Ontology Based on Non-reflexive Identity and Product Name Functor

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1. Introductory

It is curious and far from our intuition that in the standard course of logic we are taught to take it for granted and evident that there is a basic and strict difference between the predicative “is” and the “is” in the role of identity. This logical discrepancy is expressed by schematically different form. The former is expressed in the form of juxtaposition of a predicate symbol and a name, like “ $F(a)$ ” whose reading is “ a is F ”. The predicative “is” is absorbed into “ F ” as its constitutive part. In this case, “is” never appears at the surface level in its own right. In the latter case, the verb “is” has its own symbolism expressed usually by “=”, like “ $a = b$ ” whose reading is “ a is-identical-with b ”. This clear-cut distinction between predicative “is” and “is” in the role of identity evidently dates from the philosophical presupposition, on which Frege based his logical system.⁽¹⁾

It was without doubt Leśniewski who, after the construction and popularization of Frege-Russellian logical systems, rejected this widely accepted view. Indeed, it can be shown that in Leśniewski’s system the “is” in the role of identity is reducible to the “is” of predication. In fact, in his system the “is” in the role of identity can be defined by means of its primitive symbol “ ϵ ” for singular sentence formation. “ $a \epsilon b$ ” is true if and only if “ a ” refers to just one object and it falls in the extension of “ b ”. Hence, the conjunction “ $a \epsilon b. b \epsilon a$ ” clearly plays the role of identity. Thus “ $a = b$ ” is definable as “ $a \epsilon b. b \epsilon a$ ”.

At the same time, it should be noticed that interestingly enough some logicians are recently getting aware of the untenability of this Fregean dogma.⁽²⁾

The purpose of this paper is to put forward a system of Leśniewski’s logical Ontology, hereafter OL, whose primitive notions are “is” in the role of identity and the product name forming functor.⁽³⁾

Both notions, i.e. identity and compound name, are relatively easy to understand, and according to Lejewski and Clay there is not yet such a system based on these notions.⁽⁴⁾ What is more, as for identity, we seem to make much use of that notion in everyday reasoning. Categorical reasonings may be taken to be strongly

concerned with the “is” in the role of identity. Even though this view seems to be not very popular, nevertheless the fact seems to be so. This point was suggested for example by Hintikka in his [1982], though he just stated the idea, without going further into detail.

I myself arrived to a very similar view some ten years ago when I was occupied with the analysis of (ontologically free) identity along the line and in the spirit of OL. To state something about this matter, a sentence of the form “every A is B” can be restated as “every A is-identical-with some B corresponding to the chosen A,” the paraphrasing of which is “(every A) is-identical-with (some B [corresponding to the chosen A]). In the same manner, a sentence of the form “some A is B” can be restated as “some A is-identical-with some B corresponding to the chosen A,” the paraphrasing of which is “(some A) is-identical-with (some B [corresponding to the chosen A]).

Thus, if we take the position that every term appearing in a categorical sentence is *quantified*, every categorical sentence can be viewed as an identity-statement. And this is indeed the core of the strategy taken by Sommers [1982].

While the notion of identity seems to be closely connected with our reasoning at the intuitive level, still it seems that there is room for a more close analysis of the notion of identity, if we deal with this notion from a purely logical point of view.

Usually the following four conditions are required of the notion of identity :

- 1) $(a) (a = a)$
- 2) $(a, b) (a = b. \supset. b = a)$
- 3) $(a, b, c) (a = b. b = c. \supset. a = c)$
- 4) $(a, b) (a = b. \supset. : F(a). \equiv. F(b))$

What is troublesome about them for logicians as well as for philosophers (but very probably not for mathematicians) is condition 1). For, in the standard reading of quantifiers, i.e. Quinian referential reading of them, we are forced to conclude the thesis :

- 5) $(Ex) (x = x),$

which states that there exists an object that is identical with itself. But since logic in its proper and traditional sense should not be concerned with any existence claim, 5) is not desirable from a purely logical viewpoint, while in so far as the thesis 1) and referential reading are accepted, we are, as Russell somewhere complained, logically forced to a non-logical thesis claiming the existence of objects, which is, properly speaking, a matter belonging to *metaphysica sive ontologia*.⁽⁵⁾

2. Categorical Reading of Quantifiers and Ontological Commitment

Any theory concerned with (some part of) reality must have some device by means of which it is related with (the very part of) reality. Usually, this is a matter

presupposed in advance by a theory, so that we need not bother ourselves about the device of ontological commitment of the theory in question.

But this is not so once we are concerned with a logical system. If one is not a perfect formalist, he must pay attention to the vehicle of ontological commitment of the theory he is occupied with. And I am anything but a perfect formalist. A system of logic I have in mind is in some way concerned with reality or should involve in itself a capability to be concerned with it.

The device I propose as a vehicle for expressing the ontological commitment of the system I am going to develop hereafter is *not* connected with quantifiers directly, as is the case with Quine. If quantifiers have something to do with a device for the vehicle of ontological commitment of a theory, it is only indirectly. And even if the ontological commitment is conveyed by quantifiers, it is not so one-sided as in the Quinian reading, i.e. referential reading, nor so weak as the usual substitutional reading, which is so weak that we cannot have any ontological vehicle.

I go inbetween. In short, ontological commitment of a theory is not a task that quantifiers do. It is a matter of stipulation concerning (the types of) expressions of a language or a theory. Every (type of) expression has its own logico-ontological role. The role varies from (type of) expressions to (type of) expressions. It may be purely syncategorematic, while some specific (type of) expressions may be concerned with reality. I call the role every expression of a type plays the categorial convention of the type, and the role an expression plays the categorial convention of the expression.

The general reading of quantifiers I accept is the following:

Read “ $(x) (\dots x \dots)$ ” as “for some expression ‘ x ’ whose categorial convention is such and such: $\dots x \dots$ ”, and read “ $(Ex) (\dots x \dots)$ ” as “for some expression ‘ x ’ whose categorial convention is such and such: $\dots x \dots$ ”

I call this reading categorial reading. In that reading, it is the categorial conventions that owe the burden of ontological commitment, and not quantifiers. If they play the role of ontological commitment, they do it only via categorial conventions.

At this point of this paper, we introduce a particle “*is*” of the semantic category s/n , n about which I make the following categorial convention:

$C(is)$: if “ a *is* b ” holds, then “ a ” refers to an individual object and it is a b . This is a good example for how quantifiers become the vehicle of ontological commitment of a language or a theory. From this categorial convention about “*is*”, we immediately obtain that the a quantifier is referential if it quantifies a variable which stands in the subject place of a sentence consisting of two names by means of “*is*”. Thus in a sentence “ $(Ex) (x \text{ is } a)$ ”, the reading of quantifier is indeed referential but merely because of the categorial convention $C(is)$ with respect to the particle “*is*”. Quantifier can have a referential function indirectly by the categorial

It is easy to see that such “*is*” owes the meaning of “is-identical-with” in cooperation with the categorial convention $C(is)$, so that the thesis T1 characterizes “is” in the role of identity in the presence of the categorial convention $C(is)$

4. Basic properties of “=”

This chapter is concerned with some basic properties of “=”. The main point is that this identity is transitive and symmetric, but *not universally reflexive*.

$$\text{T9. } (a, b) (a = b. b = c. \supset. a = c), \quad [\text{A1}]$$

thus it is transitive.

$$\text{T10. } (a, b) (a = b. \supset. b = a), \quad [\text{T8}]$$

thus it is symmetric.

But “=” is in general not reflexive, i.e.,

$$a = a$$

does not hold universally.

This can be proved in the following way.⁽⁷⁾

We show that T1 is inferentially equivalent to ;

$$\text{T11. } (a, b) (a = b. \equiv. (Ec) (c = a. c = b)),$$

which, in turn, is inferentially equivalent to the conjunction of :

$$\text{T12(Sym) } (a, b) (a = b. \supset. b = a),$$

and,

$$\text{T13(Tr) } (a, b, c) (a = b. b = c. \supset. a = c).$$

That T11 is inferentially equivalent to the conjunction of Sym and Tr is easy to check.

$$\text{T14. } \text{T1. } \supset. \text{Sym.} \quad [\text{T8}]$$

$$\text{T15. } \text{T1. } \supset. \text{Tr.} \quad [\text{A1}]$$

$$\text{Proof. } 1. \text{ T1.} \quad (\text{sup.})$$

$$2. \quad a = b. b = c \quad (\text{sup.})$$

$$3. \quad (y) (y = b. \supset. a = y) \quad (1, 2)$$

$$4. \quad c = b \quad (2, \text{T14})$$

$$5. \quad a = c \quad (3, 4)$$

$$\text{T16. } \text{Sym. Tr. } a = b. \supset. (Ex) (x = a)$$

$$\text{T17. } \text{Sym. Tr. } a = b. x = b. y = b. \supset. x = y$$

$$\text{T18. } \text{Sym. Tr. } a = b. x = a. \supset. X = b$$

$$\text{T19. } \text{Sym. Tr. } (Ex) (x = a). (x) (x = a. \supset. x = b). \supset. a = b$$

$$\text{Proof. } 1. \text{ Sym. Tr. } (Ex) (x = a). (x) (x = a. \supset. x = b) \quad (\text{sup.})$$

$$2. \quad (Ex) (x = a). (x) (x = a. \supset. x = b) \quad (1)$$

$$3. \quad x^* = a. x^* = a. \supset. x^* = b \quad (2)$$

$$4. \quad x^* = a. x^* = b \quad (3)$$

$$5. \quad a = b \quad (1, 4)$$

T20. Sym. Tr. $\equiv$. T1.

[T14–T19]

so that A1. can be shortened to T11, which is the axiom Lejewski uses as the axiom of non-reflexive identity.⁽⁸⁾

As was shown, the reflexivity does not hold universally. And since “ $a = a$ ” or “ $(Ex) (x = a)$ ” is taken generally to express *esse simpliciter* of the object $a^{(9)}$, contrary to Quine’s famous dictum ‘everything exists’⁽¹⁰⁾, we would have to say that something may not exist.

To explain the situation more intuitively, if the name “ a ” is either non-singular or non-designative, “ $a = a$ ” and “ $(Ex) (x = a)$ ” that are equivalent to each other evidently do not hold. This reminds us of a well-known logical problem often discussed in the medieval age. “Pegasus is Pegasus” is along the line of our analysis a false statement. Since “Pegasus is Pegasus” is, written in the standard notation, “Pegasus = Pegasus”, for “Pegasus” is intended to be a singular name, and this is in turn equivalent to “ $(Ex) (x = \text{Pegasus})$ ”, and as was explained, we are allowed to read the particular quantifier in the referential manner in this case, so that “ $(Ex) (x = \text{Pegasus})$ ” becomes false, with the result that “Pegasus is Pegasus” becomes also false, despite of its apparent tautological form. This type of tautology is a good case of *sophismata*, on which much has been argued by the medieval logicians.⁽¹¹⁾ Thus for the medieval logicians it was by no means evident that identity holds universally. The discussions about this sophisma seems to have gone more and more on decline with the rise of modern logic.

T20 is of great theoretical concern. According to this, the analysis of identity leads us only to such a relation that is just transitive and symmetric. But naturally not any relation that is transitive and symmetric is the relation of identity. Then what is that which makes this analysis valid as an analysis of identity? Evidently the categorial convention C(*is*). This is one of the most crucial points in our analysis. The whole sense of our analysis rests upon the categorial convention C(*is*).

Now some more theses about “ $=$ ”.

T21. $(a, b) (a = b. \supset. a = a)$ [T9, T10]

T22. $(a, b) (b = a. \supset. a = a)$ [T21, T10]

T23. $(a, b) (a = b. \equiv. b = a)$ [T10]

T24. $(a) ((Ex) (a = x). \supset. a = a)$ [T21]

T25. $(a) ((Ex) (x = a). \supset. a = a)$ [T22]

T26. $(a) (a = a. \equiv. (Ex) (a = x))$ [T24]

T27. $(a) ((Ex) (x = a). \equiv. a = a)$ [T25]

5. On Product Name

Let us consider in this chapter a system which has as its primitive symbols “ $=$ ”

for non-reflexive identity and the functor “ $\cap$ ” for product name. It is supposed that there hold between these two functors the following relations :

- T28(A2) $(a) (a = a. \supset. a = a \cap a)$
 T29(A3) $(a, b, c) (a = b \cap c. \supset. a = c \cap b)$
 T30(A4) $(a, b, c) (a = b \cap (c \cap d). \equiv. a = (b \cap c) \cap d)$
 T31(A5) $(a, b, c) (a = b \cap c. \supset. a = a \cap b. a = a \cap c)$
 T32(A6) $(a, b, c) (a = b \cap c. b = d. \supset. a = d \cap c)$

These will be referred to as A2, A3, A4, A5, A6 respectively.

In addition to them, the following relations which regulate further the logical behaviour of “ $=$ ” are supposed :

- T33(A7) $(a, b) (a = a \cap b. b = b. \supset. b = b \cap a)$
 T34(A8) $(a) ((Ex) (x = x \cap a). (y, z) (y = y \cap a. z = z \cap a. \supset. y = z). \supset. a = a)$

and will be referred to as A7, A8.

The reason for the need of introducing T20 and T21 as axioms becomes clear if one once recognizes that the axioms so far accepted, i.e. A1–A6 can be satisfied under the interpretation of “ $=$ ” as the extentional identity, so that it need not express the identity of objects.⁽¹²⁾ We shall see that A7 and A8 narrow down the identity between extensions to the identity of objects.

This comes from the fact that the analysis of identity of object was carried out and is at work only in cooperation with the categorial convention C(*is*).

Some theses concerning T27–T31 are now mentioned :

- T35. $(a) (a = a. \equiv. a \cap a)$ [A2, T27]
 T36. $(a, b, c) (a = b \cap c. c = d. \supset. a = b \cap d)$ [A6, A3]
 T37. $(a, b, c) (a = a \cap (b \cap c). \equiv. a = a \cap b. a = a \cap c)$

Let us add the following thesis as a definition :

- T38(D) $(a, b) (a < b. \equiv. a = a \cap b),$

which will be referred to as D.

I will show that the functor “ $<$ ” introduced by the above definition D is the “*is*” for forming singular propositions, or in other words “ $<$ ” satisfies the axiom of Leśniewski’s Ontology OL.

Making use of D, we have :

- T39. $(a, b, c) (a < b \cap c. \equiv. a < b. a < c)$ [T37]
 T40. $(a, b, c) (a < b. b = b. \supset. b < a)$ [A7, D]
 T41. $(a) ((Ex) (x < a). (y, z) (y < a. z < a. \supset. y < z). \supset. a = a)$ [A8, D]
 T42. $(a, b) (a < b. \equiv. (Ec) (a = b \cap c))$
 Proof. 1.1. $a < b$ (sup.)
 1.2. $a = a \cap b$ (1.1, D)
 1.3. $a = b \cap a$ (1.2, A3)
 1.4. $(Ec) (a = b \cap c)$ (1.3)
 1. $a < b. \supset. (Ec) (a = b \cap c)$ (1.1, 1.4)

- 2.1. $(Ec) (a = b \cap c)$ (sup.)
 2.2. $a = a \cap b$ (2.1, A5)
 2.3. $a < b$ (2.2, D)
 2. $(Ec) (a = b \cap c). \supset. a < b$ (2.1, 2.4)
 T43. $(a, b, c) (a < b. b < c. \supset. a < c)$
 Proof. 1. $a < b$ (sup.)
 2. $b < c$ (sup.)
 3. $a = a \cap b$ (1, D.)
 4. $b = b \cap c$ (2, D.)
 5. $a = b \cap a$ (4, A3)
 6. $b = c \cap b$ (4, A3)
 7. $a = (c \cap b) \cap a$ (5, 6, A6)
 8. $a = c \cap (b \cap a)$ (7, A4)
 9. $(Ed) (a = c \cap d)$ (8)
 10. $a < c$ (9, T22)
 T44. $(a, b) (a < b. \supset. a = a)$ [D, T21]
 T45. $(a, b) (a = b. \supset. a < b)$
 1. $a = b$ (sup.)
 2. $a = a \cap a$ (1, T26, A2)
 3. $a = a \cap b$ (2, T22)
 4. $a < b$ (3, D)
 T46. $(a, b) (a < b. \supset. a < a)$ [T44, T45]
 T47. $(a, b) (a < b. \supset. (Ex) (x < a))$ [T46]
 T48. $(a, b, x, y) (a < b. x < a. y < a. \supset. x < y)$
 Proof. 1. $a < b$ (sup.)
 2. $x < a$ (sup.)
 3. $y < a$ (sup.)
 4. $a = a$ (1, T44)
 5. $a < y$ (3, 4, T40)
 6. $x < y$ (2, 5, T43)
 T49. $(a, b, x) (a < b. x < a. \supset. x < b)$ [variant of T43]
 T50. $(a, b) (a < b. \supset. (Ex) (x < a). (x, y) (x < a. y < a. \supset. x < y). (x) (x < a. \supset. x < b))$ [T47, T48, T49]
 T51. $(a, b) ((Ex) (x < a). (x, y) (x < a. y < a. \supset. x < y). (x) (x < a. \supset. x < b). \supset. a < b)$
 Proof. 1. $(Ex) (x < a)$ (sup.)
 2. $(x, y) (x < a. y < a. \supset. x < y)$ (sup.)
 3. $(x) (x < a. \supset. x < b)$ (sup.)
 4. $a = a$ (1, 2, T41)
 5. $a < a$ (4, T25)

$$6. \quad a < b \quad (5, 3)$$

$$\text{T51.} \quad (a, b) (a < b. \equiv. (Ex) (x < a). (x, y) (x < a. y < a. \supset. x < y). (x) \\ (x < a. \supset. x < b)) \quad [\text{T50, T51}]$$

Hence :

Theorem 1 : the functor “<” satisfies the axiom of OL.

6. System OIP

Let us consider a system of which the axioms are :

$$\text{A1} \quad (a, b) (a = b. \equiv. (Ex) (x = a). (x, y) (x = b. y = b. \supset. x = y). (x) \\ (x = a. \supset. x = b))$$

$$\text{A2} \quad (a) (a = a. \supset. a = a \cap a)$$

$$\text{A3} \quad (a, b, c) (a = b \cap c. \supset. a = c \cap b)$$

$$\text{A4} \quad (a, b, c) (a = b \cap (c \cap d). \equiv. a = (b \cap c) \cap d)$$

$$\text{A5} \quad (a, b, c) (a = b \cap c. \supset. a = a \cap b. a = a \cap c)$$

$$\text{A6} \quad (a, b, c) (a = b \cap c. b = d. \supset. a = d \cap c)$$

$$\text{A7} \quad (a, b) (a = a \cap b. b = b. \supset. b = b \cap a)$$

$$\text{A8} \quad (a) ((Ex) (x = x \cap a). (y, z) (y = y \cap a. z = z \cap a. \supset. y = z). \supset. a = a)$$

To these we add the following thesis as a definition :

$$\text{D} \quad (a, b) (a \varepsilon b. \equiv. a = a \cap b)$$

Concerning the rule of ontological definition, it is supposed that it is stated in terms of the introduced functor.⁽¹³⁾ OIP is the system the primary theses of which are $\langle \text{A1, A2, A3, A4, A5, A6, A7, A8, D} \rangle$ with appropriate rule of definition.

Now it is evident from the Theorem 1 of the preceding chapter that the axiom of OL holds in OIP, so that :

Theorem 2 : Every theorem of OL is a theorem of OIP.

Moreover it is easy to check that the axioms A1-9 as well as D are theses of OL, so that :

Theorem 3 : Every theorem of OIP is a theorem of OL.

Combining them, we get :

Theorem 4 : OL and OIP are inferentially equivalent to each other.

7. On the Simplification of OIP

It is well-known that in the presence of the Axiom of Extensionality ;

$$\text{AE.} \quad (a, b, F) (a o b. \supset. F(a). \equiv. F(b)),^{(14)}$$

AI is inferentially equivalent to AI* where :

$$\text{AI*} \quad (a, b) (a \varepsilon b. \equiv. (Ec) (a \varepsilon c. c \varepsilon b)).$$

On this point, cf. Sobociński [1934].

In OIP we have A7, A8. But in the presence of the Axiom of Extensionality

(AE), we can do away with A7, A8 in constructing OL. This is the point we aim at in this chapter.

Let us consider the subsystem of OIP the primary theses of which are :

$\langle A1, A2, A3, A4, A5, A6, D, AE \rangle$

Let us call this subsystem OIP*

TO1. $(a, b) (a \varepsilon b. \supset. (Ec) (a \varepsilon c. c \varepsilon b))$

Proof. 1. $a \varepsilon b$ (sup.)
 2. $a = a \cap b$ (1, D)
 3. $a = a \cap a. a = a \cap b$ (2, A5)
 4. $a \varepsilon a. a \varepsilon b$ (3, D)
 5. $(Ec) (a \varepsilon c. c \varepsilon b)$ (4)

TO2. $(a, b) ((Ec) (a \varepsilon c. c \varepsilon b). \supset. a \varepsilon b)$

Proof. 1. $(Ec) (a \varepsilon c. c \varepsilon b)$ (sup.)
 2. $a \varepsilon c. c \varepsilon b$ (1)
 3. $a = a \cap c. c = c \cap b$ (2, D)
 4. $a = a \cap (c \cap b)$ (3, A3, A6)
 5. $a = (a \cap c) \cap b$ (4, A4)
 6. $a = a \cap b$ (3, 5, A6)
 7. $a \varepsilon b$ (6, D)

TO3. $(a, b) (a \varepsilon b. \equiv. (Ec) (a \varepsilon c. c \varepsilon b))$ [TO1, TO2]

TO4. $(a, b) (a = b. \supset. a \varepsilon b. b \varepsilon a)$

Proof. 1. $a = b$ (sup.)
 2. $a = a$ (1, T21)
 3. $a = a \cap a$ (2, A2)
 4. $a = a \cap b$ (3, 1, A3, A6)
 5. $a \varepsilon b$ (4, D)
 6. $b \varepsilon a$ (cf. 1-5)
 7. $a \varepsilon b. b \varepsilon a$ (5, 6)

TO5. $(a, b) (a \varepsilon b. b \varepsilon a. \supset. a = b)$

Proof. 1. $a \varepsilon b. b \varepsilon a$ (sup.)
 2. $a = a \cap b. b = b \cap a$ (1, D)
 3. $a = b$ (2, A3, TR., Sym)

TO6. $(a, b) (a = b. \equiv. a \varepsilon b. b \varepsilon a)$ [TO4, TO5]

TO7. $(a, b, c) (a \varepsilon b \cap c. \supset. a \varepsilon b. a \varepsilon c)$

Proof. 1. $a \varepsilon b \cap c$ (sup.)
 2. $a = a \cap (b \cap c)$ (1, D)
 3. $a = (a \cap b) \cap c$ (2, A4)
 4. $a = a \cap (a \cap b). a = a \cap c$ (3, A5)
 5. $a = (a \cap a) \cap b. a = a \cap c$ (4, A4)
 6. $a = a$ (2, T21)

7. $a = a \cap b. a = a \cap c$ (5, 6, A6)
 8. $a \varepsilon b. a \varepsilon c$ (7, D)
 TO8. $(a, b, c) (a \varepsilon b. a \varepsilon c. \supset. a \varepsilon b \cap c)$
 Proof. 1. $a \varepsilon b. a \varepsilon c$ (sup.)
 2. $a = a \cap b. a = a \cap c$ (1, D)
 3. $a = (a \cap b) \cap c$ (2, A6)
 4. $a = a \cap (b \cap c)$ (3, A4)
 5. $a \varepsilon b \cap c$ (4, D)
 TO9. $(a, b, c) (a \varepsilon b \cap c. \equiv. a \varepsilon b. a \varepsilon c)$ [TO7, TO8]

Thus the functor “ ε ” defined by D. satisfies the original axiom of OL (cum AE), and the functor “ $=$ ” and “ $\cap$ ” have the same meaning as in the original system OL.
 (15)

Hence ;

Theorem 5 : OIP* is inferentially equivalent to OL (cum Axiom of Extensionality).

Notes

- (1) Cf. for example, Sommers [1982], p. 124.
 (2) Cf. e.g., Hintikka [1983], Sommers [1982].
 (3) About the system OL, cf. e.g. Lejewski [1958], Slupecki [1955]. An informal representation is in Henry [1972].
 (4) This was suggested to the author by Prof. Lejewski in Manchester, April, 1983, and by Prof. Clay in Tokyo, July, 1986. Both in private conversation.
 (5) About the relation between logic and ontology, cf. Lejewski [1981], Doig [1967], Waragai [198x]. Among the works by the medieval thinkers, Aquinas' *In Meta.* is of great interest.
 (6) Cf. Waragai[1979], Gauchet[1985]
 (7) Cf. Lejewsk [1976], [1981]
 (8) Cf. (7)
 (9) Cf. Existence and Quantification, *Ontological Relativity and Other Essays*, 1969. 91-113.
 (10) On What There is, *From a Logical Point of View*.
 (11) Cf. A Chimera is Chimera, A Medieval Tautology. *Journal of the Philosophy of Ideas*, 21, 273-278. Roberts [1960].
 (12) In OL, we can define the functor “o” for extensional identity in the following way :
 $a o b. \equiv. (x) (x \varepsilon a. \equiv. x \varepsilon b)$.
 This functor satisfies all the conditions of identity. Moreover, A1-A6 are all satisfied by it.
 (13) On ontological definition, cf. Henry, Lejewski, Slupecki.
 (14) On the functor “o”, cf. (12)
 (15) Notice that the functors “ $=$ ” and “ $\cap$ ” are defined in OL in the following way :
 $(a, b) (a = b. \equiv. a \varepsilon b. b \varepsilon a)$
 $(x, a, b) (x \varepsilon a \cap b. \equiv. x \varepsilon a. x \varepsilon b)$

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