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研究論文

232. An Analysis of Dynamic Characteristics of Plutonium- and/or Thorium-Recycled Reactor, (II)

Analytical approach to the stability

プルトニウム、トリウムリサイクル系における
原子炉動特性の解析, (II)

原子炉安定解析

By Itaru MICHİYOSHI* and Kunihiko UEMATSU*

The analytical approach to the stability of linear and nonlinear reactor kinetic equations which had been derived in the first paper of this serial report was made in this paper for both cases of constant power removal and Newton's law of cooling model. From the comparison of the results between linear and nonlinear analyses, the stability criterions obtained in terms of temperature coefficient and other thermal properties of the reactor materials were

$$r_1\alpha_1n_0 - r_2\alpha_2n_0 > 0 \quad \text{or} \quad r_1\alpha_1n_0b_2 - r_2\alpha_2n_0b_1 > 0$$

for constant power removal or Newton's law of cooling model, respectively.

It was proved that the delayed neutron fraction did not have any effect on the stability criterion and on the equilibrium values of power and temperature. The delayed neutron fraction did have an effect on reducing the maximum power and on the behavior of approaching the equilibrium values.

I. INTRODUCTION

In the first paper of this serial report, the author pointed out that the plutonium-and/or thorium-recycled reactor might have two temperature coefficients of reactivity, fuel and moderator, of the opposite sign, and the use of the effective delayed neutron fraction, $\beta_{eff}^{(1)}$, for the mixture of uranium, plutoniums and thorium reduced the complexity of the power excursion analysis of such reactor containing two or more species of the

fissile atoms. Using these reactor kinetic equations derived in the first paper of this serial report, the authors analyzed the stability or boundedness of reactor power and temperature for both linearized case of reactor kinetic equations which represent the small power excursion and nonlinear case which represent the large power excursion. The criterions of the reactor stability are reached in terms of temperature coefficient of reac-

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tivity and other thermal properties of the reactor materials.

The reference reactor used for this analysis is a high temperature gas-cooled, graphite-moderated reactor⁽²⁾ and the heat removal models used are constant power removal and Newton's law of cooling.

II. LINEARIZED CONSTANT POWER REMOVAL

The solutions of the linearized reactor kinetic equations have been rather completely analyzed by many researchers and their results can be seen in many reports and books of the reactor theory. For an example, a report of Ergen and Weinberg⁽³⁾ can be named as a classical one. However, the most of those reports have been concerned with the case of a reactor which has only one temperature coefficient of reactivity, so called overall temperature coefficient. The authors analyzed the reactor kinetic equations containing two temperature coefficients of reactivity which derived in the first paper of this serial report.

These reactor kinetic equations are linearized with a proper approximation of which Weinberg and Wigner⁽⁴⁾ did in their book. The coupling term of temperature and power, xy , which makes the reactor kinetic equations nonlinear is replaced by the coupling term of the constant initial temperature and power. This assumption implies a neglect of second order term only and renders the equation linear in temperature and power. The linearized equations are, then,

$$\begin{aligned}\frac{dx}{dt} &= Ax - Cy + D \\ \frac{dy}{dt} &= x.\end{aligned}\quad (1)$$

This set of equations is simple first order simultaneous differential equation and can be reduced to

$$\begin{aligned}\frac{d^2x}{dt^2} - A\frac{dx}{dt} + Cx &= 0 \\ \frac{dy}{dt} &= x,\end{aligned}\quad (2)$$

and then can be easily solved for a general case with a proper set of the initial conditions.

$$\begin{aligned}x &= 0, \quad \frac{dx}{dt} = D \\ y &= 0, \quad \frac{dy}{dt} = 0\end{aligned}\quad \text{at } t = 0.$$

However, solving Eq. (2) is somewhat tedious for only obtaining the stability criterion of this system. The simpler technique to reach this criterion is to use the Laplace Transform of Eq. (1) and its special characteristics. The Laplace Transform of Eq. (1) are,

$$\begin{aligned}s\mathcal{L}(x) &= A\mathcal{L}(x) - C\mathcal{L}(y) + \frac{D}{s} \\ s\mathcal{L}(y) &= \mathcal{L}(x).\end{aligned}\quad (3)$$

Then,

$$\begin{aligned}\mathcal{L}(x) &= \frac{D}{s^2 - As + C} \\ \mathcal{L}(y) &= \frac{D}{s(s^2 - As + C)}.\end{aligned}\quad (4)$$

The clues of obtaining the stability criterion from Eq. (4) can be reached from the characteristics of the Inverse Laplace Transform and they are summarized as follows;

- (1) If the denominator of Eq. (4) have a set of roots which are all negative and real or all with negative real part, Eq. (1) is stable, since the solutions of Eq. (1) are exponentially decreasing functions with time.
- (2) If the roots of the denominator of Eq. (4) are all negative and real, the solutions are aperiodic.
- (3) If the roots of Eq. (4) are all complex conjugates, the solutions are periodic.

The roots of the denominator of Eq. (4) are

$$s_{1,2} = \frac{A \pm \sqrt{A^2 - 4C}}{2}.\quad (5)$$

Therefore, the stability criterion of Eq. (1), which satisfies the condition (1) above mentioned, is

$$\begin{aligned}A < 0 \quad \text{or} \quad \Delta k < \frac{\beta}{1 - \beta} \\ C > 0 \quad \text{or} \quad r_1 a_1 n_0 - r_2 a_2 n_0 > 0,\end{aligned}\quad (6)$$

regardless of the periodic or aperiodic solution. The criterion of a periodic solution is

$$A^2 < 4C\quad (7)$$

from the condition (3). If Eq. (7') is satisfied, the periodic solution only exists.

$$\frac{\beta}{2\ell} < \sqrt{\frac{r_1 a_1 n_0 - r_2 a_2 n_0}{\ell}} \quad (7')$$

For the existence of an aperiodic solution,

$$A^2 > 4C$$

or
$$\frac{\beta}{2\ell} > \sqrt{\frac{r_1 a_1 n_0 - r_2 a_2 n_0}{\ell}} \quad (8)$$

have to be satisfied from the condition (2). The summary of these results is shown in Table 1. The criterion of an aperiodic solution (8) is similar to Weinberg and Wigner's condition⁽⁴⁾ which is applicable to one temperature coefficient of reactivity case.

The assumption, of course, made for the linearization of the reactor kinetic equations has to be taken account into the stability criterion (6). Assuming that x and y equal

to an initial temperature and an initial power, respectively, the coupling term between temperature and power becomes zero, *i.e.*,

$$xy = 0. \quad (9)$$

Therefore, the power rise, x , and temperature rise, y , must be very small to satisfy this assumption which is the case of a small excess reactivity, Δk . Thus, one has to realize that the linearized analysis applies only for a small perturbation of the reactor power.

When the power excursion is stable, power and temperature asymptotically approach the equilibrium values

$$x_{\infty} = 0, \quad y_{\infty} = \frac{\Delta k}{r_1 a_1 n_0 - r_2 a_2 n_0}. \quad (10)$$

Table 1 Stability conditions (Constant power removal, linearized)

$\Delta k < \frac{\beta}{1-\beta}$ $r_1 a_1 n_0 - r_2 a_2 n_0 > 0$	$\frac{\beta}{2\ell} < \sqrt{\frac{r_1 a_1 n_0 - r_2 a_2 n_0}{\ell}}$	Stable ; Periodic solution only exists
	$\frac{\beta}{2\ell} > \sqrt{\frac{r_1 a_1 n_0 - r_2 a_2 n_0}{\ell}}$	Stable ; Aperiodic solution exists
$\Delta k < \frac{\beta}{1-\beta}$ $r_1 a_1 n_0 - r_2 a_2 n_0 = 0$		Power converges but temperature diverges
$r_1 a_1 n_0 - r_2 a_2 n_0 < 0$		Unconditionally unstable

III. CONSTANT POWER REMOVAL (Nonlinear)

The necessary condition for the boundedness of the power excursion in a linearized reactor system was obtained in the former section. However, it should be emphasized that the linearized analysis is not adequate to estimate the boundedness of the power excursion which may be occurred at the introduction of a large excess reactivity. The nonlinear reactor kinetic equations must be investigated for the large power excursion analysis.

There are several analytical methods investigating nonlinear systems; numerical method, the phase plane method, describing function method, *etc.* The method applied in this section is similar to the phase plane method which is one of the most powerful

analytic tools presently available. The phase plane method is the analysis of the phase portrait consists of a number of phase trajectories in the dx/dt vs. x plane or in some plane related to this plane through a convenient transformation of variables. The reactor kinetic equations to be considered here are

$$\begin{aligned} \frac{dx}{dt} &= Ax - Bxy - Cy + D \\ \frac{dy}{dt} &= x. \end{aligned} \quad (11)$$

Therefore, Eq. (11) is not the real phase plane equations but they can be treated as if they were phase plane equations. From Eq. (11)

$$\frac{dx}{dy} = \frac{Ax - Bxy - Cy + D}{x}, \quad (12)$$

Eq. (12) has a singular point at

$$x = 0, \quad y = \frac{\Delta k}{r_1 a_1 n_0 - r_2 a_2 n_0}. \quad (13)$$

The investigation of the nature of this singu-

lar point gives the information on the prediction of stability and of oscillations. Poincaré's method⁽⁵⁾ may be the best approach to this point for this specific problem. It is found that the characteristic equation of Eq. (12) is

$$\lambda^2 - \left(A - \frac{BD}{C}\right)\lambda + C = 0. \quad (14)$$

Table 2 Characteristics of the singular point

	Roots	Comments
$\frac{\beta^2}{4\ell} \geq r_1 a_1 n_0 - r_2 a_2 n_0 > 0$	Both are real and negative	Stable node ; Aperiodic
$r_1 a_1 n_0 - r_2 a_2 n_0 > \frac{\beta^2}{4\ell}$	Both are complex conjugates with negative real part	Stable focus ; Periodic
$r_1 a_1 n_0 - r_2 a_2 n_0 < 0$	Real and opposite sign	Saddle ; Diverges
$r_1 a_1 n_0 - r_2 a_2 n_0 = 0$	Eq. (11) becomes linear	See Table 1

Therefore, the stability criterion reached here is

$$r_1 a_1 n_0 - r_2 a_2 n_0 > 0 \quad (16)$$

for any given excess reactivity, Δk . The condition of an aperiodic oscillation is the same as the results of the linearized analysis.

$$\frac{\beta}{2\ell} > \sqrt{\frac{r_1 a_1 n_0 - r_2 a_2 n_0}{\ell}} \quad (17)$$

It is found that power and temperature asymptotically approach the equilibrium values

$$x_{\infty} = 0, \quad y_{\infty} = \frac{\Delta k}{r_1 a_1 n_0 - r_2 a_2 n_0} \quad (18)$$

in case of a stable power excursion. These equilibrium values are the function of the excess reactivity, Δk , and temperature coefficients, r_1 and r_2 , but the delayed neutron fraction, β . However, it was concluded from the same kind of analysis⁽⁶⁾ that gradient, α , of phase trajectories approaching the equilibrium value, temperature at the maximum power, y at $x_{\max}$, and the maximum power, $x_{\max}$, are the function of the delayed neutron fraction, β , which is an important parameter to evaluate the difference of the reactor dynamics between plutonium recycled and non-plutonium recycled reactor.

$$\alpha = -\beta/\ell$$

$$x_{\max} \cong \frac{\beta}{\beta - \Delta k} \quad \text{for small } \Delta k, \quad (19)$$

$$y \text{ at } x_{\max} \cong \frac{1}{r_1 a_1 n_0 - r_2 a_2 n_0} \left[\Delta k - \frac{\beta}{1 - \beta} \right] \quad \text{for large } \Delta k.$$

The roots of this characteristic equation are

$$\lambda_{1,2} = \frac{-\beta \pm \sqrt{\beta^2 - 4\ell(r_1 a_1 n_0 - r_2 a_2 n_0)}}{2\ell}. \quad (15)$$

The results of the investigation of the singular point through the Poincaré's stability criterion are summarized in Table 2.

Fig. 1 shows a schematic representation of these results.

Fig. 1 Schematic representation of phase trajectories for same Δk but different β .

IV. NEWTON'S LAW OF COOLING (Linearized)

Constant power removal model was investigated in Chaps. II and III of this paper and those results showed that the reactor was stable unless $r_1 a_1 n_0 - r_2 a_2 n_0 \leq 0$. However, as is

stated in Part I of this serial report, constant power removal model is not the adequate model to represent the heat transfer characteristics of a reactor. It is tolerable only for a time short before power and temperature reaching the significant values. Newton's law of cooling is much more proper representation of heat transfer model of a reactor. Unfortunately this model leads to the complicated reactor kinetic equations which are usually consisted of more than two differential equations which may be nonlinear in temperature and power.

$$\begin{aligned}\frac{dx}{dt} &= Ex - Fxy + Gxz - Hy + Kz + L \\ \frac{dy}{dt} &= x - b_1 y \\ \frac{dz}{dt} &= x - b_2 z\end{aligned}\quad (20)$$

The linearized analysis of Eq. (20) would not yield much difference between constant power removal and Newton's law of cooling model for a very small Δk . Poincaré's method has no value on the analysis of Eq. (20) except for a special case of $b_1 = b_2 \equiv b$ which is very unlikely in the ordinary thermal reactor.

There are two cases of the linearized form of Eq. (20), which are $b_1 = b_2$ and $b_1 \neq b_2$, to be investigated. The former case leads to

$$\begin{aligned}\frac{dx}{dt} &= Ex - (H - K)y + L \\ \frac{dy}{dt} &= x - by.\end{aligned}\quad (21)$$

Applying the Laplace Transform on Eq. (21),

$$\begin{aligned}\mathcal{L}(x) &= \frac{L(s+b)}{s[s^2 - (E-b)s + (H-K)]} \\ \mathcal{L}(y) &= \frac{L}{s[s^2 - (E-b)s + (H-K)]}.\end{aligned}\quad (22)$$

The same rules, which have been mentioned in the Chap. II, are true again here. The real part of all roots of the denominator of Eq. (22) to be negative, (23) has to be satisfied from the Hurwitz's theorem.

$$\begin{aligned}E - b < 0 \quad \text{or} \quad \Delta k < \frac{\beta}{1-\beta} + \frac{b\ell}{1-\beta} \\ H - K > 0 \quad \text{or} \quad r_1 a_1 n_0 - r_2 a_2 n_0 > 0\end{aligned}\quad (23)$$

Therefore, Eq. (23) is the stability criterion of Eq. (21). The necessary condition of the existence of an aperiodic solution of Eq. (21)

is to satisfy Eq. (24) besides Eq. (23).

$$(E-b)^2 - 4(H-K) > 0$$

or

$$\frac{\beta + b\ell}{2\ell} > \sqrt{\frac{r_1 a_1 n_0 - r_2 a_2 n_0}{\ell}}\quad (24)$$

If there exist any equilibrium values of x and y at $t = \infty$, the excess reactivity, k_{ex} , should be zero at $t = \infty$. Thus,

$$\begin{aligned}x_{\infty} &= \frac{\Delta k b}{r_1 a_1 n_0 - r_2 a_2 n_0} \\ y_{\infty} &= \frac{\Delta k}{r_1 a_1 n_0 - r_2 a_2 n_0}.\end{aligned}\quad (25)$$

If $b_1 \neq b_2$, the linearized form of Eq. (20) becomes

$$\begin{aligned}\frac{dx}{dt} &= Ex - Hy + Kz + L \\ \frac{dy}{dt} &= x - b_1 y \\ \frac{dz}{dt} &= x - b_2 z.\end{aligned}\quad (26)$$

Using the Laplace Transform,

$$\begin{aligned}\mathcal{L}(x) &= \frac{L(s+b_1)(s+b_2)}{s(s^3 + Ps^2 + Qs + R)} \\ \mathcal{L}(y) &= \frac{L(s+b_2)}{s(s^3 + Ps^2 + Qs + R)} \\ \mathcal{L}(z) &= \frac{L(s+b_1)}{s(s^3 + Ps^2 + Qs + R)}\end{aligned}\quad (27)$$

where

$$\begin{aligned}P &= b_1 + b_2 - E \\ Q &= b_1 b_2 - E(b_1 + b_2) + (Hb_2 - Kb_1) \\ R &= (Hb_2 - Kb_1) - Eb_1 b_2.\end{aligned}$$

From the Hurwitz's theorem, the necessary and adequate condition of all roots having negative real part are

$$\begin{aligned}P > 0, \quad Q > 0, \quad R > 0, \\ PQ - R > 0.\end{aligned}\quad (28)$$

Therefore, if Eq. (28) is satisfied, Eq. (26) is stable. Rewriting Eq. (28) with $r_1, r_2, \beta, \ell, b_1$ and b_2 into a very general form is not easy and they are all the limitations imposed on the value of the initial step reactivity, Δk , or on E .

$$\begin{aligned}E < b_1 + b_2, \quad E < \frac{Hb_2 - Kb_1}{b_1 b_2}, \\ E < \frac{b_1 b_2 + (Hb_2 - Kb_1)}{b_1 + b_2}, \\ (b_1 + b_2)(E - b_1)(E - b_2) \\ - [E + 1 - (b_1 + b_2)](Hb_2 - Kb_1) > 0.\end{aligned}\quad (29)$$

If there exist any equilibrium values of x, y and z at $t = \infty$, they are

$$\begin{aligned}x_{\infty} &= \frac{\Delta k b_1 b_2}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1} \\y_{\infty} &= \frac{\Delta k b_2}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1} \\z_{\infty} &= \frac{\Delta k b_1}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1},\end{aligned}\quad (30)$$

and they should be positive numbers. Therefore,

$$r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1 > 0 \quad (31)$$

is a necessary condition should be satisfied.

The damping oscillation may occur on the solution of Eq.(26), when the denominator of Eq.(27) has a set of complex conjugates roots and Eq.(29) is satisfied at the same time. The condition of a set of complex conjugates roots being exist can be obtained from the discriminator of the denominator of Eq.(27) and it is

$$-4Q^3 + P^2Q^2 - 27R^3 + 18PQR - 4P^3R < 0. \quad (32)$$

V. NEWTON'S LAW OF COOLING (Nonlinear)

The nonlinear reactor kinetic equations of Newton's law of cooling model are quite complex and difficult to solve without having the help of digital or analogue computer. The only analytical approach to this problem is the Liapunov's second method⁽⁷⁾. However, the requirement of existence of a Liapunov's function may be over restricting for a practical use. Gyptopoulos⁽⁸⁾ of Massachusetts Institute of Technology handsomely applied the Liapunov's second method on the analysis of the reactor kinetic equations in which he considered two temperature coefficients of reactivity, Newton's law of cooling and linear approximation near at the equilibrium point and reached the stability criterion of the system. The authors tried a much simpler approach to get this stability criterion.

If there exists any equilibrium point within physically feasible domain of power and temperature to which power and temperature approach, periodically or aperiodically, at the infinite time after the introduction of a step reactivity and if the criterion for the existence of this equilibrium point can be reached, this must be the necessary criterion for the sta-

bility of the system in question.

Eq.(20) and a reactivity equation

$$k_{\text{ex}} = \Delta k - r_1 a_1 n_0 y + r_2 a_2 n_0 z \quad (33)$$

are the system to be considered here. If there exists any equilibrium point at $t = \infty$, k_{ex} must be zero at such point. Otherwise, the system can not be stable. Then,

$$\Delta k = r_1 a_1 n_0 y_{\infty} - r_2 a_2 n_0 z_{\infty} \quad \text{at } t = \infty. \quad (34)$$

Also, dx/dt , dy/dt and dz/dt have to be zero at $t = \infty$ being the system stable. Then, from Eq.(20),

$$\begin{aligned}y_{\infty} &= \frac{x_{\infty}}{b_1} = \frac{\Delta k b_2}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1} \\z_{\infty} &= \frac{x_{\infty}}{b_2} = \frac{\Delta k b_1}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1}.\end{aligned}\quad (35)$$

Using Eqs.(34) and (35),

$$x_{\infty} = \frac{\Delta k b_1 b_2}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1}. \quad (36)$$

To give any physically feasible meaning to this equilibrium point, Eqs.(35) and (36) must be positive. Since Δk , b_1 and b_2 are always positive, the criterion of the stability of Eq.(20) is

$$r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1 > 0. \quad (37)$$

This is a necessary condition of the stability but might not be an adequate condition. Since this necessary condition is reached on the assumption of the equilibrium values being exist for any given Δk , the hypothesis on Δk must be proved for Eq.(37) being an adequate condition. Fortunately, Gyptopoulos already proved this hypothesis in his paper⁽⁸⁾. Therefore, Eq.(37) can be a necessary and adequate condition of the stability.

VI. SUMMARY

The stability criterions of the linear and nonlinear reactor kinetic equations with two temperature coefficients of reactivity and delayed neutron were obtained for both constant power removal and Newton's law of cooling model. The results of these analyses were summarized in Table 3.

The universal stability criterion was

$$\begin{aligned}\text{or} \quad & r_1 a_1 n_0 - r_2 a_2 n_0 > 0 \\& r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1 > 0\end{aligned}\quad (38)$$

Table 3 Stability criterion

	Stability condition	Aperiodic condition	Equilibrium values
Constant power removal model	Linear $\Delta k < \frac{\beta}{1-\beta}$ $r_1 a_1 n_0 - r_2 a_2 n_0 > 0$ necessary and adequate condition	$\frac{\beta}{2\ell} > \sqrt{\frac{r_1 a_1 n_0 - r_2 a_2 n_0}{\ell}}$	$x_\infty = 0$ $y_\infty = \frac{\Delta k}{r_1 a_1 n_0 - r_2 a_2 n_0}$
	Nonlinear any Δk $r_1 a_1 n_0 - r_2 a_2 n_0 > 0$ necessary and adequate condition	$\frac{\beta}{2\ell} > \sqrt{\frac{r_1 a_1 n_0 - r_2 a_2 n_0}{\ell}}$	$x_\infty = 0$ $y_\infty = \frac{\Delta k}{r_1 a_1 n_0 - r_2 a_2 n_0}$
Newton's law of cooling model	Linear $b_1 = b_2 \equiv b$ $\Delta k < \frac{\beta}{1-\beta} + \frac{b\ell}{1-\beta}$ $r_1 a_1 n_0 - r_2 a_2 n_0 > 0$ necessary and adequate condition	$\frac{\beta + b\ell}{2\ell} > \sqrt{\frac{r_1 a_1 n_0 - r_2 a_2 n_0}{\ell}}$	$x_\infty = \frac{\Delta k b}{r_1 a_1 n_0 - r_2 a_2 n_0}$ $y_\infty = \frac{\Delta k}{r_1 a_1 n_0 - r_2 a_2 n_0}$
	Linear $b_1 \neq b_2$ Eq. (29) $r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1 > 0$ necessary and adequate condition	Eq. (32)	$x_\infty = \frac{\Delta k b_1 b_2}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1}$ $y_\infty = \frac{\Delta k b_2}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1}$ $z_\infty = \frac{\Delta k b_1}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1}$
	Nonlinear $r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1 > 0$ necessary condition		$x_\infty = \frac{\Delta k b_1 b_2}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1}$ $y_\infty = \frac{\Delta k b_2}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1}$ $z_\infty = \frac{\Delta k b_1}{r_1 a_1 n_0 b_2 - r_2 a_2 n_0 b_1}$

for constant power removal or Newton's law of cooling, respectively. Comparison between linear and nonlinear analysis showed that even if the linearized analysis gave the unbounded solution with a large excess reactivity, this did not necessarily mean that the reactor system considered here was unbounded. This was evident from the nonlinear analysis which showed the bounded solution with the condition (38) even with a large excess reactivity which led to the unbounded solution of linearized analysis.

The condition of the existence of aperiodic solution was obtained for constant power removal model and it showed that the condition arrived here was similar to that of Weinberg and Wigner's. For Newton's law of cooling model, this condition was too complicated to be presented in a simple form.

When the system was stable, power and

temperature asymptotically approached the equilibrium values which were shown in Table 3. These equilibrium values showed no difference between linear and nonlinear analysis. The maximum temperature of fuel and moderator which were the great concern of reactor safety could be estimated from these results.

The delayed neutron fraction, β , did not have any effect on the nonlinear stability of the system nor on the equilibrium values of power and temperature. This was the proof of that the stable reactor without delayed neutron was also stable with delayed neutron. However the delayed neutron fraction, β , did have the effect on the condition of the existence of an aperiodic solution and on the reduction of the maximum power.

(Received June 14, 1962)

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[Nomenclature]

$$A : \frac{1}{\ell} [\Delta k(1-\beta) - \beta]$$

$$B : \frac{1}{\ell} (1-\beta)(r_1 a_1 n_0 - r_2 a_2 n_0)$$

$$C : \frac{1}{\ell} (r_1 a_1 n_0 - r_2 a_2 n_0)$$

$$D : \frac{\Delta k}{\ell}, \quad E : A$$

$$F : \frac{1}{\ell} (1-\beta) r_1 a_1 n_0$$

$$G : \frac{1}{\ell} (1-\beta) r_2 a_2 n_0$$

$$H : \frac{1}{\ell} r_1 a_1 n_0$$

$$K : \frac{1}{\ell} r_2 a_2 n_0, \quad L : D$$

Δk : a step excess reactivity due to the outside cause

ℓ : effective neutron life time

β : effective delayed neutron fraction which was defined in Part I of this report

r_1 : temperature coefficient of fuel (considered as a negative)

r_2 : temperature coefficient of moderator (considered as a positive)

$a_{1,2}$: coefficient of power removal in the fuel and moderator, respectively

$b_{1,2}$: inverse time constant of the temperature increase in the fuel and moderator, respectively

研究論文

233. Continuous Monitoring of Low Level Radioactive Waste Water

低レベル放射能連続監視装置

By Shuji YAMADA* and Hisashi YAMAMOTO*

A continuous β monitor was developed for transparent liquid streams using Cerenkov effect. The Cerenkov pulse was discriminated from the thermal noises of the photomultipliers by the coincidence counting method and by cooling the photomultipliers. In order to determine the optimum measuring condition, the signal to noise ratios were observed with the various operating voltages of the photomultipliers, the gains of the amplifier and the volumes of the detection container.

The minimum activity level that can be detected reliably with this apparatus is $0.8 \times 10^{-7} \mu\text{C/cc}$ for ^{90}Sr (plus ^{90}Y in equilibrium) when counting time is 5 min.

I. INTRODUCTION

In an atomic plant, it is necessary to

establish the continuous waste water monitor

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