Information Flow and Design Systems

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1. Introduction

We may say that our life is a series of our constant efforts to realize our desires. We realize our desires in the spatio-temporal world, in various forms : as machines, buildings, clothes, music, pictures, software, mathematical theorems and so on. If these processes of realization are systematically accomplished, we often call them *production*. Thus, production is a human activity expressed by an information flow from the ideal world to the real world. *Design* is another human activity expressed via information flow. *Abstract Design Theory* (ADT) is a mathematical theory that elucidates the concept of design from the point of view of information flow. It has been developed under the strong influences of Yoshikawa's *General Design Theory* (GDT) and Barwise-Seligman's *Channel Theory* (Yoshikawa [7] and Barwise-Seligman [1]). In the ADT developed in Kakuda [2, 3] and Kakuda-Kikuchi [4], the ideal world is taken to be a classification generated by a theory of *functions*, and the real world to be a classification of entities classified by their *attributes*. ADT is premised on investigated from the idealistic standpoint that we have definite classification of media and information contents, in which we are assumed to know completely whether or not an entity has an attribute. This assumption does not hold in a realistic case: we cannot deduce that a medium carries the negation of an information content when the medium does not carry the information content. Tomiyama-Yoshikawa [6] extended GDT to realistic cases by modifying an assumption in GDT. In this paper, we introduce the concept of *indefinite classification* by generalizing the notion of classification in Barwise-Seligman's Channel Theory and the framework of Tomiyama-Yoshikawa's extended GDT. We also introduce the notions of *information link* and *information field* with which we express information flow between indefinite classifications, and we prove new kinds of *completeness* and *soundness* theorems, which mean that the two definitions of information flow, one is based on information links and the other is based on the concept of channels by Barwise-Seligman, are equivalent. Based on these arguments, we clarify the concept of design in a realistic case, by taking design as a channel which makes production flow from our internal world, where our desires or

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needs are mainly concerned, to the external world, where our desires or needs are interpreted by behaviors of entities.

2. Indefinite classifications

An information has the form “a medium a carries an information content α ”. Thus, we consider a set $\text{tok}(A)$ of media, a set $\text{typ}(A)$ of information contents and a binary relation $a \models_A \alpha$ between $\text{tok}(A)$ and $\text{typ}(A)$ that a medium a carries an information content α . Mathematically, it means that we consider a concept of classifications defined as follows :

Definition 2.1. A *classification* $A = \langle \text{tok}(A), \text{typ}(A), \models_A \rangle$ consists of the following things :

- (i) A set $\text{tok}(A)$, of which elements are called *tokens* of A ,
- (ii) A set $\text{typ}(A)$, of which elements are called *types* of A , and
- (iii) A binary relation $\models_A$ between $\text{tok}(A)$ and $\text{typ}(A)$, where $a \models_A \alpha$ means “a token a has a type α ”.

Can we express a site λ where information springs, which we call an *information source*, as a classification A so that $\text{tok}(A)$ is a set of media concerning λ , $\text{typ}(A)$ is a set of information contents and $a \models_A \alpha$ is a binary relation between $\text{tok}(A)$ and $\text{typ}(A)$ that a medium a carries an information content α ? Then, what does a $a \not\models_A \alpha$ mean? $a \not\models_A \alpha$ literally means “it is not the case that a medium a carries an information content α ”. Does it contain the case that any information concerning a and α is in silence? The most appropriate explanation for this argument runs as follows : We have three relations between the set of media and the set of information contents for λ such that information “a medium a carries an information content α ” springs from λ , information “a medium a does not carry α ” springs from λ , and any information about a and α does not spring from λ . Then, considering an information “ a does not carry α ” is suppressed to spring from λ to be an indefinite information “a medium a might carry an information content α ”, we can reach the definition of *indefinite classifications*.

Definition 2.2. An *indefinite classification* $A = \langle \text{tok}(A), \text{typ}(A), \models_A^!, \models_A^\dagger \rangle$ consists of the following things :

- (i) A set $\text{tok}(A)$,
- (ii) A set $\text{typ}(A)$, and
- (iii) Two binary relations $\models_A^!$ and $\models_A^\dagger$ between $\text{tok}(A)$ and $\text{typ}(A)$ such that $a \models_A^! \alpha$ implies $a \models_A^\dagger \alpha$ for all $a \in \text{tok}(A)$ and $\alpha \in \text{typ}(A)$.

Let A be an indefinite classification. We define binary relations $\models_{\bar{A}}$, $\models_{\bar{A}}^\times$, $\models_{\bar{A}}^?$, and $\models_{\bar{A}}^\circ$ between $\text{tok}(A)$ and $\text{typ}(A)$ as follows :

- (i) $a \models_{\bar{A}} \alpha$ if and only if $a \not\models_A^1 \alpha$,
- (ii) $a \models_{\bar{\lambda}} \alpha$ if and only if $a \not\models_A^+ \alpha$,
- (iii) $a \models_{\bar{A}}^? \alpha$ if and only if $a \models_{\bar{A}}^+ \alpha$ and $a \models_{\bar{A}} \alpha$, and
- (iv) $a \models_{\bar{A}}^@ \alpha$ if and only if $a \not\models_A^? \alpha$.

Let 0 and 1 denote either +, !, −, ? or @. We say that $\{0, 1\}$ is *adequate* if 0 is neither 1 nor 1^{-1} , where $!^{-1} = -$, $-^{-1} = !$, $+^{-1} = \times$, $\times^{-1} = +$, $?^{-1} = @$, $@^{-1} = ?$. The following lemma can be easily shown.

Lemma 2.3. *Let $\{0, 1\}$ be an adequate pair. Then, we can have the following properties :*

- (1) 0, 1, 0^{-1} and 1^{-1} are different from each other.
- (2) The remainder of $\{0, 1, 0^{-1}, 1^{-1}\}$ is of the form $\{2, 2^{-1}\}$.
- (3) We can choose two different elements, 3 and 4, from $\{0, 1, 0^{-1}, 1^{-1}\}$ so that either (a), for all $a \in \text{tok}(A)$ and $\alpha \in \text{typ}(A)$, $a \models_{\bar{A}}^2 \alpha$ if and only if $a \models_{\bar{A}}^3 \alpha$ and $a \models_{\bar{A}}^4 \alpha$; or (b), for all $a \in \text{tok}(A)$ and $\alpha \in \text{typ}(A)$, $a \models_{\bar{A}}^2 \alpha$ if and only if $a \not\models_{\bar{A}}^3 \alpha$ or $a \not\models_{\bar{A}}^4 \alpha$.

An information source λ is said to be represented by an indefinite classification A if $\text{tok}(A)$ and $\text{typ}(A)$ are the set of media and the set of information contents concerning λ respectively, and $a \models_{\bar{A}}^! \alpha$ and $a \models_{\bar{\lambda}} \alpha$ represent that information “a medium a carries an information content α ” and information “ a does not carry α ” springs from λ respectively. In this case, $a \models_{\bar{A}}^+ \alpha$ represents that a medium a might carry α as an information content. $a \models_{\bar{A}}^- \alpha$ represents that a medium a might not carry α as an information content. $a \models_{\bar{A}}^@ \alpha$ represents that an information concerning a and α springs from λ . $a \models_{\bar{A}}^? \alpha$ represents that no information concerning a and α springs from λ .

Example 2.4. Let us consider information about the meanings of sentences. Let Σ be the set of, say, English sentences, and A be the set of our concepts. We can have, in principle, the classification $\langle A, \Sigma, \models \rangle$ such that $a \models \alpha$ if and only if “a meaning of a is α ”. If “information...springs” is represented by “I know that...”, we have the indefinite classification $\langle A, \Sigma, \models^+, \models^! \rangle$ such that $a \models^! \alpha$ if and only if I know that a meaning of a is α , and $a \models^\times \alpha$ if and only if I know that a meaning of a is not α . $a \models^+ \alpha$ is defined by that I do not know that a meaning of a is not α . Therefore, $a \models^+ \alpha$ means that I think that a meaning of a might be α . $a \models^? \alpha$ if and only if I do not know whether a means α . In other words, I could think that a meaning of a might be α and might not be α simultaneously.

Example 2.5. Let Σ be a set. Let S be a set of disjoint pairs of subsets of Σ . An indefinite classification A can be defined by :

- (1) $\text{tok}(A) = S$,

- (2) $\text{typ}(\mathcal{A}) = \Sigma$,
- (3) $\langle \Gamma, \Delta \rangle \models_{\mathcal{A}}^! \alpha$ if $\alpha \in \Gamma$, and
- (4) $\langle \Gamma, \Delta \rangle \models_{\mathcal{A}}^{\dagger} \alpha$ if $\alpha \in \Sigma \setminus \Delta$.

We call the above $\mathcal{A}$ the *indefinite classification generated by S* . An important example for design theory is the case that Σ is the set of our desires or needs. Then, a specification concerning Σ can be expressed by a pair of disjoint subsets of Σ . Let $\mathcal{A}$ be the indefinite classification generated by a set S of specifications concerning Σ . Then, $\langle \Gamma, \Delta \rangle \models_{\mathcal{A}}^! \alpha$ means that the specification $\langle \Gamma, \Delta \rangle$ requires α , $\langle \Gamma, \Delta \rangle \models_{\mathcal{A}}^{\times} \alpha$ that the specification avoids α , and $\langle \Gamma, \Delta \rangle \models_{\mathcal{A}}^? \alpha$ that the specification $\langle \Gamma, \Delta \rangle$ does not describe α at all.

Example 2.6. Let B be the set of entities, and Θ be the set of behaviors of entities. We have a classification $\langle B, \Theta, \models \rangle$ such that $b \models \beta$ means “an entity b behaves in β ”. However, we do not know b behaves in β for all entities b and behaviors β . We only say that “ b behaves in β ” from our knowledge. Thus, we can have the indefinite classification $\mathcal{B} = \langle \text{tok}(\mathcal{B}), \text{typ}(\mathcal{B}), \models_{\mathcal{B}}^!, \models_{\mathcal{B}}^{\dagger} \rangle$, such that $\text{tok}(\mathcal{B}) = B$, $\text{typ}(\mathcal{B}) = \Theta$, $b \models_{\mathcal{B}}^! \beta$ if and only if we know that b behaves in β , and $b \models_{\mathcal{B}}^{\dagger} \beta$ if and only if we do not know that b does not behave in β , or, equivalently, we could think that b might behave in β . We say that $\mathcal{B}$ is the indefinite classification with respect to the behaviors (by our knowledge).

Definition 2.7. Let $\mathcal{A}$ be an indefinite classification. A pair $\langle \mathcal{A}, \Sigma \rangle$ of a subset $\mathcal{A}$ of $\text{tok}(\mathcal{A})$ and a subset Σ of $\text{typ}(\mathcal{A})$ is said to be *definite* in $\mathcal{A}$ if $a \models_{\mathcal{A}}^{\circ} \alpha$ for all $a \in \mathcal{A}$ and $\alpha \in \Sigma$. In other words, $\langle \mathcal{A}, \Sigma \rangle$ is definite if $a \models_{\mathcal{A}}^{\dagger} \alpha$ implies $a \models_{\mathcal{A}}^! \alpha$ for all $a \in \mathcal{A}$ and $\alpha \in \Sigma$.

If $\langle \text{tok}(\mathcal{A}), \text{typ}(\mathcal{A}) \rangle$ is definite in an indefinite classification $\mathcal{A}$, we say that $\mathcal{A}$ is a *definite classification*. A definite classification $\mathcal{A}$ is to be an ordinary classification by taking $a \models_{\mathcal{A}} \alpha$ for $a \models_{\mathcal{A}}^{\dagger} \alpha$, since $a \models_{\mathcal{A}}^! \alpha$ is equivalent to $a \models_{\mathcal{A}}^{\dagger} \alpha$. From now on, we will mean a classification as an indefinite classification in this paper. An ordinary classification $\mathcal{A} = \langle \text{tok}(\mathcal{A}), \text{typ}(\mathcal{A}), \models_{\mathcal{A}} \rangle$ will be identified with the definite classification $\langle \text{tok}(\mathcal{A}), \text{typ}(\mathcal{A}), \models_{\mathcal{A}}^!, \models_{\mathcal{A}}^{\dagger} \rangle$, such that $\models_{\mathcal{A}}^{\dagger} = \models_{\mathcal{A}}^! = \models_{\mathcal{A}}$.

Example 2.8. Let $\mathcal{A}$ be the classification generated by a set S of disjoint pairs of subsets of a set Σ . $\langle \mathcal{A}, \Sigma' \rangle$ is a definite pair in $\mathcal{A}$ if and only if $\langle \Gamma \cap \Sigma', \Delta \cap \Sigma' \rangle$ is a partition of Σ' for every $\langle \Gamma, \Delta \rangle$ in $\mathcal{A}$. Henceforth, $\mathcal{A}$ is a definite classification if and only if $\text{tok}(\mathcal{A}) = S$ is a set of partitions of Σ . In this case, every specification $\langle \Gamma, \Delta \rangle$ in S is *complete* in the sense that $\langle \Gamma, \Delta \rangle$ requires or avoids every desire α . On the contrary, suppose that $\langle \Gamma, \Delta \rangle \models_{\mathcal{A}}^? \alpha$ for all $\langle \Gamma, \Delta \rangle \in S = \text{tok}(\mathcal{A})$ and $\alpha \in \Sigma =$

$\text{typ}(\mathbf{A})$. Then, $\alpha \in \Sigma \setminus \Gamma$ and $\alpha \in \Sigma \setminus \Delta$ for all $\alpha \in \Sigma$. This means that $\Gamma = \emptyset$ and $\Delta = \emptyset$. Thus, $S = \{\langle \emptyset, \emptyset \rangle\}$. In this case, S does not specify anything.

Let λ and μ be information sources represented by classifications $\mathbf{A}$ and $\mathbf{B}$ respectively. Suppose that there exists a quadruple relation $P(a, b, \alpha, \beta)$ among $a \in \text{tok}(\mathbf{A})$, $b \in \text{tok}(\mathbf{B})$, $\alpha \in \text{typ}(\mathbf{A})$ and $\beta \in \text{typ}(\mathbf{B})$. Suppose that we notice that, for any a, b, α and β , such that $P(a, b, \alpha, \beta)$, information “ b carries β ” springs from μ whenever information “ a carries α ” springs from λ . Mathematically, we have :

$$\forall a \forall b \forall \alpha \forall \beta (P(a, b, \alpha, \beta) \Rightarrow (a \models_{\mathbf{A}} \alpha \Rightarrow b \models_{\mathbf{B}} \beta))$$

Under the above situation we perceive that $P(a, b, \alpha, \beta)$ *undergoes a metamorphosis* from information “ a carries α ” to information “ b carries β ”. That is, we perceive that P makes a certain *information flow* from λ to μ . If P can be represented by a pair $\langle R, S \rangle$ of a binary relation R between $\text{typ}(\mathbf{A})$ and $\text{typ}(\mathbf{B})$, and a binary relation S between $\text{tok}(\mathbf{B})$ and $\text{tok}(\mathbf{A})$, so that

$$\alpha R \beta \wedge b S a \iff P(a, b, \alpha, \beta)$$

then $\alpha R \beta$ denotes the relation that α is *interpreted into* β by the information flow, and $b S a$ denotes the relation that a is a *sender* and b is an *acceptor* of the information flow.

Definition 2.9. Let $\mathbf{A}$ and $\mathbf{B}$ be ordinary classifications. A pair $\langle R, S \rangle$ of correspondences $R : \text{typ}(\mathbf{A}) \rightarrow \text{typ}(\mathbf{B})$ and $S : \text{tok}(\mathbf{B}) \rightarrow \text{tok}(\mathbf{A})$ is said to be an *information link*¹ from $\mathbf{A}$ to $\mathbf{B}$ if it satisfies the condition

$$(1) \quad \forall a \forall b \forall \alpha \forall \beta (\alpha R \beta \wedge b S a \Rightarrow (a \models_{\mathbf{A}} \alpha \Rightarrow b \models_{\mathbf{B}} \beta))$$

Let $\mathbf{A}$ be a classification. Then, we denote by $\mathbf{A}^*$ the definite classifications $\langle \text{tok}(\mathbf{A}), \text{typ}(\mathbf{A}), \models_{\mathbf{A}} \rangle$, where $*$ is $!$, $\times$, $+$, $-$, $?$ or $@$.

Definition 2.10. Let $\mathbf{A}$ and $\mathbf{B}$ be classifications. A $(0, 1)$ -*link* $\langle R, S \rangle$ from $\mathbf{A}$ to $\mathbf{B}$ is an information link from $\mathbf{A}^0$ to $\mathbf{B}^1$, where 0 and 1 denote $!$, $\times$, $+$, $-$, $?$ or $@$.

We simply say that $\langle R, S \rangle$ is a *link from a classification $\mathbf{A}$ to a classification $\mathbf{B}$* , in symbol $\langle R, S \rangle : \mathbf{A} \rightrightarrows \mathbf{B}$, if $\langle R, S \rangle$ is a pair of a correspondence $R : \text{typ}(\mathbf{A}) \rightarrow \text{typ}(\mathbf{B})$ and a correspondence $S : \text{tok}(\mathbf{B}) \rightarrow \text{tok}(\mathbf{A})$.

Example 2.11. Let $\mathbf{A}$ and $\mathbf{B}$ be the classification $\mathbf{A}$ generated by a set S of specifications and the classification with respect to the behaviors, respectively. We can consider a link $\langle R, S \rangle : \mathbf{A} \rightrightarrows \mathbf{B}$ as a certain production system (though it is

¹ The notion of information links was introduced by L. Moss and J. Seligman in [5]. However, the notion was also independently conceptualized by the Mathematical Foundation of Design research group at Kobe University as the notion of $*$ -conditional morphisms.

another question whether $\langle R, S \rangle$ can be really called a production system). Let $\alpha \in \text{typ}(A)$, $\beta \in \text{typ}(B)$, $\langle \Gamma, \Delta \rangle \in \text{tok}(A)$ and $b \in \text{tok}(B)$ be such that $\alpha R \beta$ and $b S \langle \Gamma, \Delta \rangle$.

- (1) $\langle R, S \rangle$ is a $(!, !)$ -link: Then, $\alpha \in \Gamma$ implies $b \models_B^! \beta$. That is, we know that b behaves in β if $\langle \Gamma, \Delta \rangle$ requires α .
- (2) $\langle R, S \rangle$ is a $(+, +)$ -link: Then, $\alpha \notin \Delta$ implies $b \models_B^+ \beta$. That is, we think b might behave in β if $\langle \Gamma, \Delta \rangle$ does not avoid α . By taking the contraposition, $\langle \Gamma, \Delta \rangle$ avoids α if we know that b does not behave in β .
- (3) $\langle R, S \rangle$ is a $(+, !)$ -link: Then, $\alpha \notin \Delta$ implies $b \models_B^! \beta$. That is, we know that b behaves in β if $\langle \Gamma, \Delta \rangle$ does not avoid α .
- (4) $\langle R, S \rangle$ is a $(!, +)$ -link: Then, $\alpha \in \Gamma$ implies $b \models_B^+ \beta$. That is, we think that b might behave in β if $\langle \Gamma, \Delta \rangle$ requires α .

Example 2.12. Take the classification with respect to the meanings of sentences in Example 2.4 as B in Example 2.11. Then, a link $\langle R, S \rangle: A \rightleftharpoons B$ denotes a certain composite way of sentences we want to express. For example, suppose that $\langle R, S \rangle$ is a $(!, !)$ -link. Then, I know that b is a meaning of β if $\langle \Gamma, \Delta \rangle$ requires α whenever $\alpha R \beta$ and $b S \langle \Gamma, \Delta \rangle$. That is, if our desire α is interpreted by a sentence β and a specification $\langle \Gamma, \Delta \rangle$, and what we want to express corresponds to a concept b through a composite way $\langle R, S \rangle$, then “ $\langle \Gamma, \Delta \rangle$ requires α ” implies “I know that b is a meaning of β ”. If $\langle R, S \rangle$ is $(+, +)$, then “ $\langle \Gamma, \Delta \rangle$ does not avoid α ” implies “I think that b might be a meaning of β ”. If $\langle R, S \rangle$ is $(+, +)$ and $(-, -)$, then “ $\langle \Gamma, \Delta \rangle$ does not avoid α ” implies “I think that b might be a meaning of β ”, and “ $\langle \Gamma, \Delta \rangle$ does not require α ” implies “I think that b might not be a meaning of β ”.

Example 2.13. Let λ and μ be two people. Let A and B be the set of English sentences and the set of Japanese sentences respectively, and Σ be the set of our concepts. We define the classification $A = \langle A, \Sigma, \models_A^+, \models_A^! \rangle$ such that $a \models_A^! \alpha$ if and only if λ knows that the meaning of a is α and $a \models_A^+ \alpha$ if and only if λ knows that the meaning of a is not α . $B = \langle B, \Sigma, \models_B^+, \models_B^! \rangle$ can be defined by the same way with respect to μ . A link $\langle R, S \rangle: A \rightleftharpoons B$ can be considered as an interpretation system from λ to μ . Let a, b, α , and β be such that $\alpha R \beta$ and $b S a$. Suppose that $\langle R, S \rangle$ is $(!, !)$. Then, “ λ knows that a is a meaning of α ” implies “ μ knows that b is a meaning of β ”. That is, if α is interpreted by β and a concept a corresponds to a concept b through the interpretation system $\langle R, S \rangle$, then “ λ knows that a is a meaning of α ” implies “ μ knows that b is a meaning of β ”. If $\langle R, S \rangle$ is $(+, +)$, then “ λ thinks that a might be a meaning of α ” implies “ μ thinks that b might be a meaning of β ”. If $\langle R, S \rangle$ is $(+, +)$, then “ λ thinks that a might be a meaning of β ” implies “ μ thinks that b might be a meaning of β ”. If $\langle R, S \rangle$ is $(-, -)$, then “ λ thinks that a might not be a meaning of α ” implies “ μ thinks that

b might not be a meaning of β ".

Lemma 2.14. *Let $\langle R, S \rangle$ be a link from A to B .*

- (i) $\langle R, S \rangle : A \rightrightarrows B$ is a $(-, -)$ -link if and only if $\langle R^{-1}, S^{-1} \rangle : B \rightrightarrows A$ is a $(!, !)$ -link.
- (ii) $\langle R, S \rangle : A \rightrightarrows B$ is a $(\times, \times)$ -link if and only if $\langle R^{-1}, S^{-1} \rangle : B \rightrightarrows A$ is a $(+, +)$ -link.
- (iii) $\langle R, S \rangle : A \rightrightarrows B$ is a $(-, \times)$ -link if and only if $\langle R^{-1}, S^{-1} \rangle : B \rightrightarrows A$ is a $(+, !)$ -link.
- (iv) $\langle R, S \rangle : A \rightrightarrows B$ is a $(\times, -)$ -link if and only if $\langle R^{-1}, S^{-1} \rangle : B \rightrightarrows A$ is a $(!, +)$ -link.

Definition 2.15. Let $\langle R, S \rangle$ be a link from a classification A to a classification B .

- (1) $\langle R, S \rangle : A \rightrightarrows B$ is a $(0, 1)$ -information link if $\langle R, S \rangle$ is a $(0, 0)$ -link and a $(1, 1)$ -link from A to B , where $\{0, 1\}$ is an adequate pair.
- (2) $\langle R, S \rangle : A \rightrightarrows B$ is a definite link if $\langle R, S \rangle$ is a $(+, !)$ -link.
- (3) $\langle R, S \rangle : A \rightrightarrows B$ is an attenuate link if $\langle R, S \rangle$ is a $(!, +)$ -link.

Information links have the same type if they are $(0, 1)$ -information links for the common adequate pair $\{0, 1\}$.

Definition 2.16. Let $\langle R, S \rangle : A \rightrightarrows B$ be a link. $\langle R, S \rangle$ is said to be an *information bilink* from A to B if both $\langle R, S \rangle : A \rightrightarrows B$ and $\langle R^{-1}, S^{-1} \rangle : B \rightrightarrows A$ are $(!, +)$ -information links. $\langle R, S \rangle$ is said to be a *definite (resp. attenuate) bilink* from A to B if both $\langle R, S \rangle : A \rightrightarrows B$ and $\langle R^{-1}, S^{-1} \rangle : B \rightrightarrows A$ are definite (resp. attenuate) links.

We have the following lemma by the above definition. (1) is easily proved from Lemma 2.3.

Lemma 2.17. (1) $\langle R, S \rangle : A \rightrightarrows B$ is an information bilink if and only if $\langle R, S \rangle : A \rightrightarrows B$ and $\langle R^{-1}, S^{-1} \rangle : B \rightrightarrows A$ are information links with the same type.

- (2) $\langle R, S \rangle : A \rightrightarrows B$ is an information bilink if and only if $\langle R, S \rangle : A \rightrightarrows B$ is a $(*, *)$ -link for all $*$, where $*$ denotes $!, +, \times, -, ?$ or $@$.
- (3) If $\langle R, S \rangle : A \rightrightarrows B$ is a definite link, then it is a $(+, !)$ -information link.
- (4) If $\langle R, S \rangle : A \rightrightarrows B$ is a $(+, !)$ -information link, then it is an attenuate link.
- (5) If $\langle R, S \rangle : A \rightrightarrows B$ is an information bilink, then it is an attenuate bilink.

Lemma 2.18. $\langle R, S \rangle : A \rightrightarrows B$ is a definite bilink if and only if it is a $(+, -)$ -information link from A to B and $\langle \text{dom}(R), \text{range}(S) \rangle$ and $\langle \text{range}(R), \text{dom}(S) \rangle$ are

definite in A and B respectively.

Proof. Suppose that $\langle R, S \rangle$ is a definite bilink. Let a, b, α , and β be such that $\alpha R \beta$ and $b S a$. Assume that $a \models_A^! \alpha$. Since $\langle R, S \rangle$ is a $(+, !)$ -link, we have $b \models_B^! \beta$. Since $\langle R, S \rangle: A \rightrightarrows B$ is a $(-, \times)$ -link, we have $b \models_A^{\times} \beta$. This is a contradiction. Thus, $\langle \text{dom}(R), \text{range}(S) \rangle$ is definite in A . In the same fashion, we can prove that $\langle \text{range}(R), \text{dom}(S) \rangle$ is definite in B . Conversely, suppose that $\langle \text{dom}(R), \text{range}(S) \rangle$ and $\langle \text{range}(R), \text{dom}(S) \rangle$ are definite. Suppose that $\alpha R \beta$ and $b S a$. If $a \models_A^+ \alpha$, then $b \models_B^+ \beta$ by the fact that $\langle R, S \rangle: A \rightrightarrows B$ is a $(+, +)$ link, and so $b \models_B^! \beta$ by the fact that $\langle \text{range}(R), \text{dom}(S) \rangle$ is definite. Thus, $\langle R, S \rangle$ is a $(+, !)$ -link. In the same way, we can show that $\langle R, S \rangle: A \rightrightarrows B$ is a $(-, \times)$ -link.

□

Let $\langle R, S \rangle: A \rightrightarrows B$ be a link. Let α, β, a , and b be such that $\alpha R \beta$ and $b S a$. Suppose that $\langle R, S \rangle$ is an information bilink. Then, information “ a carries α ” springs from λ ($a \models_A^! \alpha$) if and only if information “ b carries β ” springs from μ ($b \models_B^! \beta$), and information “ a does not carry α ” springs from λ ($a \models_A^{\times} \alpha$) if and only if information “ b does not carry β ” springs from μ ($b \models_B^{\times} \beta$). It follows that information concerning a and α is in silence ($a \models_A^! \alpha$) if and only if information concerning b and β is in silence ($b \models_B^! \beta$). Thus, a and b have the same *degree of indefiniteness* in the sense that a and b carry the common form of information, interpreting α into β . Suppose that $\langle R, S \rangle: A \rightrightarrows B$ is a definite bilink. Then, “ a carries α ” is a definite information ($a \models_A^{\oplus} \alpha$), and so is “ b carries β ” ($b \models_B^{\oplus} \beta$), and “ a carries α ” springs if and only if “ b carries β ” springs. This means that a and b are *counterparts* to each other in the sense that a and b have common information, transforming α to β . Suppose that $\langle R, S \rangle: A \rightrightarrows B$ is an attenuate bilink. Then, “ b might carry β ” ($b \models_B^{\times} \beta$) if information “ a carries α ” springs from λ ($a \models_A^! \alpha$), and “ b might not carry β ” if information “ a does not carry α ” springs from λ . Thus, $\langle R, S \rangle$ attenuates information “ a definitely carries α ” to information “ b might carry β ”, and “ a definitely does not carry α ” to information “ b might not carry β ”.

Example 2.19. We consider the meanings of the various bilinks defined in the above Example 2.13. Let a, b, α , and β be such that $\alpha R \beta$ and $b S a$. Suppose that $\langle R, S \rangle$ is an attenuate bilink. Then, μ thinks that the meaning of b might be β if λ knows that the meaning of a is α , and μ thinks that the meaning of b might not be β if λ knows that the meaning of a is not α . Thus, $\langle R, S \rangle$ is considered to be an interpretation system which makes definite meanings λ knows indefinite for μ , though the interpretation system itself is not necessarily wrong. Suppose that $\langle R, S \rangle$ is an information bilink. Then, λ knows that the meaning of a is α if and only if μ knows that the meaning of b is β , and λ knows that the meaning of a is not α if and only if μ knows that the meaning of b is not β . It follows that λ does not

know whether the meaning of a is α if and only if μ does not know whether the meaning of b is β . In this case, the translation system $\langle R, S \rangle$ is neutral in the sense that it neither increases nor decreases information. Suppose that $\langle R, S \rangle$ is a definite bilink. Then, λ definitely knows whether the meaning of a is α or not, and so does μ . Further, λ knows that the meaning of a is α if and only if μ knows that the meaning of b is β . In this case, the interpretation system $\langle R, S \rangle$ is faithful in the sense that an English concept a is a counterpart of a Japanese concept b , interpreting an English sentence α to a Japanese sentence β .

Lemma 2.20. *Let $\langle A, \Sigma \rangle$ and $\langle B, \Theta \rangle$ be definite in classifications A and B respectively. If $\langle R, S \rangle : A \rightrightarrows B$ is a information bilink, then $\langle R', S' \rangle : A \rightrightarrows B$ is a definite bilink, where*

$$R' = \{ \langle \alpha, \beta \rangle \in R \mid \alpha \in \Sigma \text{ and } \beta \in \Theta \}, \text{ and} \\ S' = \{ \langle b, a \rangle \in S \mid a \in A \text{ and } b \in B \}.$$

An infomorphism is the central notion in [1]. We generalize the notion for indefinite classifications.

Definition 2.21. Let A and B be classifications, and $\langle f^\wedge, f^\vee \rangle : A \rightrightarrows B$ be a pair of maps $f^\wedge : \text{typ}(A) \rightarrow \text{typ}(B)$ and $f^\vee : \text{tok}(B) \rightarrow \text{tok}(A)$. We say that $\langle f^\wedge, f^\vee \rangle$ is an *infomorphism* from A to B if $\langle f^\wedge, f^\vee \rangle$ is a information bilink from A to B . In other words, $\langle f^\wedge, f^\vee \rangle : A \rightrightarrows B$ is an infomorphism if and only if (i) $f^\vee(b) \models_A^+ \alpha$ if and only if $b \models_B^+ f^\wedge(\alpha)$ and (ii) $f^\wedge(b) \models_A^- \alpha$ if and only if $b \models_B^- f^\vee(\alpha)$ for $\alpha \in \text{typ}(A)$ and $b \in \text{tok}(B)$. $\langle f^\wedge, f^\vee \rangle : A \rightrightarrows B$ is a definite infomorphism if $\langle f^\wedge, f^\vee \rangle$ is a definite bilink. That is, $f^\vee(b) \models_A^+ \alpha$ implies $b \models_B^+ f^\wedge(\alpha)$, and $f^\wedge(b) \models_A^- \alpha$ implies $b \models_B^- f^\vee(\alpha)$ for $\alpha \in \text{typ}(A)$ and $b \in \text{tok}(B)$.

It is clear from Lemma 2.18 that $f : A \rightrightarrows B$ is a definite infomorphism if and only if $f : A \rightrightarrows B$ is an infomorphism and $\langle \text{typ}(A), \text{range}(f^\vee) \rangle$ and $\langle \text{range}(f^\wedge), \text{tok}(B) \rangle$ are definite. Particularly, if A and B are definite classifications, any infomorphism from A to B is a definite infomorphism.

The notion of information and definite links is associative. That is, we have the following lemma :

Lemma 2.22. *Let $\langle R, S \rangle : A \rightrightarrows B$ and $\langle R', S' \rangle : B \rightrightarrows C$ be information links with the same type (resp. definite links) respectively. Then $\langle R' \circ R, S \circ S' \rangle : A \rightrightarrows C$ is an information link with same type as $\langle R, S \rangle$ (resp. a definite link).*

3. Theories

Let $T = \langle \text{typ}(T), \vdash \rangle$ be a theory. The **Weakening, Identity, and Global Cut**

for T are the following :

Weakening : If $\Gamma \vdash \Delta$, then $\Gamma, \Gamma' \vdash \Delta, \Delta'$.

Identity : $\alpha \vdash \alpha$,

Global Cut : Let Σ be a subset of $\text{typ}(T)$. If $\Gamma, \Gamma' \vdash \Delta, \Delta'$ for every covering $\langle \Gamma', \Delta' \rangle$ of Σ , then $\Gamma \vdash \Delta$.

Remark 3.1. Global Cut in the above is a little weaker than in [1]. However, it is clear that the two are equivalent to each other under **Weakening**.

Let A be a classification. Define a theory $\text{Th}^{0,1}(A) = \langle \text{typ}(A), \vdash_A^{0,1} \rangle$ so that $\Gamma \vdash_A^{0,1} \Delta$ is defined by

$$\forall a \in \text{tok}(A) (\forall \alpha \in \Gamma (a \models_A^0 \alpha) \Rightarrow \exists \alpha \in \Delta (a \models_A^1 \alpha))$$

for $\Gamma, \Delta \subseteq \text{typ}(A)$, where 0 and 1 denote $+$, $-$, $!$, $\times$, $?$ or $@$. It is clear that **Weakening** holds for $\text{Th}^{0,1}(A)$.

Lemma 3.2.

- (i) If $a \models_A^0 \alpha$ implies $a \models_A^1 \alpha$ for all $a \in \text{tok}(A)$ and $\alpha \in \text{typ}(A)$, then **Identity** holds for $\text{Th}^{0,1}(A)$.
- (ii) If $a \models_A^1 \alpha$ implies $a \models_A^0 \alpha$ for all $a \in \text{tok}(A)$ and $\alpha \in \text{typ}(A)$, then **Global Cut** holds for $\text{Th}^{0,1}(A)$.

Proof. (i) is clear. To prove (ii), suppose that $a \models_A^1 \alpha$ implies $a \models_A^0 \alpha$ for all $a \in \text{tok}(A)$ and $\alpha \in \text{typ}(A)$. Let Σ be a subset of $\text{typ}(A)$ such that $\Gamma, \Gamma' \vdash \Delta, \Delta'$ for every covering $\langle \Gamma', \Delta' \rangle$ of Σ . Let $a \in \text{tok}(A)$ be such that $a \models_A^0 \alpha$ for all $\alpha \in \Gamma$. Set $\Gamma' = \{\alpha \in \Sigma \mid a \models_A^0 \alpha\}$ and $\Delta' = \{\alpha \in \Sigma \mid a \not\models_A^1 \alpha\}$. By our supposition, $\langle \Gamma', \Delta' \rangle$ is a covering of Σ . Henceforth, $\Gamma, \Gamma' \vdash_A^{0,1} \Delta, \Delta'$. Since $a \models_A^0 \alpha$ for all $\alpha \in \Gamma \cup \Gamma'$, $a \models_A^0 \alpha$ for some $\alpha \in \Delta \cup \Delta'$. By the definition of Δ' , $a \models_A^1 \alpha$ for some $\alpha \in \Delta$. $\square$

By Lemma 3.2, we have the following list of **Identity** and **Global Cut** :

Identity	$\text{Th}^{+,+}(A), \text{Th}^{-,-}(A), \text{Th}^{!,!}(A), \text{Th}^{\times,\times}(A), \text{Th}^{@,@}(A), \text{Th}^{!,?}(A)$ $\text{Th}^{!,+}(A), \text{Th}^{\times,-}(A), \text{Th}^{!,@}(A), \text{Th}^{\times,@}(A), \text{Th}^{!,+}(A), \text{Th}^{!,-(A)}$
Global Cut	$\text{Th}^{+,+}(A), \text{Th}^{-,-}(A), \text{Th}^{!,!}(A), \text{Th}^{\times,\times}(A), \text{Th}^{@,@}(A), \text{Th}^{!,?}(A)$ $\text{Th}^{+,!}(A), \text{Th}^{-,\times}(A), \text{Th}^{@,!}(A), \text{Th}^{@,\times}(A), \text{Th}^{+,{?}}(A), \text{Th}^{-,{?}}(A)$

Lemma 3.3. Let T be a theory with **Weakening**. Then, **Global Cut** holds for T if and only if for every $\langle \Gamma, \Delta \rangle$, such that $\Gamma \not\vdash_T \Delta$, there exists a covering $\langle \Gamma', \Delta' \rangle$ of $\text{typ}(T)$, such that $\Gamma \subseteq \Gamma'$, $\Delta \subseteq \Delta'$, and $\Gamma' \vdash_T \Delta'$.

The definite classification $\text{Cla}(T)$ is defined for a regular theory T in [1]. Here

we define two classifications $\text{Cla}^D(T)$ and $\text{Cla}^C(T)$ for a theory T with **Weakening**.

Definition 3.4. Let T be a theory with **Weakening**. A classification $\text{Cla}^C(T)$ and $\text{Cla}^D(T)$ are defined as follows:

- (1) $\text{tok}(\text{Cla}^C(T))$ is the set of coverings $\langle \Gamma, \Delta \rangle$ of $\text{typ}(T)$ such that $\Gamma \not\vdash_T \Delta$,
- (2) $\text{tok}(\text{Cla}^D(T))$ is the set of disjoint pairs $\langle \Gamma, \Delta \rangle$ of subsets Γ and Δ of $\text{typ}(T)$ such that $\Gamma \not\vdash_T \Delta$,
- (3) $\text{typ}(\text{Cla}^C(T)) = \text{typ}(\text{Cla}^D(T)) = \text{typ}(T)$,
- (4) $\langle \Gamma, \Delta \rangle \models_{\text{Cla}^C(T)}^+ \alpha$ if and only if $\alpha \in \Gamma$,
- (5) $\langle \Gamma, \Delta \rangle \models_{\text{Cla}^C(T)}^! \alpha$ if and only if $\alpha \notin \Delta$,
- (6) $\langle \Gamma, \Delta \rangle \models_{\text{Cla}^D(T)}^+ \alpha$ if and only if $\alpha \notin \Delta$, and
- (7) $\langle \Gamma, \Delta \rangle \models_{\text{Cla}^D(T)}^! \alpha$ if and only if $\alpha \in \Gamma$.

Lemma 3.5. Let T be a theory with **Weakening**. Then, we have

- (i) $T \sqsubseteq \text{Th}^{+,!}(\text{Cla}^C(T))$, and
- (ii) $T \sqsubseteq \text{Th}^{!,+}(\text{Cla}^D(T))$.

Proof. Suppose that $\Gamma_0 \vdash_T \Delta_0$. To prove $\Gamma_0 \vdash_{\text{Cla}^C(T)}^! \Delta_0$, let $\langle \Gamma, \Delta \rangle \in \text{tok}(\text{Cla}^C(T))$ be arbitrary. Assume that $\langle \Gamma, \Delta \rangle \models_{\text{Cla}^C(T)}^+ \alpha$ for every $\alpha \in \Gamma_0$ and $\langle \Gamma, \Delta \rangle \not\models_{\text{Cla}^C(T)}^! \alpha$ for every $\alpha \in \Delta_0$. Then, $\Gamma_0 \subseteq \Gamma$ and $\Delta_0 \subseteq \Delta$. By **Weakening** for T , $\Gamma \vdash_T \Delta$. This contradicts $\langle \Gamma, \Delta \rangle \in \text{Cla}^C(T)$. In the same fashion, we can show that $T \sqsubseteq \text{Th}^{!,+}(\text{Cla}^D(T))$. $\square$

Proposition 3.6. Let T be a theory. Then, we have the following (i) and (ii) :

- (i) **Weakening** and **Identity** hold for T if and only if $T = \text{Th}^{!,+}(\text{Cla}^D(T))$.
- (ii) **Weakening** and **Global Cut** hold for T if and only if $T = \text{Th}^{+,!}(\text{Cla}^C(T))$.

Proof. (i) : Suppose that **Weakening** and **Identity** hold for T . It suffices to show that $\text{Th}^{!,+}(\text{Cla}^D(T)) \sqsubseteq T$. Assume that $\Gamma_0 \vdash_{\text{Cla}^D(T)}^! \Delta_0$. If there exists an $\alpha \in \Gamma_0 \cap \Delta_0$, then $\alpha \vdash_T \alpha$ by **Identity**, and so $\Gamma_0 \vdash_T \Delta_0$ by **Weakening**. Thus we can assume that $\langle \Gamma_0, \Delta_0 \rangle$ is a disjoint pair. If $\Gamma_0 \not\vdash_T \Delta_0$, then $\langle \Gamma_0, \Delta_0 \rangle \in \text{Cla}^D(T)$, and so $\langle \Gamma_0, \Delta_0 \rangle \not\models_{\text{Cla}^D(T)}^! \alpha$ for some $\alpha \in \Gamma_0$, or $\langle \Gamma_0, \Delta_0 \rangle \models_{\text{Cla}^D(T)}^+ \alpha$ for some $\alpha \in \Delta_0$, by our assumption $\Gamma_0 \vdash_{\text{Cla}^D(T)}^! \Delta_0$. This would be impossible. The “if” part follows from the list of **Identity** and **Global Cut**.

(ii) : Suppose that **Weakening** and **Global Cut** hold for T . It suffices to show that $\text{Th}^{+,!}(\text{Cla}^C(T)) \sqsubseteq T$. Suppose that $\Gamma_0 \not\vdash_T \Delta_0$. By Lemma 3.3, there exists a covering $\langle \Gamma, \Delta \rangle$ of $\text{typ}(T)$, such that $\Gamma_0 \not\vdash_T \Delta$, $\Gamma_0 \subseteq \Gamma$, and $\Delta_0 \subseteq \Delta$. Then, $\langle \Gamma, \Delta \rangle \in \text{tok}(\text{Cla}^C(T))$. We have $\langle \Gamma, \Delta \rangle \models_{\text{Cla}^C(T)}^+ \alpha$ for any $\alpha \in \Gamma_0$ by $\Gamma_0 \subseteq \Gamma$, and $\langle \Gamma, \Delta \rangle \not\models_{\text{Cla}^C(T)}^! \alpha$ for any $\alpha \in \Delta_0$ by $\Delta_0 \subseteq \Delta$. Thus, $\Gamma_0 \not\vdash_{\text{Cla}^C(T)}^! \Delta_0$. $\square$

4. Information fields

If we perceive an information flow from an information source λ to an information source μ , this information flow is related not only with λ and μ , but also with other information sources. For example, let us consider a situation in which information flows from $\lambda = \lambda_0$ to λ_1 , from λ_1 to $\lambda_2, \dots$, and from λ_{n-1} to $\lambda_n = \mu$, successively. This situation can be expressed by considering classifications $A_{\lambda_0}, \dots, A_{\lambda_n}$, which represent information sources $\lambda_0, \dots, \lambda_n$ respectively, and information links with the same type $\langle R_0, S_0 \rangle : A_{\lambda_0} \rightleftharpoons A_{\lambda_1}$, $\langle R_1, S_1 \rangle : A_{\lambda_1} \rightleftharpoons A_{\lambda_2}, \dots, \langle R_{n-1}, S_{n-1} \rangle : A_{\lambda_{n-1}} \rightleftharpoons A_{\lambda_n} = A_{\mu}$. The composition $\langle R_{n-1} \circ \dots \circ R_0, S_0 \circ \dots \circ S_{n-1} \rangle : A_{\lambda_0} \rightleftharpoons A_{\mu}$ is an information link with the same type, and so it makes an information flow from λ to μ .

Definition 4.1. Let Λ be a set of information sources. We say that an information field for Λ is represented by a family $\{A_\lambda\}_{\lambda \in \Lambda}$ of classifications if each A_λ represents an information source λ .

Example 4.2. Let Λ be a set of people. We define an information field $\lambda = \{A_\lambda\}_{\lambda \in \Lambda}$, so that (i) $\text{tok}(A_\lambda)$ is the set of our concepts, (ii) $\text{typ}(A_\lambda)$ is the set of English sentences, and (iii) $a \models_{A_\lambda} \alpha$ means that “ λ knows that a meaning of α is a ”, and $a \not\models_{A_\lambda} \alpha$ means that “ λ does not know that a meaning of α is not a ” for each $\lambda \in \Lambda$.

Definition 4.3. Let $\mathcal{L} = \{A_\lambda\}_{\lambda \in \Lambda}$ be a family of classifications. A *path* for $\mathcal{L}$ is a map $s : \Lambda \rightarrow \bigcup_{\lambda \in \Lambda} \text{tok}(A_\lambda)$, such that $s(\lambda) \in \text{tok}(A_\lambda)$ for each $\lambda \in \Lambda$.

Let λ and μ be different elements in Λ . We say that $\langle R, S \rangle$ is a link from λ to μ in $L = \{A_\lambda\}_{\lambda \in \Lambda}$ if $\langle R, S \rangle$ is a link from A_λ to A_μ .

Definition 4.4. Let $\mathcal{F}$ be a set of links in Λ .

- (i) Let λ, μ be information sources in Λ . We say that $\mathcal{F}$ *transforms* $\alpha \in \text{typ}(A_\lambda)$ to $\beta \in \text{typ}(A_\mu)$ if there exists a finite sequence $\alpha_0 \in \text{typ}(A_{\lambda_0}), \dots, \alpha_n \in \text{typ}(A_{\lambda_n})$ of types, and a finite sequence

$$\langle R_0, S_0 \rangle : A_{\lambda_0} \rightleftharpoons A_{\lambda_1}, \dots, \langle R_{n-1}, S_{n-1} \rangle : A_{\lambda_{n-1}} \rightleftharpoons A_{\lambda_n}$$

of links in $\mathcal{F}$, such that $\lambda_0 = \lambda$, $\lambda_n = \mu$, $\alpha_0 = \alpha$, $\alpha_n = \beta$, and $\alpha_{i-1} R_{i-1} \alpha_i$ for $1 \leq i \leq n$.

- (ii) A path s for L is said to be a *propagation path* for $\mathcal{F}$ if $s_\mu S s_\lambda$ for every $\langle R, S \rangle : A_\lambda \rightleftharpoons A_\mu$ in $\mathcal{F}$.

For each set $\mathcal{F}$ of links in $\mathcal{L} = \{A_\lambda\}_{\lambda \in \Lambda}$, and for each $\lambda \in \Lambda$, $D_\lambda^\mathcal{F}$ will denote the set of types α of A_λ , such that $\mathcal{F}$ transforms α to itself, and $S_\lambda^\mathcal{F}$ will denote the set

$\{s(\lambda) \mid s \text{ is a propagation path for } \mathcal{F}\}$. We say that $\mathcal{F}$ *definitely transforms* $\alpha \in \text{typ}(A_\lambda)$ to $\beta \in \text{typ}(A_\mu)$ if we can take a finite sequence α_i ($0 \leq i \leq n$) in the above definition from $D_\lambda^\mathcal{F}$ ($0 \leq i \leq n$).

Definition 4.5. A set $\mathcal{F}$ of links in $\mathcal{L}$ is said to be a $(0, 1)$ -*propagator* in $\mathcal{L}$ for an adequate pair $\{0, 1\}$ if, for each propagation path s for $\mathcal{F}$, and each $\langle R, S \rangle : A_\lambda \rightleftharpoons A_\mu$ in $\mathcal{F}$, $s(\lambda) \models_{A_\lambda}^0 \alpha$ implies $s(\mu) \models_{A_\mu}^0 \beta$, and $s(\lambda) \models_{A_\lambda}^1 \alpha$ implies $s(\mu) \models_{A_\mu}^1 \beta$ whenever $\alpha R \beta$. $\mathcal{F}$ is said to be a *definite propagator* if, for each propagation path s for $\mathcal{F}$, and each $\langle R, S \rangle : A_\lambda \rightleftharpoons A_\mu$ in $\mathcal{F}$, $s(\lambda) \models_{A_\lambda}^1 \alpha$ implies $s(\mu) \models_{A_\mu}^1 \beta$ whenever $\alpha R \beta$.

It is clear that a set of $(0, 1)$ -information links (resp. definite links) in $\mathcal{L}$ is a $(0, 1)$ -propagator (resp. a definite propagator) in $\mathcal{L}$. We simply say that $\mathcal{F}$ is a propagator if $\mathcal{F}$ is a $(0, 1)$ -propagator for some adequate pair $\{0, 1\}$.

Lemma 4.6. Let $\mathcal{F}$ be a definite propagator in $\mathcal{L} = \{A_\lambda\}_{\lambda \in \Lambda}$. Then, $\langle S_\lambda^\mathcal{F}, D_\lambda^\mathcal{F} \rangle$ is definite in A_λ for each $\lambda \in \Lambda$.

By the above lemma, for any definite propagator $\mathcal{F}$, we can define a definite classification $D^\mathcal{F}(A_\lambda)$ for each $\lambda \in \Lambda$, such that (i) $\text{tok}(D^\mathcal{F}(A_\lambda)) = S_\lambda^\mathcal{F}$, (ii) $\text{typ}(D^\mathcal{F}(A_\lambda)) = D_\lambda^\mathcal{F}$, and (iii) $s(\lambda) \models_{A_\lambda} \alpha$ if and only if $s(\lambda) \models_{A_\lambda} \alpha$.

Definition 4.7. A set $\mathcal{F}$ of links in $\mathcal{L}$ (resp. definitely) induces a link $\langle R, S \rangle : A_\lambda \rightleftharpoons A_\mu$, in symbol $\mathcal{F} \vdash_{\mathcal{L}} \langle R, S \rangle$ (resp. $\mathcal{F} \vdash_{\mathcal{L}}^d \langle R, S \rangle$), if the following conditions are fulfilled:

- (i) If s is a propagation path with respect to $\mathcal{F}$, then $s_\mu S s_\lambda$.
- (ii) If $\alpha R \beta$, α is (resp. definitely) transformed to β by $\mathcal{F}$.

For sets $\mathcal{F}$ and $\mathcal{G}$ of links in Λ , $\mathcal{F} \vdash_{\mathcal{L}} \mathcal{G}$ is defined by $\mathcal{F} \vdash_{\mathcal{L}} \langle R, S \rangle$ for all $\langle R, S \rangle \in \mathcal{G}$.

Example 4.8. Consider Example 4.2. A set $\mathcal{F}$ of links in $\mathcal{L}$ can be considered as a communication system between each other in Λ . Let $\langle R, S \rangle : A_\lambda \rightleftharpoons A_\mu$ be a link. $\langle R, S \rangle$ can be considered as a communication way between λ and μ . Suppose that $\mathcal{F} \vdash_{\mathcal{L}} \langle R, S \rangle$. Let $\alpha \in \text{typ}(A_\lambda)$, $\beta \in \text{typ}(A_\mu)$, $a \in \text{tok}(A_\lambda)$, and $b \in \text{tok}(A_\mu)$ be such that $\alpha R \beta$ and $b S a$. Though $\mathcal{F}$ transforms α to β , $\mathcal{F}$ does not make a communication between λ and μ . Suppose that $\mathcal{F}$ is a propagator, say a $(!, +)$ -propagator and there exists a propagation path s for $\mathcal{F}$ such that $s(\lambda) = a$ and $s(\mu) = b$. In this case, definite information “ a is a meaning of α ” in λ is propagated in the form of definite information “ b is a meaning of β ” in μ by s through $\mathcal{F}$. Indefinite information “ a might be a meaning of α ” and “ b might be a meaning of β ” follow similarly.

The following definition is a generalization of the notion of (dual) invariants in [1].

Definition 4.9. Let A be a classification. A pair $\langle R, S \rangle$ of a binary relation R on $\text{typ}(A)$ and a subset S of $\text{tok}(A)$ is said to be a *dual* $(0, 1)$ -semi-invariant on A for an adequate pair $\{0, 1\}$ if, for every $a \in S$, $a \models_A^0 \alpha$ implies $a \models_A^0 \alpha'$, and $a \models_A^1 \alpha$ implies $a \models_A^1 \alpha'$, whenever $\alpha R \alpha'$. $\langle R, S \rangle$ is said to be *definite* if, for every $a \in S$, $a \models_A^+ \alpha$ implies $a \models_A^! \alpha'$, whenever $\alpha R \alpha'$.

It is clear that $\langle R, S \rangle$ is a dual $(0, 1)$ -semi-invariant (resp. a dual definite semi-invariant) if and only if $\langle R, \text{id}_S \rangle : A \rightrightarrows A$ is a $(0, 1)$ -information (resp. definite) link.

We define the direct sum $\Sigma_{\lambda \in \Lambda} A_\lambda$ of $\mathcal{L} = \{A_\lambda\}_{\lambda \in \Lambda}$ as follows :

- (1) $\text{tok}(\Sigma_{\lambda \in \Lambda} A_\lambda) = \prod_{\lambda \in \Lambda} \text{tok}(A_\lambda) = \{s \mid s \text{ is a path for } \mathcal{L}\},$
- (2) $\text{typ}(\Sigma_{\lambda \in \Lambda} A_\lambda) = \Sigma_{\lambda \in \Lambda} \text{typ}(A_\lambda) = \{\langle \alpha, \lambda \rangle \mid \lambda \in \Lambda, \alpha \in \text{typ}(A_\lambda)\},$
- (3) $s \models_{\Sigma_{\lambda \in \Lambda} A_\lambda}^+ \langle \alpha, \lambda \rangle$ if and only if $s \models_{A_\lambda}^+ \alpha,$
- (4) $s \models_{\Sigma_{\lambda \in \Lambda} A_\lambda}^! \langle \alpha, \lambda \rangle$ if and only if $s \models_{A_\lambda}^! \alpha.$

It is clear that ΣA_λ is definite if each A_λ is definite. For each $(0, 1)$ -propagator $\mathcal{F}$ in $\mathbf{L}$, we define a dual $(0, 1)$ -semi-invariant $\langle R_{\mathcal{F}}, S_{\mathcal{F}} \rangle$ on $\Sigma_{\lambda \in \Lambda} A_\lambda$ as follows :

- (1) $\langle \alpha, \lambda \rangle R_{\mathcal{F}} \langle \beta, \mu \rangle$ if and only if there exists a link $\langle R, S \rangle : A_\lambda \rightrightarrows A_\mu$ in $\mathcal{F}$ such that $\alpha R \beta,$
- (2) $S_{\mathcal{F}} = \{s \in \text{tok}(\Sigma_{\lambda \in \Lambda} A_\lambda) \mid s \text{ is a propagation path for } \mathcal{F}\}.$

It is clear that $\langle R_{\mathcal{F}}, S_{\mathcal{F}} \rangle$ is definite if $\mathcal{F}$ is definite.

Definition 4.10. Let $J = \langle R, S \rangle$ and $J' = \langle R', S' \rangle$ be dual semi-invariants on an indefinite classification A . We say that J *depends* J' , in symbol $J' \vdash_A J$, if

- (1) there exists a finite sequence $\alpha = \alpha_0, \dots, \alpha_n = \beta$ such that $\alpha_{i-1} R' \alpha_i$ for $1 \leq i \leq n$ whenever $\alpha R \beta$, and
- (2) $S' \subseteq S.$

Lemma 4.11. Let $\mathcal{F}$ and $\mathcal{G}$ be propagators in $\mathcal{L}$. Then, $\mathcal{F} \vdash_{\mathbf{L}} \mathcal{G}$ if and only if $J_{\mathcal{F}} \vdash_{\Sigma_{\lambda \in \Lambda} A} J_{\mathcal{G}}$, where $J_{\mathcal{F}} = \langle R_{\mathcal{F}}, S_{\mathcal{F}} \rangle$ and $J_{\mathcal{G}} = \langle R_{\mathcal{G}}, S_{\mathcal{G}} \rangle.$

Definition 4.12. Let $\{0, 1\}$ be an adequate pair. A $(0, 1)$ -ordered classification $\mathfrak{U} = \langle \text{tok}(\mathfrak{U}), \text{typ}(\mathfrak{U}), \models_{\mathfrak{U}}^!, \models_{\mathfrak{U}}^+, \leq_{\mathfrak{U}} \rangle$ consists of the followings :

- (1) A classification $\text{cla}(\mathfrak{U}) = \langle \text{tok}(\mathfrak{U}), \text{typ}(\mathfrak{U}), \models_{\mathfrak{U}}^!, \models_{\mathfrak{U}}^+ \rangle,$
- (2) A partial ordered set $\text{ord}(\mathfrak{U}) = \langle \text{typ}(\mathfrak{U}), \leq_{\mathfrak{U}} \rangle,$ such that $\alpha \leq_{\mathfrak{U}} \beta$ implies

$$\begin{aligned} \forall a \in \text{tok}(\mathfrak{U}) (a \models_{\mathfrak{U}}^0 \alpha \Rightarrow a \models_{\mathfrak{U}}^0 \beta), \\ \forall a \in \text{tok}(\mathfrak{U}) (a \models_{\mathfrak{U}}^1 \alpha \Rightarrow a \models_{\mathfrak{U}}^0 \beta). \end{aligned}$$

An *ordered definite classification* $\mathfrak{U}$ is a $(+, !)$ -ordered classification such that $\text{cla}(\mathfrak{U})$

is a definite classification. Henceforth, in this case, the condition for $\leq_{\mathfrak{U}}$ can be written as

$$\alpha \leq_{\mathfrak{U}} \beta \Rightarrow \forall a \in \text{tok}(\mathfrak{U})(a \models_{\mathfrak{U}} \alpha \Rightarrow a \models_{\mathfrak{U}} \beta).$$

We simply say that $\mathfrak{U}$ is an ordered classification if $\mathfrak{U}$ is a $(0, 1)$ -ordered classification for some adequate pair $\{0, 1\}$.

The following theorem is a generalization of the existence of quotient of a classification in [1].

Theorem 4.13. *Let $J = \langle R, S \rangle$ be a dual $(0, 1)$ -semi-invariant on a classification A with an adequate pair $\{0, 1\}$. Then, there exists a pair $\langle \mathfrak{U}, f \rangle$ of a $(0, 1)$ -ordered classification $\mathfrak{U}$ and an infomorphism $f : A \rightrightarrows \text{cla}(\mathfrak{U})$ with the following properties :*

- (a) $\alpha R \beta$ implies $f(\alpha) \leq_{\mathfrak{U}} f(\beta)$, and $f(c) \in S$ for $c \in \text{tok}(\mathfrak{U})$,
- (b) If $\langle \mathfrak{U}', f' \rangle$ is a pair of a $(0, 1)$ -ordered classification $\mathfrak{U}'$ and an infomorphism $f' : A \rightrightarrows \text{cla}(\mathfrak{U}')$ having the property (a), then there exists a unique infomorphism $h : \mathfrak{U} \rightrightarrows \mathfrak{U}'$ such that $f' = h \circ f$.

Moreover, the unique infomorphism h preserves the orderings of $\mathfrak{U}$ and $\mathfrak{U}'$.

Proof. Let $\langle R, S \rangle$ be a dual $(0, 1)$ -semi-invariant on a classification A . Then,

$$(2) \quad \forall a \in S((a \models_A^0 \alpha \Rightarrow a \models_A^0 \alpha') \wedge (a \models_A^1 \alpha \Rightarrow a \models_A^1 \alpha'))$$

Let $\lesssim_R$ be the least preordering on $\text{typ}(A)$ containing R . Since the binary relation between α and α' defined by (2) is a preordering containing R , then $\alpha \lesssim_R \alpha'$ satisfies (2). Let $\simeq_R$ be the equivalence relation defined by $\alpha \simeq_R \beta$ if and only if $\alpha \lesssim_R \beta$ and $\beta \lesssim_R \alpha$. By Lemma 2.3, we have

$$\forall a \in S \forall \alpha, \beta \in \text{typ}(A)(\alpha \simeq_R \beta \Rightarrow (a \models_A^! \alpha \iff a \models_A^! \beta) \wedge (a \models_A^+ \alpha \iff a \models_A^+ \beta)).$$

A $(0, 1)$ -ordered classification $\mathfrak{U}$ can be defined as follows :

- (1) $\text{tok}(\mathfrak{U}) = S$.
- (2) $\text{typ}(\mathfrak{U}) = \text{typ}(A) / \simeq_R$.
- (3) $a \models_{\text{cla}(\mathfrak{U})}^! [\alpha]_{\simeq_R}$ if $a \models_A^! \alpha$.
- (4) $a \models_{\text{cla}(\mathfrak{U})}^+ [\alpha]_{\simeq_R}$ if $a \models_A^+ \alpha$.
- (5) $[\alpha]_{\simeq_R} \leq_{\mathfrak{U}} [\beta]_{\simeq_R}$ if $\alpha \lesssim_R \beta$.

Note that the above definition is well-defined. We also should note that $a \models_{\mathfrak{E}}^* [\alpha]$ if and only if $a \models_A \alpha$ for all $a \in S$ and $\alpha \in \text{typ}(A)$, where $*$ is in $\{!, +, \times, -, ?, @\}$. From this, we can have $\leq_{\mathfrak{E}}$ is a $(0, 1)$ -ordering. We define an infomorphism $f : A \rightrightarrows \text{cla}(\mathfrak{E})$ by $f^\wedge(\alpha) = [\alpha]$ and $f^\vee(a) = a$ for $\alpha \in \text{typ}(A)$ and $a \in S$. It is clear that $\langle \mathfrak{E}, f \rangle$ satisfies the property (a). Suppose that $\langle \mathfrak{E}', f' \rangle$ is a pair of a $(0, 1)$ -ordered classification $\mathfrak{E}'$, and an infomorphism f' from A to $\text{cla}(\mathfrak{E}')$ satisfying the property (a). Define an infomorphism $h : \text{cla}(\mathfrak{E}') \rightrightarrows \text{cla}(\mathfrak{E})$, so that :

- (1) $h^\wedge([\alpha]) = f'^\wedge(\alpha),$
- (2) $h^\vee(a') = f'^\vee(a').$

Since the binary relation between α and β such that $f'(\alpha) \leq_{\mathfrak{E}'} f'(\beta)$ on $\text{typ}(A)$ is a preordering containing R , it contains $\lesssim_R$. Thus, we have

$$(3) \quad \alpha \lesssim_R \beta \text{ implies } f'(\alpha) \leq_{\mathfrak{E}'} f'(\beta).$$

It follows that $[\alpha] = [\beta]$ implies $f'(\beta) = f'(\alpha)$. Thus, $h^\wedge$ is well-defined. It is clear that h is an infomorphism. The uniqueness of h is clear from the definition of h . It follows from (3) that h preserves the orderings. $\square$

We say that the pair $\langle \mathfrak{E}, f \rangle$ in the above theorem, uniquely determined up to isomorphisms, is the pair of the *quotient* of A with respect to a dual $(0, 1)$ -semiinvariant $\langle R, S \rangle$ and the canonical surjection to the quotient.

For the case that a dual semi-invariant $\langle R, S \rangle$ is definite, we have the following theorem. Recall that $D^{\langle R, S \rangle}(A)$ is the definite classification, such that $\text{tok}(D^{\langle R, S \rangle}(A)) = S$, $\text{typ}(D^{\langle R, S \rangle}(A)) = D_A^{\langle R, S \rangle}$ is the set of types α of A such that there exists a finite $\alpha = \alpha_0, \dots, \alpha_n = \alpha$ of elements of A with $\alpha_{i-1} R \alpha_i$ ($1 \leq i \leq n$), and $a \models_{D^{\langle R, S \rangle}(A)} \alpha$ if and only if $a \models_A^! \alpha$.

Theorem 4.14. *Let $\langle R, S \rangle$ be a dual definite semi-invariant on a classification A . Then, there exists a pair $h \langle \mathfrak{E}, f \rangle$ of a ordered definite classification $\mathfrak{E}$ and an infomorphism $f : D^{\langle R, S \rangle}(A) \rightrightarrows \text{cla}(\mathfrak{E})$ with the following properties (a) and (b) :*

- (a) $\alpha R \alpha'$ implies $f(\alpha) \leq_{\mathfrak{E}} f(\alpha')$ for any $\alpha, \alpha' \in \text{typ}(D^{\langle R, S \rangle}(A))$, and $f(c) \in S$ for any $c \in \text{tok}(\mathfrak{E})$.
- (b) *If $\langle \mathfrak{E}', f' \rangle$ is another pair of an ordered definite classification $\mathfrak{E}'$ and an infomorphism $f' : D^{\langle R, S \rangle}(A) \rightrightarrows \text{cla}(\mathfrak{E}')$ with the property (a), then there exists a unique infomorphism $h : \text{cla}(\mathfrak{E}) \rightrightarrows \text{cla}(\mathfrak{E}')$ such that the diagram is commutative.*

Moreover, the unique infomorphism h preserves the orders $\leq_{\mathfrak{E}}$ and $\leq_{\mathfrak{E}'}$.

Proof. Let $\bar{R}$ be the least transitive relation on $\text{typ}(A)$ containing R . Since the binary relation $\alpha R' \alpha'$ defined by

$$(4) \quad \forall a \in S (a \models_A^+ \alpha \implies a \models_A' \alpha')$$

is a transitive relation containing R , then R' contains $\bar{R}$. It follows that $\alpha \bar{R} \alpha'$ implies (4). Note that there exists a sequence $\alpha_0, \dots, \alpha_n$ such that $\alpha = \alpha_0 R \alpha_1 R \dots R \alpha_{n-1} R \alpha_n = \alpha'$ if and only if $\alpha \bar{R} \alpha'$. $R_{=}$ is the relation on $\text{typ}(A)$ defined by $\alpha \bar{R} \alpha'$ and $\alpha' \bar{R} \alpha$. Then, $\text{dom}(R_{=}) = D_A^{\langle R, S \rangle}$. It is clear that the restriction of $R_{=}$ to $D_A^{\langle R, S \rangle}$ is an equivalence relation.

Let $\mathfrak{E}$ be as follows :

- (1) $\text{tok}(\mathfrak{E}) = S$.
- (2) $\text{typ}(\mathfrak{E}) = D_A^{\langle R, S \rangle} / R_{=}$.
- (3) $a \models_{\mathfrak{E}} [\alpha]_R$ if and only if $a \models_A' \alpha$.
- (4) $[\alpha]_R \leq_{\mathfrak{E}} [\alpha']$ if and only if $\alpha \bar{R} \alpha'$.

It is clear that 3 and 4 are well-defined. Note that $a \not\models [\alpha]_R$ if and only if $a \models_A^{\times} \alpha$. It is also clear that, for all $a \in \text{tok}(\mathfrak{E})$, $a \models_{\mathfrak{E}} [\alpha]$ implies $a \models_{\mathfrak{E}} [\alpha']$ whenever $[\alpha]_R \leq_{\mathfrak{E}} [\alpha']_R$. Thus, a type-ordered classification $\mathfrak{E} = \langle \text{tok}(\mathfrak{E}), \text{typ}(\mathfrak{E}), \models_{\mathfrak{E}}, \leq_{\mathfrak{E}} \rangle$ has been defined. We define a map $f^{\wedge} : D_A^{\langle R, S \rangle} \rightarrow \text{typ}(\mathfrak{E})$, and a map $f^{\vee} : S \rightarrow \text{tok}(A)$ by $f^{\wedge}(\alpha) = [\alpha]_R$ and $f^{\vee} = a$. Since $\text{cla}(\mathfrak{E})$ is a definite classification, it suffices to show that $f(a) \models_A^! \alpha$ if and only if $a \models_{\mathfrak{E}} f(\alpha)$ for $\alpha \in D_A^{\langle R, S \rangle}$ and $a \in S$. This is clear from the definition of $a \models_{\mathfrak{E}} [\alpha]_R$. It is also clear that $\langle \mathfrak{E}, f \rangle$ satisfies (a). Let $h : \langle \mathfrak{E}', f' \rangle$ be another pair of a type-ordered classification $\mathfrak{E}'$ and an infomorphism $f' : D^{\langle R, S \rangle}(A) \rightrightarrows \text{cla}(\mathfrak{E}')$ with the property (a). Suppose that $\alpha \bar{R} \alpha'$. Then, there exists a sequence $\alpha_0, \dots, \alpha_n$, such that $\alpha = \alpha_0 R \alpha_1 R \dots R \alpha_n = \alpha'$. Since f' satisfies (a), then $f'(\alpha) = f'(\alpha_0) \leq_{\mathfrak{E}'} \dots \leq_{\mathfrak{E}'} f'(\alpha_n) = f'(\alpha')$. It follows that $f'(\alpha) \leq_{\mathfrak{E}'} f'(\alpha')$. Now, we can define an infomorphism $h : \text{cla}(\mathfrak{E}) \rightrightarrows \text{cla}(\mathfrak{E}')$ as follows :

- (1) $h([\alpha]) = f'(\alpha)$ for $\alpha \in D_A^{\langle R, S \rangle}$.
- (2) $h(c') = f'(c')$ for $c' \in \text{tok}(\mathfrak{E}')$.

The uniqueness of h is clear from the definition of h . It is also clear that h

perseveres the orderings. □

5. Channels

In [1], Barwise and Seligman develop a theory of information flow based on the theory of channels. In this section the notion of channels will be connected with the notion of links developed in the previous section.

Definition 5.1. A *channel* $\mathcal{C} = \{f_\lambda : A_\lambda \rightrightarrows C\}_{\lambda \in A}$ is a family of infomorphisms with the common codomain C , which is called the *core* of $\mathcal{C}$. A channel $\mathcal{C} = \{f_\lambda : A_\lambda \rightrightarrows C\}$ is said to be *definite* if it is a family of infomorphisms between definite classifications. A channel $\mathcal{C}$ is said to have the *(0, 1)-ordered core* $\mathfrak{E}$ if $\mathfrak{E}$ is a (0, 1)-ordered classification and the core of $\mathcal{C}$ is $\text{cla}(\mathfrak{E})$.

We define an infomorphism $\sigma_\lambda : A_\lambda \rightrightarrows \Sigma A_\lambda$ so that $\sigma_\lambda^\wedge(\alpha) = \langle \alpha, \lambda \rangle$ and $\sigma_\lambda^\vee(s)$ for each $\lambda \in A$. The channel $\{\sigma_\lambda : A_\lambda \rightrightarrows \Sigma A_\lambda\}$ will be denoted by $\mathcal{C}_A$.

Definition 5.2. Let $\mathcal{C} = \{f_\lambda : A_\lambda \rightrightarrows C\}$ and $\mathcal{C}' = \{f'_\lambda : A_\lambda \rightrightarrows C'\}$ be channels for L . A *refinement infomorphism* r from $\mathcal{C}$ to $\mathcal{C}'$ is an infomorphism from C to C' such that $r \circ f_\lambda = f'_\lambda$ for each $\lambda \in A$. If there exists a refinement infomorphism from $\mathcal{C}$ to $\mathcal{C}'$, we say that $\mathcal{C}$ is a *refinement* of $\mathcal{C}'$.

For every channel $\mathcal{C} = \{f_\lambda : A_\lambda \rightrightarrows C\}$ for L there exists a unique refinement infomorphism from $\mathcal{C}_A$ to $\mathcal{C}$.

Definition 5.3. A channel $\mathcal{C} = \{f_\lambda : A_\lambda \rightrightarrows \mathfrak{E}\}$ with the (0, 1)-ordered core for $\mathcal{L}$ *covers* a link $\langle R, S \rangle : A_\lambda \rightrightarrows A_\mu$ if (i) $\alpha R \beta$ implies $f_\lambda(\alpha) \leq_{\mathfrak{E}} f_\mu(\beta)$ for $\alpha \in \text{typ}(A_\lambda)$ and $\beta \in \text{typ}(A_\mu)$, and (ii) $f_\mu(c) S f_\lambda(c)$ for $c \in \text{tok}(\mathfrak{E})$. A set $\mathcal{F}$ of links in $\mathcal{L}$ (0, 1)- *covers* a link $\langle R, S \rangle$ in $\mathcal{L}$, in symbol $\mathcal{F} \models_{\mathcal{L}}^{(0,1)} \langle R, S \rangle$, if $\langle R, S \rangle$ is covered by every channel $\mathcal{C}$ with the (0, 1)-ordered core, such that $\mathcal{C}$ covers $\langle R', S' \rangle$ for every link $\langle R', S' \rangle$ in $\mathcal{F}$. A set $\mathcal{F}$ of links in $\mathcal{L}$ *definitely covers* a link $\langle R, S \rangle$ in $\mathcal{L}$, in symbol $\mathcal{F} \models_{\mathcal{L}}^d \langle R, S \rangle$, if $\langle R, S \rangle$ is covered by every channel $\mathcal{C} = \{f_\lambda : D^{\mathcal{F}}(A_\lambda) \rightrightarrows \mathfrak{E}\}$ with the ordered core such that $\mathcal{C}$ covers $\langle R', S' \rangle$ for every link $\langle R', S' \rangle$ in $\mathcal{F}$.

Lemma 5.4 (Soundness). *Let $\mathcal{F}$ be a set of links in $\mathcal{L}$, and $\langle R, S \rangle : A_\lambda \rightrightarrows A_\mu$ be a link in $\mathcal{L}$. If $F \vdash_{\mathcal{L}} \langle R, S \rangle$, then $\mathcal{F} \models_{\mathcal{L}}^{(0,1)} \langle R, S \rangle$ for every adequate pair $\{0, 1\}$. If $\mathcal{F} \vdash_{\mathcal{L}}^d \langle R, S \rangle$, then $\mathcal{F} \models_{\mathcal{L}} \langle R, S \rangle$.*

Proof. Let $\mathcal{F}$ be a set of links in $\mathcal{L}$. Suppose that $\mathcal{F} \vdash_{\mathcal{L}} \langle R, S \rangle$. Let $\{0, 1\}$ be

an adequate pair. Let $\mathcal{C} = \{f_\lambda : A_\lambda \rightrightarrows \mathbb{E}\}$ be a channel with the $(0, 1)$ -ordered core which covers every link in $\mathcal{F}$. Let $\alpha \in \text{typ}(A_\lambda)$ and $\beta \in \text{typ}(A_\mu)$ be such that $\alpha R \beta$. Since $\mathcal{F}$ transforms α to β , there exist $\alpha_0 \in \text{typ}(A_{\lambda_0}), \dots, \alpha_n \in \text{typ}(A_{\lambda_n})$ and links $\langle R_i, S_i \rangle : A_{\lambda_{i-1}} \rightrightarrows A_{\lambda_i}$ ($1 \leq i \leq n$) in $\mathcal{F}$, such that $\lambda_0 = \lambda, \lambda_n = \mu, \alpha_0 = \alpha, \alpha_n = \beta$, and $\alpha_{i-1} R_i \alpha_i$ for $1 \leq i \leq n$. Since $\mathcal{C}$ covers each R_i , we have $f_{\lambda_{i-1}}(\alpha_{i-1}) \leq_{\mathbb{E}} f_{\lambda_i}(\alpha_i)$ for $1 \leq i \leq n$. It follows that $f_\lambda(\alpha) \leq_{\mathbb{E}} f_\mu(\beta)$. Let $c \in \text{tok}(\mathbb{E})$ be arbitrary. Then, $f_{\lambda'}(c) S' f_{\mu'}(c)$ for every link $\langle R', S' \rangle : A_{\lambda'} \rightrightarrows A_{\mu'}$ in $\mathcal{F}$. Therefore, $s : \lambda' \mapsto f_{\lambda'}(c)$ is a propagation path for $\mathcal{F}$. It follows that $f_\lambda(c) = s(\lambda) S s(\mu) = f_\mu(c)$. We have shown that $\mathcal{F} \models_{\mathcal{L}}^{(0,1)} \langle R, S \rangle$. We can prove that $\mathcal{F} \vdash_{\mathcal{L}}^a \langle R, S \rangle$ implies $\mathcal{F} \models_{\mathcal{L}}^a \langle R, S \rangle$ in the same way. $\square$

If $\mathcal{F}$ is a (definite) propagator in $\mathcal{L}$, we can prove the converse of Lemma 5.4. For this, we need the following lemma. This lemma is a generalization of the minimal covering lemma in [1].

Theorem 5.5 (Minimal Covering Lemma). *Let $\mathcal{F}$ be a propagator in $\mathcal{L}$. Then, there exists a channel $\mathcal{C} = \{f_\lambda : A_\lambda \rightrightarrows \mathbb{E}\}$ with the type-ordered core having the following properties (a) and (b).*

- (a) $\mathcal{C}$ covers each link in $\mathcal{F}$.
- (b) Let $\mathcal{C}' = \{f'_\lambda : A_\lambda \rightrightarrows \mathbb{E}'\}$ be another channel with the type-ordered core having the property (a). Then, there exists a unique order-preserving refinement infomorphism r from $\mathcal{C}$ to $\mathcal{C}'$.

Proof. Let $\mathcal{F}$ be a propagator in $\mathcal{L}$, and $\langle \mathbb{E}, f \rangle$ be the canonical pair for $\langle R_{\mathcal{F}}, S_{\mathcal{F}} \rangle$. Set $f_\lambda = f \circ \sigma_\lambda$ for each $\lambda \in \Lambda$, and $\mathcal{C} = \{f_\lambda : A_\lambda \rightrightarrows \mathbb{E}\}$. Let $\langle R, S \rangle : A_\lambda \rightrightarrows A_\mu$ be in $\mathcal{F}$. Suppose that $\alpha R \beta$. Then, $\langle \alpha, \lambda \rangle R_{\mathcal{F}} \langle \beta, \mu \rangle$. From the property of f , $f_\lambda(\alpha) = f(\langle \alpha, \lambda \rangle) \leq_{\mathbb{E}} f(\langle \beta, \mu \rangle) = f_\mu(\beta)$. Let c be in $\text{tok}(\mathbb{E})$. Then, $f(c) \in S_{\mathcal{F}}$. Thus, $f(c)$ is a propagation path for $\mathcal{F}$. It follows that $f_\lambda(c) = \sigma_\lambda(f(c)) S \sigma_\mu(f(c)) = f_\mu(c)$. Next, let $\mathcal{C}' = \{f'_\lambda : A_\lambda \rightrightarrows \mathbb{E}'\}$ be the another channel having the property (a). Then, there exists a unique refinement classmorphism $f' : \Sigma A_\lambda \rightrightarrows \text{cla}(\mathbb{E}')$ from $\mathcal{C}$ to $\mathcal{C}'$. Suppose that $\langle \alpha, \lambda \rangle R_{\mathcal{F}} \langle \beta, \mu \rangle$. Then, $\alpha R \beta$ for some link $\langle R, S \rangle : A_\lambda \rightrightarrows A_\mu$ in $\mathcal{F}$. It follows that $f'_\lambda(\alpha) \leq_{\mathbb{E}'} f'_\mu(\beta)$. Therefore, $f'(\langle \alpha, \lambda \rangle) = f'(\sigma_\lambda(\alpha)) = f'_\lambda(\alpha) \leq_{\mathbb{E}'} f'_\mu(\beta) = f'(\sigma_\mu(\beta)) = f'(\langle \beta, \mu \rangle)$. Let c' be in $\text{tok}(\mathbb{E}')$. Let $\langle R, S \rangle : A_\lambda \rightrightarrows A_\mu$ be any link in $\mathcal{F}$.

Then, $f'_\mu(c) \mathcal{S} f'_\lambda(c)$. Therefore, $s: \lambda \mapsto f'_\lambda(c)$ is a propagation path for $\mathcal{F}$. Since $\sigma_\lambda(f'(c)) = f'_\lambda(c) = s(\lambda)$ for every $\lambda \in \Lambda$, $f'(c) = s \in S_{\mathcal{F}}$. The pair $\langle \mathfrak{E}', f' \rangle$ has the property (a) in the lemma. Henceforth, there exists a unique classmorphism $r: \text{cla}(\mathfrak{E}) \rightrightarrows \text{cla}(\mathfrak{E}')$ such that $r \circ f = f'$. Then, $r \circ f_\lambda = r \circ (f \circ \sigma_\lambda) = (r \circ f) \circ \sigma_\lambda = f' \circ \sigma_\lambda = f'_\lambda$ for each $\lambda \in \Lambda$. For the uniqueness of r , let r' be another refinement classmorphism from $\mathcal{C}$ to $\mathcal{C}'$. We want to show that $r' \circ f = f'$. Let α be in $\text{typ}(A_\lambda)$. Then, $(r' \circ f)(\langle \alpha, \lambda \rangle) = r'(f(\sigma_\lambda(\alpha))) = r'(f_\lambda(\alpha)) = f'_\lambda(\alpha) = f'(\sigma_\lambda(\alpha)) = f'(\langle \alpha, \mu \rangle)$. Let c' be in $\text{tok}(\mathfrak{E}')$. Since $\sigma_\lambda(f \circ r'(c')) = (\sigma_\lambda \circ f)(r'(c')) = (f_\lambda \circ r')(c') = f'_\lambda(c') = \sigma_\lambda(f'(c'))$ for each $\lambda \in \Lambda$, $(f \circ r')(c') = f'(c')$. Thus, we have $r' \circ f = f'$. $\square$

The channel determined uniquely up to isomorphisms will be called *the minimal covering* for a propagator $\mathcal{F}$. We have the following theorem for the definite propagator. The proof can be carried on as in the proof of Theorem 5.5 by using Theorem 4.14.

Theorem 5.6 (Minimal Covering Lemma (Definite Case)). *Let $\mathcal{F}$ be a definite propagator in $\mathcal{L} = \{A_\lambda\}_{\lambda \in \Lambda}$. Then, there exists a definite channel $\mathcal{C} = \{f_\lambda: D^F(A_\lambda) \rightrightarrows \mathfrak{E}\}$, with the type-ordered core having the following properties (a) and (b):*

(a) $\mathcal{C}$ covers each link in $\mathcal{F}$.

(b) Let $\mathcal{C}' = \{f'_\lambda: D^{\mathcal{F}}(A_\lambda) \rightrightarrows \mathfrak{E}'\}$ be another definite channel with the typeordered core having the property (a). Then, there exists a unique orderpreserving refinement infomorphism r from $\mathcal{C}$ to $\mathcal{C}'$.

Theorem 5.7 (Completeness). *Let $\mathcal{F}$ be a $(0, 1)$ -propagator in $L = \{A_\lambda\}_{\lambda \in \Lambda}$, and $\langle R, S \rangle: A_\lambda \rightrightarrows A_\mu$ be a link. If $\mathcal{F} \models_{\mathcal{L}}^{(0,1)} \langle R, S \rangle$, then $\mathcal{F} \vdash_L \langle R, S \rangle$. For a definite propagator $\mathcal{F}$, if $\mathcal{F} \models_{\mathcal{L}}^d \langle R, S \rangle$, then $\mathcal{F} \vdash_{\mathcal{L}}^d \langle R, S \rangle$.*

Proof. Suppose that $\mathcal{F} \models_{\mathcal{L}}^{(0,1)} \langle R, S \rangle$. Let $\mathcal{C} = \{f_\lambda: A_\lambda \rightrightarrows \mathfrak{E}\}$ be the minimal covering for $\mathcal{F}$. Suppose that $\alpha R \beta$. Then, $f_\lambda(\alpha) \leq_{\mathfrak{E}} f_\mu(\beta)$. By the construction of $\mathcal{C}$, there exists a finite sequence $\langle \alpha, \lambda \rangle = \langle \alpha_0, \lambda_0 \rangle, \dots, \langle \alpha_n, \lambda_n \rangle = \langle \beta, \lambda_n = \beta, \mu \rangle$ in $\text{typ}(\Sigma A_\lambda)$, such that $\langle \alpha_{i-1}, \lambda_{i-1} \rangle R_{\mathcal{F}} \langle \alpha_i, \lambda_i \rangle$ for $1 \leq i \leq n$. Then, there exists a finite sequence $\langle R_i, S_i \rangle: A_{\lambda_{i-1}} \rightrightarrows A_{\lambda_i}$, ($1 \leq i \leq n$) in $\mathcal{F}$, such that $\alpha_{i-1} R_i \alpha_i$ for $1 \leq i \leq n$. Thus, $\mathcal{F}$ transforms α to β . Let s be a propagation path for $\mathcal{F}$. Then, $s \in \text{tok}(\mathfrak{E})$,

and so $s(\mu) = f_\mu(s) \text{ } S \text{ } f_\lambda(s) = s(\lambda)$. We have shown that $\mathcal{F} \vdash_{\mathcal{L}} \langle R, S \rangle$. The definite case will be proved in the same fashion, using the Minimal Covering Theorem for the definite case. $\square$

Corollary 5.8. *Let $\langle R, S \rangle$ be a link in $\mathcal{L} = \{A_\lambda\}_{\lambda \in A}$. If $\mathcal{F}$ is a $(0, 1)$ -propagator in $\mathcal{L}$, $\mathcal{F} \vdash_{\mathcal{L}} \langle R, S \rangle$ if and only if $\mathcal{F} \models_{\mathcal{L}}^{(0,1)} \langle R, S \rangle$. If $\mathcal{F}$ is a definite propagator in $\mathcal{L}$, $\mathcal{F} \vdash_{\mathcal{L}}^d \langle R, S \rangle$ if and only if $\mathcal{F} \models_{\mathcal{L}}^d \langle R, S \rangle$.*

6. Productions and Designs

If we consider a set Σ of our desires, then the regular theory T concerning Σ ideally arises, though it might not be actually grasped by us. Then, an ideal specification can be expressed by a disjoint pair $\langle \Gamma, \Delta \rangle$, such that $\Gamma \not\vdash_T \Delta$; that is, a T -consistent partition $\langle \Gamma, \Delta \rangle$. Similarly, we consider *idealistically* the definite classification $\mathbf{B}$ such that (i) $\text{tok}(\mathbf{B})$ is the set of entities, (ii) $\text{typ}(\mathbf{B})$ is the set of behaviors, and (iii) $b \models_{\mathbf{B}} \beta$ if b behaves in β . We *position* a definite bilink $\langle R_I, S_I \rangle$ between $\text{Cla}(T)$ and $\mathbf{B}$. Thus, $\alpha \in \Gamma$ if and only if b behaves in β whenever $\alpha R_I \beta$ and $b S_I \langle \Gamma, \Delta \rangle$. In other words, if a desire α is interpreted into a desire β , and a complete specification $\langle \Gamma, \Delta \rangle$ corresponds to an entity b , then “ $\langle \Gamma, \Delta \rangle$ requires α ” is equivalent to “ b behaves in β ”. This is the ultimate ideal production. We cannot actually attain the production. Let $(f_I : \text{Cla}(T) \rightrightarrows \mathfrak{E}_I, g_I : \mathbf{B} \rightrightarrows \mathfrak{E}_I)$ be the minimal covering of $\{\langle R_I, S_I \rangle, \langle R_I^{-1}, S_I^{-1} \rangle\}$. In this case, $\leq_{\mathfrak{E}_I}$ is the equality on $\text{tok}(\mathfrak{E}_I)$. Thus, $\mathfrak{E}_I$ is just a definite classification $\text{cla}(\mathfrak{E}_I)$. Suppose that a desire α is interpreted into a behavior β through the production $\langle R_I, S_I \rangle$. Then, $f_I(\alpha) = g_I(\beta)$. What is this $f_I(\alpha) = g_I(\beta) \in \text{tok}(\mathfrak{E}_I)$? It is what we call the function induced by a desire α and a behavior β . Suppose that a complete specification $\langle \Gamma, \Delta \rangle$ corresponds to an entity b through the production $\langle R_I, S_I \rangle$. Then, $\langle \Gamma, \Delta \rangle$ is what we call a *concept* of b concerning our desires. Choose $c \in$

$\text{tok}(\mathbb{G})$ so that $\langle \Gamma, \Delta \rangle = f_I(c)$ and $b = g_I(c)$. c is what we call an *image* which realizes the entity b in the outer world and forms the concept $\langle \Gamma, \Delta \rangle$ in the inner world. Thus, $c \models_{\mathbb{G}_I} \gamma$ means “an image c elicits a function γ ”. Now, we consider an information field $\mathcal{A}$ where we actually design a production. Let $\mathcal{L} = \{A_\lambda\}_{\lambda \in \mathcal{A}}$ represent the information field $\mathcal{A}$. We suppose that $\mathcal{A}$ contains special information sources ι and κ such that $A_\iota = \text{Cla}(T)$ and $A_\kappa = \mathbf{B}$. We may say that *designing is equivalent to considering a channel* $\mathcal{C} = \{f_\lambda : A_\lambda \rightrightarrows \mathbb{G}\}$ with the ordered core for $\mathcal{L}$. Usually, the core $\mathbb{G}$ is a definite classification, such that $\text{tok}(\mathbb{G})$ is a set of so-called blueprints or plans, $\text{typ}(\mathbb{G})$ is a set of information (contents), and $c \models_{\mathbb{G}} \gamma$ means that an information content γ is written on a blueprint c . Information contents written on a blueprint are information like lengths, weights, and shapes of entities to be produced, that is, what we call attributes. For $\gamma_1, \gamma_2 \in \text{typ}(\mathbb{G})$, $\gamma_1 \leq_{\mathbb{G}} \gamma_2$ means that an attribute γ_1 includes an attribute γ_2 . If $\mathcal{C}$ represents an appropriate design, we must have the following things :

- (1) $f_i(\alpha)$ is the attribute obtained by estimating a desire α .
- (2) $f_\kappa(\beta)$ is the attribute obtained by observing a behavior β .
- (3) If a desire α is interpreted into a behavior β by the ideal product $\langle R_I, S_I \rangle$, $f_i(\alpha) = f_\kappa(\beta)$.
- (4) $f_i(c)$ is the concept formed by a blueprint c .
- (5) $f_\kappa(c)$ is the entity produced according to a blueprint c .
- (6) $f_i(c)$ corresponds to $f_\kappa(c)$ by the ideal product $\langle R_I, S_I \rangle$.

The above means that $\mathcal{C}$ covers $\langle R_I, S_I \rangle$ and $\langle R_I^{-1}, S_I^{-1} \rangle$. Therefore, there exists

a unique refinement informorphism r from $\mathcal{C}_I$ to $\mathcal{C}$. $r(\gamma)$ is the attribute required by a function γ . $r(c)$ is the image formed by a blueprint. Consider the meaning of $f_i = r \circ f_I$ and $f_\kappa = r \circ g_I$. Let α be a desire in Σ . The attribute $f_i(\alpha)$ obtained by interpreting α through the design system $\mathcal{C}$ is the attribute $r(f_I(\alpha))$ required by the function $f_i(\alpha)$ which is induced by α . Let β be a behavior in $\mathbf{B}$. The attribute $f_\kappa(\beta)$ obtained by interpreting β through the design system $\mathcal{C}$ is the attribute $r(g_I(\beta))$ required by the function $g_I(\beta)$ which is induced by β . Let c be a blueprint in $\text{tok}(\mathfrak{E})$. The concept $f_i(c)$ formed by c is the concept $f_I(r(c))$ formed by the image $r(c)$ which is imaged by c . The entity $f_\kappa(c)$ produced by c through $\mathcal{C}$ is the entity $g_I(r(c))$ realized by the image $r(c)$ which is imaged by c .

Suppose that an (indefinite) classification A_λ , generated by a set S of specifications concerning Σ , and the (indefinite) classification A_μ with respect to behaviors of entities by our knowledge. If the specification is well-settled, each element in S must be a T-consistent specification $\langle \Gamma, \Delta \rangle$, that is, $\langle \Gamma, \Delta \rangle$ is a disjoint pair of subsets of Σ such that $\Gamma \not\models_T \Delta$. We define a link $\langle R, S \rangle : A_\lambda \rightleftharpoons A_\mu$ so that $\alpha R \beta$ if and only if $\alpha = \beta$ and $\langle \tilde{\Gamma}, \tilde{\Delta} \rangle S \langle \Gamma, \Delta \rangle$ if and only if $\langle \tilde{\Gamma}, \tilde{\Delta} \rangle$ is an extension of $\langle \Gamma, \Delta \rangle$. $\langle R, S \rangle$ is a $(!, \times)$ -information link from A_λ to $A_\mu = \text{Cla}(T)$. $f_i(c)$ is the specification denoted by a blueprint c . Then, $f_i(c)$ must be an extension of $f_\lambda(c)$. It follows that C covers $\langle R, S \rangle$. If our knowledge is justified, then an entity b behaves in β whenever we know that b behaves in β , and b does not behave in β whenever we know that b does not behave in β , that is, $b \models_{A_\mu}^+ \beta$ implies $b \models_{A_\lambda} \beta$ and $b \models_{A_\mu}^- \beta$ implies $b \models_{A_\lambda}^- \beta$ for all $b \in \text{tok}(\mathbf{B})$ and $\beta \in \text{typ}(\mathbf{B})$. Since $f_\kappa = f_\mu$, we have a link $\langle id_{\text{typ}(\mathbf{B})}, id_{\text{tok}(\mathbf{B})} \rangle : A_\mu \rightleftharpoons A_\kappa$ is a $(!, \times)$ -information link. C covers $\langle id_{\text{typ}(\mathbf{B})}, id_{\text{tok}(\mathbf{B})} \rangle : A_\mu \rightleftharpoons A_\kappa$. Actually, we should not have dotted arrows in the diagrams. We can judge whether C covers $\langle R_I, S_I \rangle$ only by experiments, or our experiences. If experiments or experiences go well, we judge that C covers $\langle R_I, S_I \rangle$, and the design system obtained by C should be appropriate.

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