Numerical Simulation of Water-Rock Interaction through an Artificial Water Circulation in a Hot Dry Rock

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Abstract

A numerical simulator was developed to analyze the coupled hydrological-thermal-chemical processes encountered in a heat extraction system used from a hot dry rock reservoir. In this system, cold water is injected into a re-injection well drilled in a hot dry rock reservoir that has an artificially created fracture, and steam or hot water is recovered from a production well located a few hundred meters from the re-injection well. Fluid, energy, and chemical transport processes in the fracture are described as a two-dimensional problem, and energy transport processes in the rock matrix as a three-dimensional one. The convection-diffusion type equations used to model these transport processes are solved numerically by a finite difference method.

The effect of temperature on the precipitation and dissolution of silica was investigated for various injected silica concentrations. Fracture aperture decreases in the low temperature region near the re-injection well due to the precipitation of silica, whereas in the high temperature region it increases due to the dissolution of silica. The temperature dependence of precipitation and dissolution rates was confirmed by the numerical simulator. Numerical analysis, however, shows that the dissolution and deposition of silica resulted in very small change in the fracture permeability after injecting water for 1,800 days.

Key Words: hot dry rock, numerical simulation, water-rock interaction, dissolution, precipitation, silica, fracture

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INTRODUCTION

One of the most important problems in developing a hot dry rock reservoir power generation is the difficulty in predicting the lifetime of the reservoir. The hot dry rock reservoir has little or no natural steam or water, and its stored heat can not be extracted by conventional geothermal power generation systems such as withdrawal of hot water and steam from a geothermal reservoir through wells. An artificial water circulation system therefore has to be developed to extract heat from the hot dry rock reservoir. In such a system, cold water is injected through an injection well drilled in the hot dry rock reservoir that has artificially created fractures and steam or hot water is recovered from a production well located a few hundred meters from the injection well. In the circulation system, transport phenomena are complicated with coupled fluid, energy, and chemical transport processes. Substantial quantities of energy as well as dissolved chemical species are carried by and interact with the flow field. Conversely, potential fields of energy and chemical species can affect the flow field. These coupled processes often show non-linear behavior and therefore require numerical solution methods. The non-isothermal processes of water dissolved with silica have been studied by Moore et al. (1981), Keith et al. (1983), Itoi et al. (1984, 1986, 1987a, 1987b, 1989), Lai et al. (1985), Malate et al. (1991), Malate & O'Sullivan (1992a, 1992b), Matsunaga et al. (1994) and Wang et al. (1996). Moore et al. (1981) conducted a series of experiments on the non-isothermal radial flow of water supersaturated with silica through cylindrical samples of granites. The variation in permeability with time was determined from the ratio of the changing mass flow rate to the constant pressure difference.
Large reductions in both permeability and flow rate occurred as a result of silica deposition over a period of one to three weeks. Silica precipitation along grain boundaries and micro fractures was responsible for this decrease in permeability. Keith et al. (1983) also examined the effect of silica precipitation on fluid flow in experiments similar to those of Moore et al. (1981). They confirmed that significant permeability reduction was controlled mainly by silica deposition. Itoi et al. (1984, 1986, 1987a, 1987b, 1989) made an experimental study of the near-isothermal flow of geothermal fluid supersaturated with silica through a column composed of a porous medium. Their experimental results showed that silica scale is formed mainly in the region where the fluid enters the column, resulting in a marked decrease in permeability. Lai et al. (1985) made a numerical simulation of the non-isothermal processes by injection of water supersaturated with silica in a single fracture and found a substantial decrease in permeability near the fracture inlet owing to silica deposition. Matlate et al. (1991) and Matlate & O'Sullivan (1992a, b) used analytical methods to solve isothermal and non-isothermal problems in the transport and deposition of silica. They obtained analytical solutions using a method of characteristic to the one-dimensional problem of constant rate injection to a packed column and to the radial symmetric problem of flow away from a re-injection well. The strong temperature dependence of the rate of silica deposition was confirmed with this model. The radial flow model has been applied to the field data from the Otake geothermal field, Japan. Matsunaga et al. (1994) analyzed chemically the water from the production well in a water circulation test made at the Hijiori hot dry rock field, Japan. They found that the concentration of silica in the water increases during the water circulation. Wang et al. (1996) made flow tests in granite samples and confirmed that silica is deposited when the water temperature decreases abruptly from 300°C to 90°C.

We have developed a numerical simulation model to analyze hydrodynamically, thermally, and chemically the coupled processes encountered in a fracture during the injection of a silica solution to a hot dry rock reservoir. Its development was based on a previous model (Akibayashi et al. (1992), Akibayashi et al. (1994)) that handles coupled fluid and energy transport processes in a fracture with heat transfer from a hot dry rock matrix. The new model was used to examine the problem of the transport, deposition and dissolution of silica in a fracture with heat and the mass transfer from a hot dry rock matrix for various silica concentrations of injected water.

**ANALYSIS**

Figure 1 shows a conceptual model for a single fracture induced artificially in a hot dry rock reservoir. The model is idealized as shown in Figure 2. The following assumptions were made to simplify the problems of energy and chemical transport in the fracture:

1. The fracture is uniform in thickness and filled with silica solution.
2. The flow in the fracture is in a quasi-steady state, and the effect of gravity on the flow

![Fig. 1 Conceptual model of a hot dry rock reservoir.](image-url)
Governing Equation

On the basis of the assumptions, flow, energy and chemical transport equations in the fracture are written

Flow equation:

\[
\nabla \left( \frac{K}{\mu} (\nabla \rho + \rho g) \right) + q = 0,
\]

where

\[
K = \frac{gb^2}{12},
\]

\( P \) is the pressure, \( q \) the volume flow rate of injection or discharge per unit fracture water volume at the injection or production point, \( K \) the fracture permeability given by the cubic law for the laminar flow of fluids through an open fracture consisting of parallel planar plates (Witherspoon et al. (1980)), \( b \) the fracture aperture, \( \mu \) the viscosity of water, \( \rho \) the density of water and \( g \) the acceleration of gravity.

Energy equation:

\[
\lambda f \nabla^2 \theta_f - (c \rho_f) V \cdot \nabla \theta_f + E_n - \frac{2E_b}{b} = 0,
\]

where \( \theta_f \) is the fracture temperature, \( \tau \) the time, \( \lambda_f \) the thermal conductivity of the water, \( (c \rho_f) \), the thermal capacity of the water, \( V \) the velocity vector \( = -\left( \frac{K}{\mu} \nabla p \right) \), \( E_n \) the heat flow per unit water volume and unit time at the injection well, and \( E_n \) the heat flow per unit fracture area and unit time from the rock matrix to the fracture (see Equation (21) and Figure 3).

Mass transport equation:

\[
D_j \nabla^2 C_f - V \cdot \nabla C_f + \frac{M_a}{\rho_f} + N = \frac{\partial C_f}{\partial \tau},
\]

where

\[
N = k \cdot (C_f - C) ,
\]

\[
k = \frac{2k_f}{b \cdot \rho_f} \quad \text{at} \quad C_f < C \quad \text{(Rimmerstid and Barns (1980))},
\]

\[
k = \frac{2k_f}{b \cdot \rho_f} \quad \text{at} \quad C_f > C \quad \text{(Rimmerstid and Barns (1980))},
\]

\[
k_f = 10^{\left(0.707 \cdot \frac{2598}{\theta_f} \right)} \times 86400
\]

\[
k_f = 10^{\left(1.174 \cdot 0.002028 \cdot \theta_f \cdot 4158.0 \theta_f \right)} \times 86400
\]
\[ \log C_i = \frac{-731}{\theta_x} + 4.52 \text{ (amorphous silica)} \]

(Fournier and Rowe (1977)). (7)

\( C \) is the concentration of silica per unit water mass, \( D \) the mass diffusivity in the binary solution of silica and water, \( M_\text{m} \) the mass flow per unit volume and unit time at the injection well, \( N \) the dissolution or precipitation rate, \( k \) the rate constant, \( k_F \) the apparent dissolution rate constant (per unit fracture area), \( k_p \) the apparent precipitation rate constant (per unit fracture area), \( C_\text{e} \) the equilibrium silica concentration, and \( \theta_x \) the absolute temperature.

If it is assumed that all precipitated silica deposits on the fracture wall, the relation between the dissolution and deposition of silica and the aperture size of the fracture is

\[ \begin{aligned}
\frac{\partial b}{\partial r} &= 2 \frac{P}{\rho} k_c (C_i - C) \quad \text{at } C_i < C \\
\frac{\partial b}{\partial r} &= 2 \frac{P}{\rho} k_c (C_i - C) \quad \text{at } C_i > C
\end{aligned} \]

(8)

where \( \rho \) is the density of silica.

The energy equation in the rock matrix is

\[ \lambda_m \nabla^2 \theta_m - \frac{A}{V_m} (c \rho)_m \frac{\partial \theta_m}{\partial t} = 0 \]

(9)

where \( \nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \)

\( \lambda_m \) is the thermal conductivity of the rock matrix, \( \theta_m \) the temperature of the rock matrix, \( (c \rho)_m \) the thermal capacity of that matrix, \( V_m \) the volume \( (=dx \, dy \, dz) \) of the unit element (Figure 3) of

the rock matrix exposed to the fracture, \( A \), the surface area \( (=dx \, dy) \) of the unit element of the rock matrix exposed to the fracture, and \( dx, dy \)

and \( dz \) the sizes of the volume element in \( x \)-, \( y \)- and \( z \)-directions, respectively, and \( x, y \) and \( z \) the Cartesian coordinates (Figure 3).

Initial conditions:

The temperature, pressure, and silica concentration in the fracture and rock matrix initially are assumed to be uniform,

(Fracture)

\[ P(x,y) = P_0, \quad \theta_f(x,y) = \theta_0, \quad C(x,y) = C_0 \]

at \( 0 \leq x \leq L, \quad -L/2 \leq y \leq L/2 \)

(10)

(Rock matrix)

\[ \theta_m(x,y,z) = \theta_0 \quad \text{at } 0 \leq x \leq L, \quad -L/2 \leq y \leq L/2 \]

(11)

where \( \theta_0 \) is the initial temperature, \( C_0 \) the initial concentration, and \( L, L \) the sizes of domain in the \( x, y \) and \( z \) directions, respectively.

Boundary conditions:

(Fracture)

\[ \begin{aligned}
\frac{\partial P(x,y)}{\partial x} &= \frac{\partial P(L,y)}{\partial x} = 0 \\
\frac{\partial \theta_f(x,y)}{\partial x} &= \frac{\partial \theta_f(L,y)}{\partial x} = 0 \\
\frac{\partial C(x,y)}{\partial x} &= \frac{\partial C(L,y)}{\partial x} = 0
\end{aligned} \]

at \( -L/2 \leq y \leq L/2 \)

(12)

(Rock matrix)

\[ \begin{aligned}
\frac{\partial P(x-L/2,y)}{\partial y} &= \frac{\partial P(x+L/2,y)}{\partial y} = 0 \\
\frac{\partial \theta_m(x-L/2,y)}{\partial y} &= \frac{\partial \theta_m(x+L/2,y)}{\partial y} = 0 \\
\frac{\partial C(x-L/2,y)}{\partial y} &= \frac{\partial C(x+L/2,y)}{\partial y} = 0
\end{aligned} \]

at \( 0 \leq x \leq L \)

(13)

The external boundaries of the fracture are assumed to be impermeable and adiabatic and the temperature distribution to be symmetric with respect to the \( x-y \) plane at \( z=0 \) as shown in Figure 3.
\[
\frac{\partial \theta_a(x,y,z)}{\partial x} - \frac{\partial \theta_a(L_x,y,z)}{\partial x} = 0
\]

at \(-L/2 \leq y \leq L/2\) and \(-b/2 \leq z \leq L/2\),

\[
\frac{\partial \theta_a(x,y,L/2,z)}{\partial y} - \frac{\partial \theta_a(x,L/2,z)}{\partial y} = 0
\]

at \(0 \leq x \leq L_x\) and \(-L/2 \leq z \leq L/2\),

\[
\frac{\partial \theta_a(x,y,h/2)}{\partial z} - \frac{\partial \theta_a(x,y,L/2)}{\partial z} = 0
\]

at \(0 \leq x \leq L_x\) and \(-L/2 \leq y \leq L/2\).

\[q = q_m \equiv \frac{Q_m}{dxdy},\]  
\[E_m = (c \rho) \left( \frac{\theta}{\theta_f} \right),\]  
\[M_m = q_m \left( C_m - C \right).\]

where \(Q_m\) is the injection flow rate, \(q_m\) the volume flow rate per unit volume at the injection point, \(\theta_m\) the temperature of the injection water and \(C_m\) the silica concentration of the injection water.

\[q = q_m \equiv \frac{Q_m}{dxdy},\]

where \(q_m\) is the volume flow rate per unit volume at the production point.

\[E_m = - \lambda \frac{\theta_m - \theta_f}{dz^2}.\]

**METHOD OF SOLUTION**

A horizontal fracture 500m by 500m with an aperture size of \(10^3\)m, induced artificially in a hot dry rock reservoir 500m long, 500m wide and 210m high, is considered (Figure 2). Initially, the fracture is assumed to be filled with water at 270°C that has an equilibrium silica concentration. Differential equations (1), (2), (3), (8) and (9) are approximated by a finite difference scheme (ADI method, (Peaceman; 1977)).

In the numerical calculation, the calculation domain is divided into equal volume elements with 10m nodal point spacing. A very small time step, \(\Delta \tau (0.005 \text{ days})\), is assigned during the calculation to obtain results with enough accuracy and avoid numerical instabilities.

The calculation scheme is

**STEP 1.** Equation (1) is repeatedly calculated until the value of \(P\) converges at each time step, and the velocity in the fracture is determined from the pressure value \(P\) and Darcy’s law.

**STEP 2.** Equation (2) is solved, and the distribution of temperature in the fracture is determined.

**STEP 3.** Equation (9) is solved, and Steps (2) and (3) are repeated until the values of \(f\) and \(m\) converge.

**STEP 4.** After the temperature fields in both the fracture and the rock matrix are determined, Equation (3) is solved.

**RESULTS AND DISCUSSION**

To examine the coupling between mass and heat flow and silica transport processes, we investigated the injection of water through a fracture heated by hot dry rock under non-isothermal conditions. Water at 50°C saturated or supersaturated with silica was injected into a fracture containing 270°C fluid saturated with silica. Calculations were made for two cases of injected silica concentrations (Figure 4). In CASE1, water at 50°C with an equilibrium silica concentration \(C_e = 1.8 \times 10^4\text{kg/kg}\) was injected. In CASE2, hot water from the production well was cooled down to 50°C and then re-injected. The water injected in CASE 2 was supersaturated with silica as shown in Figure 4. Table 1 gives the input data used in these calculations.

In CASE 1, Figure 5 shows the distribution of the fracture aperture size in the x-y plane after injecting 50°C water saturated with silica through the injection well for 800 days. Near this
well the fracture aperture expands, indicating a circular pattern of fracture aperture around the well. The distribution of fracture temperature shows the similar pattern as that of the aperture size. This means that as the temperature of the injected water increases, the water becomes under-saturated with silica, then silica in the rock matrix surrounding the fracture dissolves. This suggests that the heat and mass transfer in the fracture was approximated by the radial flow of the injected water near the injection well.

In CASE 2, the dimension-less temperature and concentration along the x-axis for elapsed times of 100 days and 800 days, are shown in Figure 6. The dimensionless temperature and the dimension-less concentration are defined as \((\theta_i - \theta_o) / (\theta_i - \theta_o)\) and \((C - C_{eq}) / (C_{eq} - C)\) respectively, where \(C_{eq}\) is the equilibrium concentration of silica at 270°C. The thermal fronts are located behind the chemical fronts, i.e., the velocities of the former are slower than those of the latter. This figure suggests that the dissolution rates in the regions \(x \leq 60\, \text{m}\) and \(260\, \text{m} \leq x \leq 340\, \text{m}\) are higher than those in the region \(60\, \text{m} \leq x \leq 260\, \text{m}\).

Figure 7 shows the concentration profiles of silica along the x-axis of the fracture in CASES 1 and 2 after 800 days of injection. A dashed line is the equilibrium concentration of silica cal-
culated on the basis of 800 days temperature profile in Figure 6. In CASE 1, the silica concentration along the fracture is under-saturated, indicating that the silica in rock matrix could dissolve all over the axis. In CASE 2, the silica concentration is under-saturated in the regions $x \leq 160m$ and $x \geq 270m$, but is supersaturated in the region $160m \leq x \leq 270m$. This means that the silica in the rock matrix could dissolve in the former fracture regions ($x \leq 160m$ and $x \geq 270m$) and that the hot water supersaturated with silica deposits its excess silica on the fracture surface in the region $160m \leq x \leq 270m$.

Figure 8 shows a comparison of the profiles of the fracture aperture size between CASES 1 and 2 along the x-axis of the fracture 800 days after injecting water at 50°C through the re-injection well. The aperture size in CASE 1 is larger the initial value all over the fracture region along the x-axis. In particular, in CASE 1 there is noticeable expansion of the aperture in the high temperature region of $x \leq 170m$. In CASE 2, similar expansion is obtained in the region $x \leq 150m$, but the aperture decreases in the low temperature region of $150m \leq x \leq 290m$. Moreover, the expansion of the fracture aperture in the region $x \geq 320m$ is slight in both the cases.

These results are explained by the temperature dependence of the dissolution and precipitation rates, $N$, defined in Equation (4). Here, $k_+, k_-$ and $C_s$ depend on the temperature, as shown in Equations (6) and (7). Figure 9 shows the dissolution and precipitation rates of silica as a function of temperature and silica concentration of injecting water, as described by Equation (4). As the temperature of silica solution rises, the precipitation rate increases. The maximum rates of precipitation for the silica concentrations $3.0 \times 10^{-4}$, $4.0 \times 10^{-4}$, and $4.6 \times 10^{-4}$kg/kg can be found at the temperatures of 65, 85 and 105°C, respectively, and the precipitation rate decreases with increasing temperature. On the other hand, the dissolution rate of silica increases as the temperature of silica solution increases. Similar findings have been reported by Rimstidt and Barnes (1980), and Malate and O’Sullivan (1992). In CASE 2, expansion of the fracture aperture in the region of $x \leq 150m$ therefore is due to acceleration of the dissolution of silica in the rock matrix.
matrix as a consequence of the temperature rise of the injected water caused by heat transfer from the surrounding rock matrix while flowing through the fracture. The decrease in the fracture aperture in the region from $x=150\text{m}$ to $290\text{m}$ is due to an increase in precipitation rate and to the decrease in the silica saturation concentration as the temperature of the re-injected water decreases. These findings agree with those of Itoi, et al. (1984), Bodvarson & Witherspoon (1985), and Malate & O'Sullivan (1992) who predicted that amount of silica deposition is greatest at the inlet of the fracture for a linear flow.

The fracture aperture sizes for CASE 2 along the $x$-axis 1500 days after injecting water through the injection well at the flow rates of 4 and $20\text{m}^3/\text{day}$ are shown in Fig.10. The fracture aperture for the flow rate of $20\text{m}^3/\text{day}$ is narrower than that for the flow rate of $4\text{m}^3/\text{day}$ in the region of $130\text{m} \leq x \leq 320\text{m}$, where the injection and production wells are located. In this region, this is because the temperature of the water in the fracture at the flow rate of $4\text{m}^3/\text{day}$ is much higher than that at the flow rate of $20\text{m}^3/\text{day}$, i.e. the precipitation rate of silica in the injected water at the former is more slow than that at the latter.

Figure 11 shows a comparison of cumulative amount of produced silica between CASES 1 and 2. The silica production in CASE 1 up to 1500 days is higher than that in CASE 2, but beyond that it is lower. This is because, as the temperature in the fracture falls in CASE 1, the dissolution rate of silica in the rock matrix decreases, reducing the amount of dissolved silica in the water flowing through the fracture.

The permeability of the fracture after injecting water for 1500 days was calculated using $K (=b^2/12)$ in Equation (1). The above fracture aperture size values are calculated from the permeability values. The change in permeability due to the dissolution and precipitation of silica was calculated to be about $\pm 2\%$, very small as compared to the initial value for fracture permeability. We believe that the above result is due to higher dissolution rate of silica than precipitation rate at temperature over $150^\circ\text{C}$ as shown in Figure 9.

**SUMMARY**

![Fracture aperture profiles for injection flow rates of 4 and $20\text{m}^3/\text{day}$ in CASE 2.](image)

**Fig. 10** Fracture aperture profiles for injection flow rates of 4 and $20\text{m}^3/\text{day}$ in CASE 2.

![Cumulative silica production in CASES 1 and 2.](image)

**Fig. 11** Cumulative silica production in CASES 1 and 2.
A numerical simulator was developed to analyze the transport, precipitation and dissolution of silica. The heat and mass transfer model was constructed for the non-isothermal problem of the injection of water dissolved with silica into a fracture formed artificially in a hot dry rock matrix. The effects of temperature on the precipitation and dissolution of silica were investigated for injecting water saturated (CASE 1) and supersaturated (CASE 2) with silica. In CASE 1, the fracture aperture size increases solely because of the dissolution of silica. In CASE 2, the size decreases in the low temperature region near the re-injection well due to the precipitation of silica, whereas it increases in the high temperature region due to the dissolution of silica. The dependence of precipitation and dissolution rates on temperature was confirmed with the numerical simulator. Very small change in fracture permeability due to the dissolution and precipitation of silica is shown by this simulator.

REFERENCES


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