EXAMINATION ABOUT THE INFLUENCE OF AFTERSHOCKS TO BUILDINGS — CONSIDERATION BY THE ENERGY SPECTRUM AND THE P-DELTA EFFECT —

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ABSTRACT: Many aftershocks were observed in the Pacific coast of Tohoku Earthquake. In view of this fact, aftershocks should be paid attention, although it is ignored in conventional design. In this paper, the effect of aftershocks to buildings is discussed focusing on energy input. From cumulative energy spectrum, it is elucidated that aftershocks recorded this time have enough energy to bring certain level of damage. P-delta effect degrades the strength of the buildings. Therefore, buildings damaged after main shock can be brought significant incremental deformation by small energy of aftershocks.

Key Words: Aftershocks, Energy spectrum, P-delta effect, Stability ratio

1. INTRODUCTION

The great earthquake, “the Pacific coast of Tohoku Earthquake” occurred at March 11, 2011 in Japan. Extremely intensive ground motion and structural responses were observed in various areas of the country. The specific characteristic of the main shock was as follows. The shock was with high acceleration, long duration and be accompanied by many aftershocks. There were aftershocks with maximum acceleration over 300 cm/s^2 . Two stage of the earthquake intensity expected in the present seismic design code are shown in Table 1. The acceleration of 300 cm/s^2 is equivalent to the “strong

earthquake” stage in the seismic design¹⁾. The effect of aftershocks is not considered in present seismic design of Japan, because aftershocks are supposed to be much smaller than the main shock. In terms of energy input, the aftershocks followed by the main shock were surveyed first in this paper.

The effect of aftershocks on the buildings damaged by main shock was considered secondary. Progress of damage by aftershock occurred successively will be considered. The P-delta effect may increase the capacity loss on the damaged structures. Horizontal displacement of the building-structure will be increased being combined with the gravity force. This phenomenon induces the decrease of resistance of the structure in the alternative view of the matter. Small energy input of aftershock may contribute to increase the inelastic deformation of the structure when the structure has damaged and decreased in the strength. Influence of the P-delta effect on increasing damage of the structure may be serious. In this paper, the effect of aftershocks on building-structures will be discussed with numerical analyses of many structure-models focusing on the energy input especially. The energy input to the structure from the earthquake could be an appropriate physical parameter which could be used to evaluate the structural design synthetically. P-delta effect can also be considered in the energy input. The progressing structural loss after main shock is examined numerically by comparing in the behavior of the structures damaged by the main shock.

Table 1 Expected earthquake intensity used in the conventional design code

	Middle earthquake (Elastic design)	Strong earthquake (Plastic design)
Range of acceleration	80~100 cm/s ²	300~400 cm/s ²
Maximum velocity	25 cm/s	50 cm/s

2. RECORD OF AFTERSHOCKS

The earthquake occurred at 14:46, March 11, 2011 was defined as main shock, in the paper. The aftershocks observed in three month after the main shock were picked up in many seismic records that had the maximum acceleration over 300cm/s², resultant of 3-dimensional record. These strong motion records are able to be downloaded from the K-NET, one of the nationwide strong motion observation networks of National Research Institute for Earth Science and Disaster Prevention.

Observation sites with the maximum acceleration over 300cm/s² at the main shock were surveyed and collected in Fig. 1. The sum of observation sites is 76. The maximum acceleration record mentioned above is resultant of 3-dimensional elements, hereafter called as the maximum acceleration simply. The frequency of aftershock incidence in the site was checked and collected in Fig. 1. In Fig. 1, it is indicated that aftershocks exceeded the maximum acceleration of 300cm/s² were observed at the half of 76 sites approximately. Observation sites examined seismically in this paper are shown in Table 2.

These sites were selected to investigate cumulative influence of aftershocks. This selection was based on the high frequency of aftershock and that the maximum acceleration over 1000cm/s² was recorded.

Fig.1 The number of aftershocks (the Pacific coast of Tohoku Earthquake)

Table 2 Sites for investigation

Earthquake name	Site name	Direction	Maximum acceleration (cm/s ²)		The number of aftershocks
			mainshock	Maximum aftershock	
The Pacific coast of Tohoku Earthquake	Hokota	NS	1762	957	5
	Hitachi	NS	1845	594	7
	Sendai	EW	1808	1085	1

3. EQUATION OF ENERGY BALANCE ²⁾

The equilibrium equation of forces on a mass system under the condition of horizontal ground motion is as follows:

$$M \ddot{y} + C \dot{y} + F(y) = F_e \quad (1)$$

M : Mass $C \dot{y}$: Damping force $F(y)$: Restoring force F_e : Earthquake force ($= -M \ddot{z}_0$)
 z_0 : Horizontal ground motion y : The displacement of the mass relative to the ground

The equation of energy balance is expressed by multiplying both sides of Eq. (1) by $y \dot{y} dt$.

$$M \int_0^t \ddot{y} y \dot{y} dt + C \int_0^t \dot{y}^2 dt + \int_0^t F(y) y \dot{y} dt = \int_0^t F_e y \dot{y} dt \quad (2)$$

Eq. (2) can be transcribed into energy terms as follows:

$$W_e + W_h + W_p = E \quad (3)$$

W_e : Elastic vibration energy W_h : Absorbed energy by damping
 W_p : Cumulative plastic strain energy E : Total input energy

The sum of energy input to a structural system is expressed as follows:

$$E = \frac{M V_E^2}{2} \left(V_E = \sqrt{\frac{2E}{M}} \right) \quad (4)$$

V_E : Equivalent velocity of total input energy

A series of V_E for the first natural period is called as energy spectrum. Correlation between the main shock and aftershocks will be examined using the energy spectrum. The energy–spectrum employed in this paper is of elastic mass system with damping factor, $h = 0.1$.

3.1 Energy spectrum

An example of energy spectrum is shown in Fig. 2. Fig. 2 shows that the case of energy spectrum for the Pacific coast of Tohoku Earthquake exceeds in the energy at short natural periods, $T \leq 0.5s$. This behavior differs strikingly from the design spectrum used in Japanese seismic design in the case of strong earthquake.

The value of V_E was higher than the design spectrum at main shock and aftershocks. The value of maximum V_E at main shock is larger than that of aftershock and the ratio is more than two in the case of Hokota in Fig. 2. The energy ratio of main shock to aftershock is fourth or more, because total input energy E is in proportion to the second power of V_E as shown in Eq. (4).

Fig. 2 Energy spectra (Hokota, NS, $h=0.1$)

Fig. 3 Energy spectra (mainshock, $h=0.1$)

Short-period ground motion ($T \leq 0.5s$) was recorded at many observation sites (e.g. Hokota and Hitachi as shown in Fig. 3) during the Pacific coast of Tohoku Earthquake. The energy spectrum of Sendai site peaks at about 0.7s, and differs from those of Hokota and Hitachi sites. The influence of difference in spectrum shape on aftershock at Sendai site will be examined later.

3.2 Cumulative energy spectra

Cumulative energy of aftershock on the input energy of main shock is called as “cumulative energy (spectrum)” hereafter. The cumulative energy spectra is expressed by V_E^* that are reduced to velocity unit and are shown in Eq. (2). On legends in the figures, the influence of aftershocks is accounted in the cumulative energy spectra for the site name with asterisk.

$$V_E^* = \sqrt{\frac{2(mE + \sum_a E)}{M}} \quad (5)$$

$_mE$: Input energy of the main shock $_aE$: Input energy of aftershocks

On elasto-plastic structure system, a stiffness of structure system will be reduced with progress of plastic deformation in seismic response. The natural period will be longer than the initial period instantaneously in this case. And the natural period concerned with energy input i.e. effective period becomes longer by 20%³⁾. The predominant periods on observational aftershocks were the same as the

case of main shock approximately. The relation of effective period on aftershocks to the natural period was still unconfirmed. Discussion may be limited to a case study only in plain comparison between energy spectra of main shock and cumulative spectra V_E^* taken into account the effect of aftershocks. So, it is assumed that aftershocks with a range of $T \pm 0.2T$ will effect on energy spectra of main shock. It is expected to generalize the discussion on influence of aftershocks. Input energy of aftershocks with a range of $T \pm 0.2T$ was calculated and the maximum was selected. The maximum energy of aftershocks was cumulated on V_E .

For example, the cumulative spectrum V_E^* at $T=0.2s$ is obtained by summing the maximum energy of aftershocks calculated in the range of $T \pm 0.2T$ on the input energy of main shock as shown in Fig.4. The range of $T \pm 0.2T$ is shown in yellow band. The cumulative spectrum V_E^* obtained in this way will be able to express the possibility of increase of input energy with aftershock energy.

The cumulative spectra V_E^* obtained at three observation sites with period range of aftershocks $T \pm 0.2T$ are shown in Fig.5. In this case, the maximum value V_E is increased to 1.2 times on Hokota site, 1.4 times on Sendai site approximately. The natural period bands with V_E over 400cm/s are 0.19–0.39s at Hokota and 0.61–0.80s at Sendai while those for V_E^* over 400cm/s are 0.15–0.44s at Hokota and 0.43–1.35s at Sendai. The effect of aftershock on cumulative energy spectra transformed in velocity V_E^* (cm/s) should vary with scale and frequency of aftershocks. The figures of spectra are different in predominant periods between main shock and aftershocks at Hokota and Sendai site. The figures of V_E and V_E^* spectrum at these sites are same approximately, because there are little difference of predominant periods at both Hokota and Sendai sites.

Fig. 4 The concept of the width of period

Fig. 5 Cumulative energy spectrum

3.3 Cumulative energy spectra of elasto-plastic system

While most existent structures have elasto-plastic behavior, the energy spectra mentioned above are on elastic structural system. Responses of elasto-plastic models were calculated to collect actual behavior of structures. The energy spectra of elastic system envelop the input energy of elasto-plastic systems²⁾. A setting of yield strength is unnecessary to analyze energy spectrum, because the spectrum will be calculated with elastic structural system. However, yield strength is required to analyze elasto-plastic response of structures. The yield strength varies with structures. If the influence of aftershock is able to be obtained from elastic cumulative energy spectrum, the effect of aftershock will be evaluated simply, excepting elasto-plastic analysis with certain yield strength of structures. So, responses of elasto-plastic structures were analyzed and examined on cumulative energy spectrum with the effect of aftershock.

Analytical conditions are shown in Table 3. Elastic-perfectly plastic models are adopted for hysteresis characteristics as typical steel structures. Variable parameter is elastic natural period T . The response was analyzed using linear acceleration method (time increment is 0.0005 second). $h=0.02$ was adopted as damping factor of steel structures. A coefficient of yield shearing force α was selected

referencing to the strength of actual structures.

Table 3 Analysis condition

Analysis model	share type single-degree-of-freedom system
Hysteresis model	elastic-perfectl plastic
Damping factor	$h=0.02$
Natural period (s)	$T=0.1\sim 2.0$
Yield shear force coefficient	$\alpha = \frac{0.2}{\sqrt{T}}$
Yield deformation (cm)	$\delta_y = \alpha g \left(\frac{T}{2\pi} \right)^2$

Energy spectra of elasto-plastic systems are shown in Fig. 6. In Fig. 6, the total input energy obtained by elasto-plastic analysis was converted to equivalent velocity V_E . Suffix “(e-p)” is used to express elasto-plastic in the figure. Cumulative energy spectra of elasto-plastic system expressed equivalent velocity V_E^* was about 1.3 times of the spectra in the case of main shock V_E both for Hokota and Sendai sites. The period band for $V_E \geq 400\text{cm/s}$ at main shock was 0.15–0.28s at Hokota site. There was no case of period T where V_E exceeded 400cm/s at Sendai site. In the case of cumulative spectra V_E^* , the period band for $V_E^* \geq 400\text{cm/s}$ was expanded to 0.12–0.35s at Hokota site, 0.37–0.90s at Sendai site respectively.

Cumulative energy spectra on two observation sites with variable α were shown in Fig. 7. The effective period elongates with the degree of plastic behavior depending on the story shear coefficient of yield strength α . In Fig. 7, variable T is still expressed as elastic natural period. In fig.7, the elasto-plastic spectra V_E^* is shifted to the left side, short period, however it is interpreted that cumulative energy spectra calculated with elasto-plastic models should be enveloped in that of elastic spectra V_E^* approximately. From consideration mentioned above, the maximum input energy accumulated the effect of aftershock will be predicted by cumulative elastic spectrum V_E^* .

Fig. 6 Energy spectra of elasto-plastic system (e-p)

Fig. 7 Comparison of the cumulative energy spectra

4. P-DELTA EFFECT

The hysteresis characteristics of steel frames are generally modeled using elastic-perfectly plastic rule. Therefore, the consideration of degradation of stiffness and strength according to development of plastic deformation by increment of energy input is not needed.

However, P-delta effect can influence their hysteresis. P-delta effect degrades the strength of structures⁴⁾. Because of that, the effect of aftershocks cannot be estimated by plain summation of energy input. The concept diagram of P-delta effect is shown in Fig. 8.

Fig. 8 Conceptual diagram of the P-delta effect

Fig. 9 Restoring force characteristics in consideration of the P-delta effect

With horizontal deformation x , structural frames lose its horizontal resistance by additional force ΔQ as the effect of vertical load Mg . ΔQ is expressed as Eq. (6)

$$\Delta Q = \frac{Mg}{H} x = k_{P\Delta} \cdot x \quad (6)$$

ΔQ : Additional shear force M : Mass g : Acceleration of gravity H : Height
 x : Horizontal displacement $k_{P\Delta}$: The stiffness degradation due to P-delta effect

As expressed in Fig. 9, the P-delta effect is considered as an elastic element with negative stiffness.

In a case that the plastic deformation is developed in one direction, energy consumption ignoring P-delta effect for ultimate horizontal deformation δ_u (the deformation corresponds to P-delta collapse) W_p is

$$W_p = \frac{Q_Y \cdot \delta_Y}{2} + (\delta_u - \delta_Y) Q_Y = Q_Y \left(\delta_u - \frac{\delta_Y}{2} \right) \quad (7)$$

Q_Y : Yield shear force δ_Y : Yield deformation δ_u : Ultimate deformation considering the P-delta effect

On the other hand, the energy consumption considering P-delta effect W_p' is expressed as Eq. (8). The stability factor θ ⁴⁾ is an index for the degree of P-delta effect defined as the ratio of P-delta negative stiffness $k_{P\Delta}$ and structural stiffness.

The difference between W_p and W_p' is loss of energy consumption capacity by P-delta effect, that is the energy stored by negative stiffness element. Total energy input is determined by total mass of structure and fundamental natural period, and is not influenced by the difference of hysteresis under P-delta effect. The influence for elasto-plastic response by P-delta effect is the uneven (positive or negative side) plastic consumption energy, the excessive plastic deformation, and the possibility of collapse by losing whole horizontal resistance.

$$W_p' = \frac{1}{2} \delta_u \cdot Q_Y (1 - \theta) \quad \left(\theta = \frac{k_{P\Delta}}{k} \right) \quad (8)$$

θ : Stability factor k : Elastic stiffness without influence of P-delta effect

4.1 Stability Factor τ ⁵⁾

In this paper, stability factor τ proposed by Yamazaki and Endo is used for the estimation of P-delta

effect. Stability factor is defined as the ratio of stability factor θ and θ_{cr} . θ_{cr} is the stability factor in case that horizontal resistance reaches zero at the end of earthquake duration with the one side plastic deformation. Thus, θ_{cr} is a function of energy input. The stability ratio of elastic-perfectly plastic system is expressed as Eq. (9). The estimation of P-delta effect considering energy input is possible by use of stability ratio.

$$\tau = \frac{\theta}{\theta_{cr}} \quad (9)$$

Where

$$\theta_{cr} = \frac{1}{2\eta_\tau + 1} \quad \eta_\tau = \frac{E_D}{Q_Y \cdot \delta_Y}$$

θ_{cr} : stability factor in case that horizontal resistance reaches zero at the end of earthquake duration

η_τ : equivalent cumulative plastic deformation factor of E_D

E_D : The input energy contributing to damage considering the P-delta effect

Fig. 10 Conceptual diagram of the stability ratio

As shown in Fig. 10, in case of $\tau = 1.0$, structure will collapse by plastic deformation developed in one side completely. However, generally structures vibrate developing plastic deformation in both sides. Thus $\tau = 1.0$ is not sufficient condition of collapse.

4.2 Analysis condition for the consideration of P-delta effect

The analysis condition for consideration of P-delta effect is shown in Table 4. Although P-delta effect is dominant in the first story, the results of multi-story analyses have inevitable deviation of energy distribution. That makes the estimation of P-delta effect more difficult. Thus, single-degree-of-freedom analyses are performed for clear estimation of P-delta effect. The parameters of analyses are determined to correspond to the property of first story of realistic multi-story frames. As for yield story shear coefficient, $\alpha = 0.4/\sqrt{T}$ is also taken considering the difference of strength of design and real strength.

A factor κ_1 is applied for the purpose of calculation of the stability factor for single-degree-of-freedom system coinciding with the first story of multi-story frames. κ_1 is defined as the ratio of elastic stiffness between the first story of multi-story frames and single-degree-of-freedom system with the same fundamental natural period. κ_1 is approximately calculated by Eq. (10)⁶⁾. By using $\kappa_1 \cdot H_0$ (H_0 is original story height of multi-story frames) for story height, the stability factor of single-degree-of-freedom system equals to that of multi-story frame. Here, H_0 is assumed as 400cm, and by estimation equation for primary natural period of steel frames $T = 0.03H$ (H is overall height of frame), the number of story N is determined corresponding to target natural period. N - T relationship and θ - T relationship are shown in Figs. 11 and 12.

$$\kappa_1 = 0.48 + 0.52N$$

N : Number of stories

(10)

Table 4 Analysis condition considering the P-delta effect

Analysis model	share type single-degree-of-freedom system
Hysteresis model	elastic-perfectly plastic
Natural period (s)	$T = 0.1 \sim 2.0$
Yield shear force coefficient	$\alpha = \frac{0.2}{\sqrt{T}}, \frac{0.4}{\sqrt{T}}$
Height (cm)	$H = \kappa_1 \cdot H_0$
Yield deformation (cm)	$\delta_y = \alpha g \left(\frac{T}{2\pi} \right)^2$

Fig. 11 $N-T$ relationship

Fig. 12 $\theta-T$ relationship

By using damage distribution law⁷⁾ expressed as Eq. (11), the energy distribution to the first story of multi-story frames is estimated. Considering damage dispersion coefficient γ_1 , energy input to first story V_E' is expressed as Eq. (12). Although standard value of p_d is proposed as $p_d = 1.185 - 0.0014N$ ⁶⁾, $p_d = 1.25$ is applied considering emphasized damage concentration by the P-delta effect. V_{E0} is obtained by cumulative energy spectrum of elasto-plastic system. The acceleration of input ground motion is factored so that total input energy will consistent with V_E' .

By these rule, parameters for single-degree-of-freedom system considered the P-delta effect are determined. Period elongation after main shock is not considered as Chapter 3.

$$\frac{W_{p1}}{W_p} = \frac{s_1 \cdot p_1^{-n}}{s_1 \cdot p_1^{-n} + \sum_{i=2}^N s_i \cdot p_d^{-n}} = \frac{1}{\gamma_1} \quad (11)$$

Where

$$s_i = \left(\sum_{j=i}^N m_j / M \right)^2 \alpha_i^{-2} (k_1 / k_i), \quad p_1 = 1.0, \quad p_d = 1.25$$

W_{p1} : Cumulative plastic strain energy of the first story

W_p : Total cumulative plastic strain energy of all stories

n : Index of damage concentration ($n=12$)

p_d : The difference between the optimum yield shear force coefficient distribution and realistic distribution

γ_1 : Damage dispersion coefficient of the first story

α_i : Optimum yield shear force coefficient distribution⁶⁾ for i-th story

m_j : The mass of j-th story

M : Total mass

k_1 : Elastic stiffness of the first story

k_i : Elastic stiffness of i-th story

$$V_E' = V_{E0} \sqrt{\frac{\kappa_1}{\gamma_1}} \quad (12)$$

V_E' : Estimated input energy

V_{E0} : Input energy of single-degree-of-freedom system

Assumptions for the simulation of multi-story frame by single-degree-of-freedom system are summarized as follows:

- (1) The fundamental natural periods of single-degree-of-freedom system and multi-story frame are equal.
- (2) The stability factors of single-degree-of-freedom system and multi-story frame are equal.
- (3) Ground motion is factored so that total input energy of single-degree-of-freedom system and first story of multi-story frame will be consistent.
- (4) Input energy of the first story of multi-story frame is approximated by damage distribution law and cumulative energy spectrum of elasto-plastic system.

The effect of aftershocks is analyzed with ground motion created by concatenation of mainshock and followed aftershocks. As for the response result of an aftershock, two typical cases are supposable as shown in Fig. 13. Upper diagram shows that strength is recovered and residual deformation decreased as the result of aftershock. On the other hand, lower diagram shows that an aftershock developed further residual deformation. In following analyses, the worst case of aftershocks will be taken. The worst case means the situation that the residual deformation is developed by every aftershock. Therefore, the sign of acceleration of aftershocks may be inverted.

Fig. 14 shows load-deformation hysteresis of an analysis case (load is divided by Mg , deformation is divided by δ_y). The vertical dashed line shows residual deformation. The residual deformation accumulates as the structure is subjected to every aftershock.

Fig. 13 Procedure for concatenation of aftershocks

Fig. 14 Load-deformation relationship (Hokota, $T=0.8s$, $h=0$, $\theta=0.01$)

4.3 P-delta effect about the pacific coast of Tohoku Earthquake

The result for $\alpha = 0.2/\sqrt{T}$ is shown in Figs. 15, 16, 17. Fig. 15 shows δ/δ_Y-T relationship. Blue, green, purple dashed lines respectively indicate the ultimate deformation δ_u , $H/30$, and $H/50$. $H/30$ and $H/50$ are realistic allowable limit in earthquake-resistant design. Fig. 15 shows that deformation exceeded $H/30$ or $H/50$ after main shock in most cases and exceeded δ_u at aftershock in some cases as for Hokota. As for Sendai, collapses occur at $T \geq 0.48s$ range, the range is expanded by aftershocks.

However, buildings collapsed by aftershocks about this disaster are not reported⁸⁾. The difference between the reality and analyses is presumably due to period elongation and the omission of the stiffness of nonstructural members. In some cases the strength must be recovered by aftershock (see the upper diagram in Fig. 13).

Fig. 15 δ/δ_Y-T relationship ($0.2/\sqrt{T}$)

Fig. 16 shows $\tau-T$ relationship, dashed line indicates $\tau=1.0$ (Collapse is impossible below $\tau=1.0$). Fig. 17 shows $\eta/\mu-T$ relationship. η is cumulative plastic deformation factor, and μ is maximum plastic deformation factor, these indices are defined as follows:

$$\eta = \frac{\delta_p}{\delta_Y} \quad (13)$$

δ_p : Cumulative plastic deformation

$$\mu = \frac{\delta_m}{\delta_Y} - 1 \quad (14)$$

δ_m : Maximum story deformation

η/μ means bias of hysteresis. $\Delta\eta/\Delta\mu$ indicated by Circle markers in Fig. 17 are the ratio of incremental η to μ . From Figs. 16 and 17, the relationship between development of damage and maximum deformation can be found. For instance, when both η/μ and τ is large, more energy is needed to collapse.

In Fig. 16, τ extremely exceeds 1.0 in some range of period. But Fig. 15 shows that collapse does not occur in the range. Large value of η/μ shown in Fig. 17 can explain the fact. In Hokota in $T \leq 0.5s$ range, large value of $\Delta\eta/\Delta\mu$ and τ indicate that biased deformation increment by aftershock is not notable. In $T \geq 0.5s$ range, $\Delta\eta/\Delta\mu$ is near to 1.0, energy input by aftershock develop deformation directly.

In Sendai, τ sharply decreased in $T \geq 0.5s$ range. The reason for this fact is that vibration system reached collapse state before energy correspond to V_E' is supplied. The difference between V_E' and actual energy input is shown in Fig. 18.

Fig. 16 τ - T relationship ($0.2/\sqrt{T}$)

Fig. 17 η/μ - T relationship ($0.2/\sqrt{T}$)

Fig. 18 Input energy (Sendai)

The result for $\alpha = 0.4/\sqrt{T}$ is shown in Figs. 19, 20, 21. Maximum deformation and value τ are decreased due to higher strength. Fig. 21 shows that $\Delta\eta/\Delta\mu$ approximately equals to η/μ in Sendai. For $\alpha = 0.4/\sqrt{T}$, the variation of $\Delta\eta/\Delta\mu$ is intensive, and the tendency approaching 1.0 diminishes.

— : mainshock — : mainshock and aftershocks - - - : Ultimate deformation (δ_u/δ_Y)
- - - : $H/30$ - - - : $H/50$

Fig. 19 δ/δ_Y - T relationship ($0.4/\sqrt{T}$)

● : mainshock ■ : mainshock and aftershocks - - - : Reference line ($\tau=1.0$)

Fig. 20 τ - T relationship ($0.4/\sqrt{T}$)

— : mainshock η/μ ● : aftershocks $\Delta\eta/\Delta\mu$ - - - : Reference line ($\eta/\mu=1.0$)

Fig. 21 η/μ - T relationship ($0.4/\sqrt{T}$)

The influences of aftershocks considering the P-delta effect are summarized as follows:

The response drastically changes depending on assumed strength. That may be a reason for the difference between analyses and realistic phenomena. As shown in Fig. 12, in longer period range, large stability factor causes tendency of biased deformation, η/μ then approaches 1.0. Although the degree depends on period range, aftershocks cause the development of maximum deformation. In shorter

period range, large η/μ and τ cause small deformation.

In Sendai, it is clear that the deformation is developed by aftershocks. The case that damage is developed by aftershocks is actually reported⁸⁾ (Although tilt by the damage of piles is included). Although the P-delta effect can be diminished by stiffness after yield, that is ignored in this study.

5. THE STUDY BY THE AFTERSHOCKS PATTERN

As shown in Chapter 3, the magnitude of input energy of aftershocks cannot be ignored. Further, as shown in Chapter 4, buildings have possibility of collapse in the worst case of aftershocks considering the P-delta effect. However, for the time being, because of the lack of record, examination using actual aftershock records is limited. Therefore, in order to assume various aftershocks, aftershocks patterns are used as follows.

In the aftershocks pattern, aftershocks are obtained by multiplying a factor by main shock record, and summation is made for some events. With the main shock and aftershock pattern, it is possible to investigate for various aftershocks.

5.1 The analysis conditions by the aftershock pattern

Table 5 shows an aftershock pattern to perform the study. Because the magnitude of aftershocks is not available quantitatively, it was assumed that the aftershocks have the following aftershock pattern: Earthquake of the same magnitude as the main shock: (1.0×1) . three times an earthquake with half magnitude of the main shock: (0.5×3) . The (0.5×3) was set in order to consider the development of the damage by consecutive small aftershocks, in other words, to examine the progress of the damage by the number of aftershocks. Residual deformation increases by aftershocks as shown in Fig. 13. The Pacific coast of Tohoku Earthquake is coupled earthquake, because of long earthquake duration, the influence of damping is large. Therefore, response is easy to fluctuate even under same amount of total input energy depending on the contribution of damping.

Therefore, ground motions observed at The Pacific coast of Tohoku Earthquake is not used in the study. Instead, two records commonly used in examination are adopted for following study. Table 6 shows analytical conditions for aftershocks pattern. Height H is set to match stability factor θ . The acceleration of main shock is factored to set stability ratio to $\tau = 0.5$. Thus, the damage development by aftershocks is able to be examined from equivalent damage of main shock regardless of the period.

The stability factor $\theta = 0.04$ is set as the value of first story of multi-story frames. This is the case for the 1/120-yield deformation angle with yield shear force coefficient $\alpha = 0.2$. Also, the case $\theta = 0.01$ is examined in order to consider the difference of the impact of negative stiffness. $\tau = 0.5$ is set as energy input that can occur when the damage is concentrated on only the first story in consideration of the P-delta effect. Also, it means half amount of energy that can be absorbed until the collapse when the deformation is biased to one side. In the case of $\theta = 0.04$, when it becomes $\tau = 0.5$, η is 5.75. It means the minor⁹⁾ or more damage occurred by main shock. Natural period in the analysis is not assumed as meaning of the range of building size. The purpose is to investigate the characteristics of the ground motion at a constant stability factor. Investigation with aftershocks pattern aims to examine response characteristics (strength decreased after main shock) under variable magnitude of aftershocks.

Table 5 Aftershocks pattern

Seismic waves	Signage of pattern	Aftershocks		Concatenated seismic waves
		Magnification	Number of times	
El Centro (NS) JMA-Kobe (NS)	mainshock	-	-	mainshock
	1.0×1	1.0	1	mainshock and aftershock
	0.5×3	0.5	3	mainshock and three times aftershocks

Table 6 Analysis condition (Aftershocks pattern)

Analysis model	share type single-degree-of-freedom system
Hysteresis model	elastic-perfectly plastic
Damping factor	$h=0.02$
Natural period (s)	$T=0.5\sim 2.0$
Yield shear force coefficient	$\alpha = \frac{0.2}{\sqrt{T}}$
Satability factor	$\theta=0.01, 0.04$
Height (cm)	$H = \frac{Mg}{k\theta}$
Stability ratio at the end of main shock	$\tau=0.5$

5.2 Analytical result

Fig. 22 is a result of the aftershock pattern. Similar to Fig. 17, Markers of “1.0 × 1” and “0.5 × 3” indicate $\Delta\eta/\Delta\mu$. Although it varies with the building period, η/μ of aftershocks is close to 1.0. Moreover, this trend is prominent when the stability factor is large. Markers at $\eta/\mu = 0$ means that almost all input energy from aftershocks is absorbed by damping, so damage did not proceed. In addition, the input energy of the first aftershock of (0.5 × 3) is small compared to (1.0 × 1). Therefore, the number of times of yield is less. For example, in case that the system yields only one time, cumulative plastic deformation equals to maximum plastic deformation, so η/μ is 1.0. Therefore, η/μ in (0.5 × 3) have more intensive tendency to approach 1.0 than under that in (1.0 × 1).

In most period range, the maximum plastic deformation of (0.5 × 3) is smaller than that of (1.0 × 1). However, in some cases, maximum deformation for (0.5 × 3) is the same as or higher than that for (1.0 × 1). Therefore, the damage can be developed by consecutive small aftershocks, and bring some substantial influences. However, when aftershocks are smaller than a certain level, additional input energy is absorbed by damping and do not develop further damage. Although responses under JMA Kobe have more intensive variation than El Centro, the trend under both ground motions is similar.

Fig. 23 shows the value of μ at the end of each earthquake. By rough estimation, deformation is developed constantly after each aftershock. However, as shown in Fig. 24, $\Delta\mu/\mu_0$ (μ_0 is μ value after main shock) increases gradually after each aftershock. This is because the strength degradation is proceeded by aftershocks. As a result, the larger deformation is developed under the same amount of energy.

Fig. 22 The result of the aftershocks pattern (El Centro)

Fig. 23 The value of μ at the end of each earthquake (El Centro, $\theta=0.04$, 0.5×3)

Fig. 24 The increase rate of μ (El Centro, $\theta=0.04$, 0.5×3)

Fig. 23 shows the value of μ at the end of each earthquake. By rough estimation, deformation is developed constantly after each aftershock. However, as shown in Fig. 24, $\Delta\mu/\mu_0$ (μ_0 is μ value after main shock) increases gradually after each aftershock. This is because the strength degradation is proceeded by aftershocks. As a result, the larger deformation is developed under the same amount of energy.

6. CONCLUSIONS

In this paper, the effect of aftershocks on buildings is investigated with focusing on input energy. Findings are summarized as follows:

- 1) Through cumulative energy spectrum, the possibility of deformation increment and damage development is demonstrated. In addition, it is possible to obtain the maximum response value of the cumulative energy input of aftershocks in the elasto-plastic system from the cumulative energy spectrum of the elastic system.
- 2) Residual deformation of aftershocks is increased by the P-delta effect. Collapse and the development of damage to the building by aftershocks may occur. In Hokota, τ is large relative to η/μ , the responses tend to accumulate damage and small maximum deformation.
- 3) In this study, it is assumed that every aftershock develops maximum deformation and damage is concentrated on the first story of multi-story frame as the worst situation. With degradation of

strength on the first story, the accumulation of severe damage is possible. In a disaster report ⁸⁾, many cases of major damage in piloti-type buildings have been reported. Detailed reports about damage development by aftershocks are scarce. Therefore, though it is difficult to assertively estimate the result of analyses corresponding to actual damage, it can be considered that non-structural members may be damaged by main shock and deformation is developed by aftershocks.

- 4) The P-delta effect degrades strength of buildings, and biased deformation can be developed by small energy of aftershocks with the effect. However, for quite small aftershocks, most of the input energy is absorbed by damping. Depending on the relationship between the natural period of buildings and the characteristics of ground motion, cumulative deformation for (0.5×3) aftershocks pattern can be greater than that for (1.0×1) pattern. Thus, the effect of consecutive small aftershocks is sometimes significant.

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REFERENCES

- 1) Architectural Institute of Japan : Ultimate Strength and Deformation Capacity of Buildings in Seismic Design, Oct., 1990 (in Japanese)
- 2) Kato, B. and Akiyama, H. : ENERGY INPUT AND DAMAGES IN STRUCTURES SUBJECTED TO SEVERE EARTHQUAKES, Transactions of the Architectural Institute of Japan, Sep., 1975 (in Japanese)
- 3) Ministry of Land, Infrastructure, Transport and Tourism (Japan) : Notification No. 631, Jun., 2005
- 4) Akiyama, H. : P - δ effect in earthquake resistant design for share type multi-story frames, Journal of structural and construction engineering, Architectural Institute of Japan, No. 617, Jul., 2007, pp.87-94 (in Japanese)
- 5) Yamazaki, S. and Endo, K. : STABILITY RATIO AND DYNAMIC P - Δ EFFECTS IN INELASTIC EARTHQUAKE RESPONSE, Journal of structural and construction engineering, Architectural Institute of Japan, No.527, Jan., 2001, pp.71-78 (in Japanese)
- 6) Akiyama, H. : Earthquake-Resistant Design Method for Buildings Based on energy balance, Gihodoshuppan, Nov., 1999 (in Japanese)
- 7) Kato, B. and Akiyama, H. : DAMAGE DISTRIBUTION LAW OF SHEAR-TYPE MULTI-STORY STRUCTURES UNDER EARTHQUAKES, Journal of structural and construction engineering, Architectural Institute of Japan, No. 270, Aug., 1978, pp.61-68 (in Japanese)
- 8) Architectural Institute of Japan : Preliminary Reconnaissance Report of the 2011 Tohoku-Chiho Taiheiyo-Oki Earthquake, Jul. , 2011 (in Japanese)
- 9) Shimizu, K., Chiba, Y., and Teramoto, T. : Performance of Steel Structure Buildings on Hyogo-Ken-Nanbu Earthquake –Part3–, Summaries of Technical papers of annual meeting, Architectural Institute of Japan, Sep., 2000, pp.871-872 (in Japanese)

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