研究論文**216. *Trapping of Thermal Neutrons, (I)******Criteria of highly trapping media*****熱中性子の捕捉, (I)****捕捉物質の条件**

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Concerning thermal neutron trap reactors, a comparative study of trapping media is made by an analytical approach especially to derive criteria of highly trapping media. Spherical and cylindrical shapes of media are analyzed in the two-group diffusion theory. A criterion is that the thermal neutron diffusion coefficient of a trapping medium must be much smaller than each of the slowing-down length and the thermal neutron diffusion length, provided that both lengths should be modified in case of a finite cylinder. A small size of trap reactors can be realized by a medium whose characteristic lengths are both short.

Calculated values of the trapping effectiveness¹⁾ and the optimum radius of each ordinary moderator are given. Although the comparison with experiments is not yet possible for lack of experimental information under the conditions assumed in this theoretical study, it is suggested that the trapped thermal neutron flux cannot be definitely larger than the flux at the periphery of a trapping medium by a factor of ten.

INTRODUCTION

A fissile material is a strong absorber of thermal neutrons as well as a strong source of fast neutrons. It is reasonable therefore that the so-called flux trap reactor, devised to give higher density of thermal neutrons, consists of a moderator enclosed by a multiplying or active medium and an outer reflector in addition. The dimension of the internal moderator will be comparable to its slowing-down length.

It has been pointed out by Wigner^(*) that the internal moderator, *i.e.*, the trapping medium must have two properties: the diffusion coefficient must decrease with decreasing energy, and the absorption cross-section for thermal neutrons must be very small. Ordinary water satisfies the first criterion best, heavy water the second. These two criteria

were derived from the following idea. Fast neutrons diffusing into a medium in which the diffusion coefficient (and therefore the mean free path of thermal neutrons) is much smaller than that of fast neutrons will remain trapped once they become thermal. If, furthermore, the absorption of thermal neutrons in the medium is small, their density will grow to a high level.

The end of this paper is to make the fundamental features of thermal neutron trapping clear. We idealize the situation by assuming the active shell to be black for thermal neutron flow. Owing to this simplification the trapping effect can be treated separately from the criticality problem. We define a trapping factor as a measure of trapping and set up improved criteria of highly trapping media.

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1. Trapping in spherical geometry

Let us consider a reactor consisting of a spherical trapping medium surrounded by an active shell. The analysis is made in the two-group diffusion approximation.

In the steady state the neutron flux in the internal medium must satisfy the following diffusion equations :

$$\begin{aligned} D_f \Delta \phi_f - \Sigma_f \phi_f &= 0 \\ D_t \Delta \phi_t - \Sigma_t \phi_t + \Sigma_f \phi_f &= 0, \end{aligned} \quad (1)$$

where Δ is Laplacian operator. The subscripts f and t refer to fast and thermal neutrons, respectively, throughout the present paper; ϕ_f is the fast neutron flux, ϕ_t the thermal neutron flux, D_f and D_t the diffusion coefficients of fast and thermal neutrons, respectively, Σ_f the macroscopic slowing-down or removal cross-section for fast neutrons, and Σ_t the macroscopic absorption cross-section for thermal neutrons. We have assumed no neutron absorption during moderation. Solution of these equations are

$$\begin{aligned} \phi_f(r) &= \frac{c_1}{r} \sinh \frac{r}{L_f} \\ \phi_t(r) &= \frac{c_2}{r} \sinh \frac{r}{L_t} + S \frac{c_1}{r} \sinh \frac{r}{L_f}, \end{aligned} \quad (2)$$

where r is the radial distance and

$$L_f^2 = \frac{D_f}{\Sigma_f}, \quad L_t^2 = \frac{D_t}{\Sigma_t}, \quad S = \frac{D_f}{D_t} \frac{L_t^2}{L_f^2 - L_t^2}, \quad (3)$$

and c_1 and c_2 are arbitrary constants. Here L_f is the slowing-down length, L_t the thermal neutron diffusion length, and S is called the coupling constant.

An active shell surrounding the trapping medium is a fairly strong absorber of thermal neutrons. The situation is idealized so that the shell is assumed to be black for thermal neutron flow. Then the relation between constants c_1 and c_2 will be fixed, and the ratio of the thermal neutron flux at the center of the trapping medium to that at the periphery will be determined. Let us call the ratio the *trapping factor* :

$$\tau = \frac{\phi_t(0)}{\phi_t(a)} \quad (4)$$

where a is the radius of the trapping medium.

At first glance, it would seem that the

effectiveness of a trapping medium should be expressed by some other quantity such as the ratio of the central thermal neutron flux (or density) to the fast neutron current flowing into the medium. In the present ideal case (in which the following Eq. (7) holds), once a trapping medium and its size are specified, such a ratio is necessarily determined as well as the ratio given by Eq. (4). Thus these ratios are uniquely fixed whenever a reactor having a given trapping medium of a given size is made critical by an appropriate combination of an active shell and an outer reflector. The expression (4) presents, however, a more direct measure of trapping effectiveness for the following reason.

An increase in the reactor power may result in a proportional rise in the neutron flux throughout the reactor, but the thermal neutron flux in the active region must be limited to a certain level due to technical difficulties in heat removal. It should be noted that this limitation depends on the composition and structure of the active shell but is essentially independent of the internal trapping medium.

Near the interface with the trapping medium, the specific power in the active shell will be approximately given by

$$P \simeq p \Sigma_{fis} \phi_t(a), \quad p = 3.0 \times 10^{-11} \text{ joule/fission} \quad (5)$$

where Σ_{fis} is the macroscopic fission cross-section for thermal neutrons. If the maximum permissible value of P is given, say by P_m , a higher trapping factor will result in higher thermal neutron flux at the center, that is,

$$\phi_t(0) = \tau \phi_t(a) \simeq \frac{\tau P_m}{p \Sigma_{fis}} \quad (6)$$

Now, the blackness of the active shell is expressed as the boundary condition at the surface of the trapping medium :

$$j_t(a) = \frac{\phi_t(a)}{4} + \frac{D_t}{2} \frac{d\phi_t}{dr} \Big|_{r=a} = 0, \quad (7)$$

which states that no thermal neutrons flow into the medium. With the aid of Eq. (7), the ratio between c_1 and c_2 in Eqs. (2) is determined, and after a straightforward cal-

culation, the trapping factor (4) becomes

$$\tau(\lambda, x) = \frac{\mu}{2} f(\lambda, x) + g(\lambda, x), \quad (8)$$

where

$$f(\lambda, x) = \left(\frac{x}{\sinh x} - \frac{\lambda x}{\sinh \lambda x} \right) / h(\lambda, x) \quad (9)^*$$

$$g(\lambda, x) = \left[\left(\lambda \coth \lambda x - \frac{1}{x} \right) \frac{x}{\sinh x} - \left(\coth x - \frac{1}{x} \right) \frac{\lambda x}{\sinh \lambda x} \right] / h(\lambda, x) \quad (10)^*$$

$$h(\lambda, x) = \lambda \coth \lambda x - \coth x. \quad (11)$$

Here

$$x = \frac{a}{L_f}, \quad \frac{1}{\lambda} = \frac{L_t}{L_f} \quad \text{and} \quad \frac{1}{\mu} = \frac{D_t}{L_f}, \quad (12)$$

by means of which all the physical constants concerned are expressed in units of the slowing-down length L_f : the ratio D_t/L_f does not appear in the trapping factor. Values of μ and λ for ordinary moderators, computed from their physical constants in Table 1, are listed in Table 2.

Table 1 Physical constants of moderators†

	Density (g/cm³)	Fast diffusion coeff. D_f (cm)	Thermal diffusion coeff. D_t (cm)	Fast removal cross- section Σ_f (cm⁻¹)	Thermal absorption cross-section Σ_t (cm⁻¹)	Slowing down length L_f (cm)	Thermal diffusion length L_t (cm)	$\frac{D_t}{L_f}$	$\frac{D_t}{L_t}$	$\frac{D_t + D_f}{L_f + L_t}$
H₂O	1.00	1.42	0.164	0.0429	0.0220	5.75	2.73	0.0285	0.0601	0.0886
D₂O ^(a)	1.10	1.31	0.620	0.00947	0.0000770	11.8	89.7	0.0527	0.0069	0.0596
Be	1.85	0.687	0.600	0.00680	0.00124	10.1	22.0	0.0597	0.0273	0.0870
BeO	3.01	0.645	0.750	0.00571	0.000730	10.6	32.1	0.0706	0.0234	0.0940
C ^(b)	1.60	1.19	0.778	0.00282	0.000347	20.5	47.4	0.0379	0.0164	0.0543

† Densities, D_f , D_t , Σ_f and Σ_t are cited from "Reactor Physics Constants", ANL-5800 (1958), p. 160 and p. 524, provided that

(a) D₂O is assumed to contain 0.2 mol/o H₂O, and

(b) the microscopic absorption cross-section of graphite is assumed to be 4.00 mb for thermal neutrons due to impurities.

Table 2 Thermal neutron trapping in spherical geometry

	$\frac{L_f}{L_t}$ λ	$\frac{L_f}{D_t}$ μ	Opti- mum of x x_m	Maxi- mum of $f(\lambda, x_m)$	Opti- mum radius a_m (cm)	Maxi- mum trapping factor τ_{max}
H₂O	2.11	35.1	1.5	0.43	8.6	8.5
D₂O	0.131	19.0	4.6	1.12	55	11.6
Be	0.457	16.8	3.2	0.92	32	8.6
BeO	0.332	14.2	3.6	1.00	38	8.0
C	0.434	26.4	3.3	0.93	67	13.2

Curves of the functions $f(\lambda, x)$ and $g(\lambda, x)$ are illustrated in Figs. 1 and 2 for several values of λ ; every $f(\lambda, x)$ has a maximum, while every $g(\lambda, x)$ starts from 1 for $x=0$ and decreases monotonically with increasing x . The dependence of the trapping factor on the parameter μ (or on D_t) is quite simple. As μ becomes very small, the trapping factor $\tau(\lambda, x)$ approaches $g(\lambda, x)$ according to Eq.(8), so that the trapping of thermal neutrons does not take place.

To make a comparative study of trapping media, we must know the dependence of the

Fig. 1 $f(\lambda, x)$ defined by Eq. (9) vs. x

* For $\lambda = 0$, i.e., no absorption of thermal neutrons, we have

$$f(0, x) = \left(\frac{x}{\sinh x} - 1 \right) / \left(\frac{1}{x} - \coth x \right)$$

$$g(0, x) = 1.$$

For $\lambda = 1$, the expressions (9) and (10) are indefinite and should be replaced by

$$f(1, x) = \frac{x(x \cosh x - \sinh x)}{(1/2) \sinh 2x - x}$$

$$g(1, x) = \frac{(x^2 + 1) \sinh x - x \cosh x}{(1/2) \sinh 2x - x}.$$

Fig. 2 $g(\lambda, x)$ defined by Eq. (10) vs. x

maximum trapping factor on the parameters μ and λ . It is hardly possible to solve a transcendental equation $(\partial/\partial x) \tau(\lambda, x) = 0$ analytically, but a good approximation may be found in the following cases.

Since the factor $\mu/2$ of an ordinary moderator is of the order of ten as shown in Table 2, on the right-hand side of Eq. (8) the first term is predominant except a certain range of very small x . If $x_m(\lambda)$ denotes the value of x where $f(\lambda, x)$ becomes maximum, then the maximum of the trapping factor will also be found in the close vicinity of the $x_m(\lambda)$, that is,

$$\begin{aligned} \tau_{\max}(\lambda) &\approx \tau(\lambda, x_m) \\ &= \frac{\mu}{2} f(\lambda, x_m) + g(\lambda, x_m) \end{aligned} \quad (13)$$

Now, $f(\lambda, x_m)$, $g(\lambda, x_m)$ and $x_m(\lambda)$ are shown in Fig. 3 and can be approximated by the expressions:

Fig. 3 Dependence of $f(\lambda, x_m)$, $g(\lambda, x_m)$ and $x_m(\lambda)$ on λ . $f(\lambda, x)$ becomes maximum for $x = x_m(\lambda)$.

$$f(\lambda, x_m) \approx \frac{1.30}{1 + \lambda} \quad (14)$$

$$g(\lambda, x_m) \approx 0.9 \quad (15)$$

$$x_m(\lambda) \approx \frac{4.95}{1 + \lambda}, \quad (16)$$

whose derivation is given in Appendix. According to the definition of x in Eqs. (12), Eq. (16) yields the optimum radius a_m of the trapping medium to give the maximum trapping:

$$a_m = x_m L_f \approx \frac{4.95}{\frac{1}{L_f} + \frac{1}{L_t}} \quad (17)$$

Hence, a medium whose slowing-down length and thermal neutron diffusion length are both short being used, a small size of flux trap-reactors can be obtained. From this point of view, light water is most typical.

When Eqs. (14) and (15) are introduced into Eq. (13), the maximum trapping factor becomes

$$\begin{aligned} \tau_{\max}(\lambda) &\approx \frac{\mu}{2} \frac{1.30}{1 + \lambda} + 0.9 \\ &= \frac{0.65}{D_t \left(\frac{1}{L_f} + \frac{1}{L_t} \right)} + 0.9 \end{aligned} \quad (18)$$

Hence, in order to get a high trapping, a medium whose thermal neutron diffusion coefficient D_t is much smaller than each of the characteristic lengths L_f and L_t must be used. This is just an improved criterion of highly trapping media.

As mentioned in Introduction, Wigner's criteria may be written in the form that

$$(1) \quad D_t < D_f$$

and

$$(2) \quad \Sigma_t \text{ is small.}$$

From our study it follows that his first requisite must be substituted by

$$(1') \quad D_t \ll L_f$$

and the second by

$$(2') \quad D_t \ll L_t \text{ or } \Sigma_t \ll \frac{1}{D_t}.$$

The first condition, which means $\mu \gg 1$, has been noticed in regard to Eq. (8). The second states that the disadvantage of a medium due to the thermal neutron absorption should be estimated not from the absorption cross-section itself but from its relative magnitude compared with the reciprocal diffusion.

coefficient (or with the transport cross-section) for thermal neutrons. This is the reason why light water gives a fairly high trapping in spite of its rather large absorption cross-section for thermal neutrons.

Numerical data for five ordinary moderators are shown in Table 2. It is rather remarkable that, as to the maximum trapping factor, there is not a great difference among these moderators. Graphite or heavy water gives the highest trapping but a fairly large size of the moderator is required.

It is interesting to note that the maximum trapping factor of graphite is independent of the density, because it is a function of the non-dimensional quantities μ and λ only, according to Eq. (18).

2. Trapping in cylindrical geometry

Let us consider a cylindrical trapping medium which is not enclosed by an active material in the axial direction. The neutron flux in a cylindrical column of a trapping medium is given as follows:

$$\begin{aligned}\phi_j(\rho, z) &= c_1 I_0\left(\frac{\rho}{L_j'}\right) \cos \frac{\pi z}{H} \\ \phi_i(\rho, z) &= \left\{ c_2 I_0\left(\frac{\rho}{L_i'}\right) + S c_1 I_0\left(\frac{\rho}{L_j'}\right) \right\} \cos \frac{\pi z}{H},\end{aligned}\quad (19)$$

where I_0 is the modified Bessel function of order zero of the first kind, ρ and z being the radial and axial coordinates, respectively, with the origin at the center of the column. Here H is the height of the column including the extrapolation distances at the top and the bottom, and L_j' and L_i' are defined by the equations:

$$\begin{aligned}\frac{1}{L_j'^2} &= \frac{\Sigma_j}{D_j} + \left(\frac{\pi}{H}\right)^2 = \frac{1}{L_j^2} + \left(\frac{\pi}{H}\right)^2 \\ \frac{1}{L_i'^2} &= \frac{\Sigma_i}{D_i} + \left(\frac{\pi}{H}\right)^2 = \frac{1}{L_i^2} + \left(\frac{\pi}{H}\right)^2.\end{aligned}\quad (20)$$

Now, in place of $I_0(y)$, a function $u(\alpha y) \equiv (1/\alpha y) \sinh \alpha y$ ($\alpha = 1.17$) may be substituted with an error less than several percent, so long as the argument y is within the range of practical interest, i.e., between 0 and about 4.5. Then the radial dependence of the flux is written in the form

$$\phi_j(\rho) = c_1 u\left(\frac{\alpha \rho}{L_j'}\right)$$

$$\phi_i(\rho) = c_2 u\left(\frac{\alpha \rho}{L_i'}\right) + S c_1 u\left(\frac{\alpha \rho}{L_j'}\right). \quad (21)$$

If L_j'/α and L_i'/α were replaced by L_j and L_i , respectively, the above expressions would be identical with those in the spherical case, except the coupling constant S . It can be found that, if the trapping factor in cylindrical geometry is defined by

$$\tau = \frac{\phi_i(0, z)}{\phi_i(a, z)}, \quad (22)$$

where a is the radius of cylinder, the trapping factor is independent of the coupling constant similarly to the spherical case.

When x , λ and μ are defined anew by

$$x = \frac{a}{L_j'/\alpha}, \quad \frac{1}{\lambda} = \frac{L_i'/\alpha}{L_j'/\alpha} = \frac{L_i'}{L_j'} \quad (23)$$

and

$$\frac{1}{\mu} = \frac{D_i}{L_j'/\alpha},$$

the same investigation as in the previous section applies to the present case. In other words, a cylindrical medium with L_j and L_i has approximately the same trapping factor as that of a spherical one whose slowing-down length is equal to L_j'/α and the thermal neutron diffusion length equal to L_i'/α . Both media are assumed to have the same diffusion coefficient D_i for thermal neutrons. This assumption is necessary to impose the same boundary condition (7) on the thermal neutron flux in both cases.

Thus, on referring to Eqs. (17) and (18), we find that the optimum radius and the maximum trapping factor in cylindrical geometry are, to the first approximation,

$$a_{m, cyl} \simeq \frac{4.95}{\alpha \left(\frac{1}{L_j'} + \frac{1}{L_i'} \right)} \quad (24)$$

$$\tau_{max, cyl} \simeq \frac{0.65}{D_i \alpha \left(\frac{1}{L_j'} + \frac{1}{L_i'} \right)} + 0.9. \quad (25)$$

In case of an infinite cylinder, L_j' and L_i' given by Eqs. (20) are identical with L_j and L_i , so that $a_{m, cyl}$ and $\tau_{max, cyl}$ are approximately $1/\alpha$ times as large as a_m (spherical) and τ_{max} (spherical), respectively.

In case of a finite cylinder, $a_{m, cyl}$ and $\tau_{max, cyl}$ will be reduced with decrease in the height H . It is obviously due to the

neutron leakage through the top and the bottom of the cylindrical column.

Eqs. (20) imply that the leakage effect becomes fairly large, when H is comparable to either $\pi L_j'$ or $\pi L_t'$. If, for instance, a cylinder is 50 cm high, $\tau_{\max, \text{cyl}}$ will be much lower than $\tau_{\max}$ (spherical) except the case of light water. In fact, Table 3 shows that light water has the highest trapping factor in this case (Case B). Numerical results in Table 3 as well as in Table 2 have been calculated from Eq. (13) with the aid of Fig. 3.

Table 3 Thermal neutron trapping in cylindrical geometry†

A : Case of infinite cylinder						
	L_j'	L_t'	λ	μ	Optimum radius $a_{m, \text{cyl}}$ (cm)	Max. trapping factor $\tau_{\max, \text{cyl}}$
H ₂ O	5.75	2.73	2.11	30.0	7.3	7.4
D ₂ O	11.8	89.7	0.131	16.2	47	10.1
Be	10.1	22.0	0.457	14.3	28	7.5
BeO	10.6	32.1	0.332	12.1	33	6.9
C	20.5	47.4	0.434	22.6	57	11.4
B : Case of finite cylinder ($H=50$ cm)						
H ₂ O	5.41	2.69	2.01	28.2	7.1	7.2
D ₂ O	9.46	15.7	0.603	13.0	23	6.4
Be	8.50	12.9	0.659	12.1	20	5.8
BeO	8.84	14.3	0.618	10.1	21	5.1
C	12.6	15.1	0.834	13.8	27	5.9

† Here L_j' and L_t' are defined by Eqs. (20), and λ and μ by Eqs. (23).

Thus, the new criterion of highly trapping media, derived in the preceding section, can be extended into cylindrical geometry: the thermal neutron diffusion coefficient of a trapping medium must be much smaller than each of the modified characteristic lengths L_j' and L_t' defined by Eqs. (20).

Recently, a few experimental data concerning the trapping were obtained by DeVilliers, *et al.*⁽²⁾ Their experiments were made in a critical assembly which consisted of a 50.8 cm high, annular fuel region enclosing an 12.7 cm × 12.7 cm light water internal column. The external radial reflector consisted of beryllium blocks and the axial reflectors were effectively infinite light water.

The radial distribution of thermal neutron flux was measured in case of a 7.25 kg core, in which ²³⁵U (in 93.5 % enrichment), H₂O, Al and stainless steel densities were 0.073 g/cm³, 0.384 g/cm³, 1.495 g/cm³ and 0.34 g/cm³.

The trapping factor, *i.e.*, the ratio of the thermal neutron flux peak at the center to the flux at the periphery of the column was found to be about 2.8. The radius of the equivalent cylinder with the same cross-sectional area equals 7.17 cm.

Placing a 0.635 cm aluminum liner along the periphery of the light water column, they obtained a slightly higher trapping factor about 3. This implies that the optimum radius of water column must be slightly less than 7.17 cm, still we may say that the experimental value agrees well with the calculated radius (7.1 cm) in Table 3.

On the other hand, the magnitude of the trapping factor is calculated at 7.2 in Table 3 and it is more than twice the experimental value. It seems that this difference results from the fact that the core in their experiment is not so black, since the absorption mean free path of thermal neutrons is estimated to be 6.5 cm. If a more concentrated fuel is used (or if a thin thermal-neutron absorber is placed along the periphery of the trapping column), the trapping factor may be expected to be raised, but it will be associated with increment of the critical mass and therefore with reduction of the ratio between the central thermal neutron flux and the total power of the reactor. It is opposite to the aim of their project.

3. Remark

We have assumed that an active shell is black for thermal neutron flow. If we consider that the active shell is actually not perfectly black, the trapping factor may be reduced to some extent.

By increasing a coolant mass in the active region, it will be generally possible to make the specific power and the thermal neutron flux higher. This effect will compete with the depletion of the trapping factor due to the lowered blackness of the active region.

Moreover, we have used the two-group approximation so that effects of epithermal neutrons have been neglected. It is needless to say that, in a more detailed study, effects of these neutrons should be taken into account. In such a study, however, the trapping phenomena are to be entangled with the criticality condition and in turn with the combination of an active shell and an external reflector, so that it will become hardly possible to make a comparative study of trapping media by an analytical approach.

In spite of the disadvantages mentioned above, the attainment of a reasonable improvement of Wigner's intuitive criteria suggests that our simple theory is properly treating the essential features of thermal neutron trapping.

Since we have dealt with only the ratio of the central thermal neutron flux to the peripheral flux, it remains to be proved that there is no higher thermal neutron flux than the central, throughout the trapping medium.

The thermal neutron flux in a spherical medium is given as follows, according to the second of Eqs. (2) and Eqs. (7) and (12):

$$\phi_t(r) = \frac{Sc_1}{r} \left[-C \sinh \frac{r}{L_t} + \sinh \frac{r}{L_f} \right], \quad (26)$$

where

$$C = \frac{1 + \frac{2}{\mu} \left(\coth x - \frac{1}{x} \right)}{1 + \frac{2}{\mu} \left(\lambda \coth \lambda x - \frac{1}{x} \right)} \frac{\sinh x}{\sinh \lambda x}. \quad (27)$$

Near the center, it becomes

$$\phi_t(r) = \phi_t(0) + \phi_2 r^2 + O(r^4), \quad (28)$$

where

$$\phi_2 = \frac{Sc_1}{6} \left(-\frac{C}{L_t^2} - \frac{1}{L_f^2} \right). \quad (29)$$

It can be shown that, when the size of the medium ($a = xL_f$) exceeds such a value that $\phi_2 = 0$, depression in the flux will appear at the center.

If we take an assumption that $\mu \gg 1$ and $\mu \gg \lambda$ as before, the equation $\phi_2 = 0$ may be written in a simpler form

$$\frac{\lambda^3 \sinh x}{\sinh \lambda x} \approx 1. \quad (30)$$

The root of this equation, denoted by $x_c(\lambda)$, can be approximately given by

$$x_c(\lambda) \approx \frac{2.94}{\lambda - 1} \ln \lambda, \quad (31)$$

which is derived in Appendix.

Eq. (31) being compared with Eq. (16), $x_c(\lambda)$ is definitely larger than $x_m(\lambda)$ for any positive λ . Hence, so long as the size of the medium does not exceed the optimum, i.e., $x_m(\lambda)$, the thermal neutron flux has a convex distribution near the center and therefore throughout the medium, since the flux is expressed by Eq. (26).

The same conclusion will apply to the cylindrical case, if Eqs. (21) are made use of.

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[Appendix]

The Derivation of expressions for $f(\lambda, x_m)$ and others

The function $f(\lambda, x)$ given by Eq. (9) satisfies the relation :

$$f(\lambda, x) = \frac{1}{\lambda} f\left(\frac{1}{\lambda}, \lambda x\right). \quad (A1)$$

Hence, if $x_m(\lambda)$ denotes the point where $f(\lambda, x)$ becomes maximum, say $f_m(\lambda)$, then $f_m(\lambda)$ and $x_m(\lambda)$ satisfy the type of equation

$$F(\lambda) = (1/\lambda) F(1/\lambda); \quad \lambda \geq 0. \quad (A2)$$

Putting

$$\begin{aligned} F(\lambda) &= (1/\sqrt{\lambda}) G(\lambda) \\ &= (1/\sqrt{\lambda}) G_1(\Lambda) \end{aligned} \quad \Lambda = \ln \lambda$$

we have, from Eq. (A2),

$$G_1(\Lambda) = G_1(-\Lambda).$$

Hence, $f_m(\lambda)$ as well as $x_m(\lambda)$ must have the form

$$(1/\sqrt{\lambda})G(\lambda), \quad (\text{A3})$$

where $G(\lambda)$ denotes an even function of λ or $\ln \lambda$.

Now, making numerical calculations $f_m(\lambda)$ and $x_m(\lambda)$ for some λ between 0 and 1, we find that they are finite at $\lambda=0$ and decrease with increasing λ , as shown in Fig. 3. Noting that a function $1/(1+\lambda)$ belongs to the expression (A3), we can write

$$f_m(\lambda) = \frac{c_f}{1+\lambda} \{1 + \varepsilon_f(\lambda)\} \quad (\text{A4})$$

$$x_m(\lambda) = \frac{c_x}{1+\lambda} \{1 + \varepsilon_x(\lambda)\} \quad (\text{A5})$$

Here c_f and c_x are constants, and $\varepsilon_f(\lambda)$ and $\varepsilon_x(\lambda)$ are considered to be correction factors, which are some even functions of $\ln \lambda$ and normalized so that $\varepsilon_f(1) = \varepsilon_x(1) = 0$. Then c_f and c_x are equal to $2f_m(1)$ and $2x_m(1)$, respectively, that is, $c_f = 1.358$ and $c_x = 4.50$.

It is not very troublesome to fit ε_f and ε_x to empirical simple expressions. If we adopt, for instance,

$$\varepsilon_f(\lambda) = -\frac{0.02\zeta}{1+0.14\zeta}$$

$$\varepsilon_x(\lambda) = 0.22(1 - e^{-\zeta/4}),$$

where

$$\zeta = \left(\sqrt{\lambda} - \frac{1}{\sqrt{\lambda}} \right)^2 = \lambda + \frac{1}{\lambda} - 2,$$

then $f_m(\lambda)$ and $x_m(\lambda)$ can be computed with errors less than 0.5% and 2%, respectively, in the whole positive range of λ .

In the text, however, it is needless to know precise values of f_m and x_m . Taking $\bar{\varepsilon}_f = -0.04$ and $\bar{\varepsilon}_x = 0.10$ on an average, we obtain the first

approximation, from Eqs. (A4) and (A5),

$$f_m(\lambda) \simeq \frac{c_f(1+\bar{\varepsilon}_f)}{1+\lambda} = \frac{1.30}{1+\lambda} \quad (\text{A6})$$

$$x_m(\lambda) \simeq \frac{c_x(1+\bar{\varepsilon}_x)}{1+\lambda} = \frac{4.95}{1+\lambda} \quad (\text{A7})$$

The resulting error is estimated to be less than 5% in Eq. (A6) and 10% in Eq. (A7) in the range of practical interest, $0.1 < \lambda < 10$.

Another example of Eq. (A2) is offered by the root of Eq. (31), i.e., $x_c(\lambda)$. Computing $x_c(\lambda)$ for some λ between 0 and 1, we find, after some trials,

$$x_c(\lambda) = \frac{2.99 \ln \lambda}{\lambda - 1} \{1 + \varepsilon_c(\lambda)\}, \quad (\text{A8})$$

where the correction factor $\varepsilon_c(\lambda)$ is an even function of $\ln \lambda$, so that the expression (A8) belongs to the category (A3).

Taking $\bar{\varepsilon}_c = -0.017$ on an average, we have

$$x_c(\lambda) \simeq \frac{2.99 \ln \lambda}{\lambda - 1} (1 + \bar{\varepsilon}_c) = \frac{2.94}{\lambda - 1} \ln \lambda \quad (\text{A9})$$

as the first approximation with an error less than 2% in the range $0.1 < \lambda < 10$,

Now, in regard to $\vartheta(\lambda, x)$ given by Eq. (10), we find

$$\vartheta(\lambda, x_m(\lambda)) = \vartheta(1/\lambda, x_m(1/\lambda)),$$

where $x_m(\lambda)$ satisfies the type of Eq. (A2), so that $\vartheta(\lambda, x_m)$ must be an even function of $\ln \lambda$. For the purpose of the text, however, it is enough to take an expression

$$\vartheta(\lambda, x_m) = 0.92 \pm 0.06$$

in the range $0.1 < \lambda < 10$.

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