On the Fluctuation of the Rate of Increase or Decrease in the Volume of the Aquifer of the Shallow Groundwater in Paddy Fields

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Introduction

When the volume of aquifer is measured for each month, a time series of the volume of aquifer is obtained. From this series, the aquifer's volume of increase during two consecutive months is obtained for the period of rise. And also, the volume of decrease is obtained for the period of decline.

The values of the volumes of increase or decrease show a random variation in their time series. A study of the variations in the time series analysis was made.

Theory

The volume of increase or decrease between two consecutive months (dV) can be obtained by Eq. (1).

$$\left. \begin{aligned} dV_0 &= V_{\text{Mar.}} - V_{\text{Apr.}} \\ dV_1 &= V_{\text{Apr.}} - V_{\text{May}} \\ \dots &\dots \dots \dots \\ dV_{11} &= V_{\text{Jan.}} - V_{\text{Feb.}} \end{aligned} \right\} \dots\dots\dots (1)$$

where V = the volume of aquifer, $V_{\text{Mar.}}$ = the volume of V in March, $V_{\text{Apr.}}$ = the volume of V in April etc. and dV = the increased (or the decreased) volume of V between two consecutive months.

The time series of dV for a year (March to February) can be obtained if the data of V for the year is computed by using Eq. (1). The time series of dV also shows the variation of dV during the year.

The degree of the variation can be shown by the non-dimensional equation, Eq. (2).

$$r = \frac{dV}{V} \quad V = \sum dV \dots\dots\dots (2)$$

In order to express mathematically the variation of r for the full year, a harmonic analysis is needed to calculate the FOURIER coefficients for the data of r . To make the analysis, Eqs. (3) and (4) were constructed by applying the FOURIER series.

$$\begin{aligned} r_t = & \frac{a_0}{2} + a_1 \cos K \frac{2\pi}{m} + a_2 \cos 2K \frac{2\pi}{m} + \dots + a_n \cos nK \frac{2\pi}{m} \\ & + b_1 \sin K \frac{2\pi}{m} + b_2 \sin 2K \frac{2\pi}{m} + \dots + b_n \sin nK \frac{2\pi}{m} \dots\dots\dots (3) \end{aligned}$$

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$$\left. \begin{aligned} a_0 &= \frac{2}{m} \sum_{K=0}^{m-1} r_K \\ a_n &= \frac{2}{m} \sum_{K=0}^{m-1} r_K \cos nK \frac{2\pi}{m} \\ b_n &= \frac{2}{m} \sum_{K=0}^{m-1} r_K \sin nK \frac{2\pi}{m} \end{aligned} \right\} \dots\dots\dots (4)$$

The values of r obtained by Eq. (3) are called the theoretical values of r or r_t hereafter.

In practice, the values of the FOURIER coefficients are computed by using the twelve-ordinate scheme with the interval $(0, 2\pi)$ representing a year (March to February), m representing twelve months, and r_K representing the observed values of r in a month ($r_0 = r$ in March, $r_1 = r$ in April,, $r_{11} = r$ in February).

The values of r fluctuate between the positive values and the negative values. Therefore, in order to see the characteristics of the variation of r , one must estimate the variation of r in the positive values only, or in the negative values only. As one of the methods of estimating the variation of r in the positive values only or r in the negative values only, Eq. (5) was constructed.

$$R = \sum \frac{r}{\sum r} \dots\dots\dots (5)$$

R means the cumulative values of $r/\sum r$.

In order to express mathematically the curve drawn by the cumulative values of R in the positive values, Eq. (6) was constructed by applying the GOMPERTZ⁽¹⁾ curve.

$$R_{t+} = KA^{BT} \dots\dots\dots (6)$$

where K = an asymptote, or limit that the trend values approach as

BT approaches ∞ , and t = time.

In practice, in order to fit the curve to our problem, the partial total method was used.

For the cumulative curve of R in the negative values, Eq. (7) was constructed by applying the LOGISTIC⁽¹⁾ curve, and the values of constants in the equation were obtained by the selected points method.

$$R_{t-} = \frac{K}{1 + 10^{a+bt}} \dots\dots\dots (7)$$

Data

The data used in this paper was obtained from our field investigations⁽²⁾ of shallow groundwater that have been carried out on continually in the study area since July 1964.

The study area has $492.85 \times 10^4 \text{ m}^2$ in its area and is the right side region of the downstream basin of the Aya River, Kagawa Prefecture. To get the water table data, 36 observation wells⁽³⁾ ($d \leq 5 \text{ m}$, d = well depth) have used.

Results and Discussion

By using the data⁽⁴⁾, the time series of dV were obtained by Eq. (1) as listed in Table 1. From Table 1, it is seen that the volume of dV varies from $-158.95 \times 10^4 \text{ m}^3$ (October 1965)

to $186.49 \times 10^4 \text{ m}^3$ (March 1966) throughout the three years (March 1965 to February 1968).

Table 1 Values of dv (10^4 m^3) for every month from March 1965 to February 1968 and the average of them.

Month	1965 to 1966	1966 to 1967	1967 to 1968	1965 to 1968
February	0.00	0.00	0.00	0.00
March	-37.29	186.49	99.85	83.02
April	2.37	-12.03	106.15	32.16
May	137.97	-0.01	-23.32	38.21
June	101.82	78.58	-37.93	47.49
July	73.21	50.76	116.40	80.12
August	-36.57	-66.78	-31.60	-44.98
September	33.89	59.63	-26.60	22.31
October	-158.95	-124.58	-85.88	-123.14
November	-19.72	-48.24	24.95	-14.34
December	21.44	-49.28	-60.78	-29.54
January	-61.11	-54.00	-49.59	-54.90
February	-57.06	-20.54	-31.65	-36.42

Figs. 1A to 1D Variation of r . The open circles show the theoretical values calculated by Eqs. (3) and (4), and the solid points are the observed ones.

Table 2 Values of the Fourier coefficients.

	Mar. 1965 to Feb. 1966	Mar. 1966 to Feb. 1967	Mar. 1967 to Feb. 1968	Mar. 1965 to Feb. 1968
a_0	50.000	50.000	50.000	50.000
a_1	2.423	10.483	8.908	8.868
a_2	-14.348	5.798	5.581	-1.291
a_3	-4.252	8.140	16.345	7.857
a_4	8.083	18.338	-1.838	10.143
a_5	-7.772	-1.728	-7.048	-6.750
a_6	11.610	17.280	13.603	17.113
b_1	19.036	13.030	10.591	17.186
b_2	2.069	-3.624	0.761	-0.533
b_3	4.655	-2.733	8.115	3.817
b_4	-6.196	-0.163	6.182	-0.403
b_5	-3.536	1.267	0.818	-0.670

Figs. 2A to 2D Variation of R . The open circles on the solid curves show the theoretical values calculated by Eq. (6) and the solid points are the observed ones. The open circles on the dashed curves show the theoretical values calculated by Eq. (7) and the crosses are the observed ones.

By using Eq. (2), the values of r were obtained and the results of that are shown by the solid circles in Figs. 1A (March 1965 to February 1966), 1B (March 1966 to February 1967), 1C (March 1967 to February 1968) and 1D (the average of the three years). These data of r are called the observed data hereafter.

In order to obtain the values of the FOURIER coefficients for our problem, the observed data shown in Figs. 1A to 1D was used, and the values of the coefficients were obtained by Eq. (4) as listed in Table 2. By using Eq. (3) with the values of the FOURIER coefficients listed in Table 1, the values of r_t were obtained as shown by the open circles in Figs. 1A to 1D. These figures tell us the theoretical values (the open circles) are close to the observed ones (the solid circles) throughout the years.

By using Eq. (5), the values of R were obtained as shown in Figs. 2A (March 1965 to February 1966), 2B (March 1966 to February 1967), 2C (March 1967 to February 1968), and 2D (the average of three years). The solid circles show the values of R (the cumulative values of r) for r in the positive values and the crosses show the values of R for r in the negative values. These data of R are called the observed data of R hereafter.

Table 3 Values of A, B and K

Period	A	B	K
March 1965 to February 1966	0.013207	0.35743	99.271
March 1966 to February 1967	3.2007	1.09153	95.898
March 1967 to February 1968	0.31148	0.52306	100.4
March 1965 to February 1968	0.34403	0.57051	101.29

with these observed data (the solid circles) the values of A, B and K were obtained as listed in Table 3. Using these values of A, B and K , the values of R_{t+} were obtained by Eq. (6) as shown by the open circles on the solid curve in Figs. 2A to 2D. From these graphs, it is plain that the values of R_{t+} are closed to the ones.

Table 4 Values of a, b, and K .

Period	a	b	K
March 1965 to February 1966	0.169	-0.2995	94.199
March 1967 to February 1967	1.2889	-0.2275	117.986
March 1967 to February 1968	0.9591	-0.1776	138.993
March 1965 to February 1968	0.3413	-0.2821	98.988

Using the observed data (the crosses) shown in Figs. 2A to 2D, the values of K , a and b were obtained as listed in Table 4. By using Ep. (7) with the values of K , a and b in Table 4, the values of R_{t-} were obtained as shown by the open circles on the dashed curve in Figs. 2A to 2D. The graphs in these figures show that the theoretical values are close to the observed ones throughout the years.

Summary

A study was made to estimate the values of aquifer increasing or decreasing two consecutive

months and to find their characteristics by means of a time series analysis. The results that were obtained are as follows:

1) The values of aquifer increasing or decreasing during consecutive months, dV , were obtained by Eq. (1), and the results showed that the values of dV varied from $-158.95 \times 10^4 \text{m}^3$ to $186.49 \times 10^4 \text{m}^3$ as listed in Table 1.

2) To see the variation of dV during the period (March to February), the term of r was defined by Eq. (2), and the results are shown in Figs. 1A to 1D. To show mathematically variation of r (the curves show the variation of r), Eqs. (3) and (4) were constructed by applying the FOURIER series. The theoretical values of r , r_t , were obtained by these equations as shown in Figs. 1A to 1D. As shown in these figures, the theoretical values coincided with the observed ones.

3) The values of the cumulative r in the positive values, r_{t+} , were obtained by Eq. (6) that was constructed by applying the GOMPERTZ curve. Also the cumulative values of r in the negative, values, R_{t-} , were obtained by using Eq. (7) that was constructed by applying the LOGISTIC curve. The theoretical values obtained by these equations (the open circles) coincided with the observed ones (the solid circles and the crosses) as shown in Figs. 2A to 2D.

References

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