PROPAGATION OF TRANSIENT ELECTROMAGNETIC WAVES IN A LOSSY MAGNETOPLASMA HALF-SPACE WITH ARBITRARILY-ORIENTED MAGNETIC FIELD

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Abstract. The time-dependent reflected and transmitted waveforms for an impulsive plane wave incident normally onto a lossy magnetoplasma half-space are computed by the use of a numerical inversion method of the Laplace transform. The temporal evolution of wave packets are analyzed for a two-component collisional (cold) plasma with the Earth's magnetic field, $\mathbf{B}_0$, being allowed to have an arbitrary direction. Curves are presented which show the dependence of the reflected and transmitted electric field components on the orientation of the Earth's magnetic field. Properties of the transmitted waveforms are discussed in three frequency bands; (i) quasi-free space modes, (ii) whistler mode and (iii) hydromagnetic wave modes. It is found that the arrival of signal delay for the whistler wave is inversely proportional to the cosine of the angle that $\mathbf{B}_0$ makes with the normal to the interface and the amplitudes of the electric field components are reduced in intensity for larger inclination of $\mathbf{B}_0$. Other features for the cases (ii) and (iii) are also discussed. The overall behavior of the wave packets suggests that the result may have some applications in diagnostic studies of magnetoplasma.

1. Introduction

The transient propagation of pulses in a homogeneous loss-free plasma half-space was formulated by *Ishimaru* (1967) many years ago, but no numerical results were presented. The impulse responses from a lossy anisotropic plasma half-space of a linearly polarized plane wave at normal incidence have later been evaluated numerically by *Stanic and Skoric* (1972). *Hosono et al.* (1989) have derived expressions for the transmitted wave field components with the Earth's magnetic field, $\mathbf{B}_0$, being allowed to have an arbitrary direction, but they were concerned with very high frequency waves only and neglected the effects of ions and collisions in their analysis. Recently, *Omid et al.* (1993) have reconsidered the problem and presented curves for the time-dependent transmitted waveforms in the entire frequency band with a more sophisticated model of anisotropic region, representative of an actual F_2 plasma layer. The transient excitation by a delta function current source in a F_2 layer magnetoplasma have also been studied (*Omid and Hayakawa*, 1992, 1994) using a numerical inversion method of the Laplace transform (ILT). The same technique will be used here.

This paper is concerned with the problem of transmission /reflection of transient signals through /from a sharply bounded anisotropic medium. The medium is regarded as a two-component collisional (cold) plasma which can adequately represent the ionospheric F_2 layer with the Earth's magnetic field allowed to have an arbitrary direction. In Section 2,

the time-harmonic solutions are obtained for an assumed linearly polarized wave having only the x -component as its electric field incident normally upon a plasma boundary. After the solution has been put into the desired form, suitable for numerical analysis, the impulse responses are evaluated via ILT, in Section 3. The numerical results are discussed for three different frequency regimes, namely, (i) quasi-free space modes, (ii) whistler mode and (iii) hydromagnetic wave modes. Finally, a brief summary of the main results obtained and conclusions are given in Section 4.

2. Formulation

(a) Basic equations

Transmission and reflection of a plane wave from a homogenous magnetoplasma bounded by a plane interface are considered. The superimposed magnetic field has an arbitrary orientation. Choosing a Cartesian coordinate system (x, y, z) , the interface between the free-space ($z < 0$) and ionosphere ($z > 0$) lies at the plane $z=0$ as indicated in Fig.1. Without loss of generality the earth's magnetic field $\mathbf{B}_0$ is assumed to be in the xz -plane, making an angle ϑ with z , i.e. $\mathbf{B}_0 = B_0(\hat{\mathbf{x}}\sin\vartheta + \hat{\mathbf{z}}\cos\vartheta)$, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ are unit vectors in the positive x and z directions, respectively. In the following, the indices i , r , and t denote the incident, reflected and transmitted waves, respectively. Suppose that a linearly polarized wave is normally incident upon a plasma boundary. As a result, a reflected wave is set up in the free-space and a transmitted wave in the plasma. Within the plasma two waves are generally generated, because of the double valued nature of the refractive index. Hence, the polarization of reflected and transmitted fields may have components in both x and y directions.

To obtain the electromagnetic fields, we regard the plasma as a medium which is homogeneous in both space and time and its electric permittivity is in the form of a tensor $\hat{\epsilon}$ while the magnetic permeability is a scalar, then the two Maxwell's equations can be written as,

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \quad \nabla \times \mathbf{H} = \epsilon_0 \hat{\epsilon} \frac{\partial \mathbf{E}}{\partial t} \quad (1)$$

where μ_0 and ϵ_0 are, respectively, the permeability and the permittivity of free-space. The components of the permittivity tensor $\hat{\epsilon}$ in the rectangular frame of coordinate described above (see Fig.1) are given by a 3×3 matrix whose elements can be written as ϵ_{ij} with $i, j = x, y, z$ to indicate its components (Budden, 1985). The explicit expressions for the elements of $\hat{\epsilon}$ in a collisional two-component plasma consisting of electrons and one ion species can be found in Omid and Hayakawa (1994). Eliminating $\mathbf{H}$ from the Maxwell's equations (1), then the wave equation for the electric field vector can be expressed as,

$$\nabla \times \nabla \times \mathbf{E} + \mu_0 \epsilon_0 \hat{\epsilon} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0. \quad (2)$$

Let us now seek for a plane wave solution of the form $\mathbf{E}(\omega, \mathbf{r}) = \mathbf{E}_0 \exp[j(\omega t - \mathbf{k} \cdot \mathbf{r})]$, where $\mathbf{E}_0$ is a constant vector and $\mathbf{k}$ is the propagation vector to be determined. Since the geometry and excitation are independent of x and y coordinates, the derivatives $\partial/\partial x, \partial/\partial y$ are zero and $\partial/\partial z = -jk_z$ for all field components. Hence, the vector wave equation in

Fig. 1 The geometry of the problem. The Earth's magnetic field B_0 lying in x - z plane, makes an angle ϑ with the z -axis (propagation direction $\mathbf{k} = k\hat{\mathbf{z}}$).

Table 1 Ionospheric parameters

parameter	value	parameter	normalized to ω_{pe}
f_{pe}	10.0 MHz	ω_{pe}	$2\pi \times 10^7 \text{ sec}^{-1}$
f_{pi}	58.3 kHz	ω_{pi}	$5.83 \times 10^{-3} \omega_{pe}$
f_{ce}	1.0 MHz	ω_{ce}	$0.10 \omega_{pe}$
f_{ci}	34 Hz	ω_{ci}	$3.4 \times 10^{-6} \omega_{pe}$
f_R	10.55 MHz	ω_R	$1.055 \omega_{pe}$
f_L	9.55 MHz	ω_L	$.955 \omega_{pe}$
f_{UHR}	10.05 MHz	ω_{UHR}	$1.005 \omega_{pe}$
f_{LHR}	5.80 kHz	ω_{LHR}	$5.80 \times 10^{-4} \omega_{pe}$
ν_e	1.0 kHz	ν_e	$1.59 \times 10^{-5} \omega_{pe}$
ν_i	0.034 Hz	ν_i	$5.41 \times 10^{-10} \omega_{pe}$

Ionospheric parameters for a day-time F_2 layer plasma, dominated by O^+ , at altitude $z \simeq 300$ to 400 km, with the electron number density $n_e = 1.24 \times 10^6 \text{ cm}^{-3}$. In this table f_{pe} and f_{ce} , the electron plasma and electron cyclotron frequencies; f_{pi} and f_{ci} , the ion plasma and ion cyclotron frequencies; f_R and f_L , the cutoff frequencies for right- and left- hand polarized waves; f_{UHR} and f_{LHR} , the upper- and lower-hybrid resonance frequencies; and ν_e and ν_i , the collision frequencies of the electrons and ions with the neutral particles (molecules).

Eq.(2) for the components of the electric fields can be written in a matrix form in its simplified form as,

$$-k_z^2 \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix} + \left(\frac{\omega}{c}\right)^2 \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \quad (3)$$

where $c = (\mu_0 \varepsilon_0)^{-1/2}$ is the speed of light in free-space. Eliminating E_z from this set, we obtain

$$\begin{bmatrix} \Delta_{22} - \varepsilon_{zz} n^2 & \Delta_{12} \\ -\Delta_{12} & \Delta_{11} - \varepsilon_{zz} n^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = 0 \quad (4)$$

where $n = k_z c / \omega$ is the index of refraction and $\Delta_{ij} (i, j = 1, 2, 3)$ is the co-factor of the element ε_{ij} of the dielectric tensor $\hat{\varepsilon}$; e.g. $\Delta_{11} = \varepsilon_{yy}\varepsilon_{zz} - \varepsilon_{yz}\varepsilon_{zy}$, etc. In order to have a nontrivial solution for $\mathbf{E}$, the determinant of the left-hand matrix in Eq.(4) must vanish. The resulting equation, which determines n , is given by,

$$(\varepsilon_{zz} n^2)^2 - 2\Delta_x (\varepsilon_{zz} n^2) + (\Delta_{11}\Delta_{22} + \Delta_{12}^2) = 0 \quad (5)$$

where the abbreviation $\Delta_x = (\Delta_{11} + \Delta_{22})/2$ has been introduced. Now, the index of refraction can be found by solving Eq.(5) to give

$$n_{1,2}^2 = (\varepsilon_{zz})^{-1} [\Delta_x \pm (\Delta_x^2 - \varepsilon_{zz} \Delta)^{1/2}] \quad (6)$$

where $\Delta = \det(\hat{\varepsilon})$ and the two solutions n_1^2 and n_2^2 correspond to indices of refraction of the two characteristic modes in the cold plasma. In the xz plane the electric field vector of the two modes, E_α ($\alpha = 1, 2$) rotates on an ellipse. A relationship between $E_{x,\alpha}$ and $E_{y,\alpha}$ (wave polarization) is established in the waves of both modes from Eq.(4), that is

$$\rho_\alpha \equiv \left(\frac{E_x}{E_y}\right)_\alpha = \frac{\Delta_{12}}{\Delta_{11} - \varepsilon_{zz} n_\alpha^2} \quad (7)$$

In view of the foregoing discussion, it is obvious that the most general plane wave solution has the following form,

$$\mathbf{E}^t(\omega, z) = \hat{E}_1 \exp(-j\omega k_1 z) + \hat{E}_2 \exp(-j\omega k_2 z) \quad z > 0 \quad (8)$$

where $\hat{E}_{1,2}$ are constant vectors, and $k_{1,2} = n_{1,2} \omega / c$. The magnetic field vector can be found from Maxwell's equations (1) to be

$$\mathbf{H}^t(\omega, z) = (\omega \mu_0)^{-1} [k_1 \hat{\mathbf{z}} \times \hat{E}_1 \exp(-j\omega k_1 z) + k_2 \hat{\mathbf{z}} \times \hat{E}_2 \exp(-j\omega k_2 z)] \quad z > 0 \quad (9)$$

From Eqs.(7) through (9), the explicit expressions for $E_x^t(\omega, z)$, $E_y^t(\omega, z)$, $H_x^t(\omega, z)$, and $H_y^t(\omega, z)$ may easily be obtained to be given by

$$E_x^t(\omega, z) = \hat{E}_1 e^{-j\omega n_1 z/c} + \hat{E}_2 e^{-j\omega n_2 z/c} \quad (10)$$

$$E_y^t(\omega, z) = \rho_1 \hat{E}_1 e^{-j\omega n_1 z/c} + \rho_2 \hat{E}_2 e^{-j\omega n_2 z/c} \quad (11)$$

$$H_x^t(\omega, z) = -\frac{\rho_1 n_1}{\eta_0} \hat{E}_1 e^{-j\omega n_1 z/c} - \frac{\rho_2 n_2}{\eta_0} \hat{E}_2 e^{-j\omega n_2 z/c} \quad (12)$$

$$H_y^t(\omega, z) = \frac{n_1}{\eta_0} \hat{E}_1 e^{-j\omega n_1 z/c} + \frac{n_2}{\eta_0} \hat{E}_2 e^{-j\omega n_2 z/c} \quad (13)$$

where $\eta_0 = (\mu_0/\epsilon_0)^{1/2}$ is the characteristic impedance of free-space and the unknown coefficients $\hat{E}_{1,2}$ are to be found from the boundary conditions at the interface $z = 0$.

(b) *Field components in the free space region*

Suppose that a linearly polarized wave with its electric field having only the x -component as its electric field is normally incident upon a plasma boundary. In the free-space region, the electric and magnetic fields are specified by Maxwell's equations (1) with $\hat{\epsilon}$ replaced by a 3×3 unit matrix. The incident wave takes the following form

$$\mathbf{E}^i(\omega, z) = \hat{\mathbf{x}} E_0 e^{-j\omega z/c}, \quad \mathbf{H}^i(\omega, z) = \hat{\mathbf{y}} \frac{E_0}{\eta_0} e^{-j\omega z/c} \quad z < 0 \quad (14)$$

where E_0 is the amplitude of the incident wave. The reflected wave is composed, in general, of two elliptically polarized components with different amplitudes, and is expressed in terms of a complex vector reflection coefficient $\mathbf{R}$:

$$\mathbf{E}^r(\omega, z) = \mathbf{R} e^{+j\omega z/c}, \quad \mathbf{H}^r(\omega, z) = \frac{\mathbf{R} \times \hat{\mathbf{z}}}{\eta_0} e^{+j\omega z/c} \quad z < 0 \quad (15)$$

where the complex vector reflection coefficient $\mathbf{R}$ is necessary because of the transmitted-wave characteristics and the necessity of matching the boundary conditions at $z = 0$.

(c) *Boundary conditions*

The boundary conditions that the tangential components of the electric and magnetic field be continuous across the plane interface are imposed by matching the x and y components of $\mathbf{E}(\omega, z)$ and $\mathbf{H}(\omega, z)$ at $z = 0$, that is

$$\left. \begin{aligned} E_x^i(\omega, 0) + E_x^r(\omega, 0) &= E_x^t(\omega, 0) \\ H_y^i(\omega, 0) + H_y^r(\omega, 0) &= H_y^t(\omega, 0) \\ E_y^r(\omega, 0) &= E_y^t(\omega, 0) \\ H_x^r(\omega, 0) &= H_x^t(\omega, 0) \end{aligned} \right\} \quad z = 0 \quad (16)$$

Substituting Eqs.(10) through (15) in Eq. (16), we have

$$\hat{E}_1 = \frac{2E_0 N_1}{(1 + n_1)}, \quad \hat{E}_2 = -\frac{2E_0 N_2}{(1 + n_2)} \quad (17)$$

with

$$N_1 = \frac{\Delta_{11} - \epsilon_{zz} n_1^2}{\epsilon_{zz}(n_2^2 - n_1^2)}, \quad N_2 = \frac{\Delta_{11} - \epsilon_{zz} n_2^2}{\epsilon_{zz}(n_2^2 - n_1^2)}$$

and for the reflection coefficients $R_x = E_1 + E_2 - E_0$, $R_y = \rho_1 E_1 + \rho_2 E_2$. The final task is to determine the time-dependent fields by taking the ILT of Eqs.(10) through (13), and

(15) with the coefficients as given by Eqs.(16) and (17). The time-dependent transmitted and reflected electric field is then given in the following matrix form as,

$$\|\tilde{\mathbf{E}}(t, z)\| = \begin{bmatrix} \tilde{E}_x^t \\ \tilde{E}_y^t \\ \tilde{E}_x^r \\ \tilde{E}_y^r \end{bmatrix} = \frac{1}{2\pi j} \int_{Br} \|\mathbf{E}(s, z)\| e^{st} ds \quad (18)$$

where $s = j\omega$ is the Laplace transform variable and Br defines a Bromwich path in the complex s -plane. These integrals cannot be evaluated in a closed form, but a direct numerical integration is possible since the integrands are real functions of s for real s .

3. Numerical Results and Discussion

The transient field components $\tilde{\mathbf{E}}(t, z)$ were evaluated using the formula of numerical inversion of the Laplace transform as proposed by *Hosono* (1981). The ionosphere is modeled as a two-component collisional (cold) plasma consisting of electrons and oxygen ions, O^+ . Table 1 summarizes the plasma parameters representing the daytime F_2 layer ionosphere at a height of about 350 km which are adopted from *Al'pert* (1990). In this table f_{pe} and f_{ce} , the electron plasma and electron cyclotron frequencies; f_{pi} and f_{ci} , the ion plasma and ion cyclotron frequencies; f_R and f_L , the cutoff frequencies for right- and left- hand polarized waves; f_{UHR} and f_{LHR} , the upper- and lower-hybrid resonance frequencies; and ν_e and ν_i , the collision frequencies of the electrons and ions with the neutral particles (molecules). Based on the parameters given in Table 1, we discuss the numerical results in three different frequency bands as follows: (i) quasi-free space mode waves $|\text{Im}(s)| > \omega_L/\omega_{pe} \simeq 1.0$; (ii) whistler mode wave $|\text{Im}(s)| < \omega_{ce}/\omega_{pe} \simeq 0.1$; and (iii) hydromagnetic waves $|\text{Im}(s)| < \omega_{ci}/\omega_{pe} \simeq 3.4 \times 10^{-6}$. Particular attention will be given to the effect of oblique orientation of Earth's magnetic field $\mathbf{B}_0$ on the transient signals.

In the subsequent calculations, we introduce the dimensionless quantities $S = s/\omega_{pe}$, $\tau = \omega_{pe}t$, and $\xi_z = z\omega_{pe}/c$. All the parameters listed in Table 1 are normalized accordingly; e.g. $\Omega = \omega/\omega_{pe}$, $\Omega_{ce} = \omega_{ce}/\omega_{pe}$, $\Omega_{pi} = \omega_{pi}/\omega_{pe}$, $\bar{\nu}_e = \nu_e/\omega_{pe}$, etc. Consequently, the "vertical" axis represents a dimensionless electric field defined as $E_{x,y}(\tau, \xi_z)/\omega_{pe}E_0$, where E_0 is the amplitude of the incident wave in Eq.(6). The "horizontal" axis corresponds to the dimensionless time $\tau(=\omega_{pe}t)$. The value of $\xi_z(=z\omega_{pe}/c)$ in each figure is constant. For $f_{pe}=10$ MHz, $\tau=1$ implies that $t=1 \mu\text{s}$, while $\xi_z=200$ implies $z=955$ m, etc.

Since the dispersion results from the *transmission effect* at the interface $z = 0$ as well as *propagation effect* within the medium $z > 0$, we shall examine both cases in the followings.

3.1 Computational results of free-space wave modes ($|\text{Im}(S)| > \Omega_L \simeq 1.0$)

This range corresponds to the high frequency, quasi-free space mode waves for which the ions play a very little role due to their large mass. The numerical results in this frequency range are shown in Figs. 2 and 3. In these figures, the top panel refers to the co-polarized component, $\tilde{E}_x(\tau, \xi_z)$, and the bottom panel gives the corresponding cross-polarized component, $\tilde{E}_y(\tau, \xi_z)$. Fig.2 illustrates the time-dependent reflected/ transmitted waveforms at the interface. The waveforms for $\xi_z = 0$ yield the dispersion of the normally incident impulsive plane wave due to the transmission effect *only* which occurs at the interface. One should notice that in the absence of the magnetic field $\mathbf{B}_0$, the cross-

Quasi-free space modes, $\xi_z = 0$

(a)

(b)

Fig. 2 Time-dependent co- and cross-polarized components $\tilde{E}_{x,y}^r(\tau, \xi_z)$ of the reflected quasi-free space mode waves with $\xi_z=0$ as a function of ϑ .

Quasi-free space modes, $\xi_z = 7$

(a)

(b)

Fig. 3 Time-dependent co- and cross-polarized components $\tilde{E}_{x,y}^t(\tau, \xi_z)$ of the transmitted quasi-free space mode waves at $\xi_z=7$ as a function of ϑ .

polarized components $\tilde{E}_y^t$ and $\tilde{E}_y^r$ would be zero. On the other hand, as ω_{ce} takes on a greater value, a greater part of energy goes to the cross-polarized component. In order to examine the propagation effect, the time-dependent field components at one particular distance ($\xi_z = 7$) from the interface are illustrated in Fig.3. It can be seen that the waveforms are growing as ξ_z increases which correspond to an energy transfer from the $\tilde{E}_x^t$ component to the $\tilde{E}_y^t$ component. The major change which occurs when increasing ν_e is to reduce the amplitude of the field components, with the frequency of oscillatory response being practically unaffected as shown in Stanic and Skoric (1973).

3.2 Whistler branch ($|\text{Im}(S)| < \Omega_{ce} \simeq 0.1$)

In this frequency band only one wave is propagating, namely, whistler-mode. The computational results of the waveforms in this frequency range are shown in Fig.4. We have computed the waveforms of whistler-mode waves at one particular distance ($\xi_z=400$) from the interface, and three different ϑ values are adopted; $\vartheta=0^\circ$ (a), 25° (b) and 50° (c). In this case, $\xi_z > 0$, the transmitted waveforms are further dispersed due to the propagation effect. Since $\xi_z = 400 \gg 0$, the plasma medium can be considered as infinite. In this way, the propagation effect by itself can be examined. The most conspicuous point of Fig.4 is the rapid increase of the amplitude, which is closely related to the wave components with frequencies near a quarter of electron gyro-frequency $\Omega_{\text{nose}} = \Omega_{ce} \cos \vartheta / 4$ because the group velocity at Ω_{nose} attains its maximum value in the case of a homogeneous magnetoplasma (Omid and Hayakawa, 1994). This frequency is sometimes called a “nose frequency” in the whistler spectrum (Helliwell, 1965; Hayakawa, 1995). The subsequent modulation envelopes as seen in these figures are resulted from the interference between the wave components above and below this frequency, Ω_{nose} , whose group velocities are close to each other. Although the wave components with $\Omega > \Omega_{\text{nose}}$ are shown later to decay more quickly due to the collisional damping than those at $\Omega < \Omega_{\text{nose}}$ (Helliwell, 1965; Hayakawa, 1995), the response becomes less oscillatory and smooths out the modulation envelope, at shorter distances (smaller ξ_z) from the interface (see Fig.5 of Omid and Hayakawa, 1994) because the high-frequency wave components contain more energy than the low-frequency ones and hence dominate the response (Vidmar et al., 1983). Further effects which an oblique Earth's magnetic field has on the waveforms as inferred from Fig.4 are as follows:

(a) Longer delay in the arrival of wave packets

For constant ξ_z the maximum group delay, τ_{dm} , is proportional to $[\cos \vartheta]^{-1}$ (see Omid and Hayakawa, 1994, Eq.28) and it would be approximately equal to 6160, 6800 and 9580, respectively, at $\xi_z=400$ for $\vartheta=0^\circ$, 25° and 50° . This expectation is consistent with the larger time delay in the arrival of wave packets with respect to ϑ as indicated by the triangles in Fig.4. Notice that the ratio of ξ_z to τ in each panel corresponds to the group velocity normalized to the speed of light, i.e. $\xi_z/\tau = V_{gz}/c$. The curves to the left of the triangles in Fig.4 are whistler branch precursors, which propagate faster than V_{gm} (Vidmar, et al., 1983).

(b) More reduced amplitude for larger ϑ 's

The decrease of amplitude with respect to ϑ as found in Fig.4 can be explained in terms of the enhanced attenuation for larger ϑ values. It has been shown that the reduced intensity is related to ϑ with the relationship of $1/(\cos \vartheta)^{1.5}$ (Omid and Hayakawa, 1994).

Whistler, $\xi_z = 400$

Fig. 4 Time-dependent co-polarized component $\tilde{E}_x^t(\tau, \xi_z)$ of the transmitted whistler wave at $\xi_z=400$ as a function of ϑ . (a) $\vartheta = 0^\circ$, (b) $\vartheta = 25^\circ$ and (c) $\vartheta = 50^\circ$

3.3 Computational results of hydromagnetic wave modes

$$(|\text{Im}(S)| < \Omega_{ci} \simeq 3.4 \times 10^{-6})$$

The computational results of the waveforms in this frequency range (hydromagnetic waves) are shown in Fig.5. The co-polarized component $\tilde{E}_x^t(\tau, \xi_z)$ and cross-polarized one $\tilde{E}_y^t(\tau, \xi_z)$ at $\xi_z=20,000$ for $\theta=0^\circ$ are superimposed in Fig.5(a). The two propagating modes with opposite polarization are propagated, i.e. modified-Alfven wave (whistler-mode) and Alfven wave (ion cyclotron wave). As ω goes to zero ($t \rightarrow t_\Delta$), the modified-Alfven wave propagates nearly at the Alfven velocity v_A and the Alfven wave propagates with group velocity slightly less than v_A (Vidmar, et al., 1983). The phase difference in Fig.5(a) shows that prior to the arrival of zero frequency components (i.e. $\tau < \tau_\Delta$) we have a right-handed circular polarization (modified Alfven wave), while a left-handed circular polarization (Alfven wave) is propagating afterward (i.e. $\tau > \tau_\Delta$). After the arrival of zero frequency components, the amplitude of the waveform decreases while its frequency increases to Ω_{ci} . The characteristic dip (CD) at $\tau = \tau_\Delta$ is the consequence of interference of the two waves with opposite polarization, and for $\vartheta = 0^\circ$ this CD occurs at $\tau = \tau_\Delta$ and can be used to estimate the Alfven velocity and hence plasma parameters (Vidmar, et al., 1983). In Figs.5(b) and (c) we illustrate the waveforms for co-polarized component of $\tilde{E}_x(\tau, \xi_z)$ at $\xi_z=20,000$ with $\vartheta = 25^\circ$ and 50° , respectively. The major changes accomplished by increasing ϑ can be summarized as: (a) longer group delay in Alfven wave and (b) gradual decrease in Alfven wave amplitude. It can be shown (see, for example, Booker, 1984) that the refractive index and hence the group velocity of the modified-Alfven wave is independent of ϑ angle, and so the increase in ϑ would result in increase of frequency of oscillation only and would not have any effect on the amplitude of waveforms for constant ξ_z . At smaller ξ_z the modified-Alfven wave dominates the response. On the contrary, the behavior of Alfven wave packets is very similar to that of the whistler-mode wave. With the increase of ϑ , the initial part of the Alfven wave is more delayed, and its amplitude is reduced due to absorption just like the case of whistler propagation.

4. Conclusions

We have studied numerically the transient reflection and transmission of an impulsive plane wave without making any approximation for a half-space magnetoplasma when the earth's magnetic field is allowed to be oriented arbitrarily. The results serve as a special illustration of cold plasma theory and can be used as a plasma diagnostic tool by relating cold plasma theory to specific features in a waveform, e.g. plasma parameters such as the plasma frequency, electron and ion cyclotron frequencies, distance from the interface and wave polarization can be deduced. We can list some main effects of Earth's magnetic field orientations as follows: For the *whistler wave*, arrival of signal delay is inversely proportional to $\cos \vartheta$ and its reduced intensity is expected for larger ϑ 's. And, for the *hydromagnetic wave modes*, the two-oppositely polarized waves are excited (namely, modified-Alfven wave and Alfven wave), with the latter being very similar to the whistler case, whereas the former one, not much effected although the frequency of its oscillation increases as the ϑ increases.

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Fig. 5 (a) Time-dependent co- and cross-polarized components $\tilde{E}_{x,y}^t(\tau, \xi_z)$ of the transmitted hydromagnetic waves at $\xi_z=20,000$ and $\vartheta = 0^\circ$; (b) co-polarized component of $\tilde{E}_x^t(\tau, \xi_z)$ with $\vartheta = 25^\circ$ and (c) co-polarized component of $\tilde{E}_x^t(\tau, \xi_z)$ with $\vartheta = 50^\circ$

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