A Meta-Physical Investigation for the Special Theory of Relativity II

— Preliminary Remarks (Continued) —

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As a sequel of the preceding paper* in which the derivation of Lorentz transformation was investigated we deal with the fundamental principles of the special theory of relativity and their status in the theory. Their logical character and experimental evidences are investigated. Relations between these principles and Lorentz transformation are also examined.

§1. Introduction

Ordinarily it is considered that the special principle of relativity and the principle of the constancy of the light velocity are the two fundamental principles of the special theory of relativity, and the one of the most important results of the theory is the invariance of the physical laws under Lorentz transformation. Indeed, in Einstein's paper published in 1905 [1] it is shown that i) Lorentz transformation can be derived from these two principles and the additional assumption of the linearity of the transformation and ii) the invariance of the Maxwell equation under this transformation is also proved.

In this paper we shall investigate the character of these two principles and the relations between these principles and Lorentz transformation. Ordinarily these two principles are treated as if they have equal footing. But they stand on quite different points. When the principle of the constancy of the light velocity is proposed by Einstein, it is considered as a postulate. But the fact that the light velocity is independent of the velocity of its source is experimentally confirmed later in the 1960's when the accuracy of the measurement in high energy physics is raised sufficiently. Today, as the results of these experiment we consider this principle as a experimentally confirmed fact.

On the other hand the circumstance is not so simple in the case of the special principle of relativity. For instance, the questions whether this principle is really empirical law or not, and if so, how to verify its truth experimentally, or if it is not so, what is its character, and so forth, shall be solved.

* Y. Ueno: Ann. Japan Ass. Phil. Sci. 4 (1972) 71. Hereafter this paper is cited as [I].

The next problem we will investigate in this paper is to examine the logical structure for “the derivation”, a preliminary consideration of which has been presented in the preceding paper. We intend to enumerate exactly the necessary and sufficient conditions for the derivation in order to clarify the relations between Lorentz transformation or the Lorentz invariance of the physical laws and the two fundamental principles.

§2. The special principle of relativity.

According to Einstein the fundamental principles of the special theory of relativity are i) the special principle of relativity which is raised from the conjecture: “für alle Koordinaten-systeme, für welche die mechanischen Gleichungen gelten, auch die gleichen elektrodynamischen und optischen Gesetze gelten, . . .” to the following form of postulate, and ii) the principle of the constancy of the velocity of light which is an assumption that the light velocity is independent of the motion of its source. He “define” these principles as follows:

1. *Die Gesetze, nach denen sich die Zustände der physikalischen Systeme ändern, sind unabhängig davon, auf welches von zwei relativ zueinander in gleichformiger Translationsbewegung befindlichen Koordinatensystemen diese Zustandänderungen bezogen werden.*
2. *Jeder Lichtstrahl bewegt sich im “ruhenden” Koordinaten-systeme mit der bestimmten Geschwindigkeit V , unabhängig davon, ob dieser Lichtstrahl von einem ruhenden oder bewegten Körper emittiert ist*[1].

As was well-known Einstein derived the Lorentz transformation from these two principles together with some conditions such as the definition of the simultaneity in a moving system, the length of the moving scale, and the homogeneity of the space, and so forth. Indeed, (special) Lorentz transformation connects the two coordinate systems which move uniformly to each other and the velocity of light is invariant under this transformation. Therefore it is usually said that the Lorentz invariant physical law is consistent with these principles. Really Pauli wrote in his book published in 1921 as follows: “Wie im vorausgehenden Abschnitt dargelegt wurde, lassen sich die Postulate der Relativität und der Konstanz der Lichtgeschwindigkeit in die eine Forderung der Invarianz aller Naturgesetze gegenüber der Lorentz-Gruppe zusammenfassen”[2]. Many physicists seem to have the same view. For instance, Schweber adopted this condition as one of the axioms of the relativistic quantum field theory as follows: *The theory is invariant under inhomogeneous Lorentz transformation*[3]. Now the problem whether this form of the principle is equivalent to the original form of the two principles or not must be discussed. Then, one of the most important check points may be that Lorentz transformation is the only one which connects the uniformly moving system. Concerning this point we have obtained in the previous paper

some remarkable results in the course of our investigation of the derivation of the Lorentz transformation.

We shall begin with the investigation of the nature of the special principle of relativity. As we stated above, we have several kinds of form for the principle, namely, i) original form given by Einstein which states the invariance of physical laws under some coordinate transformation, ii) the form like the one stated by Pauli, and iii) the form given, say, by Fock. The last one is given in his text as: "*Phenomena occurring in a closed system are indifferent of any non-accelerated motion of the system as a whole*"[4]. At first we shall examine the physical implication of each form of the principle. Now let **E** be the proposition given by Einstein, **P** be that of Pauli, and **F** be Fock's one. It seems, at a glance, evident that the following logical relation holds between the above two propositions:

$$F \rightarrow E.$$

As is stated by Fock himself, the truth of **F** seems to be confirmed empirically by "*whole of our experience of the natural world*"[5]. And the relation $F \rightarrow E$ seems to hold as a logical (or mathematical) conclusion of the premise **F**. Further, let **L** be the principle of the constancy of the velocity of light. Then, ordinarily it is considered that the following relation

$$(E \wedge L) \rightarrow P$$

can be obtained by mathematical reduction. The results obtained in the preceding paper may be useful in examination of this procedure. Next, we must examine the inverse relations to the above ones:

$$P \rightarrow (E \wedge L) \rightarrow F.$$

Before we take up the above problems we had better to remind the structure of the (inhomogeneous) Lorentz group. It consists of the following elements:

- i) special Lorentz transformation,
- ii) 3-dimensional rotation,
- iii) space inversion,
- iv) time inversion,
- v) space-time inversion, and
- vi) translation by a real vector **a**.

Though there is no explicit reference of the invariance of the physical laws under the above transformations ii)-vi) in **F** and **E**, we understand that whole of these sense of invariance are included in **E** and **F**. However, for iii)-v) in case of quantum physical phenomena special care must be made. We reserve the discussions for the problem here.

Returning to the main subject, we shall examine the relation $(E \wedge L) \rightarrow P$. As was shown in the preceding paper Lorentz transformation can be derived from the

conditions schematically written by $(\text{part of } E) \wedge (L') \wedge (S)$, where (part of E) means the kinematical part relating to the coordinate system, L' means a conclusion from the results of some experiments of the propagation of light, and S is some auxiliary condition. Therefore, exactly speaking we must replace the proposition $(E \wedge L)$ by $(\text{part of } E) \wedge (L') \wedge (S)$.

The next problem is the one concerning the empirical confirmation of the special principle of relativity. Though we have much experimental evidences for (Lorentz invariant) physical laws in an inertial system (usually in laboratory system), they do not mean the empirical confirmation of the special principle of relativity. In other words to clarify the empirical meaning of the special principle of relativity we must explain how to confirm its truth empirically. Let us consider an example. Dirac's theory of hydrogen atom has been confirmed in our "laboratory system". Because the fundamental equations in the theory are Lorentz invariant, we have true solutions after transforming them into another inertial system by Lorentz transformation. This result is evident from mathematical inference. We can easily understand the physical meaning of the coordinate system after the transformation. But if the relative velocity of the system is very high, how can we build the laboratory really which moves with this coordinate system. With Reichenbach's terminology it is technically impossible, though physically not impossible. Therefore we cannot obtain direct empirical confirmation. But if we imagine to construct such a high speed laboratory and make any physical experiment in it, the same results as is calculated before mathematically will be obtained directly. This is a common sense view of many physicists. The above cited Fock's words shall be understood in the above sense. Expressing in another way, we can say that if the truth of a Lorentz invariant law can be confirmed in an inertial system practically, its truth in another system which moves relative to it with any non-accelerated motion can be confirmed by the thought experiment.

On the other hand according to Bridgman "*what it (the special principle of relativity) is saying is something about a "law of nature"; it is saying that unless the "law" is the same in the two frames of reference it is not a "law". This is a statement for a natural law*"[6]. According to his explanation this principle does not say that it can be checked by formulating the laws for different frames of reference and observing whether they are the same. In a word his opinion is that "*the first postulate thus appears to contain a concealed definition of "law of nature"*"[6]. And as a result we can conclude from his opinion that the scope of application of the special theory of relativity covers the region in which "*the law of nature*" adapts the first postulate.

We can arrive from the above consideration the following conclusion. At first it is to be remarked that we can decide mathematically whether any law of nature is Lorentz invariant or not. And if such a invariant law is an empirical

one, ordinarily there is a method for empirical confirmation in a reference system. If it is a logical or analytic one we can verify its truth by logical procedure. On the other hand we can say that the special principle of relativity is a certain kind of demarcation principle. Namely, if we can formulate a law of nature as to have a Lorentz invariant form, the law may be a special relativistic one. And on the other hand if we cannot make the law Lorentz invariant, it stand without the scope of the special theory of relativity. Demarcation principle marks a boundary around "the special relativistic region".

§3. Auxiliary conditions to derive the Lorentz transformation.

As is stated in the last section, it is to be carefully examined whether Lorentz transformation is the only meaningful one which connects two coordinate systems moving uniformly each other. We investigated in the preceding paper the so-called derivation problem for the Lorentz transformation. In the course of the derivation it is clarified that we must at first decide the units of both spatial and temporal length together with the method for measuring the geometrical quantities in order to define the uniform motion of the system. Further it is also clarified that we need the method for comparison the results of the above decided measuring procedures in any moving system with the ones in our "*rest laboratory system*". In other words we must at first be able to define (or measure) the velocity of any body and describe it as, say, 3-dimensional vector in the latter system. For this purpose we must presume some auxiliary principles. In Panofsky-Phillips's text of the electromagnetism [7] these circumstances are explained. Following the line of thought shown in this book, we shall illustrate these principles. They say that there exist two auxiliary principles implicit in the special theory of relativity in addition to Einstein's two fundamental principles. The first of the former principles is "invariance of the sense of time" and the second of them is "the invariance of proper quantities" or the "laws of reproducibility of proper quantities". Though the recognition of which is important for the necessary (relativistic) revision of physical concept, they have pre-relativistic character in their nature. Namely these principles, especially the second one, can be considered to be contained implicitly in classical theories and they are naturally consistent also with the Galilei's relativity principle. And at the same time it seems that the ignorance or misunderstanding of them is the main cause of many meaningless controversies around the relativistic physics. The first principle deals with the direction of time. We refer this problem in the last section briefly. Because the explanation of the second principle made by Panofsky-Phillips is simple, we should like to describe it in detail.

By the second principle, i.e., the principle of the invariance of proper quantities or the law of reproducibility of proper quantities, Panofsky-Phillips mean that "*whenever a measuring experiment is performed, the length of a standard (such as*

the wave length of a given spectral line or a crystal lattice spacing) and the rate of a fundamental clock (such as a radioactive decay period) shall be the same as seen by an observer at rest relative to the standard". This expression seems to me somewhat ambiguous and insufficient. We shall elaborate its contents in the following five items.

(a) Units of spatial and temporal length. As the standard of the unit of spatial and temporal length, we use ordinarily frequency and wave length of a certain spectral line of properly chosen atom. It is a common sense view of physicists that these units give constant numerical value as the standard of the length when we use it to measure the extension of object or the duration between two events in a rest system. We have thus paraphrased the original words given by Panofsky-Phillips. Next, we shall analyse the contents of this statement into the following three heads to make its meaning clear.

ai) An inertial system can be given to us. Namely, let it be well-defined by properly chosen concepts or in terms of a certain coordinative procedure such as definition or operation. In practice we can identify it with enough accuracy with our laboratory system.

a ii) In this system units of physical quantities are given as usual.

a iii) It is possible to measure any spatial and temporal length with the aid of this standard unit. (Comparison of the rate of clocks and scales at different space-time point can be made.)

(b) The structure of space and time. In an inertial system given by the condition of ai), let the measurement of the spatial length be made in a manner stated in a ii) and a iii).

bi) The space is 3-dimensional euclidean.

b ii) At any spatial point we can measure the time and the comparison method of the standard clocks at each point is given.

(c) Coordinate system in a rest inertial system. As the results of the above conditions we can layout a cartesian space coordinate system and the time coordinate system represented by a series of real numbers. Then we can obtain the following results.

ci) For any event cartesian space coordinate (x, y, z) and time coordinate (t) can be given.

c ii) The above stated method of spatial and temporal length measurement and units of them have an invariant meaning under i) the rotation, translation of the origin and reflexion of the spatial coordinate axes and also under ii) the translation of the origin and the reflexion (time reversal) of the time axis.

c iii) Using this coordinate system, we can give the following form for the uniform rectilinear motion of a body as a linear function of t .

$$x = v_x t + a_x, \quad y = v_y t + a_y, \quad z = v_z t + a_z,$$

where $v_x, \dots, a_x, \dots$ are constant and the magnitude of the velocity $\mathbf{v}$ is given as

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}.$$

Basing our argument on the above assumptions, we can construct the kinematics in this inertial system. The form of the kinematics is the same one used in the pre-relativistic physics. Next we shall ennumerate the following results as already confirmed experimentally with enough accuracy.

(d) Some physical facts already confirmed. The following results experimentally confirmed are necessary for the construction of the inertial system as a whole which are moving with uniform velocity to each other.

di) Propagation of light. Light propagates rectilinearly with a constant velocity, the value of which is independent of the kind of source and its state of motion and also of the place and time of its propagation.

dii) The transverse Doppler effect. The following well-known result is confirmed by Ives-Stilwell's experiment and also by several experiments using Mössbauer effect. The frequency ν' of a spectral line emitted transversally from the source atom which is in uniform motion is given by

$$\nu' = \nu (1 - (v/c)^2)^{1/2}, \quad (3-1)$$

where ν is the proper frequency of that spectral line and v is the velocity of the atom.

The implication of the statement di) is physically equivalent with the one of the usual expression of the "Principle of the constancy of light velocity". As is stated above Einstein takes it at first as an assumption. But today it is considered as a well confirmed fact.

Ordinarily it is believed that the Lorentz transformation can be derived from the special principle of relativity and the statement di), i.e., the principle of the constancy of light velocity. The explicit expression of the second principle takes several different forms and also the conditions taken from the first principle are quite different in each treatment. Moreover, as is remarked hitherto, some vague "metaphysical" assumptions such as the symmetricity or homogeneity of the space and time are taken into the deduction process without the reasonable foundations. In our treatment proposed in the last paper [I] such assumption the foundations of which are not clear was not adopted.

(e) The layout of the moving coordinate system. When the velocity v of the system is so high that we cannot ignore the term of the order $(v/c)^2$ compared with 1, the so-called relativistic effects play important roles. As is well known it is technically impossible to construct a real moving laboratory whose velocity v relative to our "rest laboratory" is so high that we cannot ignore the relativistic

effects. We can, however, imagine such high speed laboratory and various physical processes which take place in it. A moving coordinate system is considered as the one fixed in such an imaginary laboratory. We had proposed a method for the layout of a moving coordinate system. Following the line of thought logically, we can say as follows. At first we start from our well-known rest laboratory system. In it we can layout a coordinate system in ordinary manner, then, taking account of some well-known physical effects, we can "define" a moving coordinate system. Then our "thought experiment in an imaginary moving laboratory" can be done referring to the above defined moving coordinate system. To carry out such thought experiments we must further investigate the behaviour of the standard (or units) of several physical quantities in the moving laboratory. For example, the time unit in a moving laboratory is given, as the rate of moving clock, by the results of Ives-Stilwell's experiment as shown in [I]. This problem will be discussed further in the next section with respect to the moving clock and scale. As for the units of other physical quantities, say, that of mass, Bridgman's opinion shown in his book will be of great value in this respect. We will reserve the opinion concerning the first principle proposed by Panofsky-Phillips, because the further investigations should be necessary especially in the elementary particle physics.

§4. The length of a moving scale and the reading of a moving clock.

Using the following thought experiment, we can measure the rate of a moving clock. Let A and B be two fixed points in a rest system, and standard clock moves with a constant velocity along the straight line which passes both A and B. And let

t_A' = the reading of the moving clock when it passes A;

t_A = the reading of the clock at A when the moving clock passes A;

t_B' = the reading of the moving clock when it passes B;

t_B = the reading of the clock at B when the moving clock passes B.

Then the ratio

$$r = \frac{t_B' - t_A'}{t_B - t_A} \quad (4-1)$$

is the observed rate at which the moving clock is running with respect to the clocks on the rest system[8].

It is technically impossible for us to move an ordinary standard clock with a "relativistically high speed". But if we adopt the proper frequency of an atom as a unit of the standard clock, we can make use of the transverse Doppler effect stated above as to estimate the observed rate in the above thought experiment. According to Ives-Stilwell the following relation is hold between observed frequency of a spectral line from moving atom and that of the corresponding line from the

same kind of atom at rest. Let the former be designated as ν' and the latter as ν , the relation between them is as follows:

$$\nu' = \nu (1 - (v/c)^2)^{1/2}. \quad (4-2)$$

This result gives just the same rate of the moving clock as stated in the last paper if we regard the proper frequency of a spectral line as the unit of time.

Taking the result from Mössbauer effect into account, we have obtained the same result also in γ -ray region. As is well known the measurements of the life of radioactive substances or unstable elementary particles give consistent results with the above one.

Next we shall mention some problematic points on the above considerations of the unit of time. At first the problem concerning the interpretation of the result of Ives-Stilwell's experiment. Ordinarily the relation (4-2) between ν , ν' and v has been considered to be obtained directly from the experiment. Namely, the obtained result is confirmed within the accuracy of the experiment. The results of the experiment in themselves show, however, that there exists a term of the order $(v/c)^2$ in the expression of $(\nu' - \nu)/\nu$ and then the value of the term is $-(1/2)(v/c)^2$.² We can, however, only conclude that the value is the above stated one and higher order term can not be decided from the experiment. Namely, they had only

$$(\nu' - \nu)/\nu \simeq -(1/2)(v/c)^2 + \dots, \quad (4-3)$$

where...in the above expression means undecided higher order terms. Purely experimentally derived value is not $(\nu' - \nu)/\nu = 1 - 1/\sqrt{1 - (v/c)^2}$, but the right hand side terms of (4-3). They decided the above stated form of the relation (4-2) with the presuming theoretical estimation. We can conclude naturally that the experimental value consistent with the estimated value calculated theoretically. It seems to be open to question to conclude the functional form (4-1) pure inductively from the results of experiment only.

Further, Ives and Stilwell adopted the classical expression

$$eV = (1/2)mv^2 \quad (4-4)$$

in calculating the speed of atom, v , where e , m , V are the electric charge, mass of the atom and electrostatic potential between electrodes respectively. In practice the velocity of the atom used in this experiment is of the order 1000 km/sec or so. And if higher velocity atom will be used we have only to correct the equation relativistically. This procedure seems at first sight to contain a reasoning in a circle. This situation must be remarked especially in the axiomatization of the theory.

On the other hand with regard to the length of a moving scale it is technically impossible to measure directly the marks on the moving scale having relativistically high speed. And yet it is also technically very hard to accelerate a scale to such

a high relativistic speed relative to the laboratory. But the principle of the constancy of the light velocity will give us an important clue to measure the length of a moving scale.

It is usually understood that the velocity of light measured in a uniformly moving system is the constant value c which is the same as in the rest system. We shall examine this conclusion in detail. As is stated repeatedly we cannot perform physical experiments practically in a relativistically high speed system. So the value of the light velocity c "measured" in a moving system must be considered not as the directly obtained one, but as an estimated (or derived) value after some thought experiments. In such a thought experiment, as is pointed by Maehara [9], we can choose the two quantities freely among the following three: unit of length, unit of time and velocity of light. In the last paper [I] we decide the unit of length in a moving system so that the velocity of light in a moving system becomes the constant value c . This procedure is nothing but the so-called light geometrical one. As was remarked in [I] as to the length of a moving rigid scale we cannot conclude any more without some additional assumption, if we take this line of approach. In another way if we assume the existence of moving scale and that we can confirm the constancy of light velocity in moving system, we will have an indirect evidence for the readings of the moving rigid scale.

Several papers hitherto published took this line of thought. The authors of these papers compared the length of a moving scale which situated transversally to the direction of motion with the one of rest scale. For example, Panofsky and Phillips [10] concluded from symmetricity of space that the length of transversally placed moving scale must have the same value measured by an observer in a rest system. But this reasoning cannot be admitted. On the other hand Maehara's reasoning is as follows: moving observer adopts the unit of length so that the transversally laid scale has the same length as that of the rest scale. This is nothing but a definition and his reasoning is logically quite correct. But in this line of thought we cannot conclude that whether thus defined length is the same as the one defined by rigid standard scale calibrated by optical method in a moving system. In this respect the situation is almost the same one as in our method proposed in [I]. We should like to notice here that the transversally laid moving length in our procedure presented in the last paper becomes equal to that of the rest scale. We can deduce this result from a simple thought experiment.

§5. Scale factor (The length of a moving scale and the reading of a moving clock. II.)

We shall begin with a simple mathematical preliminary consideration. Einstein obtained the following relation between (x, y, z, t) and (x', y', z', t') in his first paper[1]:

$$\begin{aligned}
 x' &= \varphi(v) \frac{x - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}, \quad y' = \varphi(v) y, \\
 t' &= \varphi(v) \frac{t - \frac{v}{c^2} x}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}, \quad z' = \varphi(v) z,
 \end{aligned}
 \tag{5-1}$$

where $\varphi(v)$ is a factor depending only on v . As is easily seen we can obtain the ordinary Lorentz transformation, if we put $\varphi(v)=1$. Because this factor gives the ratio of the reading of the scale in each system, we shall call it scale factor. Einstein determined this factor to be 1 because of the symmetry of the space (he had no reason to insist the right-left asymmetry) and of the equivalency of the moving system and the rest system. These assumptions are founded as we emphasized several times also on the metaphysical ground.

As is stated above the physical meaning of this scale factor is after all the ratio of the moving scale and clock reading to those situated in a rest system. The considerations proposed by Panofsky-Phillips or Maehara as explained in the last section intend to give the reason to put the condition $\varphi(v)=1$ on them. Einstein's reason stated above contained similar ones as those of Panofsky-Phillips in principle. Maehara's one is in its nature the definition of the unit of the moving length. We cannot, however, find the reason that the length thus defined in a moving system coincides with the natural unit, such as a wave length of spectral line of an atom which stands still in the moving system. We had only to insist that the coincidence is based on the special principle of relativity.

Consequently, if we decide the functional form of $\varphi(v)$ basing on a theoretical assumption or definition, the experimental confirmation must follow. And on the other hand, starting from experimentally confirmed relations, we must further assume some supplementary theoretical condition such that the transformation makes group or so to decide an exact functional form of the factor $\varphi(v)$. Namely as is stated above we cannot obtain the exact form $\sqrt{1-(v/c)^2}$ experimentally to higher order, but only the form $(1-(1/2)(v/c)^2 + \dots)$, where the terms... cannot be decided. We can decide this higher order term only by some theoretical consideration, an example of which we shall present later.

We shall present in the last place some results obtained by the author and the physical meaning of which has not been given[11]. Though the results were obtained in the course of the study which was made taking another standpoint, we can interpret them anew in accordance with our present purpose. We change the notations as to fit our present investigations. Let the equations connecting the two sets of coordinates have the same form as the above (5-1). Next we shall consider the third coordinate system whose coordinates are designated as $(x'', y'',$

z'', t'') and connected to (x', y', z', t') by the following equations.

$$\begin{aligned} x'' &= \varphi(w) \frac{x' - wt'}{\sqrt{1 - (w/c)^2}}, \quad y'' = \varphi(w) y', \\ t'' &= \varphi(w) \frac{t' - \frac{w}{c^2} x'}{\sqrt{1 - (w/c)^2}}, \quad z'' = \varphi(w) z'. \end{aligned} \quad (5-2)$$

In ordinary interpretation the system (x'', y'', z'', t'') moves with the uniform velocity w relative to the second system (x', y', z', t') . From (5-1) and (5-2) we can eliminate (x', y', z', t') and obtain the following form:

$$\begin{aligned} x'' &= \varphi(v) \varphi(w) \frac{x - ut}{\sqrt{1 - (u/c)^2}}, \quad y'' = \varphi(v) \varphi(w) y, \\ t'' &= \varphi(v) \varphi(w) \frac{t - \frac{u}{c^2} x}{\sqrt{1 - (u/c)^2}}, \quad z'' = \varphi(v) \varphi(w) z, \end{aligned} \quad (5-3)$$

where $u = (v + w)/(1 + (vw/c^2))$. Then we assumed that the coordinate system (x'', y'', z'', t'') moves with uniform velocity u relative to (x, y, z, t) . The equations which connect these two system are given by

$$\begin{aligned} x'' &= \varphi(u) \frac{x - ut}{\sqrt{1 - (u/c)^2}}, \quad y'' = \varphi(u) y, \\ t'' &= \varphi(u) \frac{t - \frac{u}{c^2} x}{\sqrt{1 - (u/c)^2}}, \quad z'' = \varphi(u) z. \end{aligned} \quad (5-4)$$

The right hand side of the equations (5-3) and (5-4) should coincide each other. Then the functional form of $\varphi(v)$ must have the following form:

$$\varphi(v) = \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{c\beta/2}, \quad (5-5)$$

where $p = (d\varphi/dv)|_{v=0}$. We obtain the form $\varphi(v)$ by solving the following functional equation:

$$\varphi(u) = \varphi(v) \varphi(w) \quad \text{and} \quad u = \frac{v + w}{1 + \frac{vw}{c^2}}. \quad (5-6)$$

We can conclude from the above results that the rate of a clock moving to left differs to that of the clock moving to right. We have not yet obtain any evidence of such an asymmetric effect experimentally. But if there is any right-

left asymmetry in the physical world, we must examine further its effect (in higher order). Especially considerably long time has been passed since "the space inversion problem" had been proposed, and further the existence of a certain kind of absolute rest system is proposed recently. This problem of the right-left asymmetry must be examined in detail.

§6. Conclusion.

In the preceding paper we advocated a new method for the reconstruction of the special theory of relativity, on which we have tried to formulate the theory inductive manner standing only on a certain physical facts experimentally well confirmed. Taking this method of theory construction, we expected to arrive the unique and authentic theory. In this approach we obtained an tentative conclusion as for the derivation of Lorentz transformation. In this paper we comment further on problems concerning the nature of the fundamental principles of the special theory of relativity, their logical nature and experimental evidences for them. Though we have reserved the final conclusion for some important problem, we give our views on many situation. Among them the one concerning Bridgman's opinion to the special principle of relativity that we regard as a kind of demarcation principle or some kind of regulations for natural laws shall be remarked. And further concerning the requirement for the invariance of the natural laws under coordinate transformation Panofsky-Phillips's indication of the unit may be important. We emphasized the importance for the consideration about this problem especially for the comparison problem between the units taken in the moving system and in the rest system.

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