***A Meta-Physical Investigation for the Special
Theory of Relativity III
— Preliminary Remarks (Concluded) —***

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The physical meaning of the special principle of relativity is investigated. In this context a new method of derivation of Lorentz transformation basing on some kinematical (or geometrical) considerations is proposed. Special cares are taken in this procedure as to clarify the empiristic meaning of Lorentz transformation.

§ 1. Introduction

In the preceding papers [1], [2] we have investigated the foundations of the special theory of relativity (STR). Our standpoint taken in these papers is to survey empirically reliable bases as to construct the theory inductively. We intended to reject any hypothetical assumption the foundation of which is not clear. In this manner we attempted to show that we can adopt a theoretical system which will never be refuted by future experience. Though the main purpose of the following papers will be a presentation of the formalism of a theory having such characters, in this paper we will continue the investigation which have been taken in the preceding papers. Especially some problems concerning the special principle of relativity (SPR) and its relation to the Lorentz transformation (LT) will be considered.

In §2 we will comment on the character of SPR and its situation in STR and will represent its exact and concrete form. In §3 we will investigate the so-called derivation problem of LT in connection with the contents of §2. We propose a new method using the character of light propagation, taking this line of thought in §4.

§ 2. The scope of special theory of relativity

It is usually considered that the fundamental principles of STR are the SPR and the principle of the constancy of the light velocity. Indeed Einstein himself derived the LT on the premise that these two principles hold good. As is well-known LT had been introduced before Einstein from other standpoint. And also after Einstein various kinds of its derivation have been proposed basing on

different assumptions. As a matter of course the SPR was not assumed in the former cases, on the other hand it has been assumed in almost every papers in the latter cases, though the conditions actually adopted for the derivation are not the same. We will explain this circumstance in some detail. In many cases the principle is used as a ground for some conditions necessary for deriving LT. For example, when some condition is assumed, its basis is explained by the adequateness of SPR. But it is questionable that such conditions are really necessary for the derivation. Moreover some other assumptions, such as the linearity of the transformation, are actually introduced in many papers. As was remarked in the preceding papers Einstein himself adopted this assumption, the basis of which was explained as the homogeneity of the space and time. But this can be also explained mathematically as was remarked by Weyl [3]. On the other hand in some paper it was proved that this condition was unnecessary. Then what is the real conditions for deriving the LT? The purpose of many attempts was, more or less, to clarify this circumstance.

Several attempts to axiomatize the STR have been proposed by many authors such as Carathéodory and Reichenbach [4]. Recently Suppes, Noll and others also have proposed their systems respectively [5]. Roughly speaking, the former works were characterized by their empiricistic point of view, namely, their systems were constructed inductively by choosing some empirically evident facts as axioms. On the other hand modern approaches are characteristic by their deductive construction with axioms having rather abstract characters. In the former LT have been derived as a theorem from the axioms by logical inference, while on the other hand in the latter LT or Lorentz invariance of some physical laws is proposed as an assumption of the theory laying emphasis on the elegance or the logical simplicity of the theoretical system. Comparing these two approaches, we can understand clearly the relations between postulates and observations in the theory. In this connection we must also remark a paper proposed by Robertson in which the special interest was laid on the problem of "postulate versus observation"[6].

We can regard further that the inductive construction of the axiomatized theory are the metatheory of the deductive theory. Because the former supplies the epistemological or observational foundation for the latter theories, namely the evidence of the postulates in the latter theories can in many cases be clarified by the inductive procedure in the former theory. We can rearrange the results of this two types of the theories stressing the emphasis on the above stated relations. Paying attention to this stepwise structure of these two kinds of theories, we must remark clearly the distinction between the inductive and deductive reasoning appeared in the theory. Namely we start from empirically or experimentally evident facts and obtain some results. Then we can construct the

deductive theory basing on the above obtained results. For example, such a theory as Reichenbach or Carathéodory's preceds, then we can construct a deductive one such as Noll's. Or after some metatheoretical considerations a theory of dynamics like the one proposed by Suppes et al can be constructed.

Further we should consider the STR in more broad sense. Generally speaking we can consider that the preceding theories such as the special relativistic theory of space-time or of kinematics are the very contents of the STR. But other theories can also be considered as some part of the STR, if they are constructed on the Lorentz invariant concepts and laws. When the STR is considered in this broad sense, the STR in a narrow sense, i.e., the ones referred at the first part are considered as the metatheory of the latter theories.

In the following considerations we will investigate at first the situation of SPR in the STR of various kinds. The SPR claims, however, the invariance of the physical laws between a rest system and a uniformly moving system. Usually the so-called postulate of Lorentz invariance of physical laws is considered as equivalent to the SPR. We made a remark to this problem in the preceding paper. Taking this into consideration again, we will reexamine the character of SPR in the following section.

Next we will consider the other fundamental principle of the STR, the principle of the constancy of the light velocity. Around 1960 several experimental evidences for the so-called second principle of special relativity were reported. One of them clarified that the velocity of γ -ray emitted from high speed π^0 -mesons was the same as that of γ -ray emitted from rest π^0 -meson. From the result of such experiment it is concluded that the velocity of the electromagnetic wave is constant and independent of the velocity of the source. Together with this result and the ones already obtained concerning the so-called ether-drift, the principle is also confirmed by many physicists by means of several modern technics recently. We can state the principle of the constancy of light velocity as follows: The velocity of light (electromagnetic wave) in vacuum is constant and independent of 1) the velocity of its source, 2) the direction of its propagation, 3) the place and time measured, and 4) the kind of the source and its wave length. We can adopt these results as a evident premise of the theory because they are considered as reliable experimental results.

§3. The nature of SPR

The contents of SPR are divided into two parts. At first the existence of the so-called "*ruhende System*" and "*bewegte System*" is proposed. Then the invariance of the physical laws in both systems is asserted. Usually it is considered that LT, as the coordinate transformation between the both kinds of the coordinate systems, can be derived from the two fundamental "*postulates*" of the STR.

Indeed it is derived from the assumption of the constancy of light velocity and some kinematical conditions for the uniform motion. And also it is considered that LT is the only one coordinate transformation connecting the both kinds of coordinate systems. If this consideration is approved the last part of SPR is now rewritten as follows: The physical laws are invariant under LT.

We have investigated in [1] and [2] the verification problem of this principle. In this investigation the necessity of the reexamination of the so-called derivation problem of LT was aroused at first. We proposed a new method of the derivation basing on the empirically confirmed facts only at first sight. If this method is plausible, in the investigation of the verification problem of the SPR we have only to treat the problems concerning the invariance of the laws of physics under LT. Because in this case we may be foregone the examination of the adequacy of the LT itself. Therefore the examination of the correctness of the invariance becomes independent of that of LT, because the LT can be regarded as empirically confirmed transformation. As is usually said the logical structure of this principle is metanomological, then many complicated problems will be contained in the investigation of its verification or confirmation.

The main purpose of this investigation is the construction of a metatheory of the special relativistic theories, the fundamental principle of which is the special relativistic invariance of the laws of physics. As to construct such a theory we have two different approaches. One is the deductive one in which we assume the validity of the invariance under LT at the beginning of the theory and construct the theory such as the space-time theory or kinematics. Theories thus constructed are considered also as metatheory of various kinds of special relativistic theories in the special branches such as the fluid dynamics or the thermodynamics. The other one has an inductive character which starts from some empirically evident physical facts as the axioms of the theory. Main result of the latter type theory is to show that LT is the transformation which connects two coordinate systems relatively in uniform motion. In this connection we can regard the latter as a metatheory of the former type theories, and further we can explain this metatheory as a meta-metatheory of special "SRT". Our starting point is therefore to construct a theory having a character like the one taken by Reichenbach.

§4. Another derivation of LT.

In his derivation of LT Einstein takes up at first a coordinate system called stationary system. This system is defined as the one in which the equations of Newtonian mechanics hold good (to the first approximation). While in the theory of relativistic kinematics proposed by Bunge [7] the same system is defined by the condition that classical electrodynamics holds good relative to it. In the classical physics the term "the inertial system" was usually adopted to

represent such a system. It seems at first sight that these three kinds of stipulation for the system imply the same object. But as is easily seen that the Einstein's one permits the arbitrariness to the first order, and the Bunge's one is also undesirable because it can permit the coordinate systems to connect by the conformal transformation with the usual one.

Our first task will be, therefore, to propose a definition of "*an inertial system*". The next problem will be, then, how to construct a coordinate system which moves uniformly relative to an inertial system thus defined. As was stated above we will define the term inertial or stationary system (*ruhende System*) without adopting any physical theory as a whole or the condition of the invariance of the physical laws. Because we intend to construct a new theory, the use of ready-made theory or concept is not desirable.

In practice many physicists are working based on the definite coordinate system defined pragmatistically regarding it as an inertial system. Namely astronomers observe the motion of the heavenly bodies by telescope and determine their orbits after some calculations. In these procedure they use a coordinate system as the standard of the description of the motions. After the observations and theoretical calculations some corrections as to the determination of the coordinate systems. As is easily seen in this coordinate system thus defined pragmatistically the equations both of classical electrodynamics and mechanics hold good. This system can be regarded as practically equivalent to the ones defined by Einstein and Bunge. But in practise this system is determined by some complicated operations. The definition made in such a manner may be allowed to call pragmatistic one. In other words this definition can be regarded as the coordinative one in a wide sense in Reichenbach's terminology, though the processes of coordination are very complicated. We adopt the coordinate system thus defined as the stationary one.

Now we will explain how to construct the theory in detail. At first we can consider that a coordinative definition of "*rest*" is given because the stationary system has been given above. Then the question about a state of motion can be asked. To describe the motion we must have a method of the measurement of both spatial and temporal length. After that we define a moving coordinate system in terms of the stationary coordinate system. The method of measurement of the length in the moving system must also given coordinatively. In this construction of the coordinate system the requirement of the invariance of any physical law or the equivalence of the two kinds of coordinate system is not assumed. On the contrary to define the moving coordinate system in the usual treatments, the above stated kinds of assumptions are adopted together with some results obtained by some imaginary thought experiment. Our method given in [1] is an example of such character. The common character of such method is

as follows: the existences of 1) an observer moving uniformly relative to a stationary system and 2) the existence of the coordinate system which is rest at him and is constructed in the same way as the one adopted by an observer at rest in the stationary system are assumed. Then to obtain the relations between both kinds of coordinate system the existence of a certain invariant physical relation, such as the constancy of the light velocity, i.e. that the relation

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 = 0$$

is transformed into

$$ds'^2 = dx'^2 + dy'^2 + dz'^2 - c^2 dt'^2 = 0$$

is assumed. (As is well known some other condition is necessary to exclude some trivial transformation obtained under the above condition only.) In this method the character of the uniformly moving scale and clock is not given directly. Namely the behaviour of the moving scale and clock and the meaning of the coordinate in moving system are not explicitly defined. In this case some considerations about the operational definition of both spatial and temporal length or the rigidity of a moving scale will be necessary for the above purposes.

If we regard the SPR as a working hypothesis which has normative character and put the principle at the starting point of the metatheory, the latter procedure can be naturally adopted. The confirmation of the principle will be reduced to the indirect one obtained from the correctness of the special relativistic invariant laws. In such a case the legitimacy of both the invariance of the laws and the postulated coordinate transformation is to be tested at the same time. The situation of LT in the formalism can also be understand as a convenient working hypothesis. In our procedure the meaning of the LT can be explained separately from the requirement of the invariance of the physical laws or theory. Namely we can make clear the operational meaning of LT by such a manner of construction of the coordinate system.

As was stated in the preceding papers our method for obtaining LT has some problematic points. The one is the light geometrical definition of the length and its relation to the concept of rigidity. The other is a experimental process adopted in the presented thought experiment using light propagation. Many physicists, Brillouin [8] and others, pointed out the recoil of light when reflected by some particle must be taken into consideration. At any rate several questions may arouse if we adopt a thought experiment using light propagation. In the next section we will propose a new method which has somewhat formal character taking a stand point stated above.

§5. Construction of a coordinate system in uniform motion.

In the last section we stated that the stationary system can be defined

pragmatically and coordinatively. After constructing a stationary system in this way we will define a uniformly moving system. We will represent this procedure briefly leaving its strict treatment in the following paper.

Let a cartesian coordinate system $K(x, y, z)$ be given in a stationary system. On its any point it is assumed that the time coordinate t can be determined, i.e. there are clocks at any point. We will define uniform motion in terms of these coordinate system thus:

$$x = v_x t + a_x, \quad y = v_y t + a_y, \quad z = v_z t + a_z,$$

where (v_x, v_y, v_z) are components of velocity and their values are all constant. (a_x, a_y, a_z) are also constant and they represent the initial position of the body under uniform motion.

To define a uniformly moving system k the spatial coordinates (x', y', z') and temporal coordinate t' must be given in some definite way. At first we will begin the examination of the concept "*rest in the moving system*". As was remarked by Reichenbach [9] the rest state must be defined coordinatively, so we will take the following definition.

Definition 1. All points which are rest in a uniformly moving system k move with the same constant velocity referred to the stationary system K and any point in K which moves with the same velocity is also at rest in k . The coordinate x', y', z' of this point at rest in k does not change with time t' .

i) One dimensional case.

For simplicity's sake we take the so-called one dimensional problem. Let the velocity of uniform motion in question be $(v, 0, 0)$, that is, the motion along the x -axis is treated. Now we will consider a point P which moves with this velocity along the x -axis. The coordinate of that point is given by

$$x = vt + a, \quad y = 0, \quad z = 0, \quad (1)$$

where a represents the x coordinate of the point at $t=0$. We will assume that the origin of the coordinate system k is at $x=0$ when $t=0$, and choose the line which connects this origin and the point P as the x' axis. We can prove that x' -axis thus defined is coincide with x -axis at any time t . In k let the distance between arbitrary two points x'_1 and x'_2 be $|x'_2 - x'_1|$. Then the coordinate x' of any point can be represented by the relation:

$$x' = f(x, t). \quad (2)$$

Next, as we assumed before, the time t' at any point in k can be determined and be compared with the time coordinate t at x in K . This relation is given by

$$t' = g(x, t). \quad (3)$$

These functional relations will be determined by direct observation, if Maxwell's

demon existed. We will take another indirect way and determine these functional form by some mathematical way.

Now we will explain the nature of the function f and g . At first we do not determine how to measure the length in k . We imagine the length can be measured by some definite and now undefined method. The rest state in k must be now defined. The straightness and the definite direction, say, x' -axis has already defined. The conditions assumed to the function f is now only topological ones. Therefore we assume that the function $f(x, t)$ is a continuous, differentiable and monotonous increasing function of x at any fixed t and also of t at any fixed x . As to the function g , the rate of the clocks in k , we assumed that clocks are situated at any place in k and the rate of which is given by comparing the ones of clocks in K .

By Definition 1, a point P which moves with velocity v in the stationary system K is rest in the uniformly moving system k . The coordinate of the point is given in K by (1), whereas this point is at rest in k . Namely its velocity in k is zero. This condition can be written by

$$dx'/dt' = 0, \quad (4)$$

where x' is the coordinate of P in k . If we substitute the relation (1) into (2) and (3) and then calculate (4) using these relations, we obtain

$$\frac{dx'}{dt'} = \frac{dx'/dt}{dt'/dt} = \frac{v(\partial f/\partial x) + (\partial f/\partial t)}{v(\partial g/\partial x) + (\partial g/\partial t)} = 0. \quad (5)$$

The meaning of the denominator of (5) will be explained later, the value of which is not zero. Therefore we have from (5)

$$v(\partial f/\partial x) + (\partial f/\partial t) = 0. \quad (6)$$

The general solution of the partial differential equation (6) is given by

$$x' = f(x, t) = F(x - vt), \quad (7)$$

where F is an arbitrary function of $(x - vt)$. If we adopt an initial condition, $x' = 0$, when $x = t = 0$, we obtain $F(0) = 0$. As is easily seen from (7) the coordinate of a point which moves with the velocity v in k has the constant value of the coordinate in k and vice versa. Namely this relation satisfies the condition given by Def. 1.

As to the function $g(x, t)$ we remember the observed rate of a moving clock explained in the preceding paper. In that definition the rate is given in a finite form. But this is also given in an infinitesimal form as

$$\gamma = (dt'/dt) \Big|_{\frac{dx}{dt} = v} = v(\partial g/\partial x) + (\partial g/\partial t). \quad (8)$$

This is nothing but the denominator of (6), and is not zero in general. If we adopt

the result $\gamma = \text{const.}$ as in the preceding paper, we obtain the following relation:

$$v(\partial g / \partial x) + (\partial g / \partial t) = \gamma, \quad (9)$$

where γ is a constant given by some experiment. The solution of (9) is given by

$$t' = g(x, t) = G(x - vt) + \gamma(x + vt)/(2v), \quad (10)$$

where G is an arbitrary function of $(x - vt)$. As the result of the above consideration we have

$$\begin{aligned} x' &= F(x - vt), \\ t' &= G(x - vt) + \gamma(x + vt)/(2v). \end{aligned} \quad (11)$$

Next we will investigate the physical meaning of the functions F and G and determine their forms. If we consider a motion with a constant velocity c ($c > v$) in K , the coordinate of a particle with this velocity is given by

$$x = ct + a. \quad (12)$$

The velocity v' of the same particle in k when $a = 0$ is given by (11) as follows:

$$v' = (dx'/dt)/(dt'/dt) = \frac{(c-v)F'((c-v)t)}{(c-v)G'((c-v)t) + \gamma(c+v)/2v}, \quad (13)$$

where F' and G' are differential coefficients of F and G with respect to $(x - vt)$ respectively. If we choose F and G arbitrary, v' is not constant. Therefore the first equation of motion in classical mechanics does not hold in k .

Now let the above constant value c be the velocity of light. In this case this velocity in k is also given by the same form (13). Physical meaning of both F and G must be considered in this connection. If we put the condition $t = 0$ in (7), we have

$$x'|_{t=0} = f(x, 0) = F(x). \quad (14)$$

This gives us the relation between x' and x at the time $t = 0$. Namely in the stationary system if we observe the value of the coordinate in k at the point x , this is given by $x'|_{t=0} = F(x)$ at $t = 0$. $F(x)$ is, at the same time the distance between the origin of k and the point x' measured in k . We understand intuitively that the scale of k is a some strange one. On the other hand the meaning of G is given as follows. In the same way as the above consideration of F , if we substitute $t = 0$ in (10), we obtain

$$t'|_{t=0} = g(x, 0) = G(x) + (\gamma x / 2v). \quad (15)$$

As is seen from (9) if the rate of the moving clock is a constant value γ , then the movement of the moving clock seems not to be strange. But as is easily seen from (15) the definition of the simultaneity of the different clock in k is in some way strange. The arbitrariness contained in F and G concern the above explained

facts. We can determine the form of F and G by some plausible conditions.

Because the velocity of light in K is constant it is convenient to choose the above condition as to determine the velocity of light in k the same constant c . This condition is satisfied by taking the relation

$$dx/dt = c \rightarrow dx'/dt' = c, \quad \text{or} \quad x = ct + a \rightarrow x' = ct' + a'. \quad (16)$$

If we put the relation

$$G(x-vt) \equiv (1/c)F(x-vt) - \gamma(c+v)(x-vt)/2v(c-v), \quad (17)$$

the above condition (16) is satisfied and this is the necessary and sufficient condition for satisfying (16). Taking this condition we obtain

$$\begin{aligned} x' &= F(x-vt) \\ t' &= (1/c)F(x-vt) - \gamma(x-ct)/(c-v). \end{aligned} \quad (11-2)$$

By this "calibration" of the origin of the moving clocks at any point in k the velocity of light becomes also constant c in this moving system. Therefore in the moving system the distance between two points can be given by the time required light travels between them multiplied by c as in K . Namely if a light signal which passes two points x_1 and x_2 at the time t_1 and t_2 respectively, the following relation holds in K

$$x_2 - x_1 = c(t_2 - t_1).$$

Let the coordinates of the corresponding points in k be (x'_1, t'_1) , (x'_2, t'_2) respectively, we have also

$$x'_2 - x'_1 = c(t'_2 - t'_1), \quad (18)$$

because of the relation (11-2). Naturally this procedure of the synchronization of clocks in k is different from the one given by Einstein. At any rate in k we can determine the distance between two points at rest by means of the clocks and the light signal. But as is easily seen this method of the measurement is not the same adopted in the so-called light geometry. After a simple calculation we obtain the following relation:

$$dx' - cdt' = \gamma c(dx - cdt)/(c-v) \quad (19)$$

From this relation we can easily conclude that $dx' - dt' = 0$, if $dx - cdt = 0$. (In this case it is to be remarked that $dx/dt = -c$ can not be reduced from $dx'/dt' = -c$ or vice versa.)

Before we introduced the condition (16) the method of the length measurement had not been explained. After putting the condition (16), we can measure the length by means of light signal and clocks as was stated above.

Now a simple result from the transformation equation (11-2) will be proposed. If we observe a motion with a constant velocity w ($w < c$), the velocity of the same

motion is given in k as follows:

$$\frac{dx'}{dt'} \bigg|_{\frac{dx}{dt}=w} = \frac{(w-v)F'((w-v)t)}{(w-v)F'((w-v)t)/c - \gamma(w-c)/(c-v)} \quad (20)$$

In general this is not constant. As is easily seen the first equation of motion is not invariant under (11-2).

When the function F is linear, the derivative of F is constant, i.e., $F'=a$ (const.). Then the above formula becomes

$$\frac{dx'}{dt'} \bigg|_{\frac{dx}{dt}=w} = \frac{a(w-v)}{a(w-v)/c - \gamma(w-c)/(c-v)} \quad (21)$$

Conversely if we assume that $dx'/dt' = \text{const.}$, then the right hand side of (20) becomes constant and after that we can conclude that $F' = \text{const.}$, namely F is a linear function. Let this constant velocity in k be u , we obtain the relation:

$$a = \frac{\gamma u}{1-(u/c)} \left(\frac{c-u}{(u-v)(c-v)} \right). \quad (22)$$

Then (11-2) becomes as follows.

$$\begin{aligned} x' &= a(x-vt) \\ t' &= ((a/c) - \gamma/(c-v))x - [((a/c) - \gamma/(c-v))v - \gamma]t, \end{aligned} \quad (11-3)$$

where a is a constant given by (22).

Now in K the velocity of light which travels towards the negative direction is $-c$. If we assume the light has also the same velocity $-c$ in k , that is, when $w = -c$, then $u = -c$, we obtain from (22)

$$a = \gamma/(1-(v/c)^2)$$

After simple calculation (11-3) takes the following form.

$$\begin{aligned} x' &= \frac{\gamma(x-vt)}{1-(v/c)^2} \\ t' &= \frac{\gamma(t-(vx/c^2))}{1-(v/c)^2} \end{aligned} \quad (11-4)$$

If we put the condition $\gamma = 1/\sqrt{1-(v/c)^2}$ in (11-4), which is given by the result of Ives-Stilwell's experiment, we can obtain the special Lorentz transformation:

$$x' = \frac{x-vt}{\sqrt{1-(v/c)^2}}, \quad t' = \frac{t-(vx/c^2)}{\sqrt{1-(v/c)^2}}. \quad (11-5)$$

As is easily seen the linearity condition of F assumed above is not necessary if

we put the following condition directly. That is, if we assume that the light velocity towards the negative direction of x' axis is also $-c$ in (11-2), the following relation must hold.

$$x' = F(-(c+v)t),$$

$$t' = (1/c)F(-(c+v)t) + \gamma c(2ct)/(c-v), \text{ and } x' = -ct'.$$

Then we have

$$2F(-(c+v)t) = -2\gamma c^2 t/(c-v).$$

If we put $\eta = -(c+v)t$, then the function F is given as

$$F(\eta) = \gamma c^2 \eta / (c^2 - v^2) = \gamma \eta / (1 - (v/c)^2).$$

If we put $\gamma = (1 - (v/c)^2)^{1/2}$, we obtain (11-5) directly from (11-2).

In the above we stated that we assume the light velocity towards the negative direction of x' axis is also $-c$. This statement is in a manner usually adopted, but we had better to say that let the scale and clock regraduate so that the light velocity considered becomes $-c$.

Now we will resume the procedure of the above construction of the moving system.

1. A stationary system is defined coordinatively in pragmatistic manner.
2. Uniform motion and the rest state relative to the uniformly moving system are defined.
3. Rate of clock in a uniform motion is given by a constant value γ .
4. The straight line in a moving system is defined. The results obtained from the above conditions are given by (12).
5. If we further regraduate the scale and the clock so as to become the light velocity towards the right hand in the moving system is c . We obtain (11-2).
6. We regraduate again the scale and the clock in k so that the light towards the left hand be $-c$. We obtain (11-5).

ii) Three dimensional case.

We will omit the complete and detailed discussions from this paper because they are tedious and complicated. The main problem to be followed is the examination of the nature of the spatial geometry of the moving system thus constructed. Namely the problem whether the space thus defined is euclidean or not must be checked at first.* We have not yet introduced the concept of the four dimensional space-time. In this paper we will trace the problem intuitively without introducing the abstract concept of the space-time from the beginning.

We will adopt the relation (11-5). Now let a line $y = \text{const.}$ and $z = \text{const.}$ in K coincides the line $y' = \text{const.}$ and $z' = \text{const.}$ in k system as in the case of the one

* Ordinarily the 3-dimensional spatial geometry defined in the Minkowski space-time is not euclidean.

dimension. We will take this line as the x and x' -axes respectively. We will graduate on this line the x -scale in the same way done in the above case. We will consider that the graduation is made on x' -axis in the same way. Let the origin of it lie on the plane $x=0$ at $t=0$. The origin of the x' -axis moves with the uniform velocity v relative to K .

For simplicity's sake we will consider the axis which coincides $y=const.$ and $z=0$. We regard this line as $y'=const.$ and $z'=0$ in k . Now we will determine the constant value corresponding to y' . Now let a light signal start at the origin $x=y=z=0$, namely $x'=y'=z'=0$ at $t=t'=0$ and reach a point $x'=z'=0$ and $y=y'$ at the time t_1' . If the coordinates of this point in K are $(x, y, 0)$ and the time t_1 , they are given by

$$\begin{aligned} x &= vt_1, \\ y^2 + x^2 &= (ct_1)^2, \\ \therefore y^2 &= (c^2 - v^2)t_1^2 \end{aligned}$$

On the other hand in k system, the following relation holds:

$$y' = ct_1'.$$

Further the relation obtained in the above subsection holds on the line $y'=const.$ and $z'=0$. Therefore we have

$$\begin{aligned} t_1' &= \frac{t_1 - (vx/c^2)}{(1 - (v/c)^2)^{1/2}} = (1 - (v/c)^2)^{-1/2} t_1, \\ y' &= c(1 - (v/c)^2)^{1/2} t_1, \\ y' &= y. \end{aligned}$$

If we consider the line $y'=const.$ and $z'=const.$ instead of $z'=0$, we have in the same way $z'=z$. Further if we send another signal from $x'=y'=z'=0$ to $x'=a$, $y'=y$, $z'=0$ and calculate as the above, we can prove the Pythagorean theorem in k system. We can further prove several theorems of the Euclidean geometry in k system. So that we will arrive at last the conclusion that the spatial geometry in k is 3-dimensional euclidean.

Coordinate systems thus defined in the above are not completely equivalent concerning their coordinative character, namely, one is a rest one defined by pragmatistic procedure and the other is determined by mathematical method. But in the following treatment they can be regarded completely equivalent and any difference will not be recognized in the further treatment.

§ 6. Concluding remarks.

We have remarked on the meaning of SPR and then proposed a new method

of derivation of LT. We concluded that the contents of SPR are divided into two parts, in the one part the existence of a set of coordinate systems which are in uniform motion relatively is proposed and then the equivalence of physical laws between these systems is insisted in the other part. Our derivation method of LT proposed in the last section is characteristic because it is derived from the conditions originated from the former part of the principle. We could derive LT from the kinematical conditions only without referring the equivalence condition. Accordingly we can take the above interpretation of SPR. Namely we have not use the equivalent condition of the moving system. We use only some kinematical conditions concerning scale and clock and some other results confirmed experimentally. If we add only one assumption to our moving system, the system can be regarded identical with the usual one. This assumption is closely connected to the so-called "matter axioms" as Reichenbach stated, that is, the assumption that the identity of the equivalence of distances defined light geometrically and those by rigid scale in a moving system. This assumption can be confirmed experimentally at least in principle. Therefore we can regard it as an empiristically meaningful proposition. Roughly speaking we showed that LT can be derived using only observational terms with the coordination of them with geometrical terms. In this procedure we did not use "physical theories" and the concept of the equivalence of the coordinate systems. Therefore we can clarify the physical meanings or its foundations from the empiristic point of view.

The assumption of the covariance of the physical laws under LT was adopted historically because of the covariance of the classical electrodynamic equations together with the negative results for the experiments to detect the existence of the rest aether. This principle is normative one on some point of view, though it may be refuted by future experiments. The principle is therefore valuable from the heuristic view point and the meaning of which must now be investigated in more full detail.

After the above investigations our task to be followed will be the representation of a logically strict formal system of STR.

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