研究論文

253. *Reactivity Measurement on the Reflected Semi-Homogeneous Critical Assembly by Pulsed Neutron Technique*

パルス中性子法による反射体つき平均質臨界集合体の反応度測定

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This paper describes the measurements of the prompt neutron lifetime and of the equivalent reactivity worths of control rods and the gap between two halves of the SHE-1 (Semi-Homogeneous Critical Assembly Core-1) by pulsed neutron technique.

Even with the experimental difficulties, especially from high neutron background due to the build-up of the delayed neutrons and to the spontaneous fission of ^{238}U , the decay constant α could be measured in the range from 7.19 to 147.7 sec^{-1} within the error of $\pm 2.1\%$. Then the prompt neutron lifetime of this assembly was determined to be $1.21 \pm 0.03 \text{ ms}$ (assuming $\beta_{\text{eff}} = 0.0067$), and the equivalent reactivity worth of a Cd control rod to be $4.10 \pm 0.13 \text{ \$}$.

However, the reactivity reduced from the value of α by the conventional formula, $\$ = (\alpha - \alpha_c) / \alpha_c$ was found to have a larger error than experimental one below about $-9 \text{ \$}$ for SHE-1, because the change of the value of $k_{\text{eff}} \beta_{\text{eff}} / l$ with the degree of subcriticality is neglected. The theoretical validity of reactivity measurement in the reflected subcritical system by pulsed neutron technique is discussed in Appendix, considering the influence of the reflector on prompt neutron decay at the subcritical state.

I. INTRODUCTION

A set of pulsed neutron experiments has been performed to get the value of the prompt neutron lifetime and the reactivity of the Semi-Homogeneous Critical Assembly at the subcritical state, but there has been a lack of the accurate theory for the measurements in the reflected core by this technique. Therefore, to decide both the theoretical and practical limitations in the measurements for the reflected SHE by this technique has been a matter of interest.

II. THEORY

Strictly speaking, most of the published results of the theoretical study of kinetic behavior of pulsed subcritical system are applicable only to the bare and homogeneous system⁽¹⁾⁻⁽⁶⁾. Rigorous theoretical treatments for the reflected and semi-homogeneous system

including the space dependence are not yet developed. But the following phenomenon is explained by discussions using one group model in Appendix I. When sufficient time has elapsed after the introduction of a burst of neutrons, the neutron spatial distribution approaches to the eventual shape and the neutron density shows an exponential uniform decrease at any point of the whole system. Under these conditions, time differential of the neutron density is described as follows,

$$\frac{dn}{dt} = -\frac{1}{l} \left\{ 1 - k_{\text{eff}} (1 - \beta_{\text{eff}}) \right\} n \quad (1)$$

where k_{eff} : the effective neutron multiplication factor

l : the prompt neutron lifetime

β_{eff} : effective delayed neutron fraction

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n : neutron density.

The decay constant of n, α is defined as

$$\alpha = \frac{1}{l} \left\{ 1 - k_{\text{eff}}(1 - \beta_{\text{eff}}) \right\} = \frac{\beta_{\text{eff}} k_{\text{eff}}}{l} (1 + \$) \quad (2)$$

where $\$$: reactivity in dollar unit.

Near the critical state, the value of $\beta_{\text{eff}} k_{\text{eff}}/l$ does not much depend on the degree of subcriticality, and can be treated as a constant, so the reactivity in dollar unit can be determined, using the decay constant α , as follows;

$$\$ = \frac{\alpha - \alpha_c}{\alpha_c} \quad (3)$$

where α_c : decay constant at delayed critical state.

In the state, which is far below its critical point, care must be taken in the decision of the reactivity by pulsed neutron technique, since the value of $\beta_{\text{eff}} k_{\text{eff}}/l$ much does depend upon subcriticality due to the reflector influence. In Appendix I, is discussed the way of the theoretical treatment of the reactivity measurement in the reflected system in highly subcritical state.

III. EXPERIMENTAL APPARATUS

1. Assembly

The Semi-Homogeneous Critical Assembly was described in other reports⁽⁶⁾⁽⁷⁾, so only a brief description is given here.

The assembly consists of the two halves,

Fig. 1 Experimental arrangement for SHE and P. N. S. systems

with one being movable and the other fixed. (See Fig. 1) In each half are stacked approximately 1400 graphite hollow tubes and rods, each 120 cm long and 6.5 cm in O. D. The 397 hollow tubes in the central region of each half make up the core which is surrounded by the reflector of graphite rods.

The fuel rod contains the fuel disc of a homogeneous mixture of 20 % enriched uranium dioxide and pure graphite, (the graphite to UO_2 weight ratio, 10). Pure graphite spacer discs are also contained in the rod and a variation of the compositions of the core are obtainable by changing the combination of the fuel discs and the graphite discs. The overall carbon to ^{235}U atomic ratio of the core region is variable from 2,226 to approximately 10,000.

There are one control and three safety rods in each half of the assembly, which are usually hollow cylinders of cadmium, and can be replaced by rods of boral, boron carbide, etc.

In the present experiments, the core of SHE-1 when loaded, is approximately a right cylinder of 100 cm in length and 88 cm in diameter, with a $\text{C}/^{235}\text{U}$ atomic ratio of 5,378, and $\text{C}/^{238}\text{U}$ ratio of 1,349. The thickness of reflector is radially 80 cm, and axially 70 cm. The critical mass with this configuration was determined to be 3.4 kg of ^{235}U under the practical operating conditions. The results of this paper were obtained, when the 365 fuel rods were loaded.

2. Pulsed neutron source⁽⁸⁾⁽⁹⁾

The pulsed neutron source (JAERI-Toshiba-NS-II-PT) is a 200 kV Cockcroft type neutron generator. The pulsed ion beam is obtained by modulating both the extraction voltage at the probe of the ion source and the deflecting voltage for the accelerated ion beam. A beam analyzing magnet for the accelerated beam is available for reducing the background due to the accelerator.

The extension tube with a T target was placed horizontally in the graphite tube (MC9) (See Fig. 2), which was inserted in the reflector near the boundary with the core (See Fig. 1). To a regret, it was impossible to place the T target in the fuel region because of very limited geometry for the accelerator.

Fig. 2 SHE core arrangement for P. N. S. systems

The operating condition of the pulsed neutron source was mostly fixed in the present experiment. (See Table 1)

3. Measuring system

A BF_3 counter was placed in the mid core section of the graphite tube (5F5) in the fuel region (See Fig. 2), for the position was considered to be the best place to avoid the effect of the neutrons of higher mode, judged from a series of experimental trials. The neutron induced pulses from the counter were fed to a 20-channel time analyser system.

Table 1 The pulsed neutron source operating condition

Accelerating voltage	150 kV
Pulse width	100 μs
Peak current of pulsed ion beam	40 μA (equivalent neutron intensity, 2×10^8 n/pulse)
Repetition frequency	1~5 P.P.S. for subcritical state under -6% , 0.03~1 P.P.S. for subcritical state from -6% to -25%

This analyser consisted of a variable delay circuit (a part of the Tektronix Synchroscope type 545), a time-to-pulse height converter (domestic) and a 20-channel pulse height analyser (Atomic, Model 520, dead time = 2.4 μs). A block diagram of the measuring system is shown in Fig. 3. The conversion rate of time interval to pulse height was calibrated by simulating the detector pulses, using the same delay circuit of the measuring system, which had already been checked by a crystal-governed cycle counter. From a series of calibration results, the drift of the conversion rate didn't exceed 1% during 5 hr. Total counts in the 20 channel time analyser system amounted to more than 5×10^4 counts in each run of the experiment.

Fig. 4 shows an example of the observed decay of neutron density succeeding to the rapid build-up of thermal neutrons just after the bursts of neutrons were introduced.

Fig. 3 Block diagram for pulsed measurements

Fig. 4 Observed decay of neutron density

IV. EXPERIMENTAL RESULTS

1. Determination of the prompt neutron lifetime

Fig. 5 shows the observed α against the control rod position in the fixed half, with the another control rod in the movable half being fully pulled out. The interval from a measuring rod position to the next one was taken 12.5 mm. The measurement for the assembly which was closest to the critical point, was made at the point where the control rod was pulled out at 137.5 mm. (175.0 mm at critical point)

In the state closer to the critical point, the background due to the build-up of delayed neutrons, to the spontaneous fission neutrons from ^{238}U and to others became comparable with the transient neutron peak just after the neutron injection. This situation made it impossible to get the reliable data.

Since the decay constant measurements were made on the subcritical system only, extrapolation of the observed data was necessary to obtain α_c , which is the value of α at delayed critical under the assumption described in Sec. 2.

Fig. 5 Observed decay constant α against the control rod position near the critical point

A linear least square fitting of the data was made using IBM 650 on the base of Eq. (4).

$$\alpha = a + br \quad (4)$$

where r : the position of control rod at the fixed half.

The constants a , b and their probable errors were determined. The $\alpha_c = 5.54 \pm 0.11 \text{ sec}^{-1}$ is obtained by giving $r_0 = 175.0 \text{ mm}$ to Eq. (4).

The prompt neutron lifetime l is then determined to be $1.21 \pm 0.03 \text{ ms}$ by assuming that $\beta_{\text{eff}} = 0.0067^{(10)(11)}$.

This result shows a fairly good agreement with the value, 1.2 ms, obtained by reactor noise analysis method⁽¹²⁾.

Some discussions will be added for the background neutron level in the pulsed measurements. Bengtson⁽¹³⁾ has given the ratio of the background noise to the signal as,

$$\frac{\text{Background neutron density}}{\text{Maximum neutron density}} = \frac{1}{1 + \frac{(1 - k_{\text{eff}})^2}{2l\beta_{\text{eff}}(1 - \beta_{\text{eff}})R}} \quad (5)$$

where R : repetition rate of neutron pulse. However, it is not clear that above formula

can be applied to any subcritical state. So another method was tried in this work as follows. The ratio of the background neutron level due to the build-up of the delayed neutrons to the maximum neutron density just after the neutron injection can be estimated as, (See Appendix II)

$$\frac{\text{Background neutron density}}{\text{Maximum neutron density}} = \frac{\lambda}{\alpha} \left(1 + \frac{e^{-\lambda/R}}{1 - e^{-\lambda/R}} \right) \quad (6)$$

where λ : the average decay constant for the delayed neutrons.

Fig. 6 shows the observed decay of the neutron density and the background level, together with calculated background levels using Eqs. (5) and (6). The discrepancy between experiment and calculation using Eq. (6) can be explained by considering the background due to the multiplication of spontaneous fission neutrons from ^{238}U (calculated as 3×10^2 n/sec in whole core,) and to other external sources (mainly, the leakage neutron from the Ra-Be neutron source in the shielded coffin).

Fig. 6 Observed decay of neutron density and background at high background condition

2. The determinations of the equivalent reactivity worth of a control rod

Fig. 7 shows the observed α against the control rod position in the movable half with other control rod being fully inserted in the fixed half. The total reactivity worth of a cadmium control rod at the movable half was determined to be 4.10 ± 0.13 \$. The same experimental error percentage (≈ 3 %) is contained in the value of other equivalent reactivities (See Table 2) measured by this technique, because both α_0 and each α which must be used to determine the value of each reactivity have the error of 2.1 %.

Fig. 7 Observed decay constant α against the control rod position

3. The determination of the equivalent reactivity worth of safety rods and of the gap between the two halves

The observed decay constant α and the value of reactivity reduced by the conventional analysis are shown in Table 2 for the various configurations of the two control rods and six safety rods.

The equivalent reactivity worth of the gap between the two halves was measured both with all rods inserted and with only all control rods inserted, and is shown in Fig. 8.

Table 2 Observed α and equivalent reactivity worth for the various rod configuration

Control rod		Safety rod						Counter location	Decay constant (sec ⁻¹)	Reactivity (- β)
1	2	1	2	3	4	5	6			
■	■	□	□	□	□	□	□	5 F 5	36.3 $\pm$ 0.73	5.55 $\pm$ 0.17
■	■	□	□	□	□	□	■	5 F 5	54.5 $\pm$ 1.1	8.84 $\pm$ 0.26
■	■	□	□	■	□	□	■	5 C 5 5 F 5	71.1 $\pm$ 1.5 74.0 $\pm$ 1.5	(11.83 $\pm$ 0.36) [†] (12.36 $\pm$ 0.37) [†]
■	■	□	■	■	□	□	■	5 C 5 5 F 5	89.1 $\pm$ 1.8 90.7 $\pm$ 1.8	(15.08 $\pm$ 0.45) [†] (15.37 $\pm$ 0.46) [†]
■	■	□	■	■	□	■	■	5 C 5	110.7 $\pm$ 2.2	(18.98 $\pm$ 0.57) [†]
■	■	■	■	■	□	■	■	5 F 5	122.0 $\pm$ 2.4	(21.03 $\pm$ 0.63) [†]
■	■	■	■	■	■	■	■	5 C 5	141.8 $\pm$ 2.8	(24.60 $\pm$ 0.74) [†]

■ Rod inserted, □ Rod pulled out

SHE-1 fuel rods: 365, Carriage gap: 0.48 mm

[†] The value was reduced by conventional method for convenient, and didn't indicate the corrected results.

The calculated equivalent reactivity worths of

gap for the bare cubic reactor⁽⁶⁾ are also shown in Fig. 8.

Fig. 8 Equivalent reactivity worth of the gap between two halves

V. SUMMARY

Although there are experimental and theoretical limitations, the pulsed neutron technique is very useful for the reactivity measurements of the reflected SHE. However, the practical useful range was found to be above about -9β at present configuration, the main cause for this limitation was the errors due to the reflector effects on the prompt neutron decay.

The simplified treatment of kinetic behavior of the reflected pulsed system was intended to estimate the basic limitation, but further detailed discussion will be still necessary.

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[Appendix I]

Prompt neutron decay of reflected system

To determine the theoretical limitation of the applicability of conventional Eq.(2) in the highly subcritical reflected system, the kinetic behavior of a pulsed reflected system is analysed in the one group diffusion model. Thereby the concept of effective neutron multiplication factor and prompt neutron lifetime is introduced, which is ensured to be applicable even in a highly subcritical state. For the simplicity, the effect of delayed neutron will be neglected in further discussions.

The fundamental equation are as follows

$$\nabla^2 \phi_c(\vec{r}, t) + B_c^2 \phi_c(\vec{r}, t) = + \frac{1}{vD_c} \frac{\partial \phi_c(\vec{r}, t)}{\partial t} \quad \text{in the core,} \quad (A1)$$

$$\nabla^2 \phi_r(\vec{r}, t) - B_r^2 \phi_r(\vec{r}, t) = + \frac{1}{vD_r} \frac{\partial \phi_r(\vec{r}, t)}{\partial t} \quad \text{in the reflector,} \quad (A2)$$

$$\text{and } D_c \text{grad}_n \phi_c(\vec{r}, t) = D_r \text{grad}_n \phi_r(\vec{r}, t) \quad (A3)$$

$$\phi_c(\vec{r}, t) = \phi_r(\vec{r}, t) \quad (A4)$$

on the boundary between the core and reflector, and

$$\phi_r(\vec{r}, \phi) = 0 \quad (A5)$$

on the extrapolated surface, i.e. at a distance of $0.71 l_{tr}$ from the actual surface,

$$\text{where } B_c^2 = \frac{\Sigma_c(k_{\infty} - 1)}{D_c} \quad \text{and } B_r^2 = \frac{\Sigma_r}{D_r},$$

$\phi(\vec{r}, t)$, Σ and D being the neutron flux, the macroscopic absorption cross section, and the diffusion coefficient, respectively. The subscripts "c" and "r" denote the core and reflector respectively. " k_{∞} ", " l_{tr} " and " v " are the infinite multiplication factor in the core and the transport mean free path in the reflector and neutron velocity in one group model respectively. B_c^2 and B_r^2 are material buckling in the core and reflector respectively.

Here one introduces a set of orthonormal eigenfunctions, i.e.

$$R_0, R_1, R_2, \dots, R_m, \dots$$

$$R_n(\vec{r}) = \begin{cases} R_{cn}(\vec{r}) & \text{in the core,} \\ R_{rn}(\vec{r}) & \text{in the reflector.} \end{cases} \quad (A6)$$

These eigenfunctions satisfy the following

characteristic equations and boundary conditions;

$$\nabla^2 R_{cm}(\vec{r}) + B_{cm}^2 R_{cm}(\vec{r}) = 0 \quad \text{in the core,} \quad (\text{A7})$$

$$\nabla^2 R_{rm}(\vec{r}) + \{B_{cm}^2 - (B_c^2 + B_r^2)\} R_{rm}(\vec{r}) = 0 \quad \text{in the reflector,} \quad (\text{A8})$$

$$R_{cm}(\vec{r}) = R_{rm}(\vec{r}) \quad (\text{A9})$$

and

$$\text{grad}_n R_{cm}(\vec{r}) = \text{grad}_n R_{rm}(\vec{r}), \quad (\text{A10})$$

on the boundary between the core and reflector, and

$$R_{rm}(\vec{r}) = 0 \quad (\text{A11})$$

on the extrapolated surface of the reflector, where B_{cm}^2 is the eigenvalue corresponding to the m -th eigenfunction, $R_m(\vec{r})$.

It can be verified⁽¹⁶⁾ that these eigenfunctions form an orthonormal set which is complete in the functional space consisting of arbitrary functions differentiable to the first order and vanishing on the extrapolated surface of the reflector,

$$\int_{\text{whole volume}} R_m(\vec{r}) R_n(\vec{r}) d\vec{r} = \begin{cases} 1 & m=n \\ 0 & m \neq n \end{cases} \quad (\text{A12})$$

If the diffusion coefficient D is the same in the core and reflector*, the neutron flux, $\phi(\vec{r}, t)$, or the solution of Eqs. (A1)~(A4), can be expanded as a series of the eigenfunctions as follows,

$$\phi(\vec{r}, t) = \sum_i T_i(t) R_i(\vec{r}). \quad (\text{A13})$$

Substituting Eqs. (A13) into Eqs. (A1), (A2), one gets

$$\begin{aligned} \sum_i T_i(t) \nabla^2 R_{ci}(\vec{r}) + \sum_i T_i(t) B_{ci}^2 R_{ci}(\vec{r}) \\ = + \frac{1}{vD} \sum_i R_{ci}(\vec{r}) \frac{dT_i(t)}{dt}, \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} \sum_i T_i(t) \nabla^2 R_{ri}(\vec{r}) - \sum_i T_i(t) B_r^2 R_{ri}(\vec{r}) \\ = + \frac{1}{vD} \sum_i R_{ri}(\vec{r}) \frac{dT_i(t)}{dt}. \end{aligned} \quad (\text{A15})$$

From Eqs. (A7), (A8), (A12) and (A15) one gets immediately

$$T_m(t) (B_c^2 - B_{cm}^2) = + \frac{1}{vD} \frac{dT_m(t)}{dt}, \quad (\text{A16})$$

thus

$$T_m(t) = T_{m0} e^{-\alpha_m t}, \quad (\text{A17})$$

$$\alpha_m = vD(B_{cm}^2 - B_c^2). \quad (\text{A18})$$

In conclusion one gets, by introducing Eq. (A17) into (A13),

$$\phi(\vec{r}, t) = \sum_i T_{i0} e^{-\alpha_i t} R_i(\vec{r}) \quad (\text{A19})$$

where the coefficient T_{i0} have to be determined by requiring that the neutron flux (A19) for $t=0$ reduces to the initial spatial distribution, $\phi(\vec{r}, 0)$. Here it must be noted that the decay constant α which is measured after the eventual shape is established in experiment, is just the decay constant of fundamental mode, α_0 in Eq. (A19). The results are, using the orthonormal relation (A12),

$$T_{i0} = \int_{\text{whole volume}} \phi(\vec{r}, 0) R_i(\vec{r}) d\vec{r}. \quad (\text{A20})$$

It follows that neutrons of each mode behave independently and never excite those of the other modes.

The prompt neutron lifetime, the average time of neutron destruction, is defined here as the ratio of the total neutron importance in the reactor to a rate at which the importance is removed from the whole system. Therefore, the prompt neutron lifetime l , can be written as follows;

$$l = \frac{\int_{\text{whole volume}} \phi^*_{\text{eventual}}(\vec{r}, t) \phi_{\text{eventual}}(\vec{r}, t) d\vec{r}}{\int_{\text{whole volume}} \phi^*_{\text{eventual}}(\vec{r}, t) (-D\nabla^2 + \Sigma) \phi_{\text{eventual}}(\vec{r}, t) d\vec{r}} \times \frac{1}{v} \quad (\text{A21})$$

where $\phi_{\text{eventual}}(\vec{r}, t)$ is the eventual neutron flux distribution and $\phi^*_{\text{eventual}}(\vec{r}, t)$, the corresponding time dependent importance function of which concept was introduced by Lewins⁽¹⁴⁾⁽¹⁵⁾. In his paper it was suggested that time-dependent importance function in

* In the case that the diffusion coefficient D is not the same in the core and reflector, the following set of orthonormal eigenfunctions is provided;

$$R'_{im}(\vec{r}) = \begin{cases} R'_{cm}(\vec{r}) & \text{in the core,} \\ R'_{rm}(\vec{r}) & \text{in the reflector,} \end{cases}$$

characteristic equations

$$\nabla^2 R'_{cm}(\vec{r}) + B_{cm}^2 R'_{cm}(\vec{r}) = 0 \quad \text{in the core,}$$

$$\nabla^2 R'_{rm}(\vec{r}) - B_r^2 R'_{rm}(\vec{r}) = 0 \quad \text{in the reflector}$$

where $D_c B_{cm}^2 + D_r B_r^2 = D_c B_c^2 + D_r B_r^2$,

boundary conditions

$$R'_{cm}(\vec{r}) = R'_{rm}(\vec{r}),$$

$$D_c \text{grad}_n R'_{cm}(\vec{r}) = D_r \text{grad}_n R'_{rm}(\vec{r})$$

on the boundary between core and reflector,

$$R'_{rm}(\vec{r}) = 0$$

on the extrapolated surface of the reflector.

the one group model was a solution of the following equations.

$$\nabla^2 \phi_c^*(\vec{r}, t) + B_c^2 \phi_c^*(\vec{r}, t) = -\frac{1}{vD_c} \frac{\partial}{\partial t} \phi_c^*(\vec{r}, t) \quad \text{in the core,} \quad (\text{A22})$$

$$\nabla^2 \phi_r^*(\vec{r}, t) - B_r^2 \phi_r^*(\vec{r}, t) = -\frac{1}{vD_r} \frac{\partial}{\partial t} \phi_r^*(\vec{r}, t) \quad \text{in the reflector} \quad (\text{A23})$$

with the boundary conditions as Eqs.(A3) and (A4).

Clearly, in this case $\phi_{\text{eventual}}(\vec{r}, t)$ and $\phi^*_{\text{eventual}}(\vec{r}, t)$ are $T_{00} R_0(\vec{r}) e^{-\alpha_0 t}$ and $R_0(\vec{r}) e^{*\alpha_0 t}$ respectively, and so Eq.(A21) is rewritten as follows

$$l = \frac{\int_{\text{whole volume}} R_0^2(\vec{r}) d\vec{r}}{\int_{\text{whole volume}} R_0(\vec{r}) (-D\nabla^2 + \Sigma) R_0(\vec{r}) d\vec{r}} \cdot \frac{1}{v} \quad (\text{A24})$$

The effective multiplication factor, the ratio of production to destruction of neutrons per unit time is defined here as the creation rate of importance due to fission to the ratio of the removal rate of the importance due to the absorption and leakage, and is shown as follows,

$$k_{\text{eff}} = \frac{\int_{\text{core}} \phi^*_{\text{eventual}}(\vec{r}, t) k_{\infty} \Sigma_c \phi_{\text{eventual}}(\vec{r}, t) d\vec{r}}{\int_{\text{whole volume}} \phi^*_{\text{eventual}}(\vec{r}, t) (-D\nabla^2 + \Sigma) \phi_{\text{eventual}}(\vec{r}, t) d\vec{r}} \quad (\text{A25})$$

Eq.(A25) can be rewritten in term of $R_0(\vec{r})$ as follows,

$$k_{\text{eff}} = \frac{\int_{\text{core}} R_0(\vec{r}) k_{\infty} \Sigma R_0(\vec{r}) d\vec{r}}{\int_{\text{whole volume}} R_0(\vec{r}) (-D\nabla^2 + \Sigma) R_0(\vec{r}) d\vec{r}} \quad (\text{A26})$$

There exists an intrinsic relation between the effective multiplication factor, $k_{\text{eff}m}$, and the prompt neutron lifetime, l_m , for the m -th mode neutrons, i.e.,

$$\alpha_m = \frac{1 - k_{\text{eff}m}}{l_m} \quad (\text{A27})$$

Considering the effect of delayed neutron on neutron multiplication, k_{eff} in Eq.(A27) will be replaced by $k_{\text{eff}}(1 - \beta_{\text{eff}})$ which yields Eq.(2).

It is ensured through above discussion that

the neutron density shows an exponential decrease with the decay constant, α_0 , when sufficient time has elapsed after the introduction of a burst of neutrons to reflected system. Therefore, the fundamental mode α_0 can be reduced to the value of reactivity by the conventional Eq.(2) like in a bare system. However it can be accurate in limited range only in which the change of $\beta_{\text{eff}} k_{\text{eff}}/l$ may be negligible. This theoretical limitation must be confirmed on each situation of practical experiments.

Unfortunately, in case of the SHE-1 core, its complex geometry makes it impossible to confirm this theoretical limitation in general formulation (A21), (A25), so the fundamental equation (A1)~(A4) must be solved numerically in the view point of the check to the variation of $\beta_{\text{eff}} k_{\text{eff}}/l$. An analytical results for the similar core (cylindrical reactor with side reflector) indicates that the conventional analysis using Eq.(2), with the assumption that the factor $\beta_{\text{eff}} k_{\text{eff}}/l$ remains constant, is still valid for the pulsed experiments above -9\$ subcritical state within 3% error, of which the value equals the experimental error.

[Appendix II]

The simplified estimation of neutrons background due to pulsed neutron itself

Fultz⁽²⁾ has analysed the kinetic behavior of pulsed subcritical system using two group method, and derived a Eq.(B1) to account for the ratio of delayed neutron background to the maximum neutron density.

$$\frac{\text{Delayed neutron background}}{\text{Maximum neutron density}} = R \frac{\left(-\alpha + \frac{1}{l_1}\right)\lambda}{\frac{1}{l_1}(\alpha - \lambda)} \quad (\text{B1})$$

where l_1 : the mean lifetime for fast neutrons,
R: repetition rate of neutron pulses.

For the measurements at close points to the critical point, R becomes so small that the Eq.(B1) will give the underestimation of the ratio. It is more preferable to account for the contribution of neutron pulses train which has been injected before the newest one neutron pulse to measure, by the follow-

ing formula,

$$(e^{-\lambda/R} + e^{-2\lambda/R} + e^{-3\lambda/R} + \dots) \frac{(-\alpha + \frac{1}{l_1})\lambda}{(\alpha - \lambda)\frac{1}{l_1}} \quad (B2)$$

So, we can estimate more precisely the ratio as follows

$$\frac{\text{Delayed neutron background}}{\text{Maximum neutron density}}$$

$$= \left(1 + \frac{e^{-\lambda/R}}{1 - e^{-\lambda/R}}\right) \frac{(-\alpha + \frac{1}{l_1})\lambda}{(\alpha - \lambda)\frac{1}{l_1}} \quad (B3)$$

which leads to

$$\approx \left(1 + \frac{e^{-\lambda/R}}{1 - e^{-\lambda/R}}\right) \frac{\lambda}{\alpha} \quad (B4)$$

$$\text{for } \alpha \ll \frac{1}{l_1}.$$

研究論文

254. On the Distribution of the Slowing-down Time of Neutrons

中性子減速時間の分布

By Takashi NAKAYAMA*

The distribution of the slowing-down time of neutrons in an infinite homogeneous medium has been determined by the stochastic method. It is found that the distribution approaches that of gamma type asymptotically for a sufficiently large lethargy.

The experimental results are compared with theoretical curve, and a good agreement have been obtained.

I. INTRODUCTION

For simplicity let us assume that neutrons travel in an infinite homogeneous medium consisting of identical atomic nuclei of mass number M , that there is no absorption in the medium, and that the scattering cross section is independent of the neutron energy. Let $g(t, u)dt$ be the probability that the slowing-down time required for a neutron to reach a lethargy higher than u lies in the interval $(t, t+dt)$. The basic equation obeyed by the probability density $g(t, u)$ is mentioned in Pál and Németh's paper⁽¹⁾;

$$\frac{\partial g(t, u)}{\partial t} = -\frac{v_0}{\lambda_s} \int_0^u e^{-u'/2} \left\{ 1 - \int_0^{u-u'} w(u'') du'' \right\} \times \frac{\partial g(t, u')}{\partial u'} du', \quad (1)$$

where λ_s is the mean free path for scattering,

v_0 the velocity of pulsed neutrons with lethargy 0, and $w(u)du$ the probability that the lethargy increment of a neutron in a collision lies in the interval $(u, u+du)$, i.e.,

$$w(u) = \frac{1}{1-\alpha} e^{-u}, \quad \text{if } 0 \leq u \leq \ln \frac{1}{\alpha}, \quad (2)$$

$$w(u) = 0, \quad \text{if } u > \ln \frac{1}{\alpha} \text{ or } u < 0,$$

while

$$\alpha = \left(\frac{M-1}{M+1} \right)^2. \quad (3)$$

Here we shall explain the meaning of Eq. (1) in detail. The probability that a neutron with lethargy u will collide with the atom in the time interval $(t, t+dt)$ is

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