ON THE PEAK ELECTROMAGNETIC FIELDS RADIATED BY LIGHTNING RETURN STROKES TOWARD THE MIDDLE-ATMOSPHERE

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Abstract. The amplitude and angular distribution of the peak field and Poynting vector that a vertical return stroke radiates toward the middle-atmosphere have been computed, when the speed of the stroke, v , is a significant fraction of the speed of light, c , assuming that the channel is an infinitesimal dipole at the time the peak field is produced and that the transmission-line model for the stroke current is valid up to the time of the peak [Krider, 1992]. The effect of a large v is to increase the fields by a factor of $(1 - \beta^2 \cos^2 \theta)^{-1/2}$, where $\beta = v/c$ and θ is the polar angle measured from the vertical. When the peak current is 30 kA, a typical value for negative first strokes, the peak field that is radiated into the upper half-space is 6, 12, 21, and 64 V/m at a distance of 100 km when $\beta = 0.33$, 0.67, 0.90, and 0.99, respectively, assuming that there is no attenuation in the atmosphere or on the surface. The corresponding values of the peak power density are 0.1, 0.4, 1.1, and 11 W/m². When β is small, the amplitude of the peak field has a $\sin \theta$ distribution that peaks on the ground ($\theta = \pi/2$). When $\beta > 1/\sqrt{2}$ or $v > 0.707 c$, the polar angle of the maximum in the angular distribution is $\theta_{\max} = \sin^{-1} \sqrt{(1 - \beta^2)/\beta^2}$. Our conclusion is that return strokes in natural lightning will radiate large fields toward the middle- and upper-atmosphere and, on those occasions when the velocity is very close to the speed of light, a heavily ionizing field near the vertical.

1. Introduction

In recent years, upward discharges have been observed between the tops of thunderclouds and the lower ionosphere [Franz et al., 1990; Vaughan et al., 1992], and lightning-stimulated airglow emissions have been recorded at altitudes near 90 km [Boeck et al., 1992]. Evidence for lightning-caused heating and ionization in the lower ionosphere (70-110 km) has also been obtained [Inan et al., 1988; Dowden and Adams, 1993]. Among the mechanisms that have been proposed to explain these phenomena is a direct coupling of lightning radiation fields to the ionospheric plasma [Inan et al., 1991; Rodriguez et al., 1992; Taranenko et al., 1992, 1993]. Unfortunately, there have been very few measurements of the amplitudes of lightning fields in the middle-atmosphere in the salient frequency interval, i.e., from a few to a few hundred kilohertz [Holzworth and Chiu, 1982; Kelley et al., 1985, 1990; Holzworth et al., 1985], and there have been no measurements of the angular distributions that propagate upward. The largest sources of lightning radiation in the ELF-VLF-LF bands are the return strokes in cloud-to-ground flashes. Here, we will show that the amplitude and the angular distribution of the peak field and Poynting vector that are radiated by a vertical return stroke depend on the peak current and its speed of propagation. A detailed discussion of the model and the approximations therein has been given by Krider [1992].

Figure 1 shows typical time-domain waveforms of the radiation fields produced by the first return stroke in a cloud-to-ground flash (top) and a return stroke subsequent to the first (bottom) in the same flash. Note how the initial peak in both signatures occurs just a few microseconds after the onset of the field. Here, our basic assumption is that, at the time of the peak field, the length of the radiating channel is short relative to the distance of observation. Since the fields (and currents) peak in times that are of the order of microseconds or less, the length of the stroke at the time of the peak will be limited to hundreds of meters or less, even if the velocity is close to the speed of light. If this channel is viewed from a distance, R , that is of the order of hundreds of kilometers, it will appear to be an infinitesimal dipole. We also assume that the current is described by the simple transmission-line (TL) model, i.e., $i(z,t) = i(z - vt)$, where v , the speed of propagation, is constant [Willett et al., 1988; Nucci et al., 1990; Thottappillil and Uman, 1993]; that the surface of the earth is flat and perfectly conducting; and that there is no attenuation of the field in the atmosphere or on the surface.

Fig. 1. Typical waveforms produced by the first return stroke in a flash (top) and a subsequent return stroke in the same flash (bottom). These fields were recorded on the ground at a range of about 60 km. The abrupt transitions labeled R mark the onset of the return-stroke field.

2. Theory

The geometry of our problem is sketched in Figure 2. We choose a

Fig. 2. Spherical coordinate system with its origin at the point where the return stroke begins on the ground.

spherical coordinate system with its origin at the point where the return stroke begins. We compute the fields that are radiated by a vertical stroke propagating up the z -axis. Following Le Vine and Willett [1992], we take the direction of $\vec{v}$ to be the direction of propagation of a current front that is positive upward, $\vec{v} = v\hat{k}$. Now, the peak amplitude of the electric radiation field at radius R is:

$$\vec{E}_p\left(t_p + \frac{R}{c}\right) = Z_0 \frac{I_p(t_p)}{4\pi R} \left[\frac{\beta \sin\theta}{1 - \beta \cos\theta} + \frac{\beta \sin\theta}{1 + \beta \cos\theta} \right] \hat{\theta}, \quad (1)$$

where t_p is the time of the peak current, $Z_0 = \sqrt{\mu_0/\epsilon_0}$ is the impedance of free space, I_p is the peak current at the base of the channel, $\beta = v/c$, and $\hat{\theta}$ is the unit vector shown in Figure 2 [Krider, 1992]. The first term in (1) represents the field radiated at early times (i.e., prior to reaching the end of the channel) with a $(1 - \beta \cos\theta)^{-1}$ correction factor that is necessary when v is large relative to the speed of light, c [Rubinstein and Uman, 1990, 1991; Le Vine and Willett, 1992]. The second term in (1) is due to the image of the source in the perfectly reflecting ground [Krider, 1992]. When these terms are combined, we obtain:

$$\vec{E}_p\left(t_p + \frac{R}{c}\right) = Z_0 \frac{I_p(t_p)}{2\pi R} \left[\frac{\beta \sin\theta}{1 - \beta^2 \cos^2\theta} \right] \hat{\theta}. \quad (2)$$

Similarly, the peak magnetic radiation field is

$$\vec{H}_p\left(t_p + \frac{R}{c}\right) = \frac{I_p(t_p)}{2\pi R} \left[\frac{\beta \sin\theta}{1 - \beta^2 \cos^2\theta} \right] \hat{\phi}. \quad (3)$$

The fields vanish as θ approaches zero, and as v/c approaches unity, the fields (and radiated power) go to infinity.

Figure 3 shows a polar plot of the bracketed term in (2) and (3) for several values of β . When β is small, the largest field amplitude is where $\theta = \pi/2$, i.e., on the surface. When $\beta > 1/\sqrt{2}$ or $v > 0.707 c$, the largest amplitude is where

$$\theta_{\max} = \sin^{-1} \sqrt{\frac{1 - \beta^2}{\beta^2}}; \quad \beta > \frac{1}{\sqrt{2}} \quad (4)$$

Note that as $\beta \rightarrow 1$, the pattern of the field lifts off the ground and becomes substantially more vertical.

The Poynting vector of the peak field on a surface of constant R is

$$\begin{aligned} \vec{S}_p &= \vec{E}_p \times \vec{H}_p \\ \vec{S}_p &= Z_0 \left(\frac{I_p}{2\pi R} \right)^2 \left[\frac{\beta^2 \sin^2\theta}{(1 - \beta^2 \cos^2\theta)^2} \right] \hat{R}, \end{aligned} \quad (5)$$

where I_p is measured at t_p and $\vec{S}_p$ is measured at $t_p + R/c$. Physically, $\vec{S}$ is the electromagnetic power per unit area that is being transferred in the

direction of $\vec{S}$. It should be noted in (5) that the amplitude of $\vec{S}_p$ is proportional to both I_p^2 and β^2 .

A polar plot of the $\sin^2\theta/(1-\beta^2\cos^2\theta)^2$ factor in (5) is given in Figure 4. At small β , the angular distribution has a $\sin^2\theta$ dependence that is characteristic of a classic dipole oscillator; at large β , however, the power density that is radiated into the upper half-space is enhanced considerably. For a given I_p and $\beta > 1/\sqrt{2}$, S_p is a maximum at the polar angle θ_m .

3. Discussion

We have seen that the maximum values of E_p and S_p depend on both I_p and β . The angular distributions of the radiation and θ_m depend on β . The initial velocities of return strokes have been measured using high-speed streak cameras [Schonland, 1956; Idone and Orville, 1982] and

photoelectric systems [Mach and Rust, 1989, 1993]. The most recent two-dimensional values for natural lightning near the ground are 1.7 ± 0.7

Fig. 3. Polar plot of the $\beta \sin\theta/(1-\beta^2 \cos^2\theta)$ amplitude factor in (2) and (3). The values of β are given by each curve, and there is symmetry about the vertical axis.

$\times 10^8$ m/s for 25 negative first strokes and $1.9 \pm 0.7 \times 10^8$ m/s for 43 negative subsequent strokes [Mach and Rust, 1989]. The velocities of positive return strokes are about a factor of two less than the negative [Mach and Rust, 1993]. The three-dimensional velocities could be somewhat larger than the above values; thus, we expect the β 's corresponding to the above measurements to range from about 0.33 to 0.90, with a typical value being about 0.67.

Tables 1 and 2 show values of the maximum E_p and S_p , respectively, that would be observed at $R = 100$ km for several values of I_p and β , assuming that there is no attenuation in the atmosphere or on the ground. The polar angles of these maxima (θ_m) are also shown for each β . Both tables also list a β of 0.99 that can be regarded as an extreme. It should

Fig. 4. Polar plot of the $\sin^2\theta/(1-\beta^2 \cos^2\theta)^2$ amplitude factor in (5). The values of β are shown for each curve, and there is symmetry about the vertical axis. [Note: The β^2 factor in the numerator of (5) is not included in these plots.]

Table 1. The maximum amplitude of $\vec{E}_p$ in V/m (in the $\hat{\theta}$ -direction) that a lightning return stroke radiates into the upper half-space for various values of β and I_p . All values have been computed using Eq. (3) at a range (R) of 100 km, assuming that there is no attenuation in the atmosphere. Numbers in parentheses are percentages of negative first strokes that exceed the tabulated value of peak current [Berger et al., 1975.]

β	θ_m (degrees)	Peak Current		
		14 kA (95%)	30 kA (50%)	80 kA (5%)
0.33	90	2.8	6	16
0.67	90	5.6	12	32
0.90	29	9.8	21	56
0.99	8.2	30	64	170

Table 2. The maximum amplitude of $\vec{S}_p$ in W/m² (in the $\hat{R}$ -direction) that a lightning return stroke radiates into the upper half-space for various values of β and I_p . All values have been computed using Eq. (5) at a range (R) of 100 km, assuming that there is no attenuation in the atmosphere. Numbers in parentheses are percentages of negative first strokes that exceed the tabulated value of peak current [Berger et al., 1975.]

β	θ_m (degrees)	Peak Current		
		14 kA (95%)	30 kA (50%)	80 kA (5%)
0.33	90	0.02	0.1	0.7
0.67	90	0.08	0.4	2.8
0.90	29	0.25	1.1	8.0
0.99	8.2	2.4	11	77

Table 3. Values of the maximum peak field (E_{ps}) and the maximum peak power density (S_{ps}) from a typical first return stroke ($I_p = 30$ kA) in a horizontal plane 100 km above the surface, assuming that there is no attenuation in the lower atmosphere. θ_s is the polar angle of these maxima, and ρ_s is the radial distance to the maxima measured from the z-axis. $\vec{E}_{ps}$ is in the $\hat{\theta}$ -direction, and $\vec{S}_{ps}$ is in the $\hat{R}$ -direction (see Figure 2).

β	θ_s (degrees)	ρ_s (km)	E_{ps} (V/m)	S_{ps} (W/m ²)
0.33	43.4	94	3.0	0.02
0.67	36.6	74	8.1	0.17
0.90	23.6	44	19	0.96
0.99	8.0	14	63	10.5

be noted in Tables 1 and 2 that the radiated fields and power densities are quite large. A typical first stroke ($I_p = 30$ kA, $\beta = 0.67$) will produce a maximum peak field of about 12 V/m at a range of 100 km without attenuation. Since $\theta_m = 90$ degrees in this case, this maximum occurs at the ground.

Using the geometry of Figure 2, it is straightforward to show that, when a field described by (3) intersects a surface of constant altitude (z), the peak field (in the $\hat{\theta}$ -direction) is a maximum on this surface where the polar angle is

$$\theta_s = \tan^{-1} \sqrt{1 - \beta^2}. \quad (6)$$

Table 3 shows values of the maximum E_{ps} and S_{ps} that would be observed on a horizontal plane at an altitude of 100 km in the absence of any attenuation. The polar angles of these maxima are also given together with the transverse radii of the maxima in the plane. Note how the amplitudes of E_{ps} and S_{ps} in Table 3 increase, and how θ_s and ρ_s narrow, as β approaches unity. Note also that a typical first stroke produces a maximum peak of 8 V/m at an altitude of 100 km. Since the electron mean free path is on the order of 1 m at this altitude, this field can add significant energy to the electrons between collisions.

Taranenko et al. [1992] have recently shown that fields of the order of 5 - 10 V/m at altitudes of 90 km will produce detectable optical signatures. Rodriguez et al. [1992] have also determined that there can be significant heating and ionization in the ionospheric D-region by fields of the order of 20 V/m. Taranenko et al. [1993] have shown that single pulses of 7 - 18 V/m will enhance the ionization in the lower nighttime ionosphere by 1 to 30%. The interactions are highly non-linear, and a rapid sequence of pulses, such as from subsequent return strokes, with amplitudes of 25 V/m can produce a 300% increase in the nighttime electron density at 95 km. Both Rodriguez et al. [1992] and Taranenko et al. [1993] have demonstrated that there can be significant attenuation of large fields in the lower ionosphere and that computations of the field amplitudes and the attenuation factors need to be done in a self-consistent way.

The extreme case shown in Tables 1-3 (i.e., $\beta = 0.99$), produces very large fields and power densities in a concentrated volume near the vertical. Although such events are probably quite rare, if they do occur they will certainly produce intense ionization, and possibly even trigger a discharge between the cloud top and the lower stratosphere under favorable conditions.

In conclusion, we note again that all of our field amplitudes and angular distributions have been computed using the assumption that the channel is an infinitesimal source and that the simple TL model is valid at early times during the stroke. For times after the initial peak, the channel will grow to lengths that are not infinitesimal and eventually there will be a "turn-off" term in the radiation field [Le Vine and Willett, 1992]. Also, at later times, the simple TL model may not provide an accurate description of the return stroke current [Nucci et al., 1990; Thottappillil and Uman, 1993]. If the initial current accelerates or decelerates rapidly, if there is more than one traveling wave at the beginning of the stroke, if there are major discontinuities in the channel geometry, or if there is strong coupling between v and I , then other

velocity corrections could be required in the general field equations [Le Vine and Meneghini, 1978; Meneghini, 1984], which will change the angular distributions. Because of this, we think it would be interesting to measure the amplitudes and angular distributions of the fields that are radiated by return strokes into the upper half-space, both during their onset and at later times. Accurate measurements of these quantities could provide valuable data on the field amplitudes and power densities that are radiated into the middle-atmosphere, as well as information about the behavior of the current at the lightning source.

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