Source models for “parametric” Q–burst

A. P. Nickolaenko
Usikov Institute for Radio-Physics and Electronics
National Academy of Sciences of the Ukraine, Kharkov 61085, Ukraine

Abstract. Changes are modeled of parameters of the global electric circuit caused by the giant gamma ray burst arrived at the Earth’s dayside on December 27 2004. Modification of the air conductivity by gamma rays alters the leakage current of the Earth–ionosphere cavity and causes a discrete radio pulse. Electric parameters of the air at the tropopause–troposphere altitudes play the key role. ELF spectrum of the “parametric” current moment is ‘red’ or ‘white’ depending on the particular model. Sample spectra and waveforms are presented of the parametric Q–burst.

Key words: Earth–ionosphere cavity, Global electric circuit, Schumann resonance, Q–bursts, gamma burst.

1. Introduction

“Parametric” excitation of the Earth–ionosphere cavity was addressed by Nickolaenko (2010). The term “parametric” was introduced to underline the peculiarity of the source produced by incident gamma rays. The additional ionization of atmosphere modifies parameters of the Earth–ionosphere cavity and thus produces a specific source of electromagnetic (EM) radiation. For instance, it was assumed in the initial model that a gamma ray burst abruptly lowers the dayside of sharply bounded ionosphere, so that the source current emerged from the translation of electric charge accumulated at the ionosphere boundary. The dielectric air separated perfectly the conducting ground and ionosphere. In this case, the ionosphere height was the particular parameter responsible for emergence of the “parametric” source. The parametric source is completely different from the ordinary natural sources like lightning strokes.

We treat in what follows a more realistic parametric source model associated with the modifications of the leakage currents between the ground and ionosphere. In this model, the air resistance becomes the major varying parameter that causes the EM radiation. The predicted “parametric” pulse was found in our records of natural ELF radio signal in Japan and Sweden (Tanaka et al., 2011), and in Russia (Nickolaenko and Schekotov, 2011). A comparison of experimental and model data revealed shortcomings of the previous source model: it predicted too high amplitude of the parametric Q–burst, and it contained a wide spectral peak at frequencies around 60 Hz. We introduce here an advanced parametric source model based on properties of the global electric circuit.

The major property of ELF transient is conditioned by the great size of ionosphere modification that covers a hemisphere. Such an extended source drives only the lowest modes of the global electromagnetic (Schumann) resonance.

2. Global electric circuit

Electric activity of atmosphere charges the ionosphere to a potential of +250 kV with respect to the ground (e.g. Ogawa, 1985, Rycroft et al., 2000, 2007). The fair weather electric
field is observed of 130 V/m at the ground surface in stationary conditions. The ionosphere carries the charge of about $+2 \times 10^5$ C, and the electrostatic energy accumulated by the Earth–ionosphere capacitor is $10^{10}$ J. The capacitance is about 0.7 F, and it is coupled to the “electro-sphere” being a 6–7 km slab positioned around the tropopause where the air resistance reaches its maximum (e.g. Ogawa, 1985, Rycroft et al., 2000, 2007).

There is a leakage current from the ionosphere to the ground (the fair weather current) that closes the circuit. The current has a density of 2 pA/m$^2$, so that the cumulative resistance of the global atmosphere is 200–300 Ohms.

3. Modification of atmospheric conductivity by the gamma burst

We know from the interpretation of VLF records during the gamma burst of December 27 2004 that the lower edge of the dayside ionosphere moved from 60 to 40 km (Inan et al., 2007). The effect was observed over the whole Pacific Ocean, as it was noticed on the paths having the 60° angular distance from the sub-flare point. One may estimate modification of the air resistance by using the standard conductivity profile (Ogawa, 1985, Rycroft et al., 2000, 2007, Inan et al., 2007) and its vertical reduction by 20 km.

![Vertical profiles of atmosphere conductivity and resistance. The right plot shows the scheme of modification caused by intense gamma rays.](image)

We show two plots in Fig. 1. The left one depicts the vertical profile of air conductivity $\sigma(h)$ adopted from Rycroft et al. (2007). By using the average tilt in the 0–60 km range (thin lines), one can find that the 400-fold conductivity increase (six height scales) corresponds to
the 38.5 km altitude variation. Hence, the average height scale is \( \zeta = 38.5/6 = 6.4 \) km, and the 20 km reduction is related to the 22-fold increase in the air conductivity.

The right plot in Fig. 1 shows that a decrease in the air resistance \( \rho(h) = 1/\sigma(h) \) profile depends on the altitude of modification, provided that the profile shifts by 20 km as a whole. One may observe that the resistance decreases by a factor of 6.1 in the altitude interval of 30–50 km. The regular profile is shown by the black curve, and the red line shows its modification when gamma rays reach the \( z(t) \) altitude. Above this particular height, the resistance profile is shifted downward as a whole by \( \Delta h = 20 \) km. The profile remains regular below \( z(t) \), so that there is a discontinuity of resistance \( \Delta \rho \) at \( z(t) \). With the progress of gamma rays, the discontinuity moves from the ionosphere to the ground.

We accept that the air resistance decreases by the factor of 7. After gamma rays reach the ground, the ionosphere leakage current density \( j \) grows from its regular 2 pA/m\(^2\) to 14 pA/m\(^2\) value (\( \Delta j = 12 \) pA/m\(^2\)). The current alters during a finite time interval. The gamma rays move vertically at the disturbance epicenter (sub-flare point), so that the interval of growth is equal to

\[
\tau_g = \frac{H}{c}
\]

where \( H = 60 \) km is the regular day ionosphere height and \( c \) is the light velocity. It is easy to see that \( \tau_g = 200 \) \( \mu \)s. Modification occurs in shorter time intervals for oblique incidence of gamma rays \( (\alpha \neq 0) \):

\[
\tau_{\alpha} = \tau_g \cos \alpha = \frac{H \cos \alpha}{c}
\]

When \( \alpha = 90^\circ \), the whole vertical air column is instantly illuminated and ionized and we have a discontinuous current modification here. The onset of modification is delayed at a point with angular distance \( \alpha \) from the sub-flare point

\[
\tau_d = a \left( 1 - \frac{\cos \alpha}{c} \right)
\]

where \( a \) is the Earth’s radius. The delay \( \tau_d \) arises from the Earth’s curvature: initially, the gamma quanta enter the atmosphere at the center of disturbance. The delay \( \tau_d \) occupies the millisecond range, e.g., \( \tau_d = 21.3333 \) ms when \( \alpha = 90^\circ \).

4. Linear current modification

We postulate that the current density linearly varies with time:

\[
j(t) = j_0 + \Delta j \frac{t}{\tau_{\alpha}}
\]

where \( j_0 = 2 \) and \( \Delta j = 12 \) pA/m\(^2\). The upper panel in Fig. 2 shows relevant alterations of the current for angles \( \alpha = 0, 30, 60, \) and \( 80^\circ \). The middle panel of Fig. 2 presents temporal variations of the channel length \( L_\alpha(t) \) elapsed by ionizing radiation for the above \( \alpha \) values.

\[
L_\alpha(t) = \frac{c}{\cos \alpha} \cdot t \quad 0 < t \leq \tau_{\alpha}
\]

It is obvious that varying currents flow in the vertical channels of length \( H \), but the variations occur in different time intervals depending on \( \alpha \). The current moment of vertical column of the cross-section \( S \) is equal to:

\[
dM_C(t) = \Delta j \frac{t}{\tau_{\alpha}} SH = M_L \frac{t}{\tau_{\alpha}}
\]

where \( 0 < t \leq \tau_{\alpha} \). \( dM_C(t) \) is the variable part of the current moment of elementary source, \( M_L = \Delta j SH \). The lower plot in Fig. 2 shows temporal variations of this current moment.
The linear current variation is an extreme idealization since the gamma rays modify air conductivity, not the current. The resistance of the whole column \( R(t) \) decreases gradually, and the current varies as \( R^{-1} \). Variation of the current becomes linear in time when the air resistance is uniformly distributed along the height: \( \rho(h) = \text{const} \).

Since we plan to examine the extremely low frequency (ELF) band, the modifications might be treated as instantaneous: the particular time dependence is not significant. Indeed, the current varies in 200 \( \mu \text{s} \) or faster. Such time intervals correspond to the frequencies of a few kHz, i.e., they substantially exceed the 8 Hz basic frequency of Schumann resonance.

The spectrum of current moment depends on \( \alpha \), as its temporal variations take place in the interval \([0; \tau_a] \). Owing to the Earth’s curvature, the initiation moment \( \tau_d \) of the current change depends on \( \alpha \). Spectrum of the current moment (the source spectrum) is the Fourier transform of Eq. (6):

\[
dM_C(\omega) = \frac{M_j}{\tau_a \omega^2} \exp(-i\omega \tau_a) \left[ \exp(-i\omega \tau_a) - 1 \right]
\]

By using Eq. (7), one may estimate the current moment at ‘low frequencies’ where \( \omega \tau_a << 1 \) and obtain \( dM_C(\omega) = \frac{M_j}{i\omega} \). The sum in the brackets of Eq. (7) is responsible for the
characteristic amplitude ‘beating’ at ‘high’ frequencies (see Fig. 3). As Fig. 3 shows, the
current moment in the linear source model varies as $1/f$ when $f < 1$ kHz for all angular distances
$\alpha$. Spectral peaks appear at a few kHz frequencies and their period increases with $\alpha$.

Postulated linear temporal variations of the leakage current in the global electric circuit
result in the ‘red’ ELF spectrum of the field source. This property distinguishes from the
‘white’ source spectrum used by Nickolaenko (2010).

5. Exponential atmosphere profile

Description of the global electric circuit should account for the realistic vertical profile of
atmospheric resistance. Let us turn to the exponential profile with a constant scale height
(Fig.4):

$$\rho = \rho_0 \exp(-z/\zeta)$$  \hspace{1cm} (8)

We presume that $\rho_0 = 0.20833$ Ohm-m and $\zeta = 6$ km. Cumulative resistance of the column
of the height H is the integral of Eq. (8):

$$R = \int_0^H \rho_0 \exp(-z/\zeta)dz = \rho_0 \zeta [1 - \exp(-H/\zeta)] \approx \rho_0 \zeta$$  \hspace{1cm} (9)

One obtains $R = 1.25 \times 10^{17}$ Ohm m$^2$, which gives the density of leakage current $j = 2$
pA/m$^2$ for the ionosphere potential 250 kV. Cumulative resistance of the air in the Earth–
ionosphere cavity is found by dividing $R$ by the area of the Earth’s surface, and it is equal to
$R_\oplus = 243$ Ohm.

Gamma rays increase the air conductivity and shift the well-conducting layers downward
by $\Delta h$. At a particular time moment $t$, a discontinuity appears in the regular profile, as seen in
Fig. 4. The air resistance abruptly decreases above the ‘stair’, and the non-uniformity moves
downwards in time. It passes the distance $L(t) = c \cdot t$ and reaches the altitude $z(t) = H - L(t)$.
Figure 4 shows three successive positions of modification.

Resistance of the air column as a function of time is:
\[ R(t) = \int_0^{t-L} \rho_0 \exp(-t'/\zeta) \, dx + \int_{L-t}^{t} \rho_1 \exp(-t'/\zeta) \, dx \]  
\[ \text{where } \rho_1 = \rho_0 \exp(- \Delta h/\zeta) \]  
\[ \text{is the modified resistance at the ground surface. By integrating Eq. (10), one obtains the following time dependence:} \]
\[ R(t) = \rho_0 \zeta (1 - E_1 - E_2) \]  
\[ \text{where } E_1 = \exp[-(H-L)/\zeta \{1-\exp[-\Delta h/\zeta]\}] \text{ and } E_2 = \exp[-(H+\Delta h)/\zeta]. \]  
\[ \text{For } t = 0 \text{ we have } L = 0, \text{ so that } R = \rho_0 \zeta. \]  
\[ \text{When } t = \tau_s, \text{ we have } L = H, \text{ and cumulative resistance turns into } R = \rho_1 \zeta. \]

Fig. 4 Modification of the resistance height profile \( \rho(z) \) by the gamma ray burst. The stair-like discontinuity moves downward until it reaches the ground.

For the 1 m² cross-section of the air column, the electric current and current moment are:
\[ j(t) = \frac{V_{ION}}{R(t)} \]  
\[ M_C(t) = H \cdot j \]  

We show evolution of all parameters of the exponential model in Fig. 5. The time from the modification onset is shown on the abscissa in µs. The upper plot depicts variations of the height reached by gamma rays. The second panel shows variations of the resistance of air column having the 1 km² cross-section. The third plot depicts the current [A] in this column, and the bottom graph demonstrates the ‘variable’ current moment \( dM_C(t) \) [A·m]. The latter increases from 0 to \( 3.3 \times 10^6 \) A·m.

The current and current moment vary rather fast in the exponential profile model. One must note the key role of the last kilometers passed by gamma rays in the atmosphere (compare with Rycroft et al., 2000, 2007). The model suggests that when computing EM radiation from the modified current in the global circuit, the knowledge of exact height profile is crucial at the troposphere and tropopause altitudes. Modifications in the higher layers play a minor role. In other words, the parametric radiation originates from “electro-sphere”, the altitudes around the
upper edge of troposphere. The major part of air resistance is concentrated here, and the region determines the current of parametric source.

We already mentioned that temporal changes taking 20–30 µs might be treated as instantaneous in the Schumann resonance band. To formally validate this statement, we compare the spectra of three processes.

The first one is the discontinuity:

\[ u(t) = \begin{cases} 
0 & \text{when } t < 0 \\
1 & \text{when } t > 0
\end{cases} \quad (14) \]

It has the following spectrum:

\[ u(\omega) = \int_{-\infty}^{\infty} u(t) \exp(-i\omega t) dt = \frac{1}{i\omega} \quad (15) \]

![Fig. 5 Temporal variations of parameters of global circuit for exponential profile.](image)

The second process is the linear growth

\[ u(t) = \begin{cases} 
0 & \text{when } t < 0 \\
\frac{1}{\tau} t & \text{when } 0 \leq t \leq \tau \\
1 & \text{when } t > 0
\end{cases} \quad (16) \]

having the spectrum
The third process is the quadratic increase
\[ u(t) = \begin{cases} 
0 & \text{when } t < 0 \\
(t/\tau)^2 & \text{when } 0 \leq t \leq \tau \\
1 & \text{when } t > \tau 
\end{cases} \] (18)

Its spectrum is equal to:
\[ u_3(\omega) = \frac{2}{\omega^2 \tau^2} \left[ \exp(-i\omega\tau) + \exp(-i\omega\tau) \right] \] (19)

One may see that all three spectra are coincident with Eq. (15) at frequencies satisfying the condition \( \omega \tau \ll 1 \). The parameter is \( Z \omega \tau \ll 10 \) for the basic Schumann resonance frequency of 8 Hz. Therefore, only the magnitude of discontinuity is important, and the moment spectrum is inversely proportional to frequency:
\[ M_c(f) = \frac{\Delta j \cdot H}{i \omega \tau} \exp(-i\omega\tau_a) \] (20)

Modification of the air resistance again results in the ‘red’ source spectrum.

6. **Proportional height modification**

The above exponential model has a disadvantage of the “standard” 20 km reduction of the profile. Such a decrease seems to be excessive at the troposphere and tropopause. A more realistic model is the reduction proportional to the altitude, and such a modification is shown in Fig. 6.

Fig. 6 Proportional modifications of resistance height profile.

Formally, the following relations describe the height variations:
\[ \rho = \rho_0 \exp(-z/\zeta) \quad \text{regular profile} \] (21)
The profiles in Fig. 6 correspond to three successive time moments with the front of gamma radiation positioned at different altitudes. Transition from undisturbed profile to the disturbed one occurs at the front of gamma rays. The whole resistance profile changes its scale height from $\zeta$ to $\zeta_1$ when the ionizing radiation reaches the ground surface. We use $\zeta = 6$ km and $\zeta_1 = 4.8$ km in Fig. 7.

The complete resistance of air column of $S = 1$ m$^2$ cross-section varies in time as:

$$\rho = \rho_0 \exp\left(-z/\zeta_1 \right)$$

disturbed profile

$$R = \int_0^{t-L} \rho_0 \exp\left(-z/\zeta \right)dz + \int_{t-L}^t \rho_0 \exp\left(-z/\zeta_1 \right)dz =$$
$$= \rho_0 \zeta \left[1 - \exp\left(-H/\zeta \right)\right] - \rho_0 \zeta_1 \left[\exp\left(-H/\zeta_1 \right) - \exp\left[-(H-L)/\zeta_1 \right]\right]$$

where $L(t) = \frac{ct}{\cos\alpha}$ is measured from the 60 km ionosphere edge. Prior to modifications, the complete resistance is equal to $R = \zeta \rho_0$. When gamma rays reach the ground, the resistance reduces to $R_1 = \zeta_1 \rho_0$. The current density varies from the undisturbed $j = V_{\text{ION}}/R$ value to $j = V_{\text{ION}}/R_1$. Temporal variations are described by Eq. (22).

![Parameters of proportional model](image)

**Fig. 7** Temporal modifications in the proportional model.

Parameters of the global electric circuit are presented in Fig. 7. The time is shown on the abscissa in $\mu$s. The gamma rays enter the atmosphere (i.e., 60 km altitude) when $t = 0$. We show a set of plots corresponding to different angular distances $\alpha$ from the sub-flare point. The ‘spherical’ retard $\tau_\rho$ is not shown.
The upper frame in Fig. 7 shows the altitude reached by gamma rays. One may see that the ionization moves with the lowest speed at the sub-flare point $\alpha = 0$. The second panel presents the reduction of resistance of the vertical column of the air having the cross section of 1 square meter. Noticeable changes of the cumulative resistance $R$ occur in the 10–60 $\mu$s interval depending on $\alpha$. The current varies correspondingly: it increases from the regular to disturbed value.

We must account for an additional factor when computing the current moment. The lower edge of regular ionosphere occupies the 60 km altitude. This means that the air resistance at 60 km has reduced to a value that horizontal ‘spreading’ currents replace vertical currents of the global circuit. A particular value of $H = 60$ km is not crucial; we use it since VLF measurements by Inan et al. (2007) were associated with this boundary. The undisturbed current moment of the global electric circuit is equal to the product $j H$; it is shown at the left part of plots in the lower frame of Fig. 7.

After the gamma rays ‘leave the ionosphere and enter the atmosphere’, the effective height of the Earth–ionosphere system decreases, and the zone occupied by vertical currents shrinks in time as a linear function. It is easy to see that the air column with modified resistance acquires the new height $H_1 = H \cdot \zeta_1 / \zeta$. Initially, the complete resistance of air column does not change noticeably, and the current remains constant. However, the current moment starts to reduce. Such variations are clearly seen in the left part of plots in the lower frame of Fig. 7. The slowest reduction lasts for 60 $\mu$s at the sub-flare point. Alterations become faster with increasing $\alpha$.

Fig. 8 Spectrum of current moment in the proportional model.

Later, the ionosphere height becomes a novel constant, while the current changes are still negligible, so that the current moment is stabilized until the ionization reaches the tropopause altitudes causing an increase in the current moment. After gamma rays reach the ground, the current moment becomes a constant. It does not vary until the ionizing flux starts to fade, but we are not going to treat the closing phase of the gamma burst modification.

Since the reduced ionosphere height is $H_1 = H \cdot \zeta_1 / \zeta$, and the new resistance of air is $R_1 = \zeta_1 \rho_0$, the new current moment is exactly equal to its initial value $M_C^0 = H \cdot V_{\text{ion}} / \zeta \rho_0$. 
Thus, the current moment initially reduces linearly by the factor $\zeta_1/\zeta$. It remains practically constant during a definite interval, and finally, it increases non-linearly to the undisturbed value. Unfortunately, such a pattern does not allow for a straightforward analytical Fourier transform. Nonetheless, since we are interested in the low frequencies, we can use the simplified variations of the following form:

$$\frac{dM_C}{M_C^0} = \begin{cases} 0 & \text{when } t \leq 0 \\ -\frac{\zeta_1}{\zeta} & \text{when } 0 < t \leq \tau_a \\ 0 & \text{when } \tau_a < t < \infty \end{cases} \quad (23)$$

In distinction from the previous models, the proportional model combines two ‘stepped’ functions separated by the $\tau_a$ time interval$^1$. Spectrum of the current moment is equal to:

$$dM_C(\omega) = \frac{M_C^0}{\omega} \zeta_1 \exp(-i\omega\tau_d) \left[\exp(-i\omega\tau_a) - 1\right] \quad (24)$$

Two stepped functions of opposite sign separated in time and having the equal amplitudes modify the source spectrum. It becomes flat at low frequencies (the ‘white’ source). At frequencies around $1/\tau_a$ and higher, a distinct ‘beating’ appears in the plots of Fig. 8. Maybe, this interference has caused the VLF radio noise observed at the ‘Palmer’ Antarctic station reported by Inan et al. (2007).

Depending on particular $\zeta_1$, the parametric current moment may acquire different modifications. In our model, it is initially $M_C = 2 \text{ pA/m}^2 \times 60 \text{ km} = 1.2 \times 10^{-7} \text{ A/m}$. Then, owing to the height depression, it linearly reduces to $0.8 \times 10^{-7} \text{ A/m}$, and then rises to the undisturbed value. In this case the source has the flat spectrum at ELF.

7. Knee model

The ‘knee’ profile is regarded as the most adequate model of atmospheric resistance. A particular profile is shown in Fig. 9. Here, the abscissa shows the logarithm of air resistance and the ordinate shows the altitude in km. The black line presents the regular profile, and the red line depicts the disturbed profile.

![Fig. 9 Knee model: regular and disturbed profiles of air resistance at a particular time moment.](image)

A knee profile presents many degrees of freedom. The regular function $\rho(z)$ is characterized by different height scales: below ($\zeta = 6 \text{ km}$) and above ($\zeta_U = 2 \text{ km}$) the knee

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$^1$ In fact, variations of effective ionosphere height take place in the exponential model also, but we ignored them.
altitude $H_{\text{KNEE}}=55$ km. The resistance at the knee altitude is $\rho_{\text{KNEE}} = \rho_0 \exp(-H_{\text{KNEE}}/\zeta)$, $\rho_0 = 2.08 \times 10^{14}$ Ohm-m$^2$. The regular profile is shown in Fig. 9 by the black line:

\[ \rho = \rho_0 \exp(-z/\zeta) \quad \text{when } z < H_{\text{KNEE}} \]

\[ \rho = \rho_{\text{KNEE}} \exp\left[-\frac{(z - H_{\text{KNEE}})}{\zeta_U}\right] \quad \text{when } z > H_{\text{KNEE}} \]

(25)

The gamma burst moves the upper part of profile downward by $\Delta h$, so that the knee height of disturbed profile acquires a new value $H_{\text{NEW}} = H_{\text{KNEE}} - \Delta h$. The disturbed profile is also the combination of two exponents. Its upper part (red line in Fig. 9) has the same scale height $\zeta_U = 2$ km as the regular profile. The profile below the disturbed knee height $H_{\text{NEW}}$ has the scale height $\zeta_1 = 4.2$ km. The disturbed resistance at the ground level is $\rho_1 = 1 \times 10^{14}$ Ohm-m$^2$. The height variations of disturbed resistance are described by:

\[ \rho_d = \rho_1 \exp(-z/\zeta_1) \quad \text{when } z < H_{\text{NEW}} \]

\[ \rho_d = \rho_{\text{KD}} \exp\left[-\frac{(z - H_{\text{NEW}})}{\zeta_U}\right] \quad \text{when } z > H_{\text{NEW}} \]

(26)

where $\rho_{\text{KD}} = \rho_1 \exp(-H_{\text{NEW}}/\zeta_1)$.

The disturbed profile is shown in Fig. 9 in red. The thick blue line in Fig. 9 shows the height dependence of air resistance for a particular time moment $t \in (0; \tau_0)$ when gamma rays have reached the $z(t) \approx 25$ km altitude.

![Variations in the knee model](image)

Fig. 10 Modifications of the current moment in the knee model

The cumulative resistance is found similarly to Eq. (22):
\[ R(t) = \int_0^{\varepsilon(t)} \rho(z) \, dz + \int_{\varepsilon(t)}^{\varepsilon(t)+\delta t} \rho_d(z) \, dz \quad (27) \]

The current density and current moment of the air column are described by the same formulas.

\[ j(t) = \frac{V_{\text{ion}}}{R(t)} \quad \text{and} \quad M_C(t) = H(t) \cdot j(t) \]

Figure 10 presents temporal modifications of source parameters in the standard fashion. The major distinction of the knee model is the range of variations of the current moment (lower plot). It is easy to see that the normalized current moment is approximated by the following stepped functions for the model parameters listed above:

\[ \frac{dM}{M_0} = \begin{cases} 
0 & \text{when } t \leq 0 \\
-1/3 & \text{when } 0 < t \leq \tau_a \\
4/3 & \text{when } \tau_a < t < \infty 
\end{cases} \quad (28) \]

Therefore, the current moment of the source has the following spectrum:

\[ dM_C(\omega) = \frac{M_0}{i \omega} \exp(-i \omega \tau_a) \left[ \frac{4}{3} \exp(-i \omega \tau_a) - \frac{1}{3} \right] \quad (29) \]

where \( M_0 = H_j \).

Parameters of the resistance height profile provide the ELF pulsed amplitude close to observations (see below).

8. Model Q–burst

We use the knee model when computing the spectra and waveforms of transient Schumann resonance signals at the Moshiri observatory, Japan (44.22° N and 142.16° E). The task is performed similarly to Nickolaenko (2010). When computing the complex spectrum of a parametric pulse, we divide the illuminated hemisphere into elementary areas (sources) having different angular distances from the sub-flare point. The current moments were computed for these elementary sources and the ELF field was found as the sum of contributions from every elementary source with an account for the distance, retard, and the wave arrival angle (Nickolaenko, 2010). The parametric source occupies the whole hemisphere in our model.

The upper plots in Fig. 11 depict amplitude spectra of the horizontal magnetic field components \( H_{EW} = H_X \) and \( H_{NS} = H_Y \) measured in \( \mu A/(m \text{ Hz}^{1/2}) \). The middle plot shows the vertical electric field in \( \text{mV}/(m \text{ Hz}^{1/2}) \), and the lower plot shows the real part of the complex spectra of the Poyning vector: \( \mathbf{P} = \mathbf{E} \times \mathbf{H} \). The horizontal axes show frequency in Hz, and the ordinates depict the spectral densities.

We see that the sole first Schumann resonance mode is present in the horizontal magnetic field owing to the source ‘red’ spectrum combined with the great zone occupied by the source currents. The spectral peak around 60 Hz is also present in all field components (compare with Nickolaenko, 2010), but its amplitude is rather small. The spectrum of vertical electric field component contains the second Schumann resonance mode, which is explained by the 10.2 Mm distance from the observatory to the center of parametric source. One may observe a succession of Schumann resonance peaks in the spectra of Poynting vector together with wide peak around 60 Hz, but the first mode dominates. The Poynting vector indicates that EM wave propagates from east to west \( (\text{Re}[P_{EW}] < 0) \) and from south to north \( (\text{Re}[P_{NS}] > 0) \), which is explained by the geometry.
By applying the Fourier transform to the complex spectra of horizontal magnetic fields, we obtain the pulsed waveforms presented in Fig. 12. The time is shown here on the abscissa and the field amplitude is plotted along the ordinate in $\mu$A/m ($1 \mu$A/m = 0.4 $\pi$ pT).

![Fig. 11 Amplitude spectra of parametric pulse at Moshiri.](image)

The circular diagram in Fig. 12 shows the Poynting vector in the time interval from 0 to 0.1 s. Its orientation corresponds to the azimuth of sub-flare point of 117.2°. One may conclude that a parametric pulse is a prominent representative of so-called ‘quiet’ bursts (Q–bursts) introduced by Ogawa et al. (1966): it looks like an attenuating sinusoid of the basic frequency of the Earth–ionosphere cavity.

**9. Discussion and conclusion**

In this paper we considered a series of possible models, and these allow for important conclusions. The air ionization caused by a gamma burst results in EM radiation linked to troposphere heights, to the so-called electro-sphere. Therefore, a parametric radio emission driven by external gamma rays is less sensitive to the conductivity profile in the mesosphere and upper atmosphere. Additionally, all models predict spectral peaks at the “high” frequency of a few kHz.
We assumed that the atmosphere was horizontally uniform, so that modifications were also “uniform”. Owing to great wavelengths comparable with the Earth’s circumference, such an assumption is reasonable when modeling the Q−burst. The VLF peaks are present in all models relevant to much shorter wavelengths. Here, the horizontal structure of atmosphere might become important. For example, a system of discrete sources might emerge, which are scattered in the space, so that VLF pulses traverse different distances and have various arrival times. However, the problem of parametric VLF pulses deserves a separate treatment that we hope to perform in future.

Model ELF waveforms obtained here correspond to observations since the 60 Hz peak became very weak. The pulse amplitude obtained in the computations is close to the experimental data (Tanaka et al., 2011, Nickolaenko and Schekotov, 2011), and the oscillation period agrees with the eigen-values of Earth–ionosphere cavity with modified ionosphere (e.g. Nickolaenko et al., 2011).

The onset of parametric pulse is delayed by the time necessary for the gamma rays to reach the electro-sphere, which is approximately 150 μs. This conclusion assumes that an interaction of the atmosphere and ionizing radiation does not depend on the altitude. This is not so, as the hard solar ionizing component maintains the ionosphere F region and plasma above. By contrast, the ozone layer is produced by the long wave UV radiation.

Fig. 12 Parametric pulse computed for Moshiri.

The model indicates that the parametric radiation originates at the tropopause heights. It must depend on spectral content in the band from the soft ultra-violet to hard gamma quanta. The parametric ELF pulse will retard from the onset of ionosphere modification when the whole spectrum arrives simultaneously. However, if the soft component arrives prior to the hard one, the lag might turn into a lead. The relevant energy–arrival time scheme seems quite possible for the soft gamma repeaters (SGR). The concept of SGR explains the origin of gamma bursts by sudden accretion on the central celestial body called a magnetar. For gradually increasing accretion, the hard radiation must emerge after the softer quanta. The ‘precursory’ soft radiation might pass ‘unnoticed’ by orbital X- and gamma ray observatories, as it might have too low energy for the detectors. Simultaneously, this part of spectrum might play a crucial role in modifications of the tropospheric air conductivity. Thus, a ‘precursory’ parametric ELF burst might appear in the ELF record overlooked by gamma ray observatories.
If so, the time lead noted at Karymshino (Nickolaenko and Schekotov, 2011) might indicate interesting details of accretion on a distant magnetar. Special attention should be given in future investigations to the VLF component radiated by the parametric source. Its properties should be compared with observations. A comparison with VLF observations might show what a particular model corresponds to the reality in a better way.

Summary
We list the major results obtained in the paper.
- The spectrum of the parametric current moment is usually inversely proportional to frequency in the Schumann resonance band.
- The planetary size of modification combined with the source spectrum cause the prevalence of the first Schumann resonance maximum in the ELF records.
- Models indicate that the discrete ELF radio pulse originates at the tropopause altitudes.
- Depending on the arrival times of different parts of spectrum of ionizing radiation, the ELF pulse may lead or follow the ionosphere modification recorded with the VLF radio signals.
- The polarity of parametric source depends on the scheme of atmosphere modification.
- The parametric source has spectral maxima at frequencies 5–10 kHz. These might be responsible for an increase of radio noise reported in the VLF band.

References

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