Reality and Non-Reality of Physical Variables

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Abstract

Quantum mechanics is widely believed to be inconsistent with local realism because J.S. Bell proved the contradiction between quantum mechanics and the hidden variable theories based on reality and locality of matter. As described herein, the relation between quantum mechanics and local realism is discussed. This paper presents the question of whether any version of local realism is contradicted by quantum mechanics. The answer is negative. An interpretation is proposed by which the contradiction is explained using physical variables of two types: (1) physical variables of the first type, such as positions and angles (i.e. coordinates), are realistic variables and always possess definite values independently of measurements; (2) those of the second type, such as momenta, angular momenta, and spins (i.e., conserved quantities), are non-realistic variables, which possess no value before measurement. Value definiteness of the first type provides a reliable basis to physical reality of matter, and non-reality of the second type might solve the problem of inconsistency between quantum mechanics and local realism.

Key words: Quantum Mechanics, Statistical Interpretation, Realism

1. Introduction

Quantum mechanics is an extremely successful theory. Predictions of quantum mechanics have been confirmed using numerous experimental results. However, philosophical aspects of quantum mechanics remain controversial. This fact has provoked many physicists, philosophers, and mathematicians to debate the foundations of quantum mechanics.

The most famous debate was that between A. Einstein and N. Bohr in the 1920s and 1930s (see M. Jammer 1974, A. Fine 1996). In the debate, Einstein presented several problems related to foundations of quantum mechanics from the viewpoint of a realist. Bohr answered from the viewpoint of a non-realist. The most serious problem was given in the paper titled “Can Quantum Mechanical Description of Physical Reality Be Considered Complete?” (Einstein, Podolsky, and Rosen 1935). As described

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in this paper, Einstein, Podolsky, and Rosen (hereinafter “EPR”) discussed a thought experiment in which correlated systems possessing fixed values of the relative position and the total momentum were prepared. After the systems become sufficiently distant from one another, the position of one system and the momentum of the other system are measured. Consequently, the position and momentum of a system can be determined simultaneously, which breaks the restriction of Heisenberg’s uncertainty relation.

Based on this thought experiment, EPR claimed that quantum mechanical description is incomplete. The claim is based on the following assumptions. (1) *the EPR criterion of reality*: if, without in any way disturbing a system, the value of a physical quantity can be predicted with certainty (i.e., with probability equal to unity), then there exists an element of reality corresponding to that quantity; (2) *the EPR assumption of locality*: if two systems are sufficiently distant from one another, the measurement of one system does not directly affect the reality that pertains to the unmeasured system. Bohr responded to EPR’s paper from the viewpoint of a non-realist as follows. He first pointed out the ambiguity in the meaning of the expression of “without in any way disturbing a system” in the criterion of reality. Subsequently, he claimed nonseparability of two correlated (entangled) systems (Bohr 1935).

D. Bohm developed the EPR thought experiment (Bohm 1951). Correlated systems for which spin values are totally zero are prepared first. They fly off freely in opposite directions. Then their spin values were measured at distant positions. (Spins took the place of position and momentum in the EPR thought experiment.) Measured values were mutually anti-correlated. This is the Bohm version of the EPR thought experiment.

Using the Bohm version, J.S. Bell showed that, under a certain set of assumptions, the correlations obtained as results of the Bohm thought experiment can be expected to satisfy a particular set of inequalities: so-called Bell’s inequalities (Bell 1964). The assumptions adopted by Bell are “value definiteness” and “locality.” (1) *Value definiteness*: Systems have definite values (+1 or -1) of spins independently of measurements. (2) *Locality*: The measurement of one system does not affect the other system if two systems are sufficiently distant from one another. Any model based on these assumptions must satisfy Bell’s inequalities, but predictions of quantum mechanics do not satisfy them. Violation of Bell’s inequalities, as confirmed by numerous experiments, means that at least one of the assumptions is incorrect. That violation is usually interpreted as showing nonlocality of quantum mechanics. However, the possibility exists that quantum mechanics does not satisfy value definiteness but does satisfy locality.

This possibility is discussed in detail herein. In Section 2, the philosophical problems of quantum mechanics are divided into four groups. Section 3 shows that the ensemble interpretation solves three of the four problems. Subsequently, a mistake in

Bell's conclusion is explained in Section 4. The ensemble interpretation is developed in Section 5. The developed version introduces realistic and non-realistic variables. The realistic variables (positions, angles, etc.) always possess definite values and give a solid basis to physical reality, although the non-realistic variables (momenta, angular momenta, and spins) possess no values before measurement but instead open up the possibility of explaining the violation of Bell's inequality.

2. Four problems of quantum mechanics

Several problems are related to the philosophical aspects of quantum mechanics. They can be divided into four groups. The first is the problem of indeterminism. Quantum mechanics is a probability theory that provides statistical predictions of behaviors of a system. In this point, quantum mechanics resembles other probability theories (statistical mechanics, stochastic theory, etc.). However, probability in quantum mechanics is fundamentally different from probabilities used in other theories. In quantum mechanics, probabilistic behavior of a system is regarded as the nature of a system that cannot be ascribed to some interaction with other systems. In contrast, in the other theories the probabilistic behavior of a system is interpreted to originate from some disturbances by other systems: the nature of a system is viewed as deterministic. Quantum mechanics denies determinism as the nature of a system. The first problem is whether to accept indeterminism or not.

Secondly, the formalism of quantum mechanics differs greatly from that of other probability theories. Time evolution of probability $P(x, t)$ is not calculated directly from the basic equation, called the Schrödinger equation, but is instead derived from the square of a wave function.

$$P(x, t) = |\Psi(x, t)|^2 \quad (1)$$

Although the Schrödinger equation determines the time evolution of a wave function for the intervals between measurements, it does not describe the change of a wave function in measurement. When a certain value of a physical variable is obtained in measurement, the wave function changes instantaneously to the state of that value. This sudden change, which is known as the collapse of a wave function, cannot be explained using the Schrödinger equation. The second problem is how to explain the wave function collapse.

Thirdly, probability distribution functions $P(x, t)$ are non-additive in quantum mechanics, although they are additive in other theories. The Schrödinger equation is the equation of a wave function $\Psi(x, t)$ and is linear. For this reason, wave functions $\Psi(x, t)$ are additive but probability distribution functions $P(x, t)$ are not additive in quantum mechanics. Such non-additivity presents a difficult problem because the concept of probability is usually connected with set theory. In naive set theory, a set

is defined as an ensemble (a collection) of distinct objects; that ensemble is, of course, additive. When probability distribution functions are non-additive, we cannot understand them using the concept of a set or ensemble. In other words, non-additivity suggests that the probability distribution functions cannot be based on usual set theory (Boolean logic). Imagining any kind of ensemble is forbidden behind quantum probabilities. Consequently, the third problem is how to interpret non-additive probabilities.

The problem of non-additivity is related closely with the problem of wave-particle duality and nonlocality¹. The Schrödinger equation has a form of the wave equation, so that wave functions propagate nonlocally, as a wave. In the famous double-slit experiment, electrons ejected from a gun passed through double slits and arrived at a screen behind the slits. The distribution of electrons on the screen displays an interference pattern, showing wave-like characteristics of electrons. On the other hand, a particle-like character of electrons can be observed simultaneously. In the same experiment, each electron is detected at one point on the screen but is not observed to be spread, as a wave would be. Individual electrons are always detected as particles. Therefore, both a wave character and a particle character of electrons are recognizable. This is the wave-particle duality problem.

The wave-particle duality produces non-additivity of the probability. Consider the probability distributions $P_{1+2}(x)$, $P_1(x)$, and $P_2(x)$ on the screen: $P_{1+2}(x)$ is the probability that an electron goes through slit 1 and/or 2 and arrives at a certain position x on the screen when both slits are open; on the other hand, $P_1(x)$ ($P_2(x)$) is the probability that an electron goes through slit 1 (slit 2) and arrives at a certain position x on the screen when the other slit is closed. Because a wave character appears in the former case but disappears in the latter case, the former probability does not agree with the sum of the latter two:

$$P_{1+2}(x) \neq c_1 P_1(x) + c_2 P_2(x), \quad (\text{stochastic theory}) \quad (2)$$

where c_1 and c_2 respectively signify the ratios of electrons passing through slits 1 and 2. This kind of non-additivity reflects a wave-particle duality.

Fourth, a difference exists between quantum mechanics and the other probability theories in representation of physical variables. In quantum mechanics, when A is a physical variable, it is represented as an operator $\hat{A}$, although it is given as a function $A(x, t)$ in the other theories. For this reason, physical variables can be noncommutative in quantum mechanics but they are commutative in the other theories.

$$\hat{A}\hat{B} \neq \hat{B}\hat{A} \quad (\text{quantum mechanics})$$

¹ The nonlocality we discuss here does not include the nonlocality (nonseparability) discussed in relation to the EPR thought experiment but is limited to the nonlocality attributable to a wave character (i.e., superposition principle of a probability amplitude).

$$A(x, t)B(x, t) = B(x, t)A(x, t) \quad (\text{the other theories}) \quad (3)$$

It is known that when a pair of noncommutative variables A and B exists ($[A, B] \neq 0$), an uncertainty relation exists between the statistical deviations.

$$(\Delta A)^2(\Delta B)^2 \geq \frac{1}{4} |[A, B]|^2 \quad (4)$$

In the mathematical formulation using a Hilbert space, wave functions are unit vectors, called state vectors in a Hilbert space, and physical variables are operators acting on the state vectors. In this formulation, noncommutativity corresponds to nonorthogonality of eigenvectors of the operators. Accordingly, uncertainty relations can be derived from the nonorthogonality in exactly the same way as they are derived from the noncommutativity. Using this equivalence, Bell showed that the philosophical problem of locality and value definiteness of spins (i.e., the uncertainty relation between two spin components) are equivalent to the mathematical problem whether the value of 0 or 1 can be assigned to all points on the unit sphere in a Hilbert space under certain conditions (Bell 1966). He started from Gleason's theorem and proved that the value assignment is impossible. The result indicates that a system cannot possess definite values of spins. The values must be dependent on experimental arrangements: they are contextual. Another proof of the contextuality was given by S. Kochen and E.P. Specker (1967). Their proof was also made in the same manner as showing the impossibility of the value assignment. Therefore, the fourth problem is how to interpret the contextuality of physical variables.

The philosophical problems of quantum mechanics can be summarized as the following four. (1) indeterminism (2) the wave function collapse (3) nonlocality (wave-particle duality, non-additivity of the probability) (4) contextuality (value definiteness, noncommutativity, uncertainty).

3. Ensemble interpretation (Statistical interpretation)

The ensemble interpretation (statistical interpretation) has succeeded in solving three of the four problems presented above. The ensemble interpretation is based on Einstein's idea that the wave function is not a description of an individual system but is instead a description of an ensemble of similarly prepared systems. Actually, K.R. Popper developed Einstein's interpretation and proposed the ensemble interpretation of Heisenberg's uncertainty relations. The uncertainty relations are not restrictions on individual measurements but they do determine lower limits of standard deviations in statistical ensembles (Popper 1992). In Einstein's and Popper's interpretations, quantum mechanics is viewed as a description of a statistical ensemble. They described that all physical variables can possess definite values simultaneously. An important difference between Einstein's and Popper's interpretations is that Einstein

did not accept indeterminism, but Popper accepted it.

The ensemble interpretation was supported by L.E. Ballentine (Ballentine 1970) and others (e.g., Shirai 1998, 2005a, 2005b, 2006). On the other hand, severe criticisms have also been leveled at it. For example, T. Ishigaki pointed out difficulties in both the ensemble interpretation and in the other interpretations (Ishigaki 1994). W.M. de Muynck presented three reasons why the ensemble interpretation is impossible: (1) impossibility of joint probability, (2) Bell's theorem, and (3) the Kochen–Specker theorem (de Muynck 2002). J. Gribbin stated the present situation of the ensemble interpretation in the following way (Gribbin 1999): *“There are many difficulties with the idea, but the killer blow was struck when individual quantum entities such as photons were observed behaving in experiments in line with the quantum wave function description. The ensemble interpretation is now only of historical interest.”*

The ensemble interpretation is not supported widely for the reason that this interpretation does not agree with Bell's theorem or the Kochen–Specker theorem. However, the question persists as to whether every version of the ensemble interpretation is inconsistent with the theorems. Before discussing the question in the following section, the manner in which the ensemble interpretation solves three out of the four problems—indeterminism, wave function collapse, nonlocality, and contextuality—is explained.

The first problem (indeterminism) is solvable using Popper's version of the ensemble interpretation simply because his version accepts indeterminism. The second problem (wave function collapse) is explainable using any version of the ensemble interpretation because the ensemble interpretation regards a wave function as a description of an ensemble. The wave function collapse is regarded simply as a change of the pre-measurement ensemble to a smaller post-measurement subensemble.

The third problem (nonlocality) can also be solved by Popper's version. As Popper explained using the pin-board model, a probability distribution of a ball rolling down on a pin board is dependent on the pin arrangement (Popper 1992). The probability distribution is determined through the whole experimental arrangement. In this way, the dependence of probability on the whole arrangement is recognized not only in quantum mechanics but also in the classical pin-board model. This kind of nonlocality is not peculiar to quantum mechanics. Therefore, the third problem can be solved using Popper's pin-board model.

This model is only applicable to the nonlocal variation of wave functions obeying the Schrödinger equation, which does not imply a signal propagating with a speed exceeding that of light. Accordingly, the model is not applicable to the nonlocal variation associated with Bell's theorem. Popper's model cannot explain the violation of Bell's inequalities². It does not solve the fourth problem.

² Popper wrote the following in his book: *“I must admit that these test results have*

Therefore, the only open problem for the ensemble interpretation is the fourth problem (contextuality), which includes Bell's theorem and the Kochen–Specker theorem. This problem is so severe that most researchers probably think that the ensemble interpretation is denied completely³. On the other hand, it can also be viewed as a fruitful interpretation because it has succeeded in solving three out of the four problems at present. At least two severe problems (i.e., the wave function collapse and the contextuality) must be confronted when adopting a no-ensemble interpretation.

4. Bell's assumptions

Bell's theorem is an important difficulty accompanying the ensemble interpretation. In this section, Bell's assumptions used to derive the inequality and Bell's conclusion are both discussed. Bell adopted two assumptions to derive the inequality: one is the assumption of definiteness of values; the other is the assumption of locality. The former assumption is that the result $V_A (= \pm 1)$ of measuring the spin value of a system in the direction $\vec{a}$ by an apparatus A is determined by $\vec{a}$ and λ , where λ is an unknown parameter (or parameters) and can include the (pre-measurement) state of the system or some hidden variables.

$$V_A(\vec{a}, \lambda) = \pm 1 \quad (5)$$

The assumption of locality is that the result V_A cannot be influenced remotely by the setting $\vec{b}$ (or the result V_B) of another measurement performed sufficiently apart from the apparatus A. This assumption prohibits inclusion of $\vec{b}$ or V_B in λ .

$$V_A(\vec{a}, \lambda) = \pm 1, \quad V_B(\vec{b}, \lambda) = \pm 1 \quad (6)$$

Based on those assumptions, the expected value of the product of V_A and V_B can be expressed as shown below.

$$P(\vec{a}, \vec{b}) = \int d\lambda \rho(\lambda) V_A(\vec{a}, \lambda) V_B(\vec{b}, \lambda) \quad (7)$$

Using this expression, Bell's inequality is the following.

surprised me. When I first heard that J.F. Clauser and A. Shimony were intending to test Bell's theorem, I expected that their results would refute quantum theory. But my expectation appears to have been mistaken, since the majority of the tests have gone the other way."

³ Although most researchers identify the ensemble interpretation as a kind of hidden variable theory, which were forbidden by Bell's theorem, the ensemble interpretation is not necessarily a kind of hidden variable theory. Only when thinking that quantum systems possess definite values of all physical variables simultaneously can it be viewed as a kind of hidden variable theory.

$$\begin{aligned}
P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) &= - \int d\lambda \rho(\lambda) \{V(\vec{a}, \lambda)V(\vec{b}, \lambda) - V(\vec{a}, \lambda)V(\vec{c}, \lambda)\} \\
&= \int d\lambda \rho(\lambda) V(\vec{a}, \lambda)V(\vec{b}, \lambda) \{V(\vec{b}, \lambda)V(\vec{c}, \lambda) - 1\}
\end{aligned} \tag{8}$$

$$\begin{aligned}
|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| &\leq \int d\lambda \rho(\lambda) V(\vec{a}, \lambda)V(\vec{b}, \lambda) \{1 - V(\vec{b}, \lambda)V(\vec{c}, \lambda)\} \\
&= 1 + P(\vec{b}, \vec{c})
\end{aligned} \tag{9}$$

where we used the anticorrelation

$$V_A(\vec{a}, \lambda) + V_B(\vec{a}, \lambda) = 0 \tag{10}$$

and the definition, $V(\vec{a}, \lambda) := V_B(\vec{a}, \lambda)$.

Numerous experiments have confirmed predictions of quantum mechanics that asserts violation of the inequality. The conclusion from violation of Bell's inequality is that at least one of Bell's assumptions is incorrect. Bell's conclusion was, however, different from it. *"...there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz-invariant. Of course, the situation is different if the quantum mechanical predictions are of limited validity"* (Bell 1964). That is, Bell's conclusion is that the assumption of locality will be denied.

Bell's conclusion is misleading or incorrect because the possibility remains that the assumption of value definiteness will be denied. The correct conclusion is that at least one assumption of locality and value definiteness is incorrect. Therefore, three alternatives are obtained: (1) we deny locality, (2) we deny value definiteness, or (3) we accept neither. The first and the third alternatives deny locality. They are therefore the same as Bell's conclusion, implying a signal propagation with a speed exceeding that of light. The second is a new alternative. This alternative is discussed in detail in the following section.

5. Hypothesis of non-realistic variables

When giving up the assumption of value definiteness, how can physical variables be interpreted? Including the classical view, the following four interpretations of physical variables can be presented:

[A] A system possesses definite values of all physical variables before measurement. Physical variables are all "realistic".

[B] No value of physical variables can be assigned to a system before measurement. In this sense, all physical variables are “non-realistic”.

[C] Physical variables can have definite values under certain conditions. All physical variables are “realistic” under that condition but “non-realistic” under the other conditions.

[D] A system always possesses definite values of half of physical variables, although it has no value of the other half before measurement. Physical variables of two types exist: “realistic” and “non-realistic”.

Interpretation [A], which is adopted by Einstein and other realists (Popper, Ballentine, etc.), is expected to be the most classical view of quantum mechanics. The ensemble interpretation is almost identical to hidden-variable theories when adopting this idea. Because it is forbidden by Bell’s theorem and the Kochen–Specker theorem as stated above, an instantaneous action-at-a-distance must be introduced to explain violation of Bell’s inequality. Consequently, causality would be broken in this interpretation.

Interpretation [B], which is adopted in the so-called Copenhagen interpretation (i.e. Bohr’s view), would be the most positivistic view of quantum mechanics. In the Copenhagen interpretation, not only the assumption of value definiteness but also that of locality is abandoned. No attempt is made to interpret the physical world in this positivistic approach.

Interpretation [C] would include (or would approximate) the modal interpretation proposed by van Fraassen (1981, 1991). It was developed recently by Bub (1992) and others (e.g., Healey 1996). This interpretation separates the “value state” and the “dynamical state”; the value state specifies the values of all physical quantities, and the dynamical state determines the evolution of the system. This interpretation discusses the logical relation between propositions on the value state and on the dynamical state and gives rise to a modal logic of quantum propositions. This approach resembles the quantum logic approach because its proponents attempt to introduce a new logic. In these approaches, two different events (e.g., the event in which a certain cup is red and the event in which that cup is blue) are not necessarily mutually exclusive. In this meaning, these approaches are positivistic rather than realistic.

Interpretation [D], which is consistent with Shirai’s approach (Shirai 1998, 2005a, 2005b, 2006), is a realistic interpretation. In this interpretation, half of physical variables are regarded as realistic variables possessing definite values before measurement, whereas the other half are the non-realistic variables to which no values can be assigned before measurement. Physical reality of a system is guaranteed by possessing values of the realistic variables. It does not rely on the EPR criterion (predictability

with probability equal to unity). On the other hand, non-realistic variables might explain the inconsistency between local realism and Bell's theorem.

This is the new alternative described in Section 4. This possibility was not taken into account in Bell's conclusion. Most other interpretations have treated all physical variables similarly. In realistic approaches, such as Einstein's ensemble interpretation, Bohm's interpretation, stochastic approach, all physical variables have been regarded as realistic variables possessing definite values. In contrast, non-realistic approaches, such as quantum logic, the modal interpretation, have treated all physical variables as being non-realistic (in the meaning that different events are not mutually exclusive).

Only a few interpretations have taken the position that physical variables are divisible into two types. Bohr considered that physical variables of two types exist: space-time variables such as positions and angles provide a description using the space-time coordination; momentum-energy variables like momenta, angular momenta, and spins obey conservation laws and provide a causal description of a system. He claimed that the two descriptions are mutually exclusive and complementary. Popper also separated the concept of the space-time variables from that of the momentum-energy variables. He interpreted space-time variables to describe properties of an individual system, but interpreted the momentum-energy variables to describe properties of a statistical ensemble of similarly prepared systems. He claimed that, when interpreting a wave to describe a statistical ensemble of a system (i.e. probability) and a particle to describe an individual system, no wave-particle duality exists in quantum mechanics.

6. Toward a new interpretation

Popper's interpretation suggests that the space-time variables are more fundamental than the momentum-energy variables, because the former can describe individual systems but the latter cannot in his interpretation. Considering Popper's interpretation, we propose the following interpretation:

[D'] Actually, physical variables are of two fundamentally distinct types, with mutually different properties: space-time variables (positions, angles, etc.), and momentum-energy variables (momenta, energy, spins, etc.). The former are realistic variables and always possess definite values before measurement. They can provide solid ground for physical reality. On the other hand, the latter are non-realistic variables. No value can be assigned to them before measurement.

In this interpretation, space-time variables are regarded as being realistic and momentum-energy variables are non-realistic. Therefore, conceptual asymmetry ex-

ists between variables of these two types. Many physicists probably consider that either of the two should not be interpreted to be more fundamental than the other because there is a symmetry between the position and momentum in the formalism of quantum mechanics. Such symmetry should not be broken. (The commutation relation between position and momentum is equivalent to the Fourier transform between them in quantum mechanics; the formalism is invariant in the commutation.) However, breaking of such symmetry does not cause problems such as breaking of conservation laws. This is true because the basic equations (e.g., the Schrödinger equation, the Dirac equation, etc.) are not symmetric in commutation between position and momentum. Position and momentum are asymmetric by nature, as seen in conservation laws. The momentum-energy variables are conserved, but the space-time variables are not.

Although recognizing their fundamental difference, a problem remains which is more fundamental: the space-time description or the momentum-energy description. Three reasons why space-time variables (coordinates) are regarded as primary and the momentum-energy variables are regarded as secondary can be offered. The first is that the position at which a particle is detected is observed directly in an actual (individual) experiment. Direct observations of a wave function cannot be made. The interference pattern is apparent only in statistical results.

Bohr stated in the following way (1939). *“The significant point is that in each case some ultimate measuring instruments, like scales and clocks which determine the frame of space-time coordination — on which, in the last resort, even the definitions of momentum and energy quantities rest, must always be described entirely on classical lines, and consequently be kept outside the system subject to quantum mechanical treatment.”* What we will rely on finally would be the measuring instruments like scales and clocks, which determine the space-time coordinates.

The second reason is that spin angular momenta cannot be considered to possess definite values independently of experimental arrangements. Bell’s theorem and the Kochen–Specker theorem forbid that mode of interpretation. Spin angular momenta must be regarded as non-realistic variables to avoid conflict with these theorems. Because momenta and angular momenta are interpreted to be of the same kind as spin angular momenta, all momentum-energy variables should be regarded as non-realistic.

The third reason is that all of conservation laws of the momentum-energy variables are derived from infinitesimal transformations in the space-time variables (Noether’s theorem). Conservation laws of the space-time variables cannot be derived from infinitesimal transformations in the momentum-energy variables. In this meaning, the momentum-energy variables are secondary, and the space-time variables are prior to the momentum-energy variables conceptually.

An additional reason exists: the concepts of momentum and energy of an individual particle have not been discussed sufficiently. Historically speaking, these concepts

were first discussed in the 17–18th centuries with the conservation laws; they then changed to the concepts of wave number and frequency in the 20th century simultaneously with the construction of quantum mechanics. However, Bohr's positivistic view evacuated physicists from philosophical debate to construct their conceptual foundations. Concepts of the momentum-energy variables (as wave numbers, frequency, etc.) apparently remain controversial.

Interpretation [D'] suggests a new possibility to solve the inconsistency between Bell's theorem and local realism. The realistic variables are inferred to possess certain values that impart a solid ground of local realism. On the other hand, the non-realistic variables (including spins) are interpreted to possess no values before measurement. Only the assumption of value definiteness of spins is contradicted by Bell's theorem. To resolve the inconsistency with Bell's theorem completely, what can be explained based on this interpretation? The first is how certain values of the non-realistic variables can be obtained by measurement, although they have no values before measurement. The second is how the values violate Bell's inequality.

The first point is explainable in the following way: consider the temperature of molecules in a heat bath. An individual molecule has no value of temperature, because temperature is a property of an ensemble (collection) of molecules. However, when regarded as a sample belonging to the ensemble of molecules specified using a certain value of temperature, it can possess that value of temperature. The number of molecules is unimportant. The important point is whether to regard (or prepare) a single molecule as a sample in the ensemble of similarly prepared experiments or as an individual molecule that is independent of the experimental setting. For example, when preparing a single molecule in the heat bath with a certain temperature, its distribution is given by the Boltzmann distribution of that temperature. In this case, even if the molecule exists solely in the heat bath, it has that value of temperature. The value of temperature of a single molecule can be measured using a suitable experimental setting.

Similarly, an individual electron has no value of spin, because spin is a property of an ensemble (collection) of electrons. However, when regarded as a sample belonging to the ensemble of electrons specified by a certain value of spin, it can possess that value of spin. The number of electrons is unimportant. The important point is whether to regard (prepare) a single electron as a sample in the ensemble of similarly prepared experiments or as an individual electron that is independent of experimental setting. For example, when preparing a single electron under the repeatable experimental condition, it can be assigned the value determined by that experimental condition. (Here, the whole of the experimental condition (= "apparatus" + "environment") plays the role of a heat bath.) In this case, even if the electron exists solely in that experimental apparatus, it can possess that value of spin. In this manner, the values of spin of a single electron are measurable, although spin is a

property of a statistical ensemble of systems.

The model described above is based on Popper's "propensity" interpretation of probability. In this interpretation, the probability is regarded as a propensity to produce a relative frequency on repetition of similar conditions, so that it depends on the whole experimental arrangement. Popper stated the following: *"I propose to look upon probability statements as statements about some measure of property of the whole experimental arrangement; a measure, more precisely, of a virtual frequency; and I propose to look upon the corresponding statistical statements as statements about the corresponding actual frequency"* (Popper 1992).

This idea is developed and proposed to regard the momentum-energy variables as properties determined by the nature of a substance and the whole experimental arrangement. This resembles the situation in which temperature is a property determined not only by characteristics of molecules but also by the whole experimental arrangement (i.e. the heat bath). The momentum-energy variables are equivalent to wave parameters (wavelength, frequency, etc.), which originate from the wave character of matter. That is, from a property of an ensemble of systems. Accordingly, these variables are dependent on the whole experimental arrangement. The energy-momentum variables have no values before determining the whole experimental arrangement, because the probability—the ensemble of systems—is not determined before fixing the whole experimental arrangement.

Consequently, interpretation [D'] can explain the first point: how certain values of the non-realistic variables are obtainable by measurement, despite their lack of values before measurement. The second problem is how the values violate Bell's inequality. This problem remains open as a subject for future study.

7. Summary

The results can be summarized in the following manner.

1. Bell's conclusion from the violation of Bell's inequality is incorrect or misleading. He suggested that nonlocality would be denied. However, the correct conclusion is that at least one of Bell's assumptions of locality and value definiteness will be denied.
2. The possibility remains that value definiteness will be denied. Discussing this possibility in detail, the following interpretation was proposed herein as an alternative:

[New interpretation] Physical variables are of two types, with mutually different properties: one type is the space-time variables (positions, angles, etc.), the other is the momentum-energy variables (momenta, energy, spins, etc.). The former are realistic variables; they always possess definite values before mea-

surement. They are sufficient to provide a solid basis for physical reality. In contrast, the latter are non-realistic variables. They can be assigned no value before measurement, but certain values are measurable after the whole experimental arrangement is fixed. Value dependence on experimental settings might resolve the contradiction between Bell's theorem and local realism.

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