The load versus mid-span deflection curves for experiment and analysis are compared in Fig. 11. In the analysis, it can be observed that diagonal crack formation in the LAS beam is close to the loading points and these cracks tend to propagate beneath the loading plates, which results in a diagonal shear compression failure. In contrast, the formation of diagonal cracks in the HAS beam is far from the loading points and these diagonal cracks can easily penetrate the compression zone resulting in diagonal tension failure, Figs. 11c and d. It is likely that pre-induced stresses resulting from autogenous shrinkage not only reduce the stiffness of RC beams but also result in a shift in the stress transfer path.

Unlike the beams discussed so far, the series III beams do not have compression reinforcement and their effective depth is 500 mm. Sato and Kawakane et al. (2008) carried out three tests for these LAS and HAS beams, with each having similar properties except for small differences in compressive strength and initially induced shrinkage. Of note is that the flexural capacity of these beams is close to their shear capacity. Consistently, the LAS beams failed in flexural shear while the HAS beams failed in diagonal shear tension, both in experiment and analysis. The load versus mid-span deflection curves for experiment and analysis are shown in Fig. 12. The principal strain field plot, shown in the Fig. 12h, clearly indicates the localization of the diagonal crack and shows that this crack penetrates the compression zone in the case of HAS beams. For the LAS beams, the diagonal crack tends to propagate beneath the compression zone, ultimately leading to yielding of the tensile reinforcement. The experimental and analytical facts observed with this series indicate that pre-induced stresses due to early-age shrinkage may result in brittle failure of RC beams.

Series IV beams are similar to Series III beams but they contain compression reinforcement and have a tensile reinforcement ratio of 2.06%. The load versus mid-span deflection curves for experiment and analysis are compared in Fig. 12. These HAS beams also have a remarkably reduced shear capacity as compared to that of the LAS beams. This large difference in shear capacity and the overall load versus mid-span deflection path are
fairly well simulated by the analysis. The shift in position and sudden localization of the diagonal shear crack after its formation is evident in the HAS beams, and can be observed in the principal strain field plot in Fig. 13b. The free shrinkage strain for the HAS beams was 495\(\mu\) and the corresponding effective shrinkage strain was 275\(\mu\). Remarkably, a reduction by nearly 50\% in shear capacity can be observed in both experiment and analysis.

The final experimental series, Series V, is mainly aimed at assessing the coupled effects of size and auto-
closer to the loading point as compared to the HAS localization of diagonal cracking in the LAS beams is field plot. As can be seen in this figure, the formation and crack pattern and analytically obtained principal strain Figure 14 compression and diagonal shear tension, respectively. observed in the LAS and HAS beams: diagonal shear compression and diagonal shear tension, respectively. Figure 14 also compares the experimentally observed crack pattern and analytically obtained principal strain field plot. As can be seen in this figure, the formation and localization of diagonal cracking in the LAS beams is closer to the loading point as compared to the HAS beams. This general trend can also be observed in the experiments. All in all, the analytical results reasonably replicate the experimental results in terms of capacity, loading and unloading stiffness, crack pattern, and failure mode.

5. Sensitivity analysis

5.1 Effect of low to moderate levels of effective early-age shrinkage on shear capacity

In Section 4, the applicability of the multi-directional fixed crack model of RC with integrated constitutive models for HSC was verified. Now, in this section, the clear effects of early-age shrinkage on the shear behavior of reinforced HSC beams are presented, by keeping all parameters the same except the effective autogenous shrinkage. This is because, it less likely to control every parameter in the experiment. For the purpose of investigation, a series-I beam is considered, with a compressive strength of concrete 101MPa and tensile strength of 5.3MPa. The beam is analyzed with different effective shrinkage strain values of 0\(\mu\), 100 \(\mu\), 200 \(\mu\), 300 \(\mu\), and 400 \(\mu\). These are in the range of the effective shrinkage values induced in the RC beams investigated by Sato and Kawakane (2008). The load versus mid-span deflection curves for each effective shrinkage strain are plotted in Fig. 15a. As can be seen in the figure, a gradual increase in effective shrinkage reduces the stiffness and diagonal cracking load. Before diagonal cracking, an increase in effective shrinkage leads to more extended flexural cracks for a given load level. Besides, shrinkage increases the number of flexural cracks and henceforth their distribution over the loading span. For illustration, the principal strain plots for the beam with no shrinkage and for that with 300\(\mu\) effective shrinkage are presented in Fig. 15c. The post-peak trend is a gradual decrease in ductility with increasing shrinkage. The reduced post-peak capacity resulting from early-age shrinkage can be explained by the reduced depth of the uncracked section of concrete and the wider opening of the diagonal crack, which reduces shear transfer along the crack plane. Figure 15d shows the shear strain plot for beams with zero and 300\(\mu\) effective shrinkage at a mid-span deflection of 3.8mm. A greater concentration of shear strain at the diagonal crack can be observed for the beam with 300\(\mu\) effective shrinkage, which is a direct manifestation of crack opening and crack slip.

The above discussion of the effects of early-age shrinkage mainly focuses on beams with diagonal tension failure. However, where beams fail in shear compression, the effect of shrinkage is more pronounced. This is because shrinkage changes the failure mode, as observed in beams of series II, IV and V. Self-equilibrated stresses induced due to early-age shrinkage generate a wider distribution of flexural cracks over the loading span and result in a positional shift of diagonal cracking, towards the support. Hence, the diagonal crack can easily penetrate the compression zone, causing diagonal tension failure that leads to a remarkable reduction in shear capacity.

5.2 Effect of very high early-age shrinkage strain

As a general trend, early-age shrinkage reduces the stiffness and capacity of reinforced HSC beams. For very
high-strength concrete, with an extremely low water-to-binder ratio, larger values of effective shrinkage strain may be expected. To assess this, an extreme case with 600μ of effective strain is considered. The result for this case is shown in Fig. 15b along with the 0μ and 300μ cases. It can be observed that the capacity in the case of 600μ effective shrinkage strain is higher than in with 300μ case. This is due to the delayed formation of diagonal cracks resulting from the interaction between flexural and diagonal cracks. The importance of crack-to-crack interaction mechanism in assessing the shear performance of pre-cracked RC beams is pointed out by Pimanmas and Maekawa (2001). Thus, for extremely large values of early-age shrinkage the crack-to-crack interaction mechanism tends to prevail and may result in increased shear capacity of RC beams. This analytically observed understanding needs to be further verified through experiments.
6. Conclusions

The shear capacity and post-peak ductility of reinforced HSC beams with high autogenous shrinkage were simulated using the multi-directional fixed crack based model of RC in which appropriate constitutive models for HSC are incorporated. The following conclusions have been drawn from the results:

1. The analytical results replicated the experimentally observed behavior quite well in terms of capacity, loading/unloading stiffness, crack pattern, and failure mode, hence verifying the applicability of the multi-directional fixed crack approach as a structural concrete model even in the case of high autogenous shrinkage.

2. The effects of autogenous shrinkage in reinforced HSC beams can be addressed by introducing an effective shrinkage strain that is associated with stress. For the beams considered here, approximately 50% of the total free shrinkage of plain concrete specimens with low water to binder ratio was observed to be consistent with self-induced stresses in the RC beams and this was responsible for the remarkable reduction in shear capacity of the beams.

3. It was found that self-equilibrated stresses resulting from autogenous shrinkage alter the stress transfer path in RC beams and may alter the failure mode.

4. The effect of very high early-age shrinkage was analytically examined and the crack-to-crack interaction mechanism was confirmed to prevail, resulting in increased shear capacity. This analytically obtained prediction needs to be verified through experiments in future.

5. The authors strongly emphasize that accurate measurement of early-age autogenous shrinkage is crucial in assessing the structural performance of reinforced HSC members with regard to shear. They hope that this work will help create increased awareness of this issue.

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