THE PRELIMINARY DISCUSSION ON THE DISTRIBUTION OF LIGHTNING
STRIKING POINTS ON THE GROUND

Tosio Takeuti

Department of Electronic Engineering, Aichi College of
Technology, Gamagori, 443

Abstract. The progression of the leader of a ground discharge under the cloud base in summer seems generally to be uninfluenced by the ambient electric field and seems to depend only on the field at the tip of the streamer. Here the progression of the leader and the striking point on the ground are discussed on the basis of probability theory. The striking points are thought to have a binomial distribution around the intersection on the ground of the vertical line through the appearance point of the channel at the cloud base.

1. Introduction

Statistical analysis of many photographs of the ground discharge over a fixed area is one way to get the distribution of the striking points on the ground. However, a lot of time is required to take many photographs. Instead of this, the distribution for a summer thunderstorm has been deduced from probability theory assuming a one dimensional random walk model for the leader channel.

2. Stepped leader

Most ground discharges in summer thunderstorms begin from a stepped leader at a negative charge region in the thundercloud. As the leader reaches the ground, a return stroke current follows upward through the channel ionized by the leader. The leader stops for about 50 μ s after a progression of about 50 m. After the pause, the progression begins again. Each progression is called a step. The height of the cloud bases of summer thunderclouds over a plane range from 1 to 2 km, accordingly the number of steps under the cloud base may be roughly estimated as between 20 and 40.

During pauses of the leader, an electron avalanche appears at the tip of the leader and the avalanche produces a further new avalanche by photoelectrons from the former avalanche. A series of avalanches from such a process result in a streamer. The streamer is further developed by new avalanches appearing at the tip of the streamer if the electric field at the tip of the streamer is strong enough to produce new avalanches, that is more than 10^6 V/m. It is possible for a number of such streamers from the tip of leader to appear simultaneously. The polarity of all streamers is similar, so that every streamer moves the others in different directions ahead of the leader. One of those streamers develops about 50 m and is followed by a new step of the leader. By this process, the new step of leader may change direction.

3. The electric field under a thundercloud

The electric fields in thunderclouds measured with aircraft and balloons are of the order of 10^5 V/m (Takeuti, 1987). If one coulomb of the charge is distributed uniformly in a sphere of radius of 100 m, the electric field on the surface will be 9×10^5 V/m. Thus, the field due to the charge in the thundercloud and that due to the space charge are nearly equal to the field at the tip of streamer. This implies that the progression of the leader is influenced by those charges when the leader appears close to the charges. Actually, ground discharges with unusually complicated channel are frequently observed in winter thunderstorms. This fact can be explained as follows. The electric field on the ground under the winter thundercloud is stronger than the field under the summer one due to the lower location of the charge center in the cloud. This causes more active point discharges from the ground and more space charges which influence the progression of the leader.

The electric field on the ground under summer thunderclouds is at most 10^4 V/m. The electric field at ground level and 1 km above the ground from a point charge of 20 coulomb located 4 km above the ground, simulating the negative charge in a typical summer thundercloud, are estimated as 2.3×10^4 and 2.7×10^4 V/m respectively. Thus, the electric field between the cloud base and the ground in summer is too weak to have any effect on the progression of the leader. The channel of ground discharge with many branches seems also to indicate that the leader does not always develop in the direction of the maximum electric field.

4. Progression of the leader as deduced from probability theory

An actual lightning discharge is a three dimensional phenomenon, but it is simply discussed here as a two dimensional one. The vertical line from the appearance point of the channel from the cloud base is defined as the ordinate, and the abscissa is fixed on the ground. When the leader stops, the streamers at the tip of the leader appear tree-like ahead of the leader. One streamer chosen arbitrary from those streamers should be followed by a step of the leader of about 50 m. Such considerations might allow reliable results on the leader development to be obtained, but require complicated analysis. Therefore, it is discussed here as the one dimensional random walk model of the leader shown in Fig.1.

Suppose that every next step of the leader progresses to either right or left with same probability from the tip of the former leader. Then, every leader should deviate horizontally by a or -a from the tip of former leader. It is also assumed that the leader is composed of n steps between the cloud base and the ground.

The probability $P(0)$ for which all steps of the leader progress to the left, negative direction on the abscissa, is

$$P(0) = C \cdot (1/2)^0 \cdot (1/2)^n = C \cdot (1/2)^n \quad (1)$$

In this case, the horizontal deviation of the striking point on the ground is $-na$ from origin of the coordinate.

The probability $P(1)$ for which only one step progresses to the right, positive direction on the abscissa, is

$$P(1) = {}_nC_1 \cdot (1/2) \cdot (1/2)^{n-1} = {}_nC_1 \cdot (1/2)^n \quad (2)$$

In this case, the deviation of the striking point on the ground is $-(n-2)a$ from the origin.

Thus, the probability $P(j)$ which j steps progress to the right is

$$P(j) = {}_nC_j \cdot (1/2)^n \quad (3)$$

In this case, the horizontal deviation of the striking point on the ground is $-(n-2j)a$ from the origin.

Assuming that the height, not length, of each step and the height of the cloud base are 50 m and 1 km respectively, the number of steps is estimated to be 20. Then, the occurrence probabilities of each striking point on the ground are estimated from equation (3) as shown in Table 1.

Fig.1. The one dimensional random walk model of the stepped leader.

Table 1. Locations of the striking point on the ground and the corresponding occurrence probabilities

Number of steps to right (j)	Location of striking point on abscissa	Occurrence probability ${}_nC_j \cdot (1/2)^n$
0	-20a	0.000001
1	-18a	0.000019
2	-16a	0.000181
3	-14a	0.001087
4	-12a	0.004621
5	-10a	0.014786
6	-8a	0.036964
7	-6a	0.073929
8	-4a	0.120134
9	-2a	0.160179
10	0	0.176197
11	2a	0.160179
12	4a	0.120134
13	6a	0.073929
14	8a	0.036964
15	10a	0.014786
16	12a	0.004621
17	14a	0.001087
18	16a	0.000181
19	18a	0.000019
20	20a	0.000001

5. Comparison with photographic results

Ikeda et al. (1974) have taken many photographs of ground discharges and have obtained statistical result for the angle between a vertical line through the striking point and the inclined line drawn from appearance point of the channel at the cloud base to the striking point. Referring to this statistical result and assuming the height of the cloud base as 1 km, the horizontal deviations of the striking points from the cross point at the ground with the vertical lines through the appearing point at the cloud base are listed as Table 2.

Table 2. Horizontal deviations of the striking points on the ground from the cross point on the ground with the vertical line through the appearance point of the channel at the cloud base

Deviation (m)	Number of ground discharge	Occurrence probability
0~70	34	0.31
80~160	37	0.33
170~260	22	0.20
270~350	10	0.09
360~450	7	0.06
460~560	1	0.01

Nearly equal values of the occurrence probabilities in both Tables 1 and 2 corresponding to each other are listed in Table 3.

Table 3. Horizontal deviation of the striking points

from Table 1		from Table 2	
Deviation	Occurrence probability	Deviation (m)	Occurrence probability
4a~4a	0.74	0~160	0.64
6a~10a and -6a~-10a	0.25	170~350	0.29

The value of "a", the horizontal deviation for one step, can be estimated as about 30 m from Table 3.

6. Conclusion

The channel of a ground discharge occurring in summer is assumed as a one dimensional random walk model and the height of each step of the leader is assumed as 50 m as shown in Fig. 1.

The horizontal deviation of the each step of the leader assumed as the above model is estimated to be 30 m from the comparison between the analysis of the channel model and the results obtained from photographs of channel by Ikeda et al. (1974).

The striking points on the ground deduced from the random walk model are thought to have a binomial distribution.

This model does not include the new step of leader which progresses in the same direction as just former step of leader. In addition to this, each step of the leader is assumed to be not a straight line. The author is planning to discuss an improved model, though the procedure is not as simple as for this one dimensional random walk model. The author greatly appreciates Dr. M. Brook and Dr. N. Kitagawa for discussion on this paper.

References

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