A Dynamic Semantics for Vague Predicates

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Abstract

Since the mid-seventies, supervaluationism has been supported because of its capacity to give a conservative semantics for vague predicates and to avoid the so-called sorites paradox. As many opponents point out, however, this framework has something elusive and leaves several things unexplained. Dynamic semantics, which I present here, is introduced to interpret the elusiveness positively from an information-theoretic point of view. Thus the aim of the paper is to present a dynamic semantics for vague predicates as a revised version of supervaluationism.

1. Characteristics of Vague Predicates

Vague predicates are ubiquitous in natural languages. Thus vagueness should be paid more attention for any attempt to provide semantics for natural languages. Here I shall argue three characteristics of vague predicates to rehearse the following sections.

The first and essential characteristic is that vague predicates have borderline cases. In the case of 'heap', a grain of sand cannot make a heap whereas one million sands can. Concerning intermediate numbers of sands, however, there are borderline cases in which we cannot judge whether definitely they make a heap or definitely not. Thus vague predicates have three types of cases, namely 'positive', 'negative', and 'indeterminate' cases, which are neither positive nor negative.

The second and local characteristic is that vague predicates have a principle of tolerance. This feature of vague predicates is emphasized by Dummett [2] and Wright [12]. They introduce the notion of 'observational predicates' in order to explain it. They are predicates concerning sense data; they predicate objects based on a subject’s impressions caused by her direct observations of them. Take ‘red’ as an example in the following.1

(1) Thought Experiment on Color Spectrum 1
Condition: Suppose there is a large screen where the following color spectrum is projected, that is, the leftmost portion of it is definitely red, the rightmost one is definitely orange, and between them is a uniform and gradual transi-

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tion from one to another. The screen is divided by lots of squares, each of which has its proper hue and is too small to discriminate between its adjacent ones by our casual observations. Let us call such a sequence of objects 'sorites sequence' henceforth. In addition, all but one square is covered at any time.

Experiment: Suppose a normal subject who is standing in front of the screen has to answer definitely to an experimenter's questions asking what color she sees. First the experimenter points the leftmost square, and then asks its color to the subject. Then, hiding the square and showing its immediate right one, the experimenter asks the same question. Thereafter, the same step and question continue.

The result would be as follows. At first the subject replies, "It's red", then she repeats the same answer since she presumes that her own impressions of sensation for colors are the same. So, she seems to have a kind of tolerance on her judgment that if any pair of hues is indiscriminate, they are the same things. Dummett and Wright describe the phenomenon as follows (where $\sim_F$ stands for a relation of observational indistinguishability in respect of a vague predicate $F$, $\Rightarrow$ for a relation of classical entailment, $'$ for a successor function, $g$ for a valuation function, and $x$ for a name of the individual $x$, thereafter. See [2] pp. 114f, [12] pp. 156f, but their discussions are more informal).

(2) Principle of Tolerance
Suppose that any object $x$ and its successor $x'$ in a sorites sequence are observationally indistinguishable with respect to a vague predicate $F$, then for any object in the sequence the following holds,

$$x \sim_F x' \Rightarrow g(Fx) = g(Fx').$$

The last and global characteristic is that vague predicates have the property that can be stated as a principle of anchors. This feature is examined in Kamp [7]. In the previous experiment there is a trick that plays a role of focusing on the locality of vague predicates (i.e., that the subject is able to see only one square in the screen for being covered on the whole). Let us consider a modified experiment.

(3) Thought Experiment on Color Spectrum 2
Condition: Same as (1) except the screen's not being covered at all.
Experiment: Suppose a normal subject who is standing in front of the uncovered screen has to answer definitely to the experimenter's questions asking what color she sees. First the experimenter points the leftmost square and then asks about its color to the subject. Then, the experimenter asks the same question.
as for its immediate right one. Thereafter, the same step and question continue.

In this case, unlike in (1), a global aspect of vague predicates is considered as well. That is, the subject is aware in advance that the screen projects its transition from red to orange. Then, the result would be as follows. At first the subject answers that it is red without any hesitation, but she starts hesitating to answer. Then at some squares such answers would be returned as, "I don't know how to answer, but I dare say that it's red." or "It seems more orangey, but it's red." Finally, a reply would be returned definitely at a square such as, "It's no longer red, but orange".

The difference in results between (1) and (3) comes from that between local and global traits. Thus, on the one hand, in (1) there is a trick that makes only the principle of tolerance work, and the unacceptable conclusion follows. On the other hand, in (3) both characteristics work well, and the global one acts as a brake (or anchor) on unlimited applications to predication through the local one. Notice that when both have the same power, we are to be involved in a kind of paradox. Thus, the following principle holds globally.

(4) Principle of Anchors

For any vague predicate, it has both extensions that is definitely true and is definitely false.

Moreover, the principle would be convinced easily when the experiment is started with the rightmost square and an unnatural question 'Is this non-red?'

2. The Sorites Paradox

As the previous section implies, vague predicates seem to contain a paradox because of their tolerance. Let us take a vague predicate 'heap' for instance. It is obviously true that a single grain of sand cannot make a heap. Now, it is also true due to the principle of tolerance that for an arbitrary number of sands, if a certain number of sands cannot make a heap, then adding just one grain of sand to it cannot make a heap yet. However these premises entail that even a sufficiently large number of sands cannot make a heap, which is absurd since it is clearly against our intuition and common sense. This is the sorites paradox. The argument is made by a mathematical induction or sufficiently many applications of modus ponens (I call the former 'inductive version' and the latter 'chain version'). Similar arguments are easily applied to other vague predicates using a relevant sorites sequence; their general schema is as follows (let $x_1$ be the first object of the sorites sequence relevant
to $F$ and $x_n$ be the object late enough to apply the principle of anchors).

(5) Inductive Version

\[
\begin{array}{c}
A \\
B \\
\forall i (F_{x_i} \rightarrow F_{x_i'}) \\
\hline
C \\
F_{x_n}
\end{array}
\]

(6) Chain Version

\[
\begin{array}{c}
A \ (= C_1) \\
B_1 \\
F_{x_1} \rightarrow F_{x_2} \\
C_2 \\
F_{x_2} \\
B_2 \\
F_{x_2} \rightarrow F_{x_3} \\
C_3 \\
F_{x_3} \\
\vdots \\
C_{n-1} \\
F_{x_{n-1}} \\
B_{n-1} \\
F_{x_{n-1}} \rightarrow F_{x_n} \\
C_n \\
F_{x_n}
\end{array}
\]

Now we can state the paradoxical nature of vague predicates more accurately, that is, from premises $A$ and $B$ in the inductive version we derive conclusion $C$, but the conclusion must be false due to the principle of anchors. Thus two principles collide each other straight in the argument.

Facing with the predicament, there seems to be three possibilities to meet it.³

(7) Rejecting validities of the mathematical induction or modus ponens.
(8) Rejecting premise $B$ (or premises $B_i$)
(9) Accepting the paradoxical consequence

The essence of (7) is that any reasoning with vague predicates is no longer classical logic and that we have to have an alternative logic concerning vagueness. Thus, the approach is called ‘alternative’; various many-valued logics are typical examples.

Taking (8) means above all that we have to draw a precise boundary between positive and negative extensions of a vague predicate, which inevitably goes against our intuition. However the approach preserves all theorems of classical logic, thus it is called ‘conservative’; supervaluationism is an example.

Taking (9) is nothing more than a semantical nihilism because it accepts the impossibility of semantics for natural languages. Then, the paradox would be dissolved rather than be solved; the approach is called ‘nihilism’. However, some
of those proponents, having accepted the impossibility of a semantics for vague predicates, went further to investigate into a *pragmatics* for them. Wright is one of them (see [13]).

### 3. Supervaluationism and Its Limitations

According to the previous section, there are three ways to handle the sorites paradox. Here I shall discuss only supervaluationism because it is broadly considered as the better theory than the others and directly concerned with my own theory.⁴

Supervaluationism is a semantical framework that was formalized first by B.C. van Fraassen in the sixties, and then by Fine [3: 1975] applied to an analysis of vagueness. Introducing the concept of 'super-truth' (and 'super-falsity'), it solves the paradox treating with borderline cases as well as holding all theorems of classical logic, so that it is often said to be a conservative but elusive framework.

A rejection of premise $B$ (or premises $B_i$) means that we have to draw a precise boundary somewhere between positive and negative extensions, whereas vague predicates have their borderline cases. Therefore supervaluationism proposes such a compromise that borderline cases are a sort of semantical indeterminacy so that a precise boundary would be drawn at one place under a certain situation and at another place under another situation within limits of our admissions to draw them. Such admissible drawings of boundaries are called 'admissible specifications'. Then, the domains of being definitely true and being definitely false are regarded as those lacking any admissible specification, whereas borderline cases as those having some admissible specifications. Hence supervaluationism supposes three semantic values (i.e., truth, falsity and indeterminacy or 'truth-gaps' in an exact sense), and then gives the following definitions.

\begin{itemize}
    \item A vague sentence is 'true' $\iff$ it is true in all admissible specifications. Truth defined above is called 'super-truth'.
    \item A vague sentence is 'false' $\iff$ it is false in all admissible specifications. Falsity defined above is called 'super-falsity'.
    \item A vague sentence is indeterminate $\iff$ it is neither super-true nor super-false.
    \item An argument is valid $\iff$ if the premise(s) is (are) super-true, then the conclusion is also super-true.
\end{itemize}

Then the framework assigns 'indeterminacy' to borderline cases which conforms to our intuition. Moreover, it is known that all theorems of classical logic hold, including the law of excluded middle: $P \lor \neg P$. Accordingly, *it has an advantage* — 113 —
that it can treat with borderline cases retaining classical logic. In spite of such an advantage, however, truth definitions for complex sentences are given in a non-truth-functional way.

Based upon the above definitions, let us look at how to solve the paradox. We use the inductive version here. While evidently the premise $A$ is not only true but also super-true, the premise $B$ is super-false. This is because for each admissible specification, there is a certain object $x_k$ which makes $F_{X_k} \land \neg F_{X_k}'$ true, thus $\exists i(F_{X_i} \land \neg F_{X_i}')$ is super-true and it is logically equivalent to $\neg \forall i(F_{X_i} \rightarrow F_{X_i}')$, therefore $\forall i(F_{X_i} \rightarrow F_{X_i}')$, namely premise $B$ is super-false. Hence, it cannot derive the absurd conclusion $C$ in the argument due to the definition of validity.

Despite of the smart idea and solution for the paradox, there are lots of criticisms against supervaluationism, one of which I shall consider below. It identifies truth with super-truth, and then we accept its 'elusiveness' in Williamson's terms in [11]. While it assigns indeterminacy to borderline cases as we expect and super-truth to $P \lor \neg P$ and $\exists i(F_{X_i} \land \neg F_{X_i}')$ in the sorites argument, never does it answer such questions as whether $P$ or $\neg P$ in fact or where the boundary between $F$ and $\neg F$ lies since it merely says that in all admissible specifications hold $P \lor \neg P$ and $\exists i(F_{X_i} \land \neg F_{X_i}')$. This is the elusiveness of super-truth (see [11] sec. 5.4). As I said earlier, truth definitions for complex sentences are given in a non-truth-functional way; its very non-truth-functionality causes such elusiveness that in the sorites argument supervaluationism makes $\exists i(F_{X_i} \land \neg F_{X_i}')$ super-true without its super-true instance, and besides the situation seems to be against our understanding of an existential quantifier.

As non-truth-functionality is the essence of supervaluationism, the resultant elusiveness via super-truth should be a rejection to apply it to the analysis of vagueness, but now I leave this radical remark aside. Instead, considering the fact that supervaluationism has some benign features (e.g., the preservation of theorems of classical logic and the indeterminate evaluation for borderline cases), we might think that the elusiveness is its only defect. Therefore, it would be more desirable that supervaluationism be able to justify the fact that $\exists i(F_{X_i} \land \neg F_{X_i}')$ in the sorites argument is super-true without any super-true instance. Then, supervaluationism would explain that in any borderline case we can draw each precise boundary at any rate so that $\exists i(F_{X_i} \land \neg F_{X_i}')$ is super-true but that due to arbitrariness of drawing boundaries any instance of $\exists i(F_{X_i} \land \neg F_{X_i}')$ cannot be super-true. Yet it is inadequate for lack of any concrete explanation of how to draw a boundary. Hence, supervaluationism never justifies the concept of super-truth until it explains constructively why this boundary is drawn in this case and that one is done in that case. Without such concrete explanation, supervaluationism would be considered a mere semantics that brings the concept of super-truth in an ad hoc way just in order to solve the paradox. This is the reason why supervaluationism is elusive and has its
own limitations.

4. Dynamic Semantics: Its Idea and Formalization

A framework I shall introduce here is a revised version of supervaluationism. The framework is first suggested in Lewis [8: 1979] and formalized in Kamp [7: 1981], then followed up and developed in Pinkal [9: 1985] without a uniform name for it through their works. Moreover, a semantics which shares the basic idea with it has been developed independently and widely accepted within other areas of semantics (such as anaphora and presupposition) since the nineties. Currently, the semantics is termed dynamic semantics; my framework would be a version of dynamic semantics. Thus, the originality and contribution of my paper consist in rewriting and reinterpreting Pinkal's framework in terms of dynamic semantics (though concerning some of terminology I follow Pinkal due to the aptness (e.g., context)). I also present it as a revised version of supervaluationism, which is free from the elusiveness discussed earlier. At first, I shall give the main idea of dynamic semantics in general and a brief sketch of its application to vague predicates, and then its formalization.

A logical semantics since Frege has had a slogan that meaning of sentence equals its truth condition ([5] p. 179). It is a kind of static semantics that regards a sentence as a unit of meaning and describes a direct relation between world and language. While such a static semantics has advantages when applied to purely formal systems like mathematics, it has disadvantages as semantics for natural languages. The fact has been well known since the sixties, and many attempts have been made to overcome it. H. Kamp and I. Heim overcame the binding independently in the early eighties at last (see [5] pp. 180f in detail). Their common idea was that basic units of semantic interpretation are not sentences but discourses. Furthermore, discourses are dynamic in the sense that they are altered through utterances one after another. Then dynamic semantics, which was presented in the early nineties, has taken its drastic course along their idea and appealed to interpretation processes of discourse for dynamics of semantic interpretations. Thus it regards discourses to be series of processes in which content of the information (it is called information state) is successively updated, with possibilities of interpretations being narrowed down by utterances. In the dynamic view, a concept of meaning is expressed as follows: "$T$he meaning of a sentence is the change that an utterance of it brings about, and the meanings of non-sentential expressions consist in their contributions to this change" ([5] p. 181). Therefore, its slogan is "meaning is information change potential" ([5] p. 182) and the idea is applied to a concept of entailment as follows ([5] p. 182).
(11) Main Idea and Concept of Entailment in Dynamic Semantics

- A basic idea of dynamic semantics is the concept of information change.
- Premises entail a conclusion ⇔ updating any information state with the premises leads to an information state in which the conclusion has to be accepted.

I describe an information state in brief. It is regarded as “a set of possibilities, consisting of alternatives that are open to the information” ([5] p. 183). There are two kinds of information state. One is information about the world and the other is discourse information. The former is information about the world literally. We can get this type of information in various ways, and such a kind of information would be partial. The latter is information concerning discourse where we can exchange verbal information (its typical example is a relation of anaphora across sentence). However, the latter kind of information is not of our current interest, and I shall focus on the former hereafter (see [5] pp. 183ff for these two kinds of information in detail).

Now the interpretation process is explained as follows. A speaker begins her own partial information states at earlier situations in a discourse. Then, participants’ utterances in the discourse make her information states update and subsequent utterances make her updated ones updated further, and then the process is repeated. In this process, the more content of information the speaker has, the more specific information about the world becomes by excluding the possibilities and narrowing the interpretations. However information states are not always updated in a way that the interpretations are specified reasonably. Notice that an utterance sometimes brings about an absurd state that no longer has any possibility about the world, and then the interpretation process halts. A typical example is a flow of discourse in which a rational speaker utters, “It may rain” after her utterance, “It isn’t raining”.

Next I sketch how dynamic semantics deals with vagueness. It treats vagueness in such a way that from an information-theoretic point of view evades the paradox exploiting the three characteristics of vague predicates. The main idea is as follows. At first we assume that information states that play a role of carrying information in a discourse using vague predicates are sets of contexts that assign truth-values to sets of sentences. Updating the states with information concerning the vague predicate means that some imprecise contexts (i.e., contexts including indeterminate truth-values) are excluded through it. Then, a solution of the paradox is explained as follows. Let us think of a discourse in the color spectrum experiments earlier (regardless of (1) or (3) here), where a speaker corresponds to a subject, a participant in the discourse does to an experimenter, and sources of information for update do to ‘judgment by subject’s vision’ and ‘utterances to
respond to questions by the experimenter’. In the early stage of the discourse, the speaker has the two principles of tolerance and anchors. Having the latter in her mind in advance, the speaker applies the former to update her information states sequentially as far as the former does not interfere with the latter. Then, if the former interferes with the latter, then her application of the former will lead to an absurd state, so that she regards a sentence to update in the situation as not a result of dynamic entailment, and then eschews an utterance of the sentence. This is the reason why the paradox is avoided in the framework, and now it is obvious that a classical concept of entailment in the principle of tolerance is weakened and replaced with a dynamic one. The point is that dynamic semantics escapes the paradox by allowing the principle of anchors and weakening the principle of tolerance through the dynamic entailment.8

On the basis of this informal explanation, I shall give a formal semantics for a language with monadic vague predicates (‘LV’ for short henceforth). Let LV be the first order language with monadic vague predicate signs (‘P’, ‘Q’, ‘R’ etc.), only unary function sign ‘‘‘ (successor function sign), and individual constants as its proper signs. Based upon the vocabulary, its terms and well-formed formulas are formed recursively. At first I give definitions to some basic concepts, then a formal semantics to LV. My solution of the paradox will be proposed in the next section.

First I shall offer a definition of information states that play a role of carrying information in LV. As I mentioned earlier, information states in LV are sets of contexts that assign truth-values to sets of sentences, so that I shall define them in terms of ‘structure in LV’ and ‘assignment to the structure’.

Def. 1 Structure in LV
A structure u in LV is an ordered pair such as

\[ [D, \sim \varphi] \]

where D is a non-empty domain, \( \sim \varphi \) is \( \sim \varphi \subseteq D \times D \), and for any monadic vague predicate \( \varphi \), \( \sim \varphi \) is a relation which is reflective, symmetric and non-transitive.

On this definition, a structure u in LV describes a domain of universe structured by the relation of ‘\( \sim \varphi \)’.

Def. 2 Assignment to the Structure
An assignment of semantic values g to the structure u in LV is a quadruple \([g_t, g_+, g_-, g_f]\) satisfying the following conditions (I omit an assignment to an atomic sentence including an identity sign for convenience),

- In the case that \( \alpha \) is an individual constant \( d \), \( g_\alpha(d) \subseteq D \) (we assume that any individual has its name uniquely),

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In the case that $d'$ where 'd'' means 'a successor of d' and d is a closed term, $g_d(d') \subseteq D$ (we stipulate that in case there is no successor of d, $g_d(d) = g_d(D)$.

- In the case that $\phi$ is a monadic vague predicate, $g_+ (\phi) \subseteq D$ and $g_-(\phi) \subseteq D$.
- In the case that $\psi$ is an atomic sentence $\phi \alpha$, $g_\alpha (\phi \alpha)$
  \[
  \begin{cases}
  T : \text{if } g_\alpha (\alpha) \subseteq g_+ (\phi) \\
  F : \text{if } g_\alpha (\alpha) \subseteq g_-(\phi) \\
  I : \text{otherwise,}
  \end{cases}
  \]

- In the case that $\psi$ is $\neg A$, $g_\neg A = \begin{cases} T : \text{if } g_\neg (A) \subseteq \{F\} \\ F : \text{if } g_\neg (A) \subseteq \{T\} \\ I : \text{otherwise,} \end{cases}$

- In the case that $\psi$ is $A \land B$, $g_\land (A \land B)$
  \[
  \begin{cases}
  T : \text{if } g_\land (A) \subseteq \{T\} \text{ and } g_\land (B) \subseteq \{T\} \\
  F : \text{if } g_\land (A) \subseteq \{F\} \text{ or } g_\land (B) \subseteq \{F\} \\
  I : \text{otherwise,}
  \end{cases}
  \]

- In the case that $\psi$ is $A \lor B$, $g_\lor (A \lor B)$
  \[
  \begin{cases}
  T : \text{if } g_\lor (A) \subseteq \{T\} \text{ or } g_\lor (B) \subseteq \{T\} \\
  F : \text{if } g_\lor (A) \subseteq \{F\} \text{ and } g_\lor (B) \subseteq \{F\} \\
  I : \text{otherwise,}
  \end{cases}
  \]

- In the case that $\psi$ is $A \rightarrow B$, $g_\rightarrow (A \rightarrow B)$
  \[
  \begin{cases}
  T : \text{if } g_\rightarrow (A) \subseteq \{F\} \text{ or } g_\rightarrow (B) \subseteq \{T\} \\
  F : \text{if } g_\rightarrow (A) \subseteq \{T\} \text{ and } g_\rightarrow (B) \subseteq \{F\} \\
  I : \text{otherwise,}
  \end{cases}
  \]

- In the case that $\psi$ is $\forall x A$, $g_\forall (\forall x A)$
  \[
  \begin{cases}
  T : \text{if for all } d \in D, g_\forall (A(d/x)) \subseteq \{T\} \\
  F : \text{if for some } d \in D, g_\forall (A(d/x)) \subseteq \{F\} \\
  I : \text{otherwise,}
  \end{cases}
  \]

- In the case that $\psi$ is $\exists x A$, $g_\exists (\exists x A)$
  \[
  \begin{cases}
  T : \text{if for some } d \in D, g_\exists (A(d/x)) \subseteq \{T\} \\
  F : \text{if for all } d \in D, g_\exists (A(d/x)) \subseteq \{F\} \\
  I : \text{otherwise,}
  \end{cases}
  \]

where 'A(d/x)' stands for a formula that substitutes a name of closed term d for all x occurring as free variable in the formula A.

Then 'context', termed 'possibilities' in the standard dynamic semantics, is defined as follows.

Def. 3 Context
Let u (structure in LV) and g (assignment to the structure) be disjoint and
non-empty. Then, a context $c$ based on $u$ and $g$ is just an ordered pair $[u, g]$.

Next we define 'information state' in LV below.

**Def. 4 Information State**

Let $c$ be a context based on $u$ and $g$.
Then, information state $s$ is just a set of contexts $c$.

As to the definitions of context and information state in LV we define two orderings. They are precisification relations among information on vague predicates. Specifically, we shall consider 'determination of indeterminate sentences in contexts' (or precisification of contexts, cf. Def. 5.1) and 'elimination of contexts by precisifications' (or precisification of information states, cf. Def. 5.2), which are defined below.

**Def. 5 Precisification**

Let $c$ be context, $s$ be information state, $c \subseteq s$, $c' \subseteq s'$, $c = [u, g]$ and $c' = [u', g']$.

Then,
1. $c'$ is a precisification of $c$: $c \leq c' \iff u = u'$, $g \leq g'$,
2. $s'$ is a precisification of $s$: $s \leq s' \iff \forall c' \in s' \exists c \in s (c \leq c')$,

where '$g \leq g''$ means that for any closed term $\alpha$, predicate $\phi$, sentence $\varphi$ in LV and the quadruples of assignment $g$ and $g'$, the followings hold: $g_{\alpha}(\alpha) = g'_{\alpha}(\alpha)$, $g_{\phi}(\phi) \subseteq g'_{\phi}(\phi)$, $g_{\neg \phi}(\phi) \subseteq g'_{\neg \phi}(\phi)$, and if $g_{\varphi}(\varphi) \in \{T, F\}$, then $g_{\varphi}(\varphi) = g'_{\varphi}(\varphi)$.

According to the definition, $s'$ is a precisification of $s$ just in case definite truth-values at each $c'$ in $s'$ inherit those at each $c$ in $s$. Thus under an information state $s'$, any context which has indeterminate truth-values may be updated with assignments of definite truth-values, and then information on the vague predicate may become more precise.

As the precisification relation makes up a partial order, any context and information state make it up as well. At this point, we can classify them into some characteristic states (cf. [5] p. 189). At first, a minimal information state corresponds to a set of the whole contexts, which are merely partially ordered and in which any precisification is possible due to lack of information on vague predicates. This state is called 'state of ignorance: $s_0$'. Note that in the state the principle of anchors is not set yet. Then, we can imagine the first state where we begin a discourse and this state is called 'initial state: $s_1$'. It is identified with a(n) (actually-possible) model in LV. In the state a speaker's information is set as so-called default value in advance (in LV it corresponds to a formation of informa-
tion state by updating a state of ignorance via the principle of anchors, see (12)). Next, any maximal consistent state consists of only one possible context and is the most precise one in which the only corresponding context is possible as its information state. It is called "state of total information" (see (13)). Finally, there is an information state to which an inconsistent context corresponds (we label such a state in general as B). Therefore, the state expresses that of impossibility or truth-gluts (in LV it corresponds to a state involved in the paradox). It is called "absurd state" as already remarked. Accordingly, any other possible state is called "non-absurd state".

Now giving definitions to a couple of auxiliary concepts may be useful in further definitions.

Def. 6 Descendant and Subsistence

Let \( c \) be context, \( s \) be information state, \( s \subseteq s', c \subseteq s \) and \( c' \subseteq s' \). Then,
1. \( c' \) is a descendant of \( c \) in \( s' \) if \( c' \subseteq c \subseteq c' \),
2. \( c \) subsists in \( s' \) if \( c \) has one or more descendant in \( s' \),
3. \( s \) subsists in \( s' \) if every \( c \subseteq s \) subsists in \( s' \).

As for the definitions, we understand that a context \( c \) is subsistent in \( s' \) just in case the context \( c' \) in the precisified \( s' \) conveys the definite truth-values at \( c \). Accordingly, definite truth-values extend conservatively with precisifications.

Next I shall present update rules in LV. Note that \( s[\varphi] \) means a result of update of information state \( s \) with sentence \( \varphi \) and \( s[\varphi][\psi] \) means a result of update of \( s[\varphi] \), which is the earlier result of update of \( s \) with \( \varphi \), with \( \psi \).

Def. 7 Update Rules in LV

1. \( s[\Phi \alpha] = \{c \subseteq s| \text{ for any } \beta \subseteq c \text{ that } \alpha \models_\varphi \beta, g_\alpha(\alpha) \subseteq g_\alpha(\Phi) = g_\alpha(\beta) \subseteq g_\alpha(\Phi)\} \)
2. \( s[-\Phi \alpha] = \{c \subseteq s| \text{ for any } \beta \subseteq c \text{ that } \alpha \models_{\neg \varphi} \beta, g_\beta(\alpha) \subseteq g_\beta(\Phi) = g_\beta(\beta) \subseteq g_\beta(\Phi)\} \)
3. \( s[\varphi \land \psi] = s[\varphi][\psi] \)
4. \( s[\varphi \lor \psi] = \{c \subseteq s| c \text{ does not subsist in } s[\varphi \land \psi]\} \)
5. \( s[\varphi \land \psi] = \{c \subseteq s| c \text{ subsists in } s[\varphi] \text{ or } c \text{ subsists in } s[-\varphi][\psi]\} \)
6. \( s[\varphi \lor \psi] = \{c \subseteq s| c \text{ does not subsist in } s[\varphi \land \psi]\} \)
7. \( s[\varphi \rightarrow \psi] = \{c \subseteq s| \text{ if } c \text{ subsists in } s[\varphi], \text{ then all descendants of } c \text{ in } s[\varphi] \text{ subsist in } s[\varphi][\psi]\} \)
8. \( s[-(\varphi \rightarrow \psi)] = \{c \subseteq s| c \text{ does not subsist in } s[\varphi \land \psi]\} \)
9. \( s[\forall x \varphi] = \{c \subseteq s| \text{ for all } \bar{d} \subseteq D, c \text{ subsists in } s[\varphi(\bar{d}/x)]\} \)
10. \( s[-(\forall x \varphi)] = \{c \subseteq s| c \text{ does not subsist in } s[\forall x \varphi]\} \)
11. \( s[\exists x \varphi] = \{c \subseteq s| \text{ for some } \bar{d} \subseteq D, c \text{ does not subsist in } s[\varphi(\bar{d}/x)]\} \)
12. \( s[-(\exists x \varphi)] = \{c \subseteq s| c \text{ does not subsist in } s[\exists x \varphi]\} \)
Among them, rules 1 and 2 seem to be so peculiar that some supplementary explanation is called for. Rule 1 says that as an extension of $s[\phi \alpha]$ it takes the same as that obtained by objects which are observationally indistinguishable from $\alpha$ in respect of a positive extension of $\phi$. Rule 2 says that as an extension of $s[\neg \phi \alpha]$ it takes the same as that obtained by objects which are observationally indistinguishable from $\alpha$ in respect of a negative extension of $\phi$. Therefore, we notice that the principle of tolerance is built in those rules in advance.

From the above definitions it is obvious that the following proposition holds (see [5] p. 191).

Proposition: For all sentences $\varphi$ and information states $s$, $s \leq s[\varphi]$.

This suggests that the interpretation process in dynamic semantics for LV always leads to a new information state that precisifies the old one or the same state as the old one. The semantics with this feature is called ‘update semantics’.

Now we re-state the initial state (or model) and the state of total information based on the structure $u$ for their importance.

(12) Initial State (or Model)
An initial state $s_1$ based on the structure $u$ is just a non-empty information state that is formed by updating a state of ignorance via the principle of anchors.

(13) State of Total Information
A state of total information based on the structure $u$ is just an information state where any further precisification leads to an absurd state.

Next I shall give a few crucial definitions concerning dynamic truth-values and dynamic entailment, but we need some auxiliary concepts in order to define them.

Def. 8 Consistency and Support
Let $s$ be an information state. Then,
- $\varphi$ is consistent with $s \Leftrightarrow s[\varphi]$ exists and $s[\varphi] \neq B$,
- $\varphi$ is supported by $s \Leftrightarrow s[\varphi]$ exists and $s$ subsists in $s[\varphi]$.

These are concerned with an admissibility of a sentence in a discourse in question. The former states how $s$ permits an utterance $\varphi$ and the latter states how utterance $\varphi$ is permitted by $s$. Thus, it is obvious that a sentence is inadmissible just in case that there is no state that is consistent with it or it is not supported by any non-absurd state. Based on them, I write off important definitions concerning truth below. Notice that the definitions of truth-values are very similar to those in
supervaluationism but that the concept of entailment differs crucially (cf. (10)).

Def. 9 Dynamic Values
1. \( \varphi \) is 'true' (with respect to a structure and an initial state) \( \Leftrightarrow g_r(\varphi) \in \{T\} \) in any state of total information based on the structure.
   Truth defined above is called 'dynamic-truth'.
2. \( \varphi \) is 'false' (with respect to a structure and an initial state) \( \Leftrightarrow g_r(\varphi) \in \{F\} \) in any state of total information based on the structure.
   Falsity defined above is called 'dynamic-falsity'.
3. \( \varphi \) is indeterminate \( \Leftrightarrow \) \( \varphi \) is neither dynamically-true nor dynamically-false.

Def. 10 Dynamic Validity
An argument is dynamically valid \( \Leftrightarrow \) if the premise(s) is (are) dynamically-true, then the conclusion is also dynamically-true.

\{\varphi_1, \ldots, \varphi_n\} dynamically entails \( \psi \) with respect to a structure \( u \) in LV \( \Leftrightarrow \) for all information states \( s \) such that \( s[\varphi_1] \ldots[\varphi_n][\psi] \) exists, it holds that \( s[\varphi_1] \ldots[\varphi_n] \) supports \( \psi \).

Obviously, due to the concept of dynamic entailment the principle of tolerance is no longer supported in any context where it violates consistency in a discourse in question, and then does not hold.

5. Dynamic Semantics: Its Solution of the Paradox

Based upon the formalization in the previous section, I shall demonstrate how to solve the paradox by using two simple models in this section. Thereby it would help us to understand lucidly how the above formal semantics does work.

First we take the inductive version and then solve the paradox using a model.

(14) Structure and Model for Inductive Version
Structure \( u \) is as follows. \( D \) stands for a class of infinite sets of sands with arbitrary numbers and \( F \) for a monadic vague predicate 'is non-heap' (or 'does not make a heap'). A relation of \( \sim_F \) in \( u \) is as follows,
\[ \sim_F = \{a_1 \sim a_2 \sim a_3 \sim a_4 \sim a_5 \sim \cdots \} \]
The principle of anchors is set in as follows,
\[ g_+(F) = \{a_1, a_2, a_3, a_4, a_5, \ldots, a_{1000}\}, \quad g_-(F) = \{a_{10000}, a_{100001}, a_{10002}, \ldots\} \]
In the case, the premise \( A: Fa_{12} \) is dynamically-true and the conclusion \( C \) on a
sufficiently large number of sands: $F_{a_{100000}}$ is dynamically-false, while the premise $B: \forall i(F_{a_i} \rightarrow F_{a_i'})$ is dynamically-false due to the principle of anchors and the dynamic entailment (for which we may need still more attention). Thus, though $\exists i(F_{a_i} \land \neg F_{a_i'})$ turns out to be dynamically-true, which $a_i(1000 \leq i \leq 10000)$ a boundary between the positive extension and the negative one of $F$ is depends on information on $\sim_F$ and then is not be able to be specified in general. This way is almost similar to supervaluationism. Now it is obvious or even trivial that the inductive version never holds in the case. So the reader might think that the solution by dynamic semantics is quite same as that by supervaluationism, so that the former also accepts the elusiveness in the latter. However the difference between them will become clearer in a solution for the chain version.

(15) Structure and Model for Chain Version

Structure $u$ is as follows. Let $D$ be a set of divided squares on the color spectrum as follows,

$$D=\{a, b, c, d, e, f\}.$$

$R$ stands for a monadic vague predicate ‘is red’ and a relation of $\sim_R$ is as follows,

$$\sim_R=\{a \sim b \sim c \sim d \sim e \sim f\}.$$

A model based on the structure is Fig. 1, where the principle of anchors is set in as $g_+(R) = \{a\}$ and $g_-(R) = \{f\}$, ‘$C_n$’ is a name for context, and an arrow stands for a precisification relation which is partially ordered. Notice that I

![Fig. 1 Intial State Based on the Structure (Model)](image-url)
consider only non-absurd states for convenience.

Using Fig. 2, I shall explain how to avoid the paradox through the process of discourse under the model. Note that 's_m [P] = s_n' means that an information state s_m is updated to form s_n with a sentence P in the square brackets. An initial state is such a state that has truth-values assigned by the principle of anchors as a default value. So the extension of information state s_i becomes s_i[R_b ∧ ¬R_f] = {C_1, C_2, C_3, ..., C_15} according to (12) (in Fig. 2 it corresponds to the inside area in the largest enclosure. Notice again that in fact s_i comprises more contexts including inconsistent or glutty ones). Then, the initial state is updated with the visual information that a → b and an utterance that R_b, which is made due to the principle of tolerance (it corresponds to answering the color of the square right-adjacent to a) and modus ponens, and then it makes up a new information state s_2. Thus, its extension is s_i[R_b] = {C_2, C_3, ..., C_11} (in Fig. 2 it corresponds to the inside area in the second largest enclosure). Continuing the same process, s_2 is updated with an utterance that R_c due to the information that b → c and the principle of tolerance, then makes up s_3 and its extension is s_2[R_c] = {C_6, C_7, ..., C_11} (in Fig. 2 it corresponds to the inside area in the third largest enclosure). Next s_3 is updated with an utterance that R_d due to the information that c → d and the principle of tolerance, then makes up s_4 and its extension is s_3[R_d] = {C_9, C_10, C_11} (in Fig. 2 it corresponds to the inside area in the fourth largest enclosure).
area in the fourth largest enclosure). Furthermore, $s_4$ is updated with an utterance that $Re$ due to the information that $d \sim \neg e$ and the principle of tolerance, then makes up $s_5$ and its extension is $s_4[Re] = \{C11\}$ (in Fig. 2 it corresponds to the inside area in the smallest enclosure). Notice that square $e$ in $C9$ is very sensitive to some information in the discourse. The square is radically indeterminate between information that $Rd$ and that $\neg Rf$, so that whether $e$ is red or not depends heavily on information to be given next subsequently. In this case $s_4$ is updated to form $s_5$ with the visual information that $d \sim \neg e$, whereas $s_4$ would be updated to form, say, $s_6 (= C10)$ with the speaker's introspection or keen notice for the existence of anchor: $\neg Rf$ before being given the visual information that $d \sim \neg e$ (see [1] ch. 8 for the radical indeterminacy). Thus, generally any update depends on development of discourse and speaker's mental state. By the way, $s_5$ corresponds to the very 'state of total information', so that definite truth-values are completely assigned to the structure. Now we might update $s_5$ with $Rf$ which is entailed from the visual information that $d \sim \neg f$ and the (classical) application of the principle of tolerance. The result ends with $s_5[Rf] = B$ which is the very 'absurd state', however. Thus, any speaker who is rational enough to count consistency in a discourse would refrain from judging that $Fe \rightarrow Ff$ even if $e$ and $f$ share the same appearance, which causes the involvement in the absurd state. In terms of dynamic semantics, it means that the utterance that $Rf$ (which is concluded from $Fe \rightarrow Ff$ and $Fe$ by modus ponens) cannot be entailed dynamically because it is no longer supported by any information state earlier. Now it is palpable that the paradox is avoided also in the chain version.

6. Dynamic Semantics: Its Significance and Further Question

In this section, I shall point out the significance of dynamic semantics for vague predicates. Moreover, I shall raise a further question on it, to which I give a tentative answer.

First important thing to note about dynamic semantics is that it presents not only a theory of meaning but that of action in a formal aspect. Wright argues in [12, 13] that it is not possible to give any formal semantics for vague predicates, and then he tries to provide a pragmatic account of them instead. L.C. Burns also gives in [1] an informal pragmatic account of vague predicates, but she suspects that it is impossible to offer any formal treatment for them (see [1] p. 194). However, dynamic semantics gets pragmatics within its scope through the introduction of dynamic entailment. A typical example is that in the model (15) the speaker would take an action of eschewing utterance that $Rf$ due to a violation of consistency in the discourse. Also remember the main idea of dynamic semantics. In dynamic semantics, utterances are considered to have the potential to change the
flow of discourse, hence from the beginning dynamic semantics in general tries to bring pragmatics into formal semantics and it is the very reason why it is called 'dynamic'. Interestingly, the two authors above, who investigate pragmatics for vague predicates, share the same idea to avoid the paradox, that is, to weaken the principle of tolerance, then defend the existence of clear but somewhat whimsical boundaries under various contexts. Dynamic semantics also shares the same idea, so considering that fact, it seems to give a formal pragmatics for them as far as possible.

It is also significant that dynamic semantics has an advantage over supervaluationism because it can explain the elusiveness. As I mentioned in Section 3, supervaluationism has a stubborn trouble about elusiveness of super-truth. On the other hand, in dynamic semantics, which is an improved version of supervaluationism, the trouble is avoided for the very dynamics of vague predicates. For instance, in the model (15) $\exists i(F_{x_i} \land \neg F_{x'_i})$ is dynamically-true in Fig. 2 because there are sharp boundaries in any possible state of total information: C11, C10, C8, C5, and C15 (as equal to singleton of context, they are identified with contexts for convenience sake). In the same way as supervaluationism, the sentence is dynamically-true without exhibiting any dynamically-true instance, so that the situation is against our intuition, and then we need justification for it. Fig. 2 tells us the reason why $\exists i(F_{x_i} \land \neg F_{x'_i})$ is dynamically-true without its dynamically-true instance. That is, dynamic semantics does not tell which $x_i$ a boundary is because each boundary is changeable and sensitive about a history of update. While we, in (15), reached the state of total information $s_i (= C11)$ by updating the initial state in order of utterances: $Ra, Rb, Rc, Rd, Re$, we would reach C8 in other courses of discourse, for example (in the case six courses of discourse are possible). Thus any setting of boundaries by precisifications of information states is decided context-dependently and constructively, rather than uniquely and context-independently. This is the reason why we can say that there are necessarily boundaries without saying where they are. Furthermore, I pointed out earlier that supervaluationism has to answer adequately to the question why one boundary is drawn in this case and another one in that case; it is explained clearly and constructively in dynamic semantics. This question can be answered constructively according to the flow of information in a discourse, so that any boundary is not decided uniquely but context-dependently. Thus, whereas we cannot say where the boundary is without reference to the flow of information in the discourse, we can say there is the boundary at any rate even without reference to the flow of information in the discourse. Therefore, the elusiveness in supervaluationism has been explained by dynamic semantics.

The reader might have already noticed that there seems to be a serious problem, that is, whether dynamic semantics can deal with the genuine vagueness. It means
that vague predicates lack any kind of sharp boundary from the outset.\textsuperscript{11} She would argue that the principle of anchors is so robust and strong that it cannot fit our natural intuition about the genuine vagueness. If it is a natural concept of vagueness, it is obvious that the framework cannot explain this kind of vagueness at all. Dynamic semantics adheres to first-order vagueness in the sense that there are two sharp boundaries to classify the extensions. Faced with the trouble, we would have three alternatives left. The first one is to adjust dynamic semantics to it exploiting a precise concept of higher-order vagueness. The second one is to adjust dynamic semantics to it exploiting a concept of a vague meta-language. The last one is to defend such a first-order vagueness in a positive way. I would like to take the last one willingly, but there is no space for discussing the issue, so I shall do on another opportunity.

Notes

1. The following thought experiments, (1) and (3), are based on Kamp\cite{Kamp7} pp. 240f with some modifications for the present purposes
2. As mentioned earlier, some authors argue such consideration on the global characteristic. As far as I know, however, there is no consensus on the name of the characteristic. Though Kamp uses the term 'anchor', he does not state that having anchors has the same status as the principle of tolerance. In this connection, Horgan gives an alternative feature to mine; see [6] p. 108.
3. From a logical standpoint, there is another possibility. That is, rejection of $A$'s truth or $C$'s (or $C_n$'s) falsity. Thus this position would lead us to a kind of nihilism in the sense that there is no common thing. In fact, Williamson calls it nihilism as well as (9). See [11] ch. 6.
4. For other frameworks that I mentioned earlier and ignore here, see [11] in which (7) is discussed in ch. 4, (8) except supervaluationism in ch. 7, and (9) in ch. 6.
6. Kamp called his own model 'context sensitive model' and the semantics based on it 'practical reasoning'; while Pinkal called his revised model 'regular K-model' and the semantics 'practical semantics'. As Kamp's framework adheres to the preservation of classical logic, it lacks the consistency in the chain version, which he admits himself in the postscript. See also Burns [1] sec. 5.8 for Kamp's framework.
7. The following description of the basic idea and the formalization of dynamic semantics is largely based upon Groenendijk et al. [5], which is considered now as the standard description.
8. Pinkal also emphasizes the point; see [9] sec. 8.6. Therefore the principle of tolerance is weakened in the sense that it does not always have the classical entailment.
9. Notice that the truth definitions here do impose non-truth-functionality and context-independency on the framework, while those in Def. 2 with Def. 3 are still truth-functional and context-dependent.
10. Though it is hard to think that the extension of 'is red' is changeable for such a slight transition, the model is just made in this way for the sake of argument. As I borrow this
model from [9] sec. 8.5, figures similar to Fig. 1 and 2 are originally presented here.

11. The concept is stated explicitly in Sainsbury [10]. He calls it ‘boundarylessness’ and argues how to deal with it. See also Horgan [6].

References