Mereological Ontology and Dynamic Semantics

YASUO NAKAYAMA*

1. Introduction

A purpose of formal studies of natural languages consists in answering the question as to what objects are. Recently, this question has become actual, after the research on plural and mass terms flourished (cf. Lønning (1997), Link (1998)). There is an ontology based on individuals that considers only singular terms as constant symbols. This simple view, which I call ontology based on individuals, seems accepted implicitly in some discussions in analytical philosophy, but these discussions often ignore how to analyze complicated sentences that contain plural or mass terms. This paper proposes an alternative view, i.e. ontology based on mereology.

Frege and Russell were quite aware of problems with plural terms; actually, Frege used second-order logic that can express and characterize relations between plural objects. Lesniewski started with the study of mereology based on his own logical system, called protothetic, and his axiom of ontology. Leonard and Goodman (1940) proposed a first-order theory for mereology. Link (1983) gave an algebraic theory for mereology.

As Simons (1987) suggests, all these mereological systems do not show any fundamental difference; they are more or less varied presentations of the same idea. A mereological system can be used to treat mass terms. But, what about plural terms? In this paper, I propose how to deal with singular and plural terms based on a mereological system. Its central thesis states that objects are individuated by sortal predicates:

(1) A thesis of nominalistic ontology of natural languages
   (a) All things are made of materials and materials can be referred to by using mass terms. Water and iron are examples of materials.
   (b) The part-whole relation is applicable to any materials.
   (c) To pick up structured entities made of materials, predicates can be used. These predicates are called sortal predicates; “animal”, “cell”, “car”, etc. are sortal predicates.

*Faculty of Human Sciences, Osaka University. 1-2 Yamada-oka, Suita, Osaka 565-0871, Japan.
The aim of this paper is a construction of a unified dynamic semantics for natural languages. To realize this aim, I start from an analysis of part-whole relation. The part-whole relation is a fundamental relation and can be used as a basis for definition of other relations. I will show, then, how to combine this analysis with a description of anaphoric relations.

There is an apparent grammatical continuity between singular and plural terms, and this suggests a semantical continuity between them. For example, not only singular terms but also plural terms are used for reference; certain objects can be referred to by using a plural term like "these students". Anaphoric relations are also used for plural terms, as the following example shows: "Some students are talking in a cafeteria. They did not go to the lecture of Prof. N". Anaphoric relations are also used for mass terms. As these considerations show, plural and mass terms refer to certain objects, as singular terms do. To accept this principle means to accept mereological ontology.

2. Mereological Ontology

2.1 Extensional Mereology

I will define a system of axioms for extensional mereology (EM) by extending the system of axioms of Boolean algebra. In this paper, comma is used as conjunction and these formulas are called discourse formulas or D-formulas. To indicate axioms, I use ...A...; for definitions and theorems, ...D... and ...T... are used.

**DEFINITION 1** EM is defined within the first order logic with identity. Let \( \phi \) be an arbitrary D-formula.

**MA1.** Axioms for lattice

- \( x \sqcap x = x \).
- \( x \sqcup y = y \sqcup x \).
- \( (x \sqcap y) \sqcap z = x \sqcap (y \sqcap z) \).
- \( (x \sqcup y) \sqcup z = x \sqcup (y \sqcup z) \).
- \( x \sqcap (x \sqcup y) = x \).
- \( x \sqcup (x \sqcap y) = x \).

**MA2.** Additional axioms for Boolean algebra

- \( x \sqcup (y \sqcap z) = (x \sqcup y) \sqcap (x \sqcup z) \).
- \( x \sqcap (y \sqcup z) = (x \sqcap y) \sqcup (x \sqcap z) \).
- \( O \sqcup x = x \).
- \( 1 \sqcap x = x \).
- \( x \sqcap \text{NON}(x) = 1 \).
- \( x \sqcup \text{NON}(x) = O \).

**MA3.** \( f \in \text{fun}(x) \iff f(x) \).

**MA4.** For all Skolem function symbols \( d_k \): \( d_k(x \sqcup y) = d_k(x) \cup d_k(y) \).
Theorem 1  The following theorems are derivable from EM.
MT1. \( x \subseteq x. \) [MD1, MA1(b)]
MT2. \((x \subseteq y, y \subseteq z) \rightarrow x \subseteq z.\) [MD1, MA1(f)]
MT3. \((x \subseteq y, y \subseteq x) \rightarrow x = y.\) [MD1, MA1(d)]
MT4. \( x \neq O \rightarrow x \subseteq p x. \) [MD3, MT1]
MT5. \((x \subseteq p y, y \subseteq p z) \rightarrow x \subseteq p z.\) [MD3, MT2]
MT6. \((x \subseteq p y, y \subseteq p x) \rightarrow x = y.\) [MD3, MT3]
MT7. \( \neg(O \subseteq p x).\) [MD3]

According to this theorem, \( \subseteq \) is a partial ordering and \( \subseteq p \) is a partial ordering excluding the nothing \( O \). \( \subseteq p \) can be seen as expressing the part-whole relation and copula for material; e.g., "c is water" is expressed as \((c \subseteq p \text{WATER})\). MD4 defines the notation \( \sum(u) [\phi(u)] \), which will be read "the sum of objects that satisfy \( \phi \)".

EM is based on a Boolean algebra and can be seen as a variation of Link (1998).

2.2 Individuation by using sortal predicates

In this section, a system of extensional mereology with sortal individuation (EMSI) is defined. The basic idea of EMSI can be described as follows:

(2) All materials are already given in the world. Individuals are individuated by a use of sortal predicates; they are picked up from the world by applying sortal predicates to a mass of materials. In English, these individuals are referred to by singular terms; individual sums that are composed of the same kind of individuals are referred to by plural terms. Sortal predicates are applicable not only to individuals but also to individual sums.

Given a set of sortal predicates, EMSI can be defined as follows.

DEFINITION 2  Predicates \( F \) that satisfy the following axioms of EMSI are called sortal predicates
SA1. \( \neg F(O) \)
SA2. \((F(x), F(y)) \rightarrow F(x \cup y).\)
SA3. \((F(x), F(x \cup y), y \neq O, x \cap y = O) \rightarrow F(y).\)
SD1. \( x \subseteq p y \equiv (F(x), F(y), x \subseteq y).\)
SD2. \( \text{atom}_p(x) \equiv (F(x), \forall u(u \subseteq p x \rightarrow u = x)).\)
SD3. \( x \subseteq p y \equiv (\text{atom}_p(x), x \subseteq p y).\)
SA4. \( F(x) \rightarrow \exists u(u \subseteq p x).\)
SD4. \( x \subseteq p y \equiv (F(x), F(y), x \subseteq y).\)
SD5. \( (x = \sum p(u)[\phi(u)]) \equiv \)
((F(x), φ(x), ∀u((F(u), φ(u))→u ⊆ x)) ∨ (∀ u(F(u)→¬φ(u)), x = O)).

SD6. collective(u) [x, φ(u)] ≡ (φ(x), ∀ u((u ⊆ x, u ≠ O)→¬φ(u))).
SD7. collectiveF(u) [x, φ(u)] ≡ (φ(x), ∀ u(u ⊆ x→¬φ(u))).
SD8. distributiveF(u) [x, φ(u)] ≡ ∀ u(u ⊆ x→ φ(u)).
SD9. eachF(u) [x, φ(u)] ≡ ∀ u(x ⊆ u→ φ(u)).
SD10. nonF[G](x) ≡ ∀ u(u ⊆ x→¬G(u)), where G is a unary predicate symbol.
SD11. Functional symbols α that satisfy the condition (∃ x (α(x) ≠ O), ∀ x (α(x) ∩ x)) are called adjectives.

A sortal predicate F has the properties like F is not nothing (SA1), if x is F and y is F, then the sum of x and y is F (SA2), if x is F and the sum of x and y is F, then y is F (SA3), and if x is F, then there is a F-type individual that is included in x (SA4). x ⊆ y means that a F-type object x is a part of a F-type object y (cf. SD1), and x ⊆ x expresses that a F-type individual x is a part of a F-type object y (cf. SD3). Given sortal predicates human and animal, the sentences “Socrates is a philosopher” and “Canaries are birds” are expressed as (Socrates ∈ human PHILOSOPHER) and (CANARY ∈ animal BIRD).

From EMSI, a sentence that corresponds to Lesniewski’s axiom of ontology is derivable (cf. appendix).(2) Distributive and collective interpretations of sentences can also be expressed; distributiveF(u) [x, φ(u)] means that not only φ(x) but also all F-type parts of x satisfy φ; eachF(u) [x, φ(u)] states that all F-type individuals that are included in x satisfy φ; collectiveF(u) [x, φ(u)] expresses that φ(x) and no F-type part of x satisfies φ.

I would like to extend EMSI, so that different types of quantification become expressible. The extended theory, NRL, is a theory within two-sorted logic.

DEFINITION 3 Natural Representation Theory, NRL, consists of the following axioms and definitions.

LA0. EMSI.
LA1. A standard system of axioms for +.
LD1. (cdF(x) = 1) ≡ atomF(x).
LA2. (cdF(y) = 1, x ∩ y = O) → (cdF(x) = n ≡ cdF(x ∪ y) = n + 1).

cdF(x) expresses the cardinality of x with respect to F. It is important to relativize the notion of cardinality by a sortal predicate. Consider the following situation. b is a sum of people consisting of 5 groups, where each group has 6 members; b includes, then, 30 people. This situation can be described within NRL: (3)
We see here that $cd_{\text{group}}(b) \neq cd_{\text{human}}(b)$. It is, therefore, important to know what we count when we count.

The consistency of NRL can be shown by constructing a model based on a finite Boolean algebra (cf. appendix). Nakayama (1997) defines a framework for hypothetical reasoning based on a similar system of axioms. By extending this framework, Nakayama (1998b) defines a hypothetical reasoning equipped with learning abilities.

2.3 Problems of individuation

Link (1998) describes one of the most developed formal systems of mereology, the Logic of Plurality (LP). The fundamental difference between NRL and LP consists in the treatment of individuation. LP has a unary predicate symbol $A\text{t}$ that stands for “is an atom”. Individuation in LP is absolute, whereas individuation in NRL is relativized by sortal predicates. Without individuation, counting is impossible. However, counting presupposes sortal predicates.

It is somehow difficult to give an answer to the question “How many objects are there in this room?”, because we do not know exactly, in this context, what we should consider as objects. Normal questions have the form “How many $F$-s are there in this room?”, where $F$ is a common noun, like “desk”, based on a sortal predicate.

Russell (1903) drew the distinction between classes as one and classes as many:

"Thus classes would seem to be one in one sense and many in another. There is a certain temptation to identify the class as many with the class as one, e.g. all men and the human race. Nevertheless, wherever a class consists of more than one term, it can be proved that no such identification is permissible.” (p. 76)

Link (1998) discusses this problem in Chapter 13, but I think that LP is also faced with Russell’s problem. In NRL, these two terms refer to human that is the sum of all humans. human contains many humans, but as a species, it is one. This is expressed in NRL: $(cd_{\text{human}}(\text{human}) > 1), cd_{\text{species}}(\text{human}) = 1)$. Because LP does not respect sortal predicates, LP can count only atomic objects. In LP, human can be considered only as the fusion of many atomic objects, where it is not clear, if atomic objects are humans, cells, or molecules, because all these can be seen in some contexts as atomic.

Link treats numerals as a kind of adjective. At first sight, it seems that Russell’s problem is overcome by this treatment, but this is not the case. Consider the following example:
(3) Each of the three students went home.

LP: $\exists x([3\ \text{student}]) (x) \land \forall u (\text{individual-part}(u, x) \rightarrow u\ \text{went home})$.

NRL: $(d_1 \subseteq \text{humanSTUDENT}, cd_{\text{human}}(d_1) = 3, \forall u (u \in \text{human} d_1 \rightarrow u\ \text{went home}))$.

In this LP-translation, it is not obvious what the individual parts of the three students are. Are they humans, cells, or molecules?

Link (1998) claims that materials and individuals are different objects and cannot be the same. He discusses the example “The gold in Smith’s ring is old, but Smith’s ring is not old” (p. 22). According to Link, we should accept that the gold in Smith’s ring is not identical with Smith’s ring, to avoid inconsistency. However, this argumentation is not correct. The problem consists in the use of the word “old” that requires a presupposed supplement. The sentence can be rewritten as “Smith’s ring is not an old ring but is made of old gold”. This interpretation creates no contradiction. It can be described in NRL as $(d_1 \in \text{thingRING}, d_2 = \text{Smith}, d_1 \in \text{belong to} d_2, d_1 \in \text{thingNON(old(RING))}, d_1 \in \text{pold(GOLD)})$, where old is an adjective in the sense of SD12.

3. Things and materials

In this section, NRL is applied to analyze the relationship between mereological objects and natural languages. The following thesis indicates the starting point of this analysis.

(4) For materials, we can take several methods of individuation. For things, however, only particular methods of individuation are given. This is because things exist after individuation by using sortal predicates, whereas materials are already given and they need no individuation.

Iida (1998) studies Japanese classifiers and points out that individuative terms can be connected only with particular classifiers, whereas mass terms can be used together with various classifiers (cf. p. 5). Iida’s observations suggest that thesis (4) holds not only for European languages but also for Japanese.

3.1 Quantity of materials

Things can be counted, but materials cannot without measure. Materials can be counted, when they are put into containers; we count containers, to talk about quantity of materials. Individualization of materials depends on which containers are taken. For example, three bottles of beer—in Japanese, san bon no biiru—can be interpreted as beer filled in three bottles; two liters of whisky can be understood as whisky that can fill a two-liter container. Individualization of materials by using
containers and individuation of things can be expressed within NRL as follows:

three bottles of beer (san bon no biiru): \((d_i \subset \text{BEER}, \text{cd}_{\text{liquid-in-bottle}}(d_i) = 3)\).

three books (san satsu no hon): \((d_i \subset \text{BOOK}, \text{cd}_{\text{book}}(d_i) = 3)\).

three students (san nin no gakusei): \((d_i \subset \text{humanSTUDENT}, \text{cd}_{\text{human}}(d_i) = 3)\).

Japanese classifiers indicate how sortal predicates are used in Japanese. Classifiers, hon, satsu, and nin, suggest bottle, book, and human; for example, nin is used in Japanese only to count humans.

Many terms can be used as both individuative and mass terms. Apple is such a term (cf. Iida (1998) p. 6, Pelletier and Schubert (1989) p. 333f).

three apples (san ko no ringo): \((d_i \subset \text{THINGAPPLE}, \text{cd}_{\text{thing}}(d_i) = 3)\).

three slices of apple (san kire no ringo): \((d_i \subset \text{APPLE}, \text{cd}_{\text{sliced-thing}}(d_i) = 3)\).

It holds: \(\text{APPLE} \subset \text{APPLE}^\circ\). Apple is the sum of individual apples, whereas \(\text{APPLE}^\circ\) is the sum of materials of apple. Hence, the second includes the first.

3.2 Higher order individuation

In set theory, higher order objects can be created, when a set of objects is given (cf. Lønning (1997) p. 1041). In our mereological ontology, higher order objects indicate only a different individuation and they are not new objects. Only materials exist; sums of materials are simultaneously given, when materials are given. Things and higher order entities show only different kinds of grouping of these materials.

Think about the sentence “GG is a group of groups that consists of two student groups G1 and G2, where each group has 20 members.” This sentence is expressed within NRL:

\[(GG = G1 \cup G2, G1 \cap G2 = O, G1 \in \text{group } GG, G2 \in \text{group } GG, GG \subset \text{humanSTUDENT},
\text{each}_{\text{group}}(u)[GG, (\text{cd}_{\text{human}}(u) = 20)]).\]

Students can be seen as first order objects, groups of students can be interpreted as second order objects, and GG belongs to the third order objects. However, in mereology, 40 students, two groups of these students, and GG are all understood as the sum of these 40 students; difference comes only from the method of description that is related to difference of individuation.

Higher order individuation is also possible for materials. See the following example:

Water is a liquid. (cf. Pelletier and Schubert (1989) p. 391)

\((d_i = \text{WATER}, d_i \in \text{liquidLIQUID})\).

In this example, water is understood as the name of all the water and as a kind of liquid.
4. Semantic incompleteness of utterances

As Nakayama (1998a) states, the hearer $H$ tries to understand what the speaker $S$ intended to convey; in cases of utterances of indicative sentences, $S$ intends to convey his beliefs. By uttering a sentence, $S$ usually cannot express everything that he wanted to convey. Generally, $S$’s belief system is presupposed and a part of it becomes needed to interpret what $S$ wanted to convey. This part of $S$’s belief system that is needed for interpretation of $S$’s talk is called here presupposed information and denoted by $C_k$.

In case of direct expressions, $H$ has to construct $S$’s presupposed information to identify which object $S$ wants to refer to. A reference succeeds, when $H$ can identify what $S$ intended to refer to, i.e., when $H$ can properly combine the given information with his own belief system.

Many sentences are semantically incomplete in the sense that they do not provide enough information to determine their truth values; Frege said that such sentences express incomplete thoughts (cf. Frege 1918/19). Information given by these sentences can be combined with presupposed information; $H$ supplements the literal information with his own beliefs, and this activity can be seen as the interpretation required for understanding.

Consider the meaning of the sentence “We drank three bottles of beer”. This sentence does not provide precise information about who are referred to by the pronoun “we”, and if it is used in a collective or distributive sense. When the sentence is interpreted in a collective sense, it can be translated as the set $K_1$ of D-formulas (see (5)). Let $d_1$ be $S$’s belief concerning people whom $S$ wanted to refer to by the pronoun “we”. In this case, this sentence is interpreted as follows by giving discourse information $K_1$ and presupposed information $C_1$; explanations are given in square brackets:

(5) We drank three bottles of beer.

$K_1: \{cd_{\text{human}}(d_1) > 1, \ d_{me} \in \text{human} \ d_1, \ d_2 \subseteq \text{BEER}, \ cd_{\text{liquid-in-bottle}}(d_2) = 3, \ d_1 \ drank \ d_2\}.$

[Here, $d_1$ is a sum of humans that includes the speaker, $d_2$ is a part of beer, the cardinality of $d_2$ with respect to bottles is 3, and $d_1$ drank $d_2$.]

$C_1: \{(\phi_1(d_{me}), \ \phi_2(d_1), \ \text{collective}_{\text{human}}(u)[d_1, \ u \ drank \ d_2])\}.$

[The discourse referent for the speaker, $d_{me}$, satisfies the condition $\phi_1$, $d_1$ satisfies $\phi_2$, and the collection of $d_1$ drank $d_2$ together.]

Actually, $K_1$’s truth value can be determined. However, $K_1$’s content is not what $S$ intended to convey to $H$. $S$ intended to convey $K_1 \cup C_1$ by uttering this sentence.

When $S$ and $H$ share the same presupposed information required to interpret
the uttered sentence, $H$ can understand $S$ properly. If this is not the case, misunderstanding may take place. In the following discussion, I consider only cases where communication partners share required presupposed information.

5. Mereological framework for dynamic semantics

A mereological framework for dynamic semantics can be defined by combining the mereological system NRL and the idea of interpretation that is explained now:

(6) Three principles for dynamic interpretation

(a) use of Skolem symbols: Skolem symbols are used to refer to the objects that were introduced before. The truth of D-formulas is defined (cf. appendix):

A set $K$ of D-formulas is true in $M$ iff there is a Skolem-expansion of $M$ that makes all D-formulas in $K$ true.

(b) principle of interpretation: A discourse is interpreted by combining the expressed and the presupposed information; when $\cup K_i$ is the expressed information in a discourse and $\cup C_j$ is the presupposed information, $(\cup K_i) \cup (\cup C_j)$ is, then, the interpretation of this discourse. Here, presupposed information may contain contextual information.

(c) description of anaphoric relations: Let Skolem terms be terms that contain Skolem symbols. Let $d_n$ be the Skolem term that corresponds to an anaphoric pronoun of a new sentence. An anaphoric relation is, then, expressed, by the D-formula $(d_n = d_k)$, where $d_k$ is a Skolem term that exists already in the translation of the foregoing discourse.

Nakayama (1993, 1996) introduced Skolem symbols to express anaphoric relations. This method has advantages in that the standard inference system can be used for derivation within a discourse semantics. Principles (6a) and (6c) are related by this approach. Principle (6b) makes the idea discussed in section 4 explicit.

Now, I translate some English sentences into D-formulas and supplement them with presupposed information. Objects are represented by Skolem symbols and these objects are characterized by giving their properties and relations. Existential and universal quantifiers are rarely used. Quantified expressions are represented by relational statements between plural objects; "some $A$ are $B$", "all $A$ are $B$", "most $A$ are $B$" are expressed as $(cd_F(A \cap B) > 0)$, $(A \subseteq B)$, and $2 \cdot cd_F(A \cap B) > cd_F(B)$, where $F$ is one of the sortal predicates such that $F(A)$ and $F(B)$.

(7) John drank two liters of beer. It was very cold.

$K_1: \{(d_1 = \text{John}, \ d_2 \subseteq \text{p\_beer}, \ d_1 \ \text{drank} \ d_2, \ cd_{\text{liter}}(d_2) = 2)\}$. 

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[\(d_1 = \text{John}, d_2 \text{ is a part of beer, } d_i \text{ drank } d_2, \text{ and the cardinality of } d_i \text{ with respect to the mass liter is 2.}\)]

\[K_1: \{(d_3 \text{ was very cold})\}\]

\[C_2: \{(d_3 = d_2, \text{ distributive (u)} [d_3, u \text{ was very cold}])\}\]

\[d_3 = d_2 \text{ and any part of } d_3 \text{ was very cold.}\]

(8) The boys and the girls each sleep in a dormitory.

\[K_1: \{(d_1 \subseteq \text{humanBOY}, d_2 \subseteq \text{humanGIRL}, \text{group}(d_1), \text{group}(d_2), d_3 = d_1 \cup d_2, \text{ each} \text{group (u)}[d_3, (d_i(u) \epsilon \text{building DORMITORY, u sleep in } d_i(u))])\}\]

\[d_1 \text{ is a part of BOY, } d_2 \text{ is a part of GIRL, } d_1 \text{ and } d_2 \text{ are groups, } d_3 = d_1 \cup d_2, \text{ and each of the groups of } d_3 \text{ sleep in } d_i(u), \text{ where } d_i(u) \text{ is a dormitory for a group } u \text{ in } d_3.\]

\[C_1: \{(\phi_1(d_1), \phi_2(d_2), \text{each} \text{group (u)}[d_3, (d_i(u) \epsilon \text{building DORMITORY, distributivehuman(v)}[u, v \text{ sleep in } d_i(u)])])\}\]

\[\phi_1 \text{ characterizes the particular group of boys that is mentioned in the utterance, the same for } \phi_2, \text{ and any members of the groups of } d_3 \text{ sleep in a group's dormitory.}\]

(9) The boys carried the piano upstairs. They got a cookie as a reward.

\[K_1: \{(cd_{\text{human}(d_1)}>1, d_1 \subseteq \text{humanBOY}, d_2 \epsilon \text{thingPIANO, } d_1 \text{ carried } d_2 \text{ upstairs})\}\]

\[d_1 \text{ is a plural part of BOY, } d_2 \text{ is a piano, and } d_1 \text{ carried } d_2 \text{ upstairs.}\]

\[C_1: \{(\phi_1(d_1), \phi_2(d_2), \text{collectivehuman(u)}[d_1, u \text{ carried } d_2 \text{ upstairs})])\}\]

\[d_1 \text{ satisfies } \phi_1, d_2 \text{ satisfies } \phi_2, \text{ and } d_1 \text{ carried together } d_2 \text{ upstairs.}\]

\[K_2: \{(cd_{\text{human}(d_5)}>1, d_5 = \text{sumhuman}(u)[u \subseteq \text{human} d_4, d_4(u) \epsilon \text{thing COOKIE, } u \text{ got } d_4(u) \text{ as a reward}], d_5 = d_3\}\]

\[d_4 \text{ are humans, } d_5 \text{ is the sum of people who got a cookie as a reward, and } d_5 = d_3.\]

\[C_2: \{(d_5 = d_3, \text{eachhuman}(u)[d_5, u \text{ got } d_4(u) \text{ as a reward})])\}\]

\[d_5 = d_3 \text{ and each of } d_5 \text{ got } d_4(u) \text{ as a reward, where } d_4(u) \text{ is the cookie that } d_5 \text{'s member } u \text{ got as a reward.}\]

(10) Most farmers own a donkey. They are very cruel. They, the donkeys that at least one of the farmers own, have a bad time.

\[K_1: \{(d_1 = \text{FARMER}, d_2 = \text{sumhuman}(u)[u \subseteq \text{human} d_1, d_3(u) \subseteq \text{animalDONKEY, } u \text{ own } d_3(u)], 2 \cdot cd_{\text{human}(d_2)} > cd_{\text{human}(d_1)}\}\]

\[d_1 \text{ are whole farmers, } d_2 \text{ are farmers who own a donkey, and more than half of } d_1 \text{ is } d_2.\]

\[K_2: \{(cd_{\text{human}(d_4)}>1, d_4 \text{ be very cruel})\}\]

\[C_2: \{(d_4 = d_2, \text{distributiv humane}(u)[d_4, u \text{ be very cruel})])\}\]

\[d_4 = d_2 \text{ and any human-type part of } d_4 \text{ are very cruel.}\]
\[ K_3 : \{ (\text{cdanimal}(d_5) > 1, \text{d}_5 \text{ have a bad time}) \} \]

\[ C_3 : \{ (d_5 = d_6(d_2), \text{distributiveanimal}(u)[d_6, u \text{ have a bad time}]) \}. \]

\[ d_5 = d_6(d_2) \text{ and any animal-type part of } d_5 \text{ have a bad time, where } d_6(d_2) \text{ means the donkeys that at least one of the farmers, i.e. } d_2, \text{ own. The axiom MA4 justifies this use of } d_6(d_2). \]

Elworthy (1995) and Krifka (1996) point out difficulties in the treatment of plural anaphora in Kamp and Reyle (1993) and propose their own semantic theories to overcome them. Both theories are purely semantical and no inference system is defined. NRL is a theory based on the standard two-sorted logic that has a complete inference system; we do not need to define any new inference systems. Furthermore, NRL can handle not only plural anaphora but also “mass anaphora”, as was shown in this section.

6. Conclusion

Kamp and Reyle (1993) discussed the problem of plural anaphora. They attempted to combine DRT (discourse representation theory), GQT (generalized quantifier theory), and algebraic semantics in Link (1983). Recently, Link (1998) proposed an algebraic semantics for plural entities, mass objects, and events. This paper is closely connected with these two works. As in Link (1998), I argued that the mereological approach provides a proper ontology for the semantics of plural and mass terms.

The central theses of this paper are: 1. materials and the part-whole relation among them are ontologically fundamental; 2. things are individuated by using sortal predicates. A two-sorted theory NRL is defined and Skolem symbols are used to express anaphoric relations. A straightforward combination of mereological ontology and dynamic semantics is shown. Nakayama (1997, 1998b) proposes how NRL can be extended to a system of hypothetical reasoning that can capture knowledge representation and nonmonotonic reasoning in AI.

A mereological ontology is incomplete without analysis of events and processes. This remains as a future work.

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THEOREM 2 (A Sketch of proof of Ontology* T16, cf. note 2)

T1. \( (\text{atom}_F(x), u = x) \rightarrow u \in Fx. \) \([\text{SD1, SD3, MT1}]\)

T2. \( \text{atom}_F(x) \rightarrow \exists z(z \in Fx). \) \([\text{T1}]\)

T3. \( (\text{atom}_F(x), u \in Fx) \rightarrow u = x. \) \([\text{SD2, SD3}]\)

T4. \( \text{atom}_F(x) \rightarrow \forall u \forall v((u \in Fx, v \in Fx) \rightarrow u = v). \) \([\text{T1, T3}]\)

T5. \( \text{atom}_F(x) \rightarrow (\exists z(z \in Fx), \forall u \forall v((u \in Fx, v \in Fx) \rightarrow u \in Fv)). \) \([\text{T2, T4}]\)

T6. \( \exists z(z \in Fx) \rightarrow F(x). \) \([\text{SD1, SD2}]\)

T7. \( (u \in Fv, v \in Fu) \rightarrow u = v. \) \([\text{SD1, SD2, MT3}]\)

T8. \( \forall u \forall v((u \in Fx, v \in Fx) \rightarrow u = v) \rightarrow \forall u \forall v((u \in Fx, v \in Fx) \rightarrow u = v). \) \([\text{T7}]\)

T9. \( \text{atom}_F(x) \rightarrow x \in Fx. \) \([\text{T1}]\)

T10. \( \forall u \forall v((u \in Fx, v \in Fx) \rightarrow u = v) \rightarrow \forall u (u \in Fx \rightarrow u = x). \) \([\text{SA3}]\)

T11. \( (\exists z(z \in Fx), \forall u \forall v((u \in Fx, v \in Fx) \rightarrow u = v)) \rightarrow \text{atom}_F(x). \) \([\text{SD2, T6, T8, T10}]\)

T12. \( \text{atom}_F(x) \equiv (\exists z(z \in Fx), \forall u \forall v((u \in Fx, v \in Fx) \rightarrow u \in Fv)). \) \([\text{T5, T11}]\)

T13. \( x \in Fy \rightarrow (\text{atom}_F(x), \forall z(z \in Fx \rightarrow z \in Fy)). \) \([\text{SD3, T3}]\)

T14. \( (\text{atom}_F(x), \forall z(z \in Fx \rightarrow z \in Fy)) \rightarrow x \in Fy. \) \([\text{T9}]\)

T15. \( x \in Fy \equiv (\text{atom}_F(x), \forall z(z \in Fx \rightarrow z \in Fy)). \) \([\text{T13, T14}]\)

T16. \( x \in Fy \equiv (\exists z(z \in Fx), \forall u \forall v((u \in Fx, v \in Fx) \rightarrow u \in Fv), \forall z(z \in Fx \rightarrow z \in Fy)). \) \([\text{T12, T15}]\)

THEOREM 3 NRL is consistent.

Sketch of Proof: Let \( A \) be a finite set, \( N \) a set of natural numbers. We can construct a model \( \langle\langle \text{pow} \ (A), N \rangle, V \rangle \) for NRL based on a finite Boolean algebra. \( \phi \) must be interpreted, so that it satisfies the axiom MA4, and \( F \) must be interpreted, so that it satisfies the axioms SA1–SA4. \( \square \)

DEFINITION 4 Let \( M = \langle\langle U, N \rangle, V \rangle \) and \( K \) be a set of D-formulas.

(1) \( M^* \) is a Skolem expansion of \( M \) with respect to \( K \) iff
\[
\begin{align*}
&M^* = \langle\langle U, N \rangle, V^* \rangle & \text{[For all Skolem constant symbols} \ d_k, \\
&V^*(d_k) \subseteq U \rangle & \text{[For all n-ary Skolem function symbols} \ d_k, V^*(d_k) \text{ is a function} \\
&\in U^n \text{ into} \ U \rangle & \text{[For all unary Skolem function symbols} \ d_k, V^*(d_k)(a \cup b) = V^*(d_k)(a) \cup V^*(d_k)(b)].
\end{align*}
\]

(2) \( K \) is true in \( M, \beta \) iff
\[
\exists M^*[\langle M^* \text{ is a Skolem expansion of} \ M \text{ with respect to} \ K \rangle \& [K \text{ is true in} \ M^*, \beta]].
\]

(3) \( K \) is true in \( M \) iff for all assignment \( \beta, K \) is true in \( M, \beta \).

Notes

1) Kamp and Reyle (1993) studied plural anaphora and pointed out that interpretation of
plural terms is a serious problem for dynamic semantics. However, their treatment of plural anaphora is quite ad hoc and has been widely criticized (cf. Elworthy (1995), Nakayama (1996)).

2) Lesniewski took a different interpretation for quantification, and our presentation is, strictly speaking, not the same as Lesniewski’s ontology (cf. Simons (1987), p. 61, Simons (1992), Chap. 11). In the following, I denote first-order translation of Lesniewski’s ontology with Ontology*. There are works of Waragai clarifying Lesniewski’s ontology. Waragai’s LI+ is weaker than EM, because theorem MT3 does not follow from LI+(cf. Waragai (1990)). LI+ does not accept identity between sums of individuals. However, for treatment of plural and mass terms, identity between materials and individual sums is required. For this reason, I think that LI+ is not adequate for the study of mereological ontology of natural languages.

3) According to SD10, each $\text{group}(u) \ [b, \text{cd human}(u) = 6]$ is equivalent to $\forall u (u \in \text{group} b \rightarrow \text{cd human}(u) = 6)$.

4) Sentences of (7) and (9) are modified examples from Lønning (1997, p. 1040); (8) and (10) are modified examples from Link (1998, p. 173) and Elworthy (1995, p. 298f).

References


Frege (1918/19) “Der Gedanke”.


