1. Introduction

Based on observations made after several major earthquakes that occurred in Taiwan, China, and other parts of the world, the most seriously damaged building types were found to be elementary and secondary schools. According to a report published by the Ministry of Education of Taiwan, a total of 786 schools (1985 classrooms) were damaged in the Chi-Chi earthquake. Even in Taipei City, which is about 150 km away from the epicenter, 67 school buildings were damaged (Chung et al. 2009). As school buildings usually serve as emergency shelters following disastrous earthquake events, seismic upgrading of existing elementary and secondary school buildings is an especially pressing issue. In general, the seismic performance of a reinforced concrete (RC) building can be evaluated based on the capacity spectrum method proposed by ATC-40 (ATC 1996); then, by means of upgrading the structural strength or the ductility capacity, or a combination of both, the retrofitting seismic performance can be upgraded to meet or even exceed the code-required level. However, the cost of maintenance and damage control over the remaining service life are not considered in the seismic retrofit design, which focuses solely on safety performance.

Several models or procedures have been proposed to define and quantify multiple performances of a structure, such as safety, serviceability, economy, and repairability (Takahashi et al. 2006; Maeda and Kang 2009). Furthermore, life-cycle assessments have also been included in many proposed models of performance-based seismic design for structures in recent years (Takahashi et al. 2004 and Chiu et al. 2010). Although Padgett et al. (Padgett et al. 2010) have proposed the risk-based seismic life-cycle cost-benefit analysis method for bridge retrofit assessment, the concept of performance-based design is seldom mentioned in seismic retrofit work for existing RC buildings. This study proposes an estimating procedure that can be used to set an optimal seismic level requested in the seismic retrofit design for a low-rise RC building. Along with damage control, the cost of maintenance over the remaining service life is also included in the estimating procedure.

In the assessment of seismic structural damage for an RC building, the equivalent linearization method (Okano et al. 2002) and simplified evaluation of seismic energy demand (Manfredi 2001) were utilized to estimate the maximum deformation response and the hysteretic energy, respectively. Additionally, regarding the damage control aspect of this study, the uncertainties of ground motion and structural capacity were simulated to calculate the annual reliability index for a specified damage state.

Apart from upgrading the seismic performance, the cost of maintenance over the remaining service life should be considered in the seismic retrofit design. In this study, the cost of seismic retrofitting and repairs are both included in the cost of maintenance, and are expressed as the annual cost. A life-cycle estimating model for the repair cost of an RC building after an earthquake has
been proposed by Takahashi et al. (2006). These models and methods were adopted herein to estimate the repair cost of a low-rise RC building after an earthquake. Furthermore, the research data on the seismic retrofit cases in Taiwan were used to model the cost of the retrofit. Accordingly, the annual cost of maintenance of an RC building can be estimated based on the repair cost model and the seismic retrofit cost model, which is then expressed as the annual cost considering a specified remaining service life.

In the evaluating procedure of optimal seismic retrofit level for a low-rise RC building proposed in this study, when a seismic retrofit level defined by the ground acceleration corresponding to the ultimate deformation of the single degree of freedom (SDOF) system for a selected building is specified, several seismic retrofit methods can be used to achieve the specified seismic retrofit level. Since their respective maintenance costs differ, the average value of the annual costs of maintenance for a specified seismic retrofit level was adopted. Under the condition that the average value of the annual reliability indices of a seismic retrofit level for a specified damage state needs to be equal to or greater than the allowable annual reliability index, the seismic retrofit level corresponding to the minimal value is regarded as the optimal level. Finally, a case study was used to discuss the application of the estimating procedure for the optimal seismic retrofit level proposed here.

2. Assessment of building damage

In this section, the Park and Ang model (Park et al. 1985) was adopted to consider damage control. Although time history analysis is a common method for estimating the maximum deformation response and the hysteretic energy of a structure during an earthquake, it is not efficient for calculating seismic reliability or risk. Therefore, instead of using time history analysis, the equivalent linearization method (Okano et al. 2002) and simplified evaluation of seismic energy demand (Manfredi 2001) were utilized to estimate the maximum deformation response and the hysteretic energy, respectively. Moreover, in order to incorporate the uncertainties of ground motion and structural capacity into damage control in this study, the response spectral acceleration, the maximum deformation response, the hysteretic energy, and the limiting value for the allowable damage state were assumed using random variables to calculate the annual reliability index for the controlled damage state.

2.1 Seismic structural damage

The Park and Ang model, which is considered to be the most commonly used in the technical literature (Cosenza et al. 2009), is defined as a linear combination of the maximum deformation response and the hysteretic energy. Its damage index is expressed as:

\[ D_{PA,i} = \frac{\delta_u}{\delta_i} + \frac{\beta}{F} \int dE \]  

where \( D_{PA,i} \) is the damage index, an empirical measure of damage (\( D_{PA,i} \geq 1.0 \) indicates total damage or collapse), \( \delta_u \) is the maximum response deformation, \( \delta_i \) is the ultimate deformation under static loading, \( F \) is the calculated yield strength, \( dE \) is the absorbed incremental hysteretic energy (excluding the potential energy), and \( \beta \) is the coefficient for cyclic loading effect (a function of structural parameters and equal to 0.05 for RC buildings (Takahashi et al. 2006)).

In this study, the capacity spectrum proposed by ATC-40 was adopted to find the ultimate deformation \( \delta_u \), yielding strength \( F \), and the elastic fundamental period \( T_s \) of the SDOF system for a building. In the capacity spectrum, nonlinear static analysis (pushover analysis) can be used to simulate the capacity curve of a building, which is the relationship between the base shear force \( V \) and the displacement of the roof \( \delta_R \). The conversion of the capacity curve to the capacity spectrum is then accomplished by knowing the dynamic characteristics of the structure in terms of the first modal participation factor, and the first modal mass coefficient. After the capacity spectrum has been plotted, it is useful to approximate an equivalent bilinear capacity representation that establishes an effective yield point \( (S_{dy}, S_{uy}) \) and an effective peak inelastic limit \( (S_{dy}, S_{uy}) \). Based on these two points, it is possible to obtain the ultimate deformation \( \delta_u = \delta_u \), yielding strength \( F \), and the elastic fundamental period \( T_s \) of the SDOF system.

Apart from total damage or collapse \( (D_{PA,i} = 1.0) \), different damage states with limiting values of \( D_{PA,i} \) lower than 1.0 can be set based on research conducted in the past.

2.2 Equivalent linearization method for the maximum deformation

Generally, the iterative calculation needs to be applied in the equivalent linearization method for deformation response of the structural SDOF system. However, if the response spectrum of the acceleration, velocity, or displacement is constant, the maximum deformation response of the SDOF system can be estimated without any iterative calculations. Okano et al. (2002) simplified the equivalent linearization method and verified its application through nonlinear time history analysis.

The basic assumptions in the equivalent linearization method proposed by Okano et al. (2002) are shown in Equations (2) to (4):

\[ S_A F_A = S_{ay} \]  
\[ F_A = \frac{1.5}{1+10h_{eq}} \]
\[ h_{eq} = \gamma (1 - 1/\sqrt{\mu_0}) + 0.05 \]  

(4)

where \( S_\alpha \) is the response spectral acceleration (g), \( S_\gamma \) is the yielding acceleration of the SDOF system for a building, \( F_0 \) is the yield strength, \( \mu_0 \) is the ductility (defined by \( \delta_\mu / \delta_y \)), and \( h_{eq} \) is the equivalent damping ratio (\( \gamma \) is a modification factor of the equivalent damping ratio and equal to 0.15 (AIJ 2005)).

Under the condition of the constant response spectrum velocity (in the medium period range), which is a common condition in seismic design, the equivalent fundamental period of the SDOF system for a building \( T_{max} \), can be approximated by Equation (5). The spectral acceleration \( S_\lambda \) in Equation (2) can then be replaced by the response spectral velocity \( S_V \), as shown in Equation (6).

Accordingly, the estimation formula (Equation (7)) for the maximum deformation response of the SDOF system for a building \( \delta_M \), and the constant response spectrum of velocity, can be used to build on the basis of Equations (2) to (6):

\[
T_{max} = \sqrt{\mu \times T_\gamma} 
\] 

(5)

\[
S_\lambda = \frac{2\pi}{T_{max}} S_V 
\] 

(6)

\[
\delta_M = [a \times S_V + b] \times \delta_y 
\] 

(7)

where \( \omega_\gamma \) is the elastic fundamental frequency of the SDOF system for a building, \( \delta_y \) is the yielding deformation of the SDOF system for a building, \( a = 1.5/(1.5 + 10\gamma_\gamma) \), and \( b = 10\gamma_\gamma / (1.5 + 10\gamma_\gamma) \).

By the same approach (Handou et al. 2001 and Minegishi et al. 2001), the maximum deformation response of the SDOF system for a building \( \delta_M \), with the constant response spectrum of acceleration and displacement, can be estimated using Equation (8) and Equation (9), respectively. In addition, the response spectrum of the displacement can be approximated using Equation (10).

\[
\delta_M = \left[ \frac{1}{b} \times \frac{c \times S_\lambda}{S_\gamma} \right] \times \delta_y 
\] 

(8)

\[
\delta_M = \frac{1}{4} \left[ b + \sqrt{b^2 + \frac{4a \times S_\lambda \times \omega_\gamma^2}{S_\gamma}} \right] \times \delta_y 
\] 

(9)

\[
S_\delta = \left( \frac{T_{max}}{2\pi} \right)^2 S_\gamma (\alpha_I T_{max}) 
\] 

(10)

where \( S_\delta \) is the spectral displacement, \( c = 1.5/10\gamma_\gamma \), and \( \alpha_I \) is the modification factor for the equivalent fundamental period (= 0.82 (Okano et al. 2002)).

In this paper, we adopted these equations to estimate the mean of the maximum deformation response, \( \delta_M \), of the SDOF system for a low-rise RC building under a specified response spectral acceleration, \( S_\lambda \). The coefficient of variation of \( \delta_M \) induced by earthquake ground motion was 0.3 based on the research conducted by Okano et al. (2002).

### 2.3 Assessment of hysteretic energy

A measurement of the distribution of cycle amplitudes is \( n_{eq} \), which represents the number of cycles at the maximum deformation response \( \delta_M \) of the SDOF system that a structure could develop in order to dissipate the total amount of hysteretic energy \( E_h \). That is:

\[
n_{eq} = \frac{E_h}{F_s (\delta_M - \delta_y)} = \frac{E_h}{M S_\gamma} (\delta_M - \delta_y) 
\] 

(11)

\[
F_s = \frac{F_s}{M \times S_\gamma} 
\] 

(12)

where \( E_h \) is the total hysteretic energy and \( M \) is the effective mass of the first mode of a structure.

By combining the estimation for the number of equivalent cycles \( n_{eq} \) with \( F_s \), an expression for the hysteretic energy in terms of the ductility \( \mu_0 \) can be developed. This is expressed as Equation (13). Therefore, introducing appropriate expressions of \( F_s \) made available from previous studies (AIJ 2005), and starting from the knowledge of the response spectral acceleration \( S_\lambda \), the hysteretic energy (which is the same as \( \int dE \)) can be approximated by applying Equation (13).

\[
E_h = \frac{E_h}{M} = (\mu_0 - 1) n_{eq} \frac{S_\gamma^2}{\omega_\gamma^2} (F_s)^4 
\] 

(13)

\[
n_{eq} = 1 + 0.18 \frac{1}{F_s} (1 - 1.5^{1/3} I_D)^{1/2} 
\] 

(14)

with \( r = T_2 / T_1 \) if \( T_2 \leq T_1 \), \( r = 1 \) if \( T_2 > T_1 \)

\[
I_D = \frac{I_D}{PGA \times PGV} 
\] 

(15)

where PGA and PGV are the peak ground acceleration and the peak ground velocity, respectively, while \( I_D \) is:

\[
I_D = \int_0^{T_e} a(t)^2 dt 
\] 

(16)

where \( a(t) \) is the ground acceleration, \( T_e \) is the duration of the earthquake, and \( T_s \) is the initial period of the medium period’s range. \( I_D \) is proportional to the Arias Intensity (Arias 1970).

According to Manfredi (2001), \( n_{eq} \) is linearly dependent on the earthquake characteristics by means of the seismic index \( I_D \) (Equation (15)) in Equation (13). This can be estimated using Equation (14), and therefore this index is assumed to be an indicator of the cyclic demand of the earthquake. In Manfredi’s research, a general overview of \( I_D \) values regarding different levels
of strong motion was summarized. Impulsive records displayed low values of $I_D$ (Bucharest (3.37 in Incerc)), whereas records with a large duration effect showed high values of $I_D$ (Mexico (14.98 in SCT), Chile (36.17 in Llolleo)).

In this work, three major earthquakes that occurred in the past ten years, causing major damage in Taipei, were chosen, i.e., the 921 Chi-Chi earthquake (Richter magnitude $M_L = 7.3$, 1999), the 0614 earthquake ($M_L = 6.3$, 2001), and the 331 earthquake ($M_L = 6.8$, 2002). (The number preceding each earthquake indicates the date of occurrence.) The recorded time history data in Taipei (http://www.cwb.gov.tw/eng) was then used to estimate the seismic indices $I_D$ (Fig. 1, 2, and 3). According to the results, the values of the seismic index $I_D$ for the 331 and the 0614 earthquakes are uniformly distributed between 5 and 15, but they are between 10 and 20 for the 921 Chi-Chi Earthquake. In the case study, two uniform distributions for the seismic index $I_D$ are set up: one is in the range of 5 to 15, and the other one is in the range 15 to 25.

2.4 Uncertainties in ground motion and structural capacity

In this study, there are two kinds of uncertainties that are considered in the estimation of the annual exceedance probability of a specified damage state for an RC building. One is related to ground motion and is included in not only the maximum deformation response of the SDOF system $\delta_{M}$ and the seismic index $I_D$ stated above, but also the response spectral acceleration $S_A$. Therefore, instead of a specified response spectral acceleration $S_A$, it is necessary to take into account the earthquakes that occur during the life of a building at different locations, times, and magnitudes when estimating the annual exceedance probability of a specified damage state. The hazard curve of response spectral accelerations for a building, which is obtained from the seismic hazard analysis, is adopted to reflect the uncertainty of earthquakes, as shown in Fig. 4 (Jean et al. 2006).

The other one is related to the structural capacity and is included in the limiting value of damage index $D_{lim}$. According to the research (Park et al. 1985), the limiting value of $D_{lim}$ for the total damage or collapse—i.e., the ultimate structural capacity—was assumed to be a log-normal distribution with a mean of approximately 1.0 and a standard deviation of 0.54. However, the limiting value of $D_{lim}$ for each specified damage state may be different due to the building and structure types, or the judgment of the damage. Table 1 was constructed for low-rise RC school buildings in Taiwan through experiments (NCREE-08-023 2008). Moreover, the limiting value of $D_{lim}$ for each specified damage state is assumed to distribute uniformly in the corresponding boundary listed in Table 1 in this work.

![Fig. 1 Seismic indices of 921 Chi-Chi Earthquake ($M_L = 7.3$, 1999).](image1)

![Fig. 2 Seismic indices of 0614 Earthquake ($M_L = 6.3$, 2001).](image2)

![Fig. 3 Seismic indices of 331 Earthquake ($M_L = 6.8$, 2002).](image3)
2.5 Annual probability of exceedance for a specified damage state

As stated in section 2.1, if the estimated damage index of a building after an earthquake is larger than the limiting value of the damage index \(D_{P&A, \text{alw}}\) for a specified damage state, it can be said that the estimated damage state of the building is more serious than the specified damage state. In this work, by means of incorporating the uncertainty in the random response (\(\delta_M\) and \(\int dE\)) as well as in the structural capacity (\(D_{P&A, \text{alw}}\)), the probabilities of exceedance with a specified response spectral acceleration in different damage states \(P_f\), as shown in Equation (17), can be estimated using the Monte Carlo simulation, as follows:

\[
P_f(D_{P&A}) = P\left[D_{P&A} > D_{P&A, \text{alw}}\right] = \int_{D_{P&A, \text{alw}}}^{\infty} f(D_{P&A}) \, dD_{P&A}
\]

where \(D_{P&A, \text{alw}}\) is the limiting value of the damage index \(D_{P&A}\), \(\delta_M\) and \(\int dE / M\) are random variables (whose mean values and coefficients of variation are explained in sections 2.2 and 2.3), and \(\delta_u\), \(\delta_y\) and \(S_y\) are deterministic variables that can be obtained from the capacity spectrum.

Considering the hazard curve of response spectral accelerations \(H(S_a)\) for a selected building obtained from the seismic hazard analysis, the annual exceedance probability of a specified damage state can be calculated using Equation (18). Furthermore, if the probability is assumed to be a standard normal distribution, it is possible to calculate the annual reliability index \(R_{ia}\). When the annual reliability index, \(R_{ia}\), for a specified damage state is lower than the allowable value \(R_{alw}\), it can be deduced that the damage state for the building is controlled under the specified damage state.

3. Modeling of maintenance cost

If the seismic performance of a building is still not consistent with the code-required or specified level after the seismic evaluation, the building should be retrofitted as soon as possible. Apart from upgrading the seismic performance, the cost of maintenance over the remaining service life should also be considered in the design. In this work, the cost of the seismic retrofitting and the cost of repairs are both included in the cost of maintenance, and expressed as the annual cost.

3.1 Seismic retrofitting technologies for low-rise RC buildings

The aim of the seismic retrofit design is to upgrade structural strength and ductility capacity (defined by \(\delta_u / \delta_y\), or a combination of both (Hsiao et al. 2009 and Lee 2010)). In general, seismic retrofitting technologies that improve existing structural systems can be classified into the following categories:

1. Strengthening existing structural components, for example by enlarging existing beam or column sections, and jacketing existing beams or columns using steel plates or FRP wraps.

2. Adding extra strengthening components to existing structural systems. This is done by constructing RC...
wing walls, RC shear walls, steel braces, etc.

(3) Reducing the lateral strength demand under earthquakes, which is usually achieved by reducing structural weight, adding seismic isolators or energy dissipating devices to existing buildings.

In terms of existing low-rise RC buildings with poor seismic capacities below the code-required levels, several seismic retrofitting technologies including RC jackets, RC wing walls, and shear walls are assumed in this study. These technologies are recommended because they are effective in increasing strength, improving ductility, are economic, and have low impact on the space usage of buildings. From experiments conducted in Taiwan, they are also proven to be useful for upgrading low-rise RC buildings (Hsiao et al. 2009 and Chiu et al. 2006). Most importantly, the retrofitting materials and required technical labor are not difficult to acquire.

According to ATC-40, the inelastic response of the SDOF system for a building under a specified earthquake, i.e. acceleration and displacement, can be decided using the intersection point of its capacity spectrum and the response spectrum of the earthquake. However, iterative calculations are generally needed to work out the location of the intersection point. Instead of iterative calculations, the capacity spectrum can be transferred into the seismic performance curve using Equation (19) according to the design response spectrum modified by the equivalent damping ratio $\xi_{eq}$ and equivalent fundamental period $T_{eq}$ (secant period). This indicates the relationship between the ground acceleration and the response spectral acceleration (NCREE-08-023 2008).

Considering the definition of the ultimate state explained in section 3.2, the ground acceleration corresponding to the specified performance $A_{p}$ of a selected building can be obtained. Accordingly, the response spectral displacement for the specified performance is the same as the ultimate deformation of the SDOF system for a building, i.e. $S_{dp} = \delta_{u}$. Therefore, the ground acceleration corresponding to the ultimate deformation of the SDOF system for a building is:

$$
A_{p} = \begin{cases} 
\frac{S_{dp}}{1 + \frac{2.5}{B_{s}} \frac{T_{eq}}{2T_{s}}} & \text{for } T_{eq} \leq 0.2T_{s} \\
\frac{B_{s}S_{dp}}{2.5T_{eq}} & \text{for } 0.2T_{s} < T_{eq} \leq T_{s} \\
\frac{B_{s}S_{dp}}{2.5T_{eq}} & \text{for } T_{eq} < T_{eq} 
\end{cases}
$$

where $S_{dp}$, $T_{s}$, $B_{s}$ and $T_{eq}$ are the spectral acceleration (g) for the peak effective inelastic limit, the boundary period between the short period range and the medium period range, the modification factor of the damping ratio in terms of the equivalent damping ratio $\xi_{eq}$ and the equivalent fundamental period ($= 2\pi \sqrt{S_{dp}/(S_{g} g)}$), respectively. Notably, the ground accelerations corresponding to the ultimate deformations of the SDOF system for a building before and after seismic retrofitting are necessary for the estimation of the seismic retrofit cost stated in section 3.2.

### 3.2 Modeling of retrofit costs

The seismic retrofit costs of a building are influenced by many factors, such as the seismic retrofitting technologies available, the type and location of the building, and the seismic retrofit level. It is therefore difficult to build a general model for estimating the seismic retrofit costs. Since the differences in costs between the suggested retrofitting technologies for low-rise RC buildings were not obvious from previous investigations (NCREE-09-023 2009), the cost of seismic retrofitting was assumed to be proportional to the upgrading rate in the seismic performance in this work. Additionally, the annual seismic retrofit cost $C_{Aret}$ was proposed based on the upgrading rate in the ground acceleration corresponding to the specified ultimate state of a building, which is commonly used to represent the seismic performance of a structure in seismic design and evaluation.

In this work, the ultimate state of a low-rise RC building is defined by the maximum base shear force; in addition to this, the story drift for each floor needs to be below 2.0% in the ultimate state (NCREE-09-023). By considering the specified ultimate state, it is possible to estimate the ultimate deformation $\delta_{u}$ for the SDOF system of a building using pushover analysis. In addition, the seismic retrofit cost is normalized by the cost of replacing the building with a new one, and divided by the remaining service life to work out the annual cost $C_{Aret}$, as shown in Equation (20).

The seismic retrofittings practiced in low-rise RC school buildings in Taiwan (there are approximately 15 cases utilizing different retrofitting technologies) were adopted to generalize the factor of retrofit cost, $\lambda$, showing the relationship between the seismic retrofit costs and upgrading rate of the seismic performances. The factor of retrofit cost $\lambda$ was approximated to be 0.15 for the case study. However, for other building types and locations, the factor of retrofit cost $\lambda$ should be modified based on the investigated information.

$$
C_{Aret} = \left[ \lambda \times \left( A_{p}^{g} - A_{g} \right) / A_{g} \right] / N
$$

where $A_{p}$ is the ground acceleration corresponding to the ultimate deformation of the SDOF system for a building (g), $A_{p}^{g}$ is the ground acceleration corresponding to the ultimate deformation after the completion of the seismic retrofit (g), $\lambda$ is the factor of retrofit cost, and $N$ is the remaining service life in years.

### 3.3 Modeling of repair costs

According to the research conducted by Takahashi et al. (2006), the repair cost induced by seismic damage has a high correlation with the ductility, as shown in Equation (21). Four different types of monotonically increasing functions (Fig. 5) were then proposed to model the relationship between the damage repair index $D_{p}$, and the repair cost $C_{R}$. Generally, the repair costs of low-rise RC
buildings are characterized by damage that increases rapidly just after the maximum displacement response reaches a specific point, known as the cracking displacement. Therefore, the convex curve (Equation (22)) was chosen to estimate the repair cost of a low-rise RC building. The repair cost $C_R$ is normalized by the cost of replacing the building with a new building. When the damage repair index $D_R$ exceeds unity, the repair cost $C_R$ is assumed to be 1. When the damage index $D_R$ is below unity, the repair cost $C_R$ is calculated using Equation (22).

Notably, because the maximum deformation of the SDOF system for a building $\delta_M$ is a random variable, stated in section 2.2, the mean value of the repair costs with a specified response spectral acceleration $E[C_R(S_a)]$ would be estimated using the Monte Carlo simulation; then, considering the hazard curve of response spectral accelerations $H(S_a)$ for a selected building obtained from the seismic hazard analysis, the annual repair cost $C_{ARep}$ can be calculated using Equation (23).

$$D_R = \frac{\delta_M}{\delta_c}$$  \hspace{1cm} (21)

$$C_a = \left(1 - D_R \right) + 1$$  \hspace{1cm} (22)

$$C_{ARep} = \int_{\delta_c}^{\delta_M} E\left[C_a(S_a)\right] \times \left(-\frac{dH(S_a)}{dS_a}\right) dS_a$$  \hspace{1cm} (23)

where $\gamma$ is denoted by $\delta_M/\delta_c$ and $\delta_c$ is the cracking displacement ($= F_y/(3K_i)$, $K_i$ is the initial stiffness of the SDOF for an RC building (Takahashi et al. 2006)).

In this work, the summation of the annual seismic retrofit cost $C_{ARep}$ and annual seismic repair cost $C_{ARep}$ is regarded as the annual cost of maintenance for an RC building with seismic retrofitting.

4. Estimating procedure for the seismic retrofit level

Under the condition that the average value of the annual reliability indices of a seismic retrofit level for a specified damage state needs to be equal to or greater than over the allowable annual reliability index, the seismic retrofit level corresponding to the minimum average of the annual cost of maintenance is regarded as the optimal seismic retrofit level herein. In this section, an estimating procedure for the optimal seismic retrofit of a selected low-rise RC building is built on the basis of the minimum average of the annual cost of maintenance and allowable damage state.

4.1 Seismic performance of a retrofitted low-rise RC building

Generally, the capacity spectrum method stated in section 2.1 can be adopted to analyze the seismic performance of a building with retrofitted components. In order for the design of a seismic retrofit of a targeted building to correspond with the specified seismic retrofit level, it is necessary to have iterative calculations for the nonlinear static analysis and the capacity spectrum method. In other words, a designer has to do the seismic retrofit design several times in one project. If this conventional design procedure were adopted to work out the optimal seismic retrofit level in terms of damage control and the cost of maintenance, the computer work involved would become redundant. Therefore, instead of using the conventional design procedure for the seismic retrofit design, the upgrading rates in the yielding acceleration $\kappa$, the ductility capacity $\nu$, and the elastic fundamental frequency $\xi$ of the SDOF system for a building were introduced to upgrade its seismic performance in this paper. For instance, if steel plate jackets are applied on the column components within an RC building, the ductility capacity could be increased, even though the yielding strength and elastic fundamental frequency remain unchanged. Therefore, it can be assumed that $\kappa = 1$, $\xi = 1$, and $\nu > 1$.

After the targeted building has been retrofitted, the yielding strength $F_y'$, yielding deformation $\delta_y'$ and the ultimate deformation $\delta''_u$ of the SDOF system for the building can be calculated using Equations (24) to (26). The capacity spectrum of the building with the seismic retrofit can then be reformed based on the new structural properties; i.e., $S_{u'}$, $S_{u''} (= \delta''_y)$, $S_{u''} (= \delta''_y)$, and $T_{u''} = 2\pi \sqrt{S_{u''} / \langle S_{u''} \rangle}$, where:

$$S'_{u'} = \kappa \times S_{u}$$  \hspace{1cm} (24)

$$\delta_y' = S'_{u'} \times 980 / (\nu \xi \omega_y)$$  \hspace{1cm} (25)

$$\delta''_y = \mu_{\nu} \times \delta_y' = (\xi \times \mu_{\nu}) \times \delta_y'$$  \hspace{1cm} (26)

In addition, the annual probability of a specified damage state and cost of maintenance above can be es-
timated based on the reformed capacity spectrum.

In order to estimate the retrofit cost, the reformed capacity spectrum can also be used to find the ground acceleration corresponding to the ultimate deformation of the SDOF system.

4.2 Average value of the annual cost of maintenance for a specified seismic retrofit level

A seismic retrofit level is defined by the ground acceleration corresponding to the ultimate deformation of the SDOF system for a targeted building. When it is specified, the upgrading rates (i.e. \(k\), \(\xi\) and \(\nu\)) may be combined differently to achieve the seismic retrofit level. This means that different seismic retrofit methods can be applied to accomplish the specified level, but the cost of maintenance would not be the same. In this study, we propose an estimating procedure (Fig. 6) that can be used to calculate the average value of the annual cost of maintenance, including the repairing cost and the retrofit cost for a specified seismic retrofit level.

When a seismic retrofit level is specified, the combinations of the upgrading rates can be searched to be in accordance with the specified level by the proposed system. In addition, the annual cost of maintenance and the annual reliability index of a specified damage state for each combination are computed in order to find their averaged values. Therefore, by repeating the estimating procedure of this system, the relationship between the average value of the annual costs of maintenance and the seismic retrofit level can be established. Under the condition that the average value of the annual reliability indices of a seismic retrofit level for a specified damage state needs to be equal to or greater than over the allowable annual reliability index, the level corresponding to the minimum average of the annual costs of maintenance is regarded as the optimal seismic retrofit level herein.

5. Case study

The applicability of the system proposed in this study was investigated by analyzing the optimal seismic retrofit levels of a low-rise RC school building with one basement and three stories with different requirements, such as allowable damage states and remaining service lives. Figure 7 and 8 and Table 2 show information on a
building that is located in Taipei, in the northern part of Taiwan. The seismic hazard analysis was performed for the site by considering the seismicity, tectonic structure, and the empirical ground motion prediction model. The hazard curve is illustrated in Fig. 4. Based on the response spectrum of accelerations in terms of short and long periods (one second), the hazard curve for the selected building was obtained using formulas stated in the seismic design code of Taiwan (MOI 2005). In this case study, the upper bound of the response spectral acceleration was set to be 0.4 g (annual exceedance probability is 0.001), as shown in Fig. 9. Furthermore, we chose three earthquakes that caused major damage in Taipei as stated in section 2.3. Therefore, two uniform distributions were set for the seismic index $I_D$: one is in the range of 5 to 15, and the other is in the range of 15 to 25.

Based on the capacity spectrum stated in section 2.1, the ultimate deformation $\delta_u$, yielding acceleration $S_y$, yielding deformation $\delta_y$, and elastic fundamental period $T_y$ of the SDOF system for the targeted building were obtained through the result of the pushover analysis, which was simulated parallel to the corridor direction, as shown in Fig. 10. The relationship between the mean value of the maximum deformation responses and the response spectral acceleration for the selected building is illustrated in Fig. 4. The capacity spectrum states that the ultimate deformation $\delta_u$, yielding acceleration $S_y$, yielding deformation $\delta_y$, and elastic fundamental period $T_y$ of the SDOF system for the targeted building were obtained through the result of the pushover analysis, which was simulated parallel to the corridor direction, as shown in Fig. 10. The relationship between the mean value of the maximum deformation responses and the response spectral acceleration for the selected building is illustrated in Fig. 4.

Table 2a Structural information of the selected building.

<table>
<thead>
<tr>
<th>Location of column</th>
<th>Base floor</th>
<th>First floor</th>
<th>Second floor</th>
<th>Third floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of column (mm)</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Width of column (mm)</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>350</td>
</tr>
<tr>
<td>Clear span length of column (mm)</td>
<td>310</td>
<td>310</td>
<td>310</td>
<td>310</td>
</tr>
<tr>
<td>Thickness of concrete cover (mm)</td>
<td>40.0</td>
<td>40.0</td>
<td>40.0</td>
<td>40.0</td>
</tr>
<tr>
<td>Number and diameter of main bar (mm)</td>
<td>3-D22(D)</td>
<td>3-D22(D)</td>
<td>3-D22(D)</td>
<td>2-D22(D)</td>
</tr>
<tr>
<td>Yield stress of main bar (N/mm²)</td>
<td>280</td>
<td>280</td>
<td>280</td>
<td>280</td>
</tr>
<tr>
<td>Yield stress of stirrup (N/mm²)</td>
<td>280</td>
<td>280</td>
<td>280</td>
<td>280</td>
</tr>
<tr>
<td>Number and spacing of stirrup (mm)</td>
<td>D10@160</td>
<td>D10@200</td>
<td>D10@220</td>
<td>D10@260</td>
</tr>
<tr>
<td>Compressive strength of concrete (N/mm²)</td>
<td>15.3</td>
<td>13.6</td>
<td>11.9</td>
<td>14.6</td>
</tr>
</tbody>
</table>

Table 2b Structural information of the selected building.

<table>
<thead>
<tr>
<th>Location of floor</th>
<th>First floor</th>
<th>Second floor</th>
<th>Third floor</th>
<th>Roof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of story (m)</td>
<td>360</td>
<td>360</td>
<td>360</td>
<td>360</td>
</tr>
<tr>
<td>Vertical loading of story including weight (kg)</td>
<td>1273754</td>
<td>1272634</td>
<td>1272634</td>
<td>1158002</td>
</tr>
<tr>
<td>Area (m²)</td>
<td>1243</td>
<td>1243</td>
<td>1243</td>
<td>1243</td>
</tr>
<tr>
<td>Elastic fundamental period (sec)</td>
<td>1.04</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
was then estimated using the equivalent linearization method, shown in Fig. 11. The hysteretic energy coefficients $E_h/M$ (Equation (13)) of the targeted building according to the response spectral acceleration when the seismic index is set to 5, 15 and 25 are shown in Fig. 12. When $I_D$ is uniformly distributed from 5 to 15, the annual probabilities of exceedance within a specified response spectral acceleration for serious and total damage states can be viewed in Fig. 13. Obviously, when the response spectral acceleration is close to 0.4 g, the annual exceedance probabilities of the two damage states for the building with the elastic fundamental period of 1.04 sec approach 1.0.

To find the optimal seismic retrofit level, the allowable annual reliability index for the specified damage state stated in section 2.5 was set to 3.0. Using the estimating procedure stated in section 4.2, a curve showing the relationship between the ground acceleration corresponding to the ultimate deformation of the SDOF sys-

Fig. 9 Hazard curve of the response spectral acceleration of the selected building.

Fig. 10 Results of the capacity spectrum method of the selected building.

Fig. 11 Mean value of the maximum deformation response corresponding to the response spectral acceleration of the selected building.

Fig. 12 Hysteretic energy coefficient corresponding to the seismic index of the selected building.

Fig. 13 Annual probabilities of exceedance for specified damage states.
tem and the mean value of the annual reliability indices for a specified damage state was calculated. This is illustrated in Fig. 14. According to the regression equations as shown in Fig. 14, the lower bound of the seismic retrofit level for a specified allowable annual reliability index and damage state can be obtained, and is illustrated in Table 3. For example, if $I_D$ is in the range of 5 to 15 and $D_{alw}$ is in the range of 0.9 to 1.0 (total damage), respectively, the seismic retrofit level $A_p$ needs to be larger than 0.13 g in order to allow for the annual reliability index of 3.0. However, the budget of the seismic retrofit for a targeted building is always limited to a specified ratio of the cost for constructing a new building. Based on policies implemented by the government of Taiwan, this ratio is set to be 0.5. Therefore, based on the budgeting limit, the seismic retrofit level is limited to be below 0.48 g (upper bound) for this specific building.

When the damage state was controlled so that it would not exceed the total damage state ($D_{alw} = 0.9$ to 1.0), and the value of the seismic index was set in the range of 5 to 15, the relationship between the average value of the annual cost of maintenance and the seismic retrofit level corresponding to a specified remaining service life was obtained using this system as stated in section 4.2, as shown in Fig. 15. In addition, a polynomial equation was adopted to carry out the regression analysis for the annual repair cost in this paper, i.e. the equation $Y = 0.12 - 0.42X + 0.39X^2$, as illustrated in Fig. 15. The smooth and continuous curve of the averaged annual cost of maintenance was then drawn to find the optimal seismic retrofit level. Based on the minimum cost of maintenance, the optimal seismic retrofit level $A_{sd}$ was found for each of the remaining service lives. Simulated results for other conditions are also shown in Table 4, and the reliability indices of these optimal results are all consistent with the specified allowable reliability index. For example, when the remaining service life and the seismic index are assumed to be 10 years and uniformly distributed in the range of 5 to 15, the optimal seismic retrofit level should be 0.37 g in order to control the seismic damage for the selected building under the total damage state, with minimal maintenance cost. In order to meet the optimal seismic retrofit level of 0.37 g, combinations of upgrading rates in terms of the yielding acceleration and the ductility capacity of the SDOF system for the targeted building can be recommended using the system illustrated in Fig. 16. Because the proposed estimation model for the cost of maintenance has no correlation with the seismic index and allowable damage state, the optimal seismic retrofit level of the targeted building is only dependent on the remaining service life. However, the average annual reliability index was influenced by the remaining service life and the damage state.

In conclusion, the seismic retrofit levels obtained by

<table>
<thead>
<tr>
<th>Allowable annual reliability index</th>
<th>$I_D = 5-15$</th>
<th>$I_D = 15-25$</th>
<th>$I_D = 5-15$</th>
<th>$I_D = 15-25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{alw} = 0.6-0.9$ (Serious damage)</td>
<td>0.15</td>
<td>0.13</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td>$D_{alw} = 0.9-1.0$ (Total damage)</td>
<td>0.13</td>
<td>0.15</td>
<td>0.22</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 3 Lower bounds of the seismic retrofit level for different allowable annual reliability indices and damage states.
this system can be recommended to the designer of the retrofit. The optimal seismic retrofit level is then chosen based on the specified seismic index, damage-controlled state, and the remaining service life.

5. Conclusion

In this work, an estimating procedure that can be used to set the optimal seismic level in the seismic retrofit design for a low-rise RC building was proposed. Along with damage control, the cost of maintenance over the remaining service life was also considered in this estimating procedure. Furthermore, combinations of upgrading rates in terms of yielding acceleration and the ductility capacity of the SDOF system are also suggested to the designer of the retrofit. Although the structure in the case study was limited to a low-rise RC building in Taipei, the optimal seismic retrofit level calculated via the same procedure can be derived and utilized when making decisions about how to set the upgrading rates in the structural capacities in the seismic retrofit design based on economic considerations. By combining this system with the life-cycle earthquake scenario of a selected RC building, seismic hazards and deterioration induced by carbonation or chloride ions can also be considered when planning future maintenance.

References


Chung, L. L. et al., (2009). “Seismic Upgrading of...


