研究論文

30. Calculation of Resonance Integral in H₂O-Moderated U-Al Alloy Fuel Reactor

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An analytical calculation of the resonance escape probability of the lattice in which H₂O moderator and U-Al alloy fuel are alternately arranged is presented. An approximate formula suggested by G. M. Roe is used for $\psi(t, x)$. Two approximations, infinite mass approximation and scattering model, are used. A resonance integral for the core of swimming pool type reactor which has 20% enriched U-Al alloy fuels is numerically calculated. Finally, the comparison of numerical results by these approximations and homogeneous treatment is presented.

INTRODUCTION

In calculating the resonance escape probability, two approximations are used; infinite mass approximation and narrow resonance approximation. In a heterogeneous reactor, the narrow resonance approximation gives the same result as the scattering model suggested by Wigner⁽¹⁾, as Chernick has pointed out. Since the appropriate experimental information on resonance absorption integral for U-Al alloy fuel is not yet available, resonance escape probability of the lattice in which H₂O moderator and U-Al alloy fuel are alternately arranged must be estimated by the use of resonance parameters. This paper presents an analytical method for calculating the resonance escape probability of the aforesaid lattice by using both approximations.

The relation between resonance escape probability, p , and the resonance integral, I , is

$$-\ln p = I \quad (1)$$

The resonance integral consists of three parts: resolved, unresolved and $1/v$ contribution. Namely,

$$I = \sum_{i=1}^{27} I_i + \sum_{i=0} \gamma_i I_{uni} + I_{1/v} \quad (2)$$

The first term is derived from resonance parameters of 27 resonance levels in the resolved region and the third term represents the absorption which is inversely proportional to $\sqrt{E}$. In an infinitely dilute homogeneous mixture of ²³⁸U and moderator, $I_{uno} = 11.2$ barn, $I_{un1} = 2.8$ barn, $I_{un2} = 0.2$ barn and $I_{un3} = 0.05$ barn, according to Erginsoy⁽²⁾. Resonance integrals for $l > 3$ are neglected. γ_i in the second term represents the statistical factor, whose values for s -wave and p -wave neutron are 0.75 and 0.85 respectively, according to Chernick⁽³⁾.

If γ_2 and γ_3 are assumed to be unity, then

$$\sum_{i=0} \gamma_i I_{uni} = 11.03 = I_{uno} \quad (3)$$

The above approximation can also be derived from Tables 4 and 5 in Chernick's paper⁽⁴⁾

I. INFINITE MASS APPROXIMATION

An array of slabs, in which an Al-clad U-Al alloy fuel plate, thickness $a + 2t$, and H₂O moderator, thickness b , are alternately arranged, where t is clad thickness, is considered assuming the following conditions:

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- (1) The array consists of U-Al alloy fuel plates and homogeneous mixture of H₂O and Al which is the cladding material.
- (2) Absorption is neglected except for ²³⁸U.
- (3) Slowing down by other nuclei than hydrogen is neglected.

According to Chernick⁽⁵⁾, the resonance integral is given by:

$$I = \int_{E'}^{E_0} y_0 \frac{dE}{E} \quad (4)$$

$$y_0 = \frac{1}{P_H} \frac{1 - P_1}{1 + (1 - P_1) \left[\left(\frac{1}{P_H} - 1 \right) + \left(\frac{1}{P_a} - 1 \right) \frac{\Sigma_{tm} c}{\Sigma_{tj} a} \right]} \quad (5)$$

$$P_H = \frac{2bNH_2O\sigma_s H}{\Sigma_{tm} c}, \quad P_a = \frac{(1 - \omega) \gamma \Sigma_{av}}{\Sigma_{tj} a}$$

$$\Sigma_{tm} c = b \Sigma_{tH_2O} + 2t \Sigma_{sAl},$$

$$\Sigma_{tj} = (1 - \gamma) \Sigma_{sAl} + \gamma \Sigma_{tU}$$

$$1 - P_1 = \frac{1}{\Sigma_{tm} c} \int_0^1 \frac{[1 - \exp(-\frac{\Sigma_{tj} x}{\mu})] [1 - \exp(-\frac{\Sigma_{tm} c}{\mu})]}{1 - \exp[-\frac{\Sigma_{tj} a + \Sigma_{tm} c}{\mu}]} \mu d\mu \quad (6)$$

where Σ is the macroscopic cross section, σ the microscopic cross section and N the number of nuclei per unit volume. The suffix s refers to scattering, a absorption, t total, H₂O water, U U-Al₄, f U-Al alloy and H hydrogen. γ is the volume ratio of UAl₄ in U-Al alloy. Roe has pointed out that $(1 - P_1)$ can be approximated by the following equation⁽⁶⁾.

$$1 - P_1 = P_{ff}(1 - 2\Sigma_{tm} c P_{ff}) P_{fh} + 4P_{ff}^2 (\Sigma_{tm} c)^2 P_{fm} \quad (7)$$

$$P_{fm} = \frac{1}{\Sigma_{tm} c} \left[\frac{1}{2} - E_3(\Sigma_{tj} a) \right]$$

$$P_{ff} = \frac{1}{\Sigma_{tm} c} \left[\frac{1}{2} - E_3(\Sigma_{tm} c) \right]$$

$$P_{fh} = \frac{\Sigma_{tj} a}{\Sigma_{tj} a + \Sigma_{tm} c}$$

where

$$E_n(x) = \int_0^1 \mu^{n-2} e^{-x/\mu} d\mu.$$

The validity of this approximation can be understood by the following consideration:

When $\Sigma_{tj} a \ll \Sigma_{tm} c$, i. e., one fuel plate is immersed in moderator, $(1 - P_1)$ approaches

to P_{fm} in both Eqs. (6) and (7). When $\Sigma_{tj} a \gg \Sigma_{tm} c$, i. e., a fuel plate is very thick, $(1 - P_1)$ goes to P_{ff} , and when a and c are thin, i. e., when it is a homogeneous mixture, $(1 - P_1)$ is P_{fh} .

y_0 is maximum when $E = E_0$, i. e., neutron energy E , is equal to one of the resonance levels, E_0 , and decreases rapidly as E differs from E_0 . Since the potential scattering cross sections of U and Al are neglected in comparison with the resonance cross section of ²³⁸U at $E = E_0$, an approximate equation for y_0 is derived as follows:

$$y = P P_{vh} + Q P_{vm} \quad (8)$$

$$P = \frac{1 - 2\Sigma_{tm} c P_{ff} P_{ff}}{P_H} \left[1 - \frac{1 - P_H P_{ff}}{P_H} (1 - 2\Sigma_{tm} c P_{ff}) - 4 \frac{1 - P_H P_{ff}^2 \Sigma_{tm} c}{P_H} \right] \quad (9)$$

$$Q = 4 \frac{P_{ff}^2 (\Sigma_{tm} c)^2}{P_H} \left[1 - 2 \frac{1 - P_H P_{ff}^2 \Sigma_{tm} c}{P_H} \right]$$

The exact equation, y_0 , and the approximate equation, y , are numerically calculated for $a = 0.175$ cm, $b = 0.523$ cm, $t = 0.038$ cm,

$a = 0.175$ cm, $b = 0.523$ cm, $t = 0.038$ cm, $\gamma = 0.31$, $E_0 = 6.68$ eV

Fig. 1 Comparison of y with y_0

$\gamma = 0.31$ and $E_0 = 6.68$ eV, which is the first resonance level of ^{238}U , since I_1 resonance integral of the first resonance level is the predominant term in Eq. (2). As shown in Fig.1, the difference between y_0 and y is very small, especially in the range between $x=0$ and 20, where $x=2(E-E_0)/\Gamma$. The error in estimating the resonance integral is given by

$$\varepsilon = \frac{I_0 - I}{I_0} = \frac{\int_0^{260} y_0 dx - \int_0^{260} y dx}{\int_0^{260} y_0 dx} = 0.0506$$

Then it follows from Eq. (8) that

$$I_i = A_i f_H(t_i, \zeta_i) + B_i f_1(t_i, \eta_i) \quad (10)$$

$$A_i = \frac{\pi}{2} P \frac{\Gamma_i}{E_{0i}}, \quad B_i = \frac{\pi}{2} \frac{Q}{\sum_{imc}} \frac{\Gamma_i}{E_{0i}} \eta_i$$

$$\zeta_i = 4\pi \lambda_i^2 \frac{\Gamma_{ni}}{\Gamma_i}$$

$$\frac{(1-\omega)\gamma N_U a}{\{(1-\gamma)\sum_{iAl} + \gamma N_U(\sigma_p + 4\sigma_{sAl})\}a + \sum_{imc}}$$

$$\eta_i = 4\pi \lambda_i^2 \frac{\Gamma_{ni}}{\Gamma_i} (1-\omega)\gamma N_U a$$

$$t_i = \frac{4\mu^* k T E_{0i}}{M \Gamma_i^2}, \quad \mu^* = \frac{M}{1+M}$$

where λ_i is the neutron wave length, σ_p the potential scattering cross section of U, ω the enrichment of ^{235}U , and M mass of ^{238}U .

Two terms in Eq. (10), $f_H(t_i, \zeta_i)$ and $f_1(t_i, \eta_i)$, are functions discussed in Roe's paper⁽⁶⁾. Both functions are analytically calculated from t_i , ζ_i and η_i . The unresolved resonance integral, I_{uno} , is given as follows:

$$I_{uno} = \int_{E_{27}}^{\infty} \frac{dE}{E} \times \left(\frac{E}{D}\right) \times I_i \quad (11)$$

where D is an average level space. In the unresolved region, the values of ζ and η decrease because the neutron energy becomes higher. Therefore f_H and f_1 can be approximated to be unity.

Then,

$$I_{uno} = \frac{\pi}{2D} \left[P \Gamma_{27} \zeta_{27} + \frac{Q}{\sum_{imc}} \eta_{27} \frac{\Gamma_{27}^2}{\Gamma_{27} \Gamma_{27}} \right] \quad (12)$$

In the unresolved region, according to Dresner⁽⁷⁾.

$$t = \frac{4\mu^* k T}{M} \frac{1}{(\Gamma_{n0})^2} \quad (13)$$

Comparing Eq. (12) with the exact equation, the error for the first term of I_{uno} is found to be 2.2%.

I. SCATTERING MODEL METHOD

According to the scattering model method, which is familiar in nuclear calculations, the effective resonance integral is given by

$$\sigma_a^{eff} = A + BC \delta \frac{S}{M} \quad (14)$$

A is the mass absorption term and B is the surface absorption term. As the appropriate experimental values are not yet available for U-Al alloy fuel, A and B must be calculated. Then,

$$A = \frac{\sum_{io}}{(1-\omega)\gamma N_U} \int \frac{\sum_a dE}{\sum_i E}$$

$$B = \frac{\rho}{4(1-\omega)\gamma N_U} \int \frac{\sum_a (\sum_a + \sum_{sr}) dE}{\sum_i^2 E}$$

$$\sum_{io} = (1-\gamma)\sum_{iAl} + \gamma \sum_{iAl} A_{iAl}$$

where $\sum_{sr}$ is the resonance scattering cross section of ^{238}U . A can analytically calculated by Roe's method, and the final result is

$$A = \sum_{i=1}^{27} A_i f_H(t_i, \zeta_i) + A_{un} + A_{1/0} \quad (15)$$

$$A_i = 2\pi \lambda_i^2 \frac{\Gamma_{ni}}{\Gamma_i} \frac{\Gamma_{yi}}{E_{0i}}$$

$$A_{un} = \frac{\pi \Gamma_{27} \sum_{io}}{(1-\omega)\gamma N_U D}$$

$$\left[\sqrt{\zeta_{27} + (1+\beta)^2 + 2\frac{\beta}{\alpha}(\sqrt{1+\alpha\zeta_{27}}-1)} - (1+\beta) \right]$$

where α and β are the ones discussed in Roe's paper and are functions of t , which is defined by Eq. (13) for the unresolved region.

Therefore, A is similar to the first term of Eq. (10) except for ζ_i , which is in this case

$$\zeta_i = 4\pi \lambda_i^2 \frac{\Gamma_{ni}}{\Gamma_i} \frac{(1-\omega)\gamma N_U}{\sum_{io}} \quad (16)$$

Integral of B is represented as follows:

$$K_i = \frac{1}{\pi} \int_{-\infty}^{\infty} \left(\frac{\psi_i}{1+\zeta_i \psi_i} \right)^2 dx \quad (17)$$

where $\psi_i (\equiv \psi(t_i, x))$ is the Breit-Wigner single level resonance formula with Maxwellian distribution of scatterer velocities. Roe suggested the following approximate formula

$$K_i = \frac{2}{[(1+\beta_i)^2 + \zeta_i + 2\frac{\beta_i}{\alpha_i}(\sqrt{1+\alpha_i\zeta_i}-1)]^{\frac{1}{2}} [(1+\beta_i)^2 + \zeta_i - 2\frac{\beta_i}{\alpha_i}(\sqrt{1+\alpha_i\zeta_i}+1)]} \left\{ \frac{1}{6} - \frac{\beta_i}{3} \frac{1}{\sqrt{1+\alpha_i\zeta_i}} \right. \\ \left. + \frac{\alpha_i\beta_i}{6} \frac{\zeta_i + (1+\beta_i)^2 - \frac{\beta_i}{\alpha_i}(\sqrt{1+\alpha_i\zeta_i}+2)}{(1+\alpha_i\zeta_i)^{3/2}} - \frac{\beta_i}{\alpha_i} \frac{(1-\alpha_i)(1+\beta_i)^2}{\sqrt{1+\alpha_i\zeta_i}[(1+\beta_i)^2 + \zeta_i + 2\frac{\beta_i}{\alpha_i}(\sqrt{1+\alpha_i\zeta_i}-1)]} \right\}, \quad (18)$$

The integral for the undersolved contribution is

$$K_{un} = \int_0^{\zeta_{27}} \zeta_i K_i d\zeta_i \\ = \frac{1}{\pi} \int_{-\infty}^{\infty} \psi^2 dx \int_0^{\zeta_{27}} \frac{\zeta_i d\zeta_i}{(1+\zeta_i\psi)^2} \\ = \frac{2}{\pi} \int_0^{\infty} \log(1+\zeta_{27}\psi) dx - \zeta_{27} f_H(t, \zeta_{27}) \quad (19)$$

The first term in Eq. (19) is divided into two parts, so that

$$\int_0^{\infty} \log(1+\zeta_{27}\psi) dx = M + \int_{x_1}^{\infty} (1+\zeta_{27}\psi) dx$$

where

$$M \equiv \int_0^{x_1} \log(1+\zeta_{27}\psi) dx. \quad (20)$$

In the unresolved region, ζ_{27} is smaller and $\psi(t_i, x_1)$ is also extremely smaller than unity because of large x_1 . Therefore,

$$\int_{x_1}^{\infty} \log(1+\zeta_{27}\psi) dx \doteq \zeta_{27} \int_{x_1}^{\infty} \psi dx \\ \doteq \zeta_{27} \left[\frac{\pi}{2} - \tan^{-1} x_1 \right]$$

with $\psi \doteq \frac{1}{1+x^2}$ as $t \ll 1+x^2$.

If $x_1=300$, then

$$K_{un} = \frac{2}{\pi} M + 0.002568 \zeta_{27} - \zeta_{27} f_H(t, \zeta_{27}) \quad (21)$$

The value of M is derived from the numerical integration such as Simpson's formula. Consequently, the surface absorption integral is

$$B = \sum_{i=1}^{27} B_i K_i(t_i, \zeta_i) + B_{un} + B_{lv} \quad (22)$$

where

for $\psi(t_i, x)^*$:

$$\psi(t_i, x) = \frac{\alpha_i^2 x^2 + \alpha_i \beta_i^2}{\alpha_i^2 x^4 + \{\alpha_i^2 (1+\beta_i)^2 - 2\alpha_i \beta_i\} x^2 + \beta_i^2}$$

Using the above equation, an analytical expression for K_i can be derived;

$$B_i = \frac{\pi \rho \zeta_i^2}{4(1-\omega) \gamma N_v} \frac{\Gamma \gamma_i}{E_{0i}}$$

$$B_{un} = \frac{\pi \rho \bar{\Gamma} \gamma}{4(1-\omega) \gamma N_v D} K_{un}$$

Rearranging Eq. (14),

$$\sigma_a^{eff} = A \left(1 + \mu \frac{S}{M} \right), \quad \mu = \frac{B}{A}.$$

Comparison of calculated results of A and μ with experimental values is given in Table 1.

Table 1 Comparison of calculation and experiment (Cd cutoff=0.4 eV)

σ_a per U atom barns	A barns		μ	
	Calculation	Experiment	Calculation	Experiment†
38.5	16,471	16.0	0.9356	1.09
45.6	17,842	17.2	0.8577	0.85
49.0	18,601	18.0	0.7788	0.75

† From Fig. 1.5.20 in Handbook "Physics" (8)

It is understood from Table 1 that the calculated results are in good agreement with experimental values.

The correction factor $C^{(9)}$, and the Dancoff correction⁽¹⁰⁾, δ , in Eq. (14) are

$$C = \frac{2}{1 + \{1 + (2a \sum_{ob}^{-1})^{1/2}\}^{1/2}} \quad (23) \\ \delta = 1 - 2E_3(\sum_{ob} H_2 Ob)$$

The resonance integral is derived from scattering model;

* Ref. (13) shows that Roe's approximation for ψ is satisfactory.

$$I_s = \frac{f_r}{1 - f_r} \quad (24)$$

$$\frac{1}{f_r} = \frac{\sum a_1 b}{\sum a_0 a} \left(\frac{\kappa_0 a}{2} \coth \frac{\kappa_0 a}{2} \right) + \frac{\kappa_1 b}{2} \coth \frac{\kappa_1 b}{2}$$

$$\Sigma_{a_0} = \frac{x}{5.6} (1 - \omega) \gamma N_U \sigma_e^{eff}$$

$$\Sigma_{a1} = x \Sigma_1^*, \quad \kappa_1 = \sqrt{x} \kappa_1^*$$

where the asterisk refers to U-metal. And

$$x = \frac{(\ln E_0/E)_v}{(\ln E_0/E)_{UAI}} = \frac{\int_v a(E) \frac{dE}{E}}{\int_{UAI} a(E) \frac{dE}{E}} = \frac{9.25}{A}$$

where $(\ln E_0/E)_v$ is taken to be 5.6.

II. NUMERICAL CALCULATIONS

In carrying out the numerical calculations, the thickness of U-Al alloy fuel plate ranging from 0.09 cm to 0.41 cm is taken as the variable. The parameters are the content of U in U-Al alloy; *i.e.* 35, 4, 30.4 and 28.5 wt. % of U. Here the densities of UAl₄ and Al are assumed to be 5.7 and 2.69 g/cc respectively. The resonance parameters for 27 resonance levels are taken from *BNL-325*⁽¹¹⁾ and the best values suggested by Chernick are used as parameters in the unresolved region⁽¹²⁾; *i.e.* $D = 18.7$ eV, $\bar{\Gamma}_Y = 24$ meV and $\bar{\Gamma}_n^0 = 1.63$ m(eV)^{1/2}.

In Fig. 2, the comparisons between the values by infinite mass approximation, I_{∞} , scattering model method, I_s , and homogeneous approximation, I_h , are shown. The resonance integral by homogeneous approximation is

$$I_h = \frac{(1 - \omega) N_{\sigma}}{\xi \sum_i} \sigma_a^{eff} \quad (25)$$

where σ_a^{eff} is the function of scattering cross section per ^{238}U atom; *i.e.* $\bar{\Sigma}_s/(1-\omega)N_U$.

The numerical values of σ_a^{eff} are derived from the results which the author has previously calculated⁽¹³⁾.

It is easy to understand that I_h has the

Fig. 2 Comparison of calculations by various models

maximum value and approaches to the value of I_M as the thickness of fuel plate become thinner. That I_M is larger than I_n , agrees with Chernick's result⁽³⁾⁽¹²⁾. It is also found that I_n is approximately equal to I_M , if C and δ are equal to unity.

IV. CONCLUSION

The analytical formulas presented in this paper for the resonance integral of the lattice in which H₂O moderator and U-Al alloy fuel are alternately arranged are not so complicated, though they are approximations.

In the nuclear calculation of swimming pool type reactor the homogeneous approximation is generally used. It is shown in this paper that the homogeneous approximation causes underestimation of resonance escape probability (especially in the case of thick fuel plate), and the infinite mass approxi-

mation gives better result.

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水減速—U-Al 合金系の共鳴積分の計算

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一般に U-Al 合金の板状燃料をもつ原子炉での共鳴を逃れる確率は均質化の取扱いをする。これは ρ に対して過小評価をしていることになっている。このような取扱いをする理由の 1 つには、U-Al 合金燃料系の実効共鳴積分の実験式がないということがある。ここでは U-Al 合金の板状燃料と減速材との無限並列系での ρ を Infinite Mass 近似と Scattering Model (これは Narrow Resonance 近似である) とを用いて計算し比較した。従来の発表論文ではほとんどが数値解法をしているので、ここでは計算にあたって Roe の近似式等を用い、できる限り解析的に求めた。すなわちこの論文の目的は、

- (1) U-Al 合金系の ρ の計算が解析的に可能であること
- (2) その数値結果から Infinite Mass 近似と Scattering Model との比較を示したものである。

実際の数値計算は東芝教育訓練用原子炉の炉心構造の場合について行なわれた。