Concerning the Coefficient of Storage of Aquifer

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1. Meaning of Coefficient of Storage

The term of "Specific Yield" was proposed by Meinzer who defined it as the quantities of water ft^3 released from the column of aquifer, whose cross section was 1 ft^2 , by an unit drawdown (1 ft) of phreatic surface (also ground-water surface in broad meaning). The conception expressed by this term was applied originally to either water table well or tapped well. But later this term was redefined by Wenzel (1961)⁽¹⁾ for a tapped well and thus it was called by the term "Coefficient of Storage". Thus, the conception of "Specific Yield" was limited to a water table well only. "Specific Yield" is the same as "Effective Porosity of Aquifer", therefore its value is constant for the corresponding aquifer. However, the coefficient of storage must be variable, because it is a kind of measure to evaluate the storage of water within the corresponding aquifer. The writer will discuss this problem.

Besides, the term "Coefficient of Storage" is defined by Wenzel as the quantities of water ft^3 withdrawn from the column of aquifer, equivalent to the thickness, and of cross section 1 ft^2 , by the pressure drop corresponding to 1 ft of H_2O . But it may be defined by C.G.S. unit too, because it is non-dimensional mathematically.

2. Decreasing of Coefficient of Storage

The writer cannot approve to consider the coefficient of storage of a certain aquifer as constant, even though the Theis' non-equilibrium formula is based upon the condition. In the pumping test, the drawdown s is measured at t at the observation well which is distant r from the pumping well, and then the data are plotted on logarithmic section paper as the relation curve between s (on ordinate) and r^2/t (on abscissa) in the Theis' graphical solution. However, there is no restriction for the prescription of r^2/t in principle. In other words, which conditions answer the purpose is not restricted, that is, either when r is constant and t variable or when they are the reverse. The data curves (the relation curves) are different owing to the condition applied. This is presumed to attribute a certain imperfection contained in the Theis' Theory. But the study of the writer has not reached the stage to correct the Theis' Theory, though he published the so-called "Squeeze Theory".

Robinson and Skibitzke (1962)⁽²⁾ argued concerning the relation between the maximum drop s_{max} at an observation well being distant r from the pumping well and transmissibility T of the aquifer, under the presumption of yearly change of T . By the differentiation of the Theis' non-equilibrium formula by T and some mathematical procedures, they derived the next three equations.

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$$s_{max} = (0.647) \frac{Q}{4\pi T} \quad (1)$$

$$T = \frac{(2.30) r^2 S}{4t} \quad (2)$$

$$s_{max} = \frac{(0.281) Q}{S} \cdot \frac{t}{r^2} \quad (3)$$

here S is the coefficient of storage. In the non-equilibrium theory, it is promised that the discharge of the well is continued consuming the storage of the aquifer concerned. Therefore, the coefficient of storage must diminish consequently. On the other hand, T is considered to be invariable in the general mathematical procedure. Then it is reasonable to think that the variability of T may be only apparent due to the reflection of S .

Equ. (3) is also transformed as below in a certain separated one year, because Q is the discharge per day, that is, t is of day unit.

$$\left. \begin{aligned} S_n' &= K \frac{1}{s_{max \cdot n}} \\ K &= \frac{0.281 \times 365 \cdot Q}{r^2} \end{aligned} \right\} \quad (4)$$

here S_n' is the changed value of S at the end of a certain year of n order, when $s_{max \cdot n}$ takes a value as below.

$$s_{max \cdot n} = s_{max \cdot o} + s'_{max} \cdot t_n \quad (5)$$

here s'_{max} is the increasing rate of s_{max} and t_n year symbol of n order. Then S_n' of zero year when the pumpage began is denoted by S_o' and the next equations are given respectively.

$$S_o' = K \frac{1}{s_{max \cdot o}} \quad (6)$$

$$s_{max \cdot o} = s_{max \cdot o} + s'_{max} \cdot t_o \quad (7)$$

If S_o' is computed by the pumping data of $t=1$ day, t_o should be zero practically, and the second term of the right side of Equ. (7) is resumed to be zero. Thus, the next equation may be derived.

$$\frac{S_n'}{S_o'} = \frac{s_{max \cdot o}}{s_{max \cdot o} + s'_{max} \cdot t_n} \quad (8)$$

The figure shows the relation curves between S_n'/S_o' and t making s'_{max} as the parameter. This is no more than an idea. Then the problem does not reach its conclusion and the study is reserved hereafter. In this example, the data are $Q=2,000\text{m}^3/\text{d}$, $r=20$ m and $s_{max \cdot o}=5\text{m}$. So, from Equ. (6), S_o' is computed as 0.281.

3. Consideration of Squeeze from Confining Beds

In general ground-water development, discharge always exceeds recharge plus the so-called supplement recharge. Consequently the phreatic surface of the well drops gradually. In the case of tapped well this phenomenon means the pressure decrease of water within the aquifer concerned. In other words, so far as the discharge of the well continues constantly, the grade of compression of the water within the aquifer decreases gradually, even though the aquifer is being saturated. In a few cases, the specific capacity of a little old tapped well recovers to the same value by sufficient dredging, but in almost cases it diminishes year by year, because the grade of expansion of water withdrawn in open air decreases. This is a result of the decreasing of ground-water storage and that is the same as the decrease of coefficient of storage.

Recently Shibazaki and Shimura (1967)⁽³⁾ published a paper concerning the determination of safe yield for a ground-water basin. The result is very respectable and the arguments agree with our experiences. But in their argument, the coefficient of storage is replaced by the specific yield. So far as concerns the piestic aquifer, the argument is incorrect, because unsaturated pores equivalent to discharged water never occurs. To interpretate the inconsistency, it may be most reasonable to consider the water squeezing out from the confining beds. The compaction of confining beds by the squeeze is irreversible. Then, from the standpoint of water-releasing, the way of thinking is analogous to that of specific yield in which water releases actually from the water table aquifer.

References

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