研究論文

246. Estimation of the Buckling of ^{239}Pu Fueled System from the Data of Composite ^{239}Pu - ^{235}U Fueled Fast Critical Assembly

^{239}Pu - ^{235}U 燃料高速臨界集合体のデータから

^{239}Pu 燃料系のバックリングを評価する方法

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^{239}Pu - ^{235}U fueled fast critical assemblies are of interest to estimate properties of ^{239}Pu fueled assemblies because of safety considerations and/or limited plutonium availability. Numerical analysis for bare and reflected homogeneous cores and reflected Two-region cores are previously reported. It is concluded that the errors in the calculated critical size for a 5.0 v/o ^{239}Pu fueled system reduce more than a factor of about 10 to 20, if an accurate experimental value of critical size of a critical assembly fueled by 0.5 v/o ^{239}Pu and 4.5 v/o ^{235}U is known.

In this report, a general mathematical formula, which is useful to explain the above conclusions more clearly, is obtained for bare homogeneous mixtures using multigroup diffusion approximation.

In order to predict the critical radius for the bare homogeneous ^{239}Pu system in an accuracy, 1 %, the errors for the experimental value of critical radius must be less than 0.1 % for $V^{\text{Pu}}/V^{\text{Fuel}}=0.05$ and 0.2 % for $V^{\text{Pu}}/V^{\text{Fuel}}=0.1$, where V^{Pu} and V^{Fuel} are the volume fractions of ^{239}Pu and fuel ($^{239}\text{Pu}+^{235}\text{U}$) respectively. For the reflected Two-region cores, the experimental errors required for the above accuracy must be less than 0.5 % for the ratio of central ^{239}Pu core region volume to total core volume of 0.1.

I. INTRODUCTION

In order to design a large fast breeder reactor, fast critical assemblies requiring large amount of fuel will be necessary, since the fast reactor with large power output contains much fertile, coolant and structural materials, and consequently the dilute fuel core have a fairly large volume. It is not an easy problem to get the procurement of large amount of fuel, ^{235}U or ^{239}Pu , considering the inventory cost and safety problem.

Due to the uncertainties of cross sections, especially of ^{239}Pu , it may be difficult to predict reliably the nuclear characteristics of a plutonium-fueled fast critical system from critical experiments fueled partially by ^{239}Pu . For instance, the deviation of critical mass

for a representative reflected full ^{239}Pu system is about 7 % due to the difference of capture cross section for ^{239}Pu corresponding to standard $\alpha = \sigma_c/\sigma_f$ and low α .

The numerical analysis for the problem is previously reported by the authors⁽¹⁾. In all fast systems studied, the volume fraction of fuel, fertile, structural and coolant materials are kept constant, and the fuel is a mixture of ^{239}Pu and ^{235}U corresponding to various volume ratios, but the total volume fraction of fuel is retained at a constant value. For example, if critical experiments on a system having a homogeneous core fueled by 0.5 v/o ^{239}Pu and 4.5 v/o ^{235}U give an accurate experimental value for material buckling, and the

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cross sections of ^{235}U , ^{238}U , Fe and Na allow accurate calculation of the material buckling for a corresponding homogeneous core fueled by $5\text{ v/o}^{235}\text{U}$, one might then extrapolate the curve for material buckling as a function of ^{239}Pu concentration to the critical system fueled by $5\text{ v/o}^{239}\text{Pu}$. The extrapolation of the two experimental values of bucklings for the ^{235}U system and the ^{239}Pu - ^{235}U system, depends upon knowledge of the ^{239}Pu cross sections, and methods of calculations.

Analysis of fast reactor systems depends on solutions of the Boltzmann equation. A solution based on a transport approximation, the discrete Sn code, *i. e.*, DSN, IBM-704 code is used, for spherical geometry⁽²⁾⁽³⁾. A second method based on diffusion theory is adequate to describe the nuclear characteristics of large dilute fast breeders. Multigroup diffusion theory is fairly accurate in predicting both flux distribution and critical mass for the larger assemblies of primary design interest.

In this report, using multigroup diffusion approximation, a general mathematical formulation is obtained, which reveals the effect of multigroup constants on bucklings of bare homogeneous systems. It is shown that the bucklings are linear functions of ^{239}Pu volume fraction neglecting the second order corrections, and the deviations due to the independent variations of each group constant for ^{239}Pu , are also approximately linear functions of ^{239}Pu volume fraction, except for the case of the variations of transport cross sections for ^{239}Pu . Numerical calculations for bare and reflected homogeneous cores and reflected Two-region cores are reported⁽⁴⁾. The effects of uncertainties in ^{239}Pu group constants are shown to agree with the prediction of general formulation for bare homogeneous cores.

II. THEORY

In this chapter, the fast critical system is considered as a bare homogeneous mixture of fuel (^{239}Pu and ^{235}U), fertile (^{238}U), structural (stainless steel), and coolant (sodium) materials. The fuel is a mixture of ^{239}Pu and ^{235}U with various volume ratios, but the total volume fraction of the fuel is kept constant. As the composition of the fast critical sys-

tem is changed, the group constants are assumed to vary linearly with the volume fraction of each composite material. It is also assumed that the sets of multigroup cross sections for ^{235}U , ^{238}U , Fe and Al (Na is replaced by Al, because sodium is approximately equivalent to aluminum at 0.58 density) are accurate and that the main uncertainty in multigroup constants comes from that of ^{239}Pu . In order to estimate the effects of ^{239}Pu multigroup constants on the critical mass, the buckling of bare homogeneous mixtures fueled by ^{239}Pu and ^{235}U are calculated for several values of ^{239}Pu volume fraction or concentration. As far as the relative variations of bucklings are concerned, the simple fundamental mode calculations give sufficient information. The multigroup diffusion theory can be used for this analysis, because it can afford to predict the criticality of large dilute fast critical systems with relatively small errors.

The multigroup treatment in the diffusion approximation can be expressed in the form of a set of equations,

$$(D_j \nabla^2 - \Sigma_{aj}) \phi_j + \sum_{k \neq j} (\Sigma_{ln, k \rightarrow j} + \Sigma_{er, k \rightarrow j}) \phi_k + \chi_j \sum_{k=1}^N (\nu \Sigma_{fj})_k \phi_k = 0, \quad (1)$$

where $j=1, 2, \dots, N$ labels the energy group,

$$\Sigma_{aj} = \Sigma_{fj} + \Sigma_{cj} + \Sigma_{ln, j} + \Sigma_{er, j},$$

total removal cross section
from group j ,

$$\Sigma_{ln, j} = \sum_{k \neq j} \Sigma_{ln, j \rightarrow k},$$

inelastic removal cross section
from group j ,

$$\Sigma_{er, j} = \sum_{k \neq j} \Sigma_{er, j \rightarrow k},$$

elastic removal cross section
for group j ,

$$\chi_j = \text{fraction of fission neutrons born into group } j,$$

$$\Sigma_{fj} = \text{fission cross section for group } j,$$

$$\Sigma_{cj} = \text{capture cross section for group } j,$$

and

$$D_j = \text{diffusion coefficient for group } j, \text{ related to } \Sigma_{tr, j} \text{ by some prescription; e.g.}$$

$$D_j = 1/3 \Sigma_{tr, j}.$$

The group macroscopic cross sections Σ_{fj} , Σ_{cj} , $\Sigma_{ln, j \rightarrow k}$, and $\Sigma_{er, j \rightarrow k}$ can be obtained from the corresponding microscopic group cross sections for individual materials, *e.g.* for reaction q in material z ,

$$\Sigma_{qj} = \sum_i N^i \sigma^i_{qj},$$

where

$$N^i = \text{atoms/cm}^3 \text{ of material } i.$$

The group constants can also be written as

$$\Sigma_{qj} = \sum_i V^i \Sigma^i_{qj},$$

where

$$V^i = \text{volume fraction of material } i.$$

The adjoint of the set of diffusion equations (1) is

$$(D_j \nabla^2 - \Sigma_{aj}) \phi_j^* + \sum_{k>j} (\Sigma_{ln, j+k} + \Sigma_{er, j+k}) \phi_k^* + (\nu \Sigma_f)_j \sum_{k=1}^N \chi_k \phi_k^* = 0. \quad (2)$$

As the volume fraction of ^{239}Pu is changed, the group macroscopic cross section for reaction q , Σ_{qj} , is expressed as a linear function of ^{239}Pu volume fraction, V^{Pu} ,

$$\Sigma_{qj} = \sum_i V^i \Sigma^i_{qj} = \Sigma_{qj,0} + (\Sigma^{\text{Pu}}_{qj} - \Sigma^{\text{U}}_{qj}) V^{\text{Pu}},$$

where $\Sigma_{qj,0}$ is the macroscopic cross section of the ^{235}U fueled fast critical system. If the group constants for ^{239}Pu have some uncertainties, $\delta \Sigma^{\text{Pu}}_{qj}$,

$$\Sigma_{qj} + \delta \Sigma_{qj} = \Sigma_{qj,0} + (\Sigma^{\text{Pu}}_{qj} + \delta \Sigma^{\text{Pu}}_{qj} - \Sigma^{\text{U}}_{qj}) V^{\text{Pu}} = \Sigma_{qj} + \delta \Sigma^{\text{Pu}}_{qj} V^{\text{Pu}}.$$

The solutions of Eqs. (1) and (2) for a bare homogeneous system, can be written in the form,

$$\phi_j(x) = \phi_j F(x), \quad \phi_j^*(x) = \phi_j^* F(x),$$

where $F(x)$ is a function of space coordinates, x , expressing the fundamental mode of flux distribution, and ϕ_j is a constant corresponding to the j -th component of neutron spectrum in the fast system, and ϕ_j^* is also a constant.

The multigroup diffusion equations (1) and its adjoint (2) are combined into a matrix form

$$A \phi = B^2 D \phi, \quad \phi = \begin{pmatrix} \phi^1 \\ \phi^2 \\ \vdots \\ \phi_N \end{pmatrix} \quad (1a)$$

$$A^* \phi^* = B^2 D \phi^*, \quad \phi^* = \begin{pmatrix} \phi^*_1 \\ \phi^*_2 \\ \vdots \\ \phi^*_N \end{pmatrix} \quad (2a)$$

where A is a matrix with its elements a_{jk} :

$$a_{jj} = -\Sigma_{aj} + \chi_j (\nu \Sigma_f)_j$$

$$a_{jk} = \begin{cases} \Sigma_{ln, k+j} + \Sigma_{er, k+j} + \chi_j (\nu \Sigma_f)_k & k < j \\ \chi_j (\nu \Sigma_f)_k & k > j, \end{cases}$$

A^* is the adjoint of A , with its elements a^*_{jk} :

$$a^*_{jk} = a_{kj},$$

D is a diagonal matrix, having diagonal elements $D_1, D_2, \dots, D_N$, and B^2 is an eigenvalue corresponding to the buckling of the system. The elements of matrices, a_{jk} and a^*_{jk} , are linear in V^{Pu} , ^{239}Pu volume fraction, and the reciprocal of each diagonal element in D is also linear in V^{Pu} ,

$$a_{jk} = a_{jk,0} + a_{jk,1} \lambda, \quad \lambda = V^{\text{Pu}},$$

$$D_j = \frac{1}{d_{j,0} + d_{j,1} \lambda},$$

where

$$d_{j,0} = 3 \Sigma_{tr, j,0}$$

$$d_{j,1} = 3 (\Sigma^{\text{Pu}}_{tr, j} - \Sigma^{\text{U}}_{tr, j}),$$

so that A and A^* are given by

$$A = A_0 + A_1 \lambda,$$

and

$$A^* = A^*_0 + A^*_1 \lambda,$$

where

$$A_0 = (a_{jk,0}),$$

$$a_{jj,0} = -\Sigma_{aj,0} + \chi_j (\nu \Sigma_f)_{j,0},$$

$$a_{jk,0} = \begin{cases} \Sigma_{ln, k+j,0} + \Sigma_{er, k+j,0} + \chi_j (\nu \Sigma_f)_{k,0}, & k < j, \\ \chi_j (\nu \Sigma_f)_{k,0} & k > j, \end{cases}$$

$$A_1 = (a_{jk,1}),$$

$$a_{jj,1} = -(\Sigma^{\text{Pu}}_{aj} - \Sigma^{\text{U}}_{aj})$$

$$+ \chi_j \{ (\nu \Sigma_f)_j^{\text{Pu}} - (\nu \Sigma_f)_j^{\text{U}} \},$$

$$a_{jk,1} = \begin{cases} (\Sigma^{\text{Pu}}_{ln, k+j} - \Sigma^{\text{U}}_{ln, k+j}) + (\Sigma^{\text{Pu}}_{er, k+j} - \Sigma^{\text{U}}_{er, k+j}) - \Sigma^{\text{U}}_{er, k+j} + \chi_j \{ (\nu \Sigma_f)_k^{\text{Pu}} - (\nu \Sigma_f)_k^{\text{U}} \}, & k < j, \\ -(\nu \Sigma_f)_k^{\text{U}}, & k < j, \\ \chi_j \{ (\nu \Sigma_f)_k^{\text{Pu}} - (\nu \Sigma_f)_k^{\text{U}} \}, & k > j. \end{cases}$$

Making A'_1 and D' stand for the matrix A_1 and D , respectively, which contain some uncertainties, $\delta \Sigma^{\text{Pu}}_{qj}$, in the group constants for ^{239}Pu ,

$$A'_1 = (a'_{jk,1}),$$

$$a'_{jj,1} = -(\Sigma^{\text{Pu}}_{aj} + \delta \Sigma^{\text{Pu}}_{aj} - \Sigma^{\text{U}}_{aj})$$

$$+ \chi_j \{ (\nu \Sigma_f)_j^{\text{Pu}} + \delta (\nu \Sigma_f)_j^{\text{Pu}} - (\nu \Sigma_f)_j^{\text{U}} \},$$

$$a'_{jk,1} = \begin{cases} (\Sigma^{\text{Pu}}_{ln, k+j} + \delta \Sigma^{\text{Pu}}_{ln, k+j} - \Sigma^{\text{U}}_{ln, k+j}) + (\Sigma^{\text{Pu}}_{er, k+j} + \delta \Sigma^{\text{Pu}}_{er, k+j} - \Sigma^{\text{U}}_{er, k+j}) - \Sigma^{\text{U}}_{er, k+j} + \chi_j \{ (\nu \Sigma_f)_k^{\text{Pu}} + \delta (\nu \Sigma_f)_k^{\text{Pu}} - (\nu \Sigma_f)_k^{\text{U}} \}, & \text{for } k < j, \\ \chi_j \{ (\nu \Sigma_f)_k^{\text{Pu}} + \delta (\nu \Sigma_f)_k^{\text{Pu}} - (\nu \Sigma_f)_k^{\text{U}} \}, & \text{for } k > j, \end{cases}$$

and D'_j , the elements of diagonal matrix D'

are given by

$$D'_{j,1} = \frac{1}{d_{j,0} + d'_{j,1}\lambda},$$

$$d'_{j,1} = 3(\Sigma_{tr,j}^{Pu} + \delta \Sigma_{tr,j}^{Pu} - \Sigma_{tr,j}^{238U}).$$

Differentiating equation (1) or (1a) with respect to λ , we obtain

$$\frac{\partial A}{\partial \lambda} \phi + A \frac{\partial \phi}{\partial \lambda} = \frac{\partial (B^2)}{\partial \lambda} D \phi + B^2 \frac{\partial D}{\partial \lambda} \phi + B^2 D \frac{\partial \phi}{\partial \lambda}.$$

Multiplication of the above equation with ϕ^+ gives

$$\phi^+ \frac{\partial A}{\partial \lambda} \phi + \phi^+ A \frac{\partial \phi}{\partial \lambda} = \frac{\partial (B^2)}{\partial \lambda} \phi^+ D \phi + B^2 \phi^+ \frac{\partial D}{\partial \lambda} \phi + B^2 \phi^+ D \frac{\partial \phi}{\partial \lambda}.$$

Using the relation:

$$\phi^+ A \frac{\partial \phi}{\partial \lambda} = B^2 \phi^+ D \frac{\partial \phi}{\partial \lambda},$$

we have

$$\phi^+ \frac{\partial A}{\partial \lambda} \phi = \frac{\partial (B^2)}{\partial \lambda} \phi^+ D \phi + B^2 \phi^+ \frac{\partial D}{\partial \lambda} \phi,$$

or

$$\phi^+ A_1 \phi = \frac{\partial (B^2)}{\partial \lambda} \phi^+ D \phi + B^2 \phi^+ \frac{\partial D}{\partial \lambda} \phi. \quad (3)$$

Therefore,

$$\frac{\partial (B^2)}{\partial \lambda} = \frac{\phi^+ A_1 \phi}{\phi^+ D \phi} - B^2 \frac{\phi^+ \frac{\partial D}{\partial \lambda} \phi}{\phi^+ D \phi}. \quad (4)$$

The result is exact in the multigroup diffusion approach.

Expanding B^2 , ϕ and ϕ^+ into power series in λ ; $0 \leq \lambda \leq V^{\text{Fuel}} \ll 1$ ($V^{\text{Pu}} + V^{238\text{U}} = V^{\text{Fuel}}$),

$$B^2 = B_0^2 + B_1^2 \lambda + B_2^2 \lambda^2 + \dots$$

$$\phi = \phi_0 + \phi_1 \lambda + \phi_2 \lambda^2 + \dots$$

$$\phi^+ = \phi_0^+ + \phi_1^+ \lambda + \phi_2^+ \lambda^2 + \dots,$$

and substituting these series into Eq. (3), we obtain

$$B_1^2 = \frac{\phi_0^+ A_1 \phi_0 - B_0^2 \phi_0^+ \left(\frac{\partial D}{\partial \lambda} \right)_0 \phi_0}{\phi_0^+ D_0 \phi_0} \equiv \frac{\phi_0^+ A_1 \phi_0}{\phi_0^+ D_0 \phi_0} \quad (5)$$

$$B_2^2 = \frac{1}{2 \phi_0^+ D_0 \phi_0} \left[\phi_1^+ A_1 \phi_0 + \phi_0^+ A_1 \phi_1 - B_0^2 \left\{ \phi_0^+ \left(\frac{\partial D}{\partial \lambda} \right)_0 \phi_1 + \phi_1^+ \left(\frac{\partial D}{\partial \lambda} \right)_0 \phi_0 + \phi_0^+ \left(\frac{\partial^2 D}{\partial \lambda^2} \right)_0 \phi_0 \right\} - B_1^2 \left\{ \phi_0^+ D_0 \phi_1 + \phi_1^+ D_0 \phi_0 + 2 \phi_0^+ \left(\frac{\partial D}{\partial \lambda} \right)_0 \phi_0 \right\} \right]$$

$$\equiv \frac{\phi_1^+ A_1 \phi_0 + \phi_0^+ A_1 \phi_1}{2 \phi_0^+ D_0 \phi_0} - \frac{B_0^2 \phi_0^+ \left(\frac{\partial D}{\partial \lambda} \right)_0 \phi_0}{\phi_0^+ D_0 \phi_0}, \quad (6)$$

where D and D_0 are diagonal matrices with

elements, D_j , and $D_{j,0}$, respectively

$$D_j = \frac{1}{d_{j,0} + d_{j,1}\lambda}$$

$$D_{j,0} = \frac{1}{d_{j,0}}$$

and

$$\left(\frac{\partial D}{\partial \lambda} \right)_0 = \left(-\frac{d_{j,1}}{d_{j,0}^2} \right) = \left\{ -\frac{(\Sigma_{tr,j}^{Pu} - \Sigma_{tr,j}^{238U})}{3 \Sigma_{tr,j,0}^2} \right\}$$

$$\left(\frac{\partial^2 D}{\partial \lambda^2} \right)_0 = \left(\frac{2 d_{j,1}^2}{d_{j,0}^3} \right) = \left\{ \frac{2 (\Sigma_{tr,j}^{Pu} - \Sigma_{tr,j}^{238U})^2}{3 \Sigma_{tr,j,0}^3} \right\},$$

A numerical calculation in 4-group shows that B^2 is a linear function of $\lambda = V^{\text{Pu}}$, ^{239}Pu volume fraction (Fig. 1).

Fig. 1 Buckling, B^2 , as a function of ^{239}Pu volume fraction for homogeneous bare fast critical systems

This means that the first order expansion in λ gives a fairly good approximation, e.g.

$$B^2 \approx B_0^2 + B_1^2 \lambda.$$

If there are any uncertainties in ^{239}Pu cross section sets, the variation of buckling, $\delta(B^2) = B^2 - B_{std}^2$, where B_{std}^2 is the buckling given by the standard cross section set for ^{239}Pu , is given by

$$\delta(B^2) = B^2 - B_{std}^2 = (B_1^2 - B_{1,std}^2) \lambda + (B_2^2 - B_{2,std}^2) \lambda^2 + \dots = \delta(B_1^2) \lambda + \delta(B_2^2) \lambda^2 + \dots$$

$$= \frac{\phi_0^* \delta A_1 \phi_0 - B_0^2 \phi_0^* \delta \left(\frac{\partial D}{\partial \lambda} \right)_0 \phi_0}{\phi_0^* D_0 \phi_0 + \delta(B_0^2) \lambda^2, \dots,} \lambda \quad (7)$$

where

$$\begin{aligned} \delta A_1 &= (\delta a_{jk,1}) = A'_1 - A_1 \\ \delta a_{jj,1} &= -\delta \Sigma_{0j}^{Pu} + \chi_j \delta (\nu \Sigma_j)^{Pu} \\ \delta a_{jk,1} &= \begin{cases} \delta \Sigma_{0n,k \rightarrow j}^{Pu} + \delta \Sigma_{0r,k \rightarrow j}^{Pu} + \chi_j \delta (\nu \Sigma_j)^{Pu} & k < j \\ \chi_j \delta (\nu \Sigma_j)^{Pu}, & k > j \end{cases} \end{aligned}$$

and

$$\begin{aligned} \delta \left(\frac{\partial D}{\partial \lambda} \right)_0 &= \left(\frac{\partial D'}{\partial \lambda} \right)_0 - \left(\frac{\partial D}{\partial \lambda} \right)_0 \\ &= \left(-\frac{d'_{j,1}}{d_{j,0}^2} \right) - \left(-\frac{d_{j,1}}{d_{j,0}^2} \right) \\ &= \left(-\frac{\delta \Sigma_{tr,j}^{Pu}}{3 \Sigma_{tr,j,0}^2} \right). \end{aligned}$$

It is shown that the variations of buckling, $\delta(B^2)$, due to the uncertainties of ^{239}Pu cross sections, Σ_{0j}^{Pu} , $\Sigma_{tr,j}^{Pu}$, $\Sigma_{0n,k \rightarrow j}^{Pu}$, $\Sigma_{0r,k \rightarrow j}^{Pu}$, and ν_j^{Pu} ,

Fig. 2 Effects of variations of multigroup constants for ^{239}Pu on $\delta(B^2)$ as functions of ^{239}Pu volume fraction

are proportional to $\lambda = V^{Pu}$ neglecting the λ^2 term. The effect due to the uncertainties of all group constants for ^{239}Pu , is the superposition of the effects due to independent variations of individual group constants for ^{239}Pu . The variations of buckling due to each group constant for ^{239}Pu , are given, to the first order, by

$$\begin{aligned} \delta(B^2) &= \sum_{j=1}^N \left\{ -\phi_{j,0}^* \phi_{j,0} \delta \Sigma_{0j}^{Pu} + (\sum_i \chi_i \nu_j^{Pu} \phi_{i,0}^* \phi_{j,0} \right. \\ &\quad - \phi_{j,0}^* \phi_{j,0}) \delta \Sigma_{tr,j}^{Pu} + (\phi_{j,0}^* \phi_{k,0} - \phi_{k,0}^* \phi_{j,0}) \\ &\quad \times \delta \Sigma_{0n,k \rightarrow j}^{Pu} + (\phi_{j,0}^* \phi_{k,0} - \phi_{k,0}^* \phi_{j,0}) \delta \Sigma_{0r,k \rightarrow j}^{Pu} \\ &\quad + \sum_i \chi_i \Sigma_{tr,i}^{Pu} \phi_{i,0}^* \phi_{j,0} \delta \nu_j^{Pu} \\ &\quad + (\phi_{j,0}^* \phi_{j,0} / 3 \Sigma_{tr,j,0}^2) (B_0^2 + B_1^2 \lambda) \\ &\quad \times \delta \Sigma_{tr,j}^{Pu} \left. \right\} \lambda / \phi_0^* D_0 \phi_0. \quad (8) \end{aligned}$$

The last term in Eq. (8) contains the second order correction which can not be neglected. In Fig. 2, the results of numerical

Fig. 3 Effects of variations of multigroup constants for ^{239}Pu on $\delta(B^2)/B^2$ as functions of ^{239}Pu volume fraction

calculations are shown in the 4-group diffusion approach. It is seen that $\delta(B^2)$ is proportional to $\lambda = V^{Pu}$, except the curves e and e' due to $\delta\Sigma_{tr,j}^{Pu}$. The variation of buckling, $\delta(B^2)$, due to $\delta\Sigma_{tr,j}^{Pu}$, is not proportional to V^{Pu} , but $\delta(B^2)/B^2$ is linear with respect to V^{Pu} as shown in Fig. 3. From the above expression for $\delta(B^2)$, it can be seen that the effects of variations due to $\Sigma_{in,k+j}^{Pu}$, $\Sigma_{tr,k+j}^{Pu}$, and $\Sigma_{tr,j}^{Pu}$ are smaller about a factor of 10, compared to the effects due to the other group constants, $\Sigma_{f,j}^{Pu}$ and ν_j^{Pu} .

III. DISCUSSIONS

Theoretical treatment in Chap. II is limited only to the case of bare homogeneous cores in the multigroup diffusion approach. Analyses for reflected homogeneous cores and Two-region cores are difficult except by digital computer. The calculated results by the discrete SN, i.e., DSN, IBM 704 code^(2/3) in S_4 approximation was reported previously⁽¹⁾. Some of the figures are reproduced in this report.

1. Bare homogeneous cores

The buckling, B^2 , for bare homogeneous cores computed by 4-group diffusion theory is shown in Fig. 1. The three curves corresponding to standard, high and low capture cross section sets are linear in V^{Pu} , ^{239}Pu volume fraction, as predicted by the theory in the previous chapter. From Fig. 2, it can be seen that $\delta(B^2)$ caused by the variations of $\sigma_{c,j}^{Pu}$, ν^{Pu} and $\sigma_{in,1-2}^{Pu}$, are linear in V^{Pu} , as shown in Eq. (8). The deviation of the curves e and e' from the straight line in Fig. 2 is to be explained reasonably by considering the last term in Eq. (8).

The variations of buckling in per cent, $\delta(B^2)/B^2$ are shown in Fig. 3, corresponding to the same variations of cross sections for ^{239}Pu as in Fig. 2. It can be seen that $\delta(B^2)/B^2$ caused by the variations of $\sigma_{c,j}^{Pu}$, ν^{Pu} and $\sigma_{in,1-2}^{Pu}$, have the similar shape as a function of V^{Pu} . On the other hand, the curves e and e' due to the variation of $\sigma_{tr,1-2}^{Pu}$ for ^{239}Pu , is proportional to V^{Pu} , as can be seen from Eq. (8).

In the discussions in this chapter, it is

assumed that the cross sections of ^{235}U , fertile (^{238}U), structural (stainless steel), and coolant (sodium) materials, are given accurately and that the main uncertainty in multigroup constants exists in that of ^{239}Pu . Two types of errors can be considered in the extrapolation from the two experimental values of material bucklings for the ^{235}U system and the ^{239}Pu - ^{235}U system, to predict the buckling for the full ^{239}Pu system. One is the error due to the uncertainty of ^{239}Pu cross sections, which is inevitable even if an accurate experimental value for buckling, B_{exp}^2 , is obtained for a given $V^{Pu} = V_{exp}^{Pu}$. Another one is the error due to the experimental errors, $\pm\delta(B_{exp}^2)$, for a given $V^{Pu} = V_{exp}^{Pu}$. The actual error, $\pm\delta(B^2)$, in the prediction of buckling for the full ^{239}Pu system, is approximately the sum of both errors.

From Eq. (8), it is concluded that for small variations of multigroup constant, the magnitude of $\delta(B^2)/B^2$ at a given V^{Pu} is proportional to the fractional change of the multigroup constant, and the combined effect of any two or more variations of multigroup constants is the sum of each $\delta(B^2)/B^2$ caused by each independent variation of multigroup constant at least in the first order approximation.

Given an accurate experimental value of buckling, B_{exp}^2 , for V_{exp}^{Pu} by critical experiments, one can adjust a cross section set for ^{239}Pu which will give the same buckling as B_{exp}^2 for V_{exp}^{Pu} . There are infinite number of other cross section sets that will give the same buckling, B_{exp}^2 , for V_{exp}^{Pu} . All the cross section sets with different $\sigma_{c,j}^{Pu}$ and adjusted ν^{Pu} will give approximately the same buckling value for the fully ^{239}Pu fueled critical system as given by the standard cross section set, because $\delta(B^2)/B^2$ due to fractional changes of $\sigma_{c,j}^{Pu}$ and ν^{Pu} have similar shapes as a function of V^{Pu} , and $\delta(B^2)/B^2$ will cancel out for all ranges of ^{239}Pu concentration, at least in the first order approximation.

The same discussions can be applied to fractional changes of any combinations of $\sigma_{c,j}^{Pu}$, $\sigma_{tr,j}^{Pu}$, $\sigma_{in,j}^{Pu}$, $\sigma_{tr,j}^{Pu}$, ν^{Pu} of ^{239}Pu , and all the cross section sets, adjusted to give the same buckling, B_{exp}^2 , would give very close values for

the buckling of the all ^{239}Pu fueled system, as far as the fractional changes of multigroup constants remain in reasonable ranges.

Therefore, knowledge of the experimental buckling at 0.5 v/o fraction of ^{239}Pu results in general in an error reduction of more than a factor of 10 in the extrapolation to the buckling of the fully ^{239}Pu fueled system compared to the error if only the buckling of the analogous fully ^{235}U fueled system were known. The fractional error in the extrapolated critical mass is correspondingly also decreased by the same factor.

The effect of σ_{tr}^{Pu} on $\delta(B^2)/B^2$ is different from the other multigroup constants, because $\delta(B^2)/B^2$ due to fractional change in σ_{tr}^{Pu} is proportional to V^{Pu} as shown in Fig. 3. The effect on $\delta(B^2)/B^2$ due to compensating variations of σ_{tr}^{Pu} and ν^{Pu} will not cancel as uniformly as in other cases, however, the amount of $\delta(B^2)/B^2$ due to $\delta\sigma_{tr}^{Pu}$ is fortunately very small. A realistic possible error in transport cross section may not be at all sufficient for adjustment to compensate $\delta(B^2)/B^2$ due to the uncertainty in the other cross sections.

Therefore, the results obtained by the above discussions can be applied generally.

When the experimental value of the buckling, B_{exp}^2 , for a ^{239}Pu volume fraction, V_{exp}^{Pu} , has errors, $\pm\delta(B_{exp}^2)$, the error, $\pm\delta(B^2)$, in the prediction of the buckling for the full ^{239}Pu system due to this experimental uncertainty, will be about $\pm\delta(B_{exp}^2)(V^{Fuel}/V_{exp}^{Pu})$, neglecting the second order error due to the differences between the adjusted cross section sets for ^{239}Pu . This can be seen if one considers the error $\pm\delta(B_{exp}^2)$ as an analogous equivalent error in the ^{239}Pu cross sections.

For example, from the above discussions, in order to predict the buckling for the full ^{239}Pu system in an accuracy, 2 %, assuming extrapolated error due to cross section inaccuracies to be negligible, the errors for the experimental value of buckling, $\delta(B_{exp}^2)/B_{exp}^2$, must be less than 0.2 % for $V_{exp}^{Pu}=0.0025$ and 0.4 % for $V_{exp}^{Pu}=0.005$. $V_{exp}^{Pu}=0.0025$ and 0.005 correspond respectively to $V^{Pu}/V^{Fuel}=0.05$ and 0.1.

The relative values of $|\delta R/R|$ normalized to unity at the ^{239}Pu fueled system are shown

in Fig. 4 for the various cross section sets. It can be seen that the curves due to the variations of cross sections other than σ_{tr}^{Pu} have rather similar shapes, and the compensation of calculational errors can be expected for all ranges of ^{239}Pu concentration. Fig. 4 also shows that in order to predict the critical radius for the bare homogeneous ^{239}Pu system in an accuracy, 1 %, the errors for the experimental value of critical radius must be less than 0.1 % for $V^{Pu}/V^{Fuel}=0.05$ and 0.2 % for $V^{Pu}/V^{Fuel}=0.1$.

Fig. 4 Relative deviations of extrapolated radii for homogeneous bare cores

2. Reflected Two-region cores

Two-region spherical core reflected by a thick blanket is calculated for various cross section sets. The core composition of the central ^{239}Pu region is 5 v/o ^{239}Pu , 45 v/o ^{238}U , 16.67 v/o Fe and 19.3 v/o Al, and that of the outer ^{235}U core region is 5 v/o ^{235}U , 45 v/o ^{238}U , 16.67 v/o Fe and 19.3 v/o Al. The ratio of central region volume to total core volume is changed corresponding to various ^{239}Pu contents.

The relative values of $|\delta R/R|$ normalized to unity at the ^{239}Pu fueled core are shown in Fig. 5. The compensation of errors due to uncertainties in cross section other than σ_{tr}^{Pu} , can be expected similarly as in the bare homogeneous case. Since the effect of σ_{tr}^{Pu} on the critical size is very small, the results obtained above can be applied generally. From Fig. 5, it can be seen that in order to

Fig. 5 Relative deviations of core radii for reflected spherical Two-region cores

predict the critical radius for the reflected ^{239}Pu core in an accuracy, 1 %, the error for the experimental value of critical radius must be less than 0.5~0.6 % for the ratio of central core region volume to total core volume of 0.1. The critical experiment in the reflected Two-region core is recommended compared to the homogeneous core, because the required accuracy for the experimental critical radius is easier to attain, and the effect of uncertainties in cross sections can be more easily detected and hence adjusted.

IV. CONCLUSIONS

Using multigroup diffusion theory, the effect of multigroup constants on bucklings is investigated for bare homogeneous cores. The results of calculation are compared with the theoretical formula, which is useful to explain numerical results. Theoretical analysis for the reflected Two-region cores are complicated even in the diffusion approximation.

It is concluded that the errors in the calculated critical radius for a 5.0 v/o ^{239}Pu fueled system reduce more than a factor of about 10 to 20, if an accurate experimental value of critical size of a critical assembly fueled by 0.5 v/o ^{239}Pu and 4.5 v/o ^{235}U , is known. The conclusion above is proved numerically even in the case of reflected Two-region cores by the S_n method.

In order to predict the critical radius for the bare homogeneous ^{239}Pu system in an accuracy, 1 %, the errors for the experimental value of critical radius must be less than 0.2 % for $V^{\text{Pu}}/V^{\text{Fuel}}=0.1$. For the reflected Two-region cores, the experimental errors required for the above accuracy must be less than 0.5 % for the ratio of central ^{239}Pu core region volume to total core volume of 0.1.

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