Analysis of Kinematics and Dynamics for a Novel Hybrid Kinematics Machine*

Ying HU**, Bing LI**, Zhixing WANG**, Xiaojun YANG** and Hong HU**
**Shenzhen Graduate School, Harbin Institute of Technology
HIT Campus, Shenzhen University Town, Xili, Shenzhen, 518055, P R China
E-mail: libing.sgs@hit.edu.cn

Abstract
In this paper a novel 4 Degree-of-Freedom (DOF) parallel mechanism with the configuration of 4PUS-1RPU is first proposed, the mechanism is composed of four symmetrical driving chains and one central passive subchain. Based on the proposed 4-DOF parallel mechanism a 5-axis Hybrid Kinematics Machine (HKM) scheme is developed with an additional linear motion is combined, which can perform the 5-axis machining for freeform parts. The inverse kinematics of the 4-DOF parallel mechanism and the direct kinematics for the serial central sub-chain are then presented. The dexterity and the workspace analysis for the mechanism are also carried out. The dynamic characteristics of the PKM are investigated in detail. The simulation results show that the first four fundamental frequencies are the dominant factors that influence the dynamic characteristics of the developed PKM. The research provides the basis for the further parametric design with consideration of dynamic performances.

Key words: Dynamic Modeling; Parallel Kinematic Machine; Finite Element Model

1 Introduction
The recent trend towards high speed machining has motivated research and development of the novel type of machine tools called Parallel Kinematics Machines (PKM). The PKM is based on the kinematic architecture of parallel manipulator, which makes it suitable for high-speed machine tool with the characteristics of high stiffness and low inertia. A typical 6 Degree-of-Freedom (DOF) PKM structure consists of six legs in fully parallel and a moving platform supported by ball joints over legs within adjustable lengths. The spindle is located in the center of moving platform. With the installation of a variety of existed spindle tools, the PKM can perform various functions, such as milling, drilling, grinding, etc (1, 2, 3). However, The PKM based on 6-DOF parallel mechanism suffers from the disadvantages of complicated direct kinematics, coupled position and orientation as well as small workspace. Recently the limited-DOF parallel mechanisms have been widely investigated. In the family of parallel mechanisms with limited-DOF, the 3-DOF group has been developed intensively, with the typical examples such as DELTA, TRICEPT, etc (4,5). It is quite a challengeable task to carry out research on parallel mechanisms with 4 DOFs as is an acknowledged difficult research topic worldwide.

In 1999 Pierrot and Company proposed a 4-DOF parallel mechanism called H4, which is based on DELTA robot and provides one additional rotational DOF (6); Rolland proposed two kinds of 4-DOF parallel mechanisms called Kanuk and Manta in 1999, which are used for handling (7). In the above mentioned three 4-DOF parallel mechanisms, each includes a spherical 4-bar mechanism. Zlatanov and Gosselin proposed a new parallel mechanism with three translational DOF's and one rotational DOF in 2001 (8). In 2002 Chen and Zhao...
proposed a non-symmetrical parallel mechanism which can provide two translational DOFs and two rotational DOFs \(^9\). As most of the actual tasks require 4 DOFs, it is significant to carry out further investigation on the analysis and synthesis of 4-DOF parallel mechanisms.

The investigation on dynamics of PKMs has been attractive as it directly affects the performance of the machine tools. Currently the most common approaches in dynamic modeling for parallel manipulators are based on Lagrange formulations, the application of Hamilton’s principle and the direct application of the Newton’s equations of motion etc \(^10, 11, 12\), all of which are based on the principles of rigid multi-body dynamics. Due to the differences of stiffness of each component of PKM, i.e., the stiffness of rods and supporting beams is rather smaller than that of the platform, there exists kinematic coupling between the elastic links and the machine tool, therefore the kinematic deformation of these links of PKM can not be neglected. It is very significant to study the dynamic characteristics of the PKM with consideration of the flexibility of PKM’s components. The modal parameter is one of the most important indices which indicate the PKM’s dynamic performance, and it is of great importance that the influence of the PKM’s configuration and structural parameters with regard to the modal parameters can be further studied in order to help in optimizing the configuration of the machine tools so as to improve the dynamic performance of the PKM. This paper focuses on the kinematics and the finite element based dynamic modeling and the modal analysis for a novel Hybrid Kinematic Machine (HKM).

This paper is organized as follows: in section 2, a novel 4-DOF parallel mechanism is proposed, the novel HKM configuration scheme based on the proposed parallel mechanism is described; the dexterity analysis and workspace analysis are carried out in section 3; a dynamic model based on finite element method is developed in section 4; the static analysis and the model analysis based on the dynamic model are developed in section 5, ending with a summary in the last section.

2 The novel parallel mechanism and its kinematics

2.1 The novel parallel mechanism

In this paper, a novel parallel mechanism with the configuration of 4PUS-1RPU is proposed as shown in Fig.1(a). It is composed of a moving platform, four actuated legs in symmetrical layout on the fixed base, and one central passive subchain connecting the centers of the fixed base and the platform. Each actuated leg is composed of a constant-length link connecting moving platform by spherical-joint and connecting a slider by a U-joint, the slider can move along the guide rail which is fixed vertically to the fixed base. The actuators are driven by the ball-screw pairs. The passive subchain is made of two links connected by prismatic joint whose one end is connected with the moving platform by a U-joint, and the other end is connected with the fixed base by a revolute joint. Each PUS actuator provides six DOFs and has no constraint imposed on the moving platform, the constraints of the mechanism only come from the central passive subchain. There are two
constraints imposed on the moving platform. Therefore, the proposed parallel mechanism provides four DOFs including translations along Z-axis and Y-axis and rotations about X-axis and Y-axis.

In general a PKM should have 5 DOFs kinematically to fulfill the capability of 5-axis machining, thus based on the proposed 4-DOF parallel mechanism an additional X-axis is needed when applying it as HKM in this paper. A new 5-axis HKM scheme is developed, the machining simulation of the HKM is presented using MSC/ADAMS as shown in Fig.1(b). To the majority of the existed PKMs, the spindle is mounted in the center of the platform and the multi-axis machining movements are provided by the spindle. However, in this paper the workpiece is located at the center of the platform, the spindle is mounted on the X-axis, in the machining process the movement of the workpiece/platform provides 4 DOFs and the movement of spindle provides additional linear DOF so that the HKM can perform 5-DOF machining operation.

A brief comparison between the existed Stewart platform and the proposed novel HKM is carried out, it exhibits the following merits for the mechanism scheme proposed in this paper: (1) the structure of the proposed scheme is more simpler as only five links included; (2) As a revolute pair is used to connect the central subchain and the fixed base, the range of the orientational angle is much larger than that of the Stewart platform; (3) As a linear X-axis is serially combined with the 4-DOF parallel mechanism, this make the machining range in X-axis can be much bigger. In addition, the HKM scheme with central passive subchain is also convenient for the self-calibration of the machine tool system. When the sensors are installed properly the actual positions of the four sliders of the HKM can be obtained, the calibration can be performed by comparison of the difference of the actual and theoretical positions of the sliders.

2.2 Inverse Kinematics of the 4-PUS Actuators

As can be seen from Fig.1(a), a fixed coordinate frame $O_A - X_A Y_A Z_A$ is attached to the fixed base and a local coordinate frame $O_B - X_B Y_B Z_B$ is attached to the moving platform. Based on the vector-loop equation the inverse kinematics of each actuator can be written as:

$$L_i = \left\| R_{b_i} - R_{a_i} - H_i h_i \right\| = \left\| Q R_{b_i} + O_B - R_{a_i} - H_i h_i \right\| \quad (i = 1, 2, 3, 4)$$  \hspace{1cm} (1)

where $R_{b_i}$ and $R_{a_i}$ stand for the position vectors of the joints $b_i$ and joints $A_i$ in the fixed coordinate frame; $R_{b_i}$ denotes the position vectors of joints $b_i$ in the local coordinate frame; $O_B$ refers to the position vector of the center point of the platform $O_B$ in the fixed coordinate frame; $H_i$ refers to the length of the $i$th constant-length rod; $h_i$ refers to the direction vector of the $i$th constant-length rod; $Q$ represents the transformation matrix describing the orientation of the moving platform with respect to the fixed base.

2.3 Kinematics of the 1- RPU Passive Leg

As the passive subchain is a 4-DOF open-loop chain, its posture can be completely described by D-H convention (13). The D-H coordinate frames for the passive subchain are established as shown in Fig. 2. The corresponding D-H parameters are given in Table 1.

![Fig. 2 D-H coordinate frames for the central passive subchain](image)
Table 1  D-H parameters of the central subchain

<table>
<thead>
<tr>
<th>link</th>
<th>( a_{i-1} )</th>
<th>( \alpha_{i-1} )</th>
<th>( d_i )</th>
<th>( \theta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \theta_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( a_2 )</td>
<td>0</td>
<td>0</td>
<td>( \theta_2 )</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>( \pi/2 )</td>
<td>0</td>
<td>( \theta_3 )</td>
</tr>
</tbody>
</table>

The transformation from the fixed base to the moving platform can be written as:

\[
\frac{^4bT}{^0bT} = \frac{^3bT}{^2bT} \frac{^2T}{^1T} = \begin{bmatrix} Q & O_b \end{bmatrix}
\]

(2)

The D-H transformation matrix \( ^{-i}T^i \) of the links can be given as follows:

\[
^4T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & c_\theta_1 & -s_\theta_1 & 0 \\
0 & s_\theta_1 & c_\theta_1 & 0 \\
0 & 0 & 0 & 1 \end{bmatrix}, \quad ^3T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & c_\theta_2 & -s_\theta_2 & 0 \\
0 & s_\theta_2 & c_\theta_2 & a_1 \\
0 & 0 & 0 & 1 \end{bmatrix}, \quad ^2T_3 = \begin{bmatrix} 0 & -c_\theta_3 & s_\theta_3 & 0 \\
1 & 0 & 0 & 0 \\
0 & s_\theta_3 & c_\theta_3 & 0 \\
0 & 0 & 0 & 1 \end{bmatrix}
\]

(3)

where ‘c’ stands for ‘cos’ and ‘s’ stands for ‘sin’. Substituting the D-H transformation matrices into Eq. (2), yields:

\[
Q = \begin{bmatrix} c_\theta_1 (c_\theta_2 c_\theta_3 - s_\theta_2 s_\theta_3) - s_\theta_1 (c_\theta_1 c_\theta_2 - s_\theta_1 s_\theta_2) \\
- s_\theta_1 (c_\theta_1 c_\theta_2 - s_\theta_1 s_\theta_2) c_\theta_1 s_\theta_3 + s_\theta_1 c_\theta_3 \\
- s_\theta_2 (s_\theta_1 c_\theta_2 + c_\theta_1 s_\theta_2) s_\theta_1 s_\theta_3 - c_\theta_1 c_\theta_3 \\
0 \end{bmatrix}, \quad O_b = \begin{bmatrix} 0 & -a_1 s_\theta_1 & a_1 c_\theta_1 \end{bmatrix}
\]

(4)

where the elements of matrix \( Q \) are shown by RPY expression. Let \( \alpha, \beta, \gamma \) be the angles that the moving platform rotates about X-axis, Y-axis and Z-axis in the fixed coordinate frame. The position and orientation of the center point of the platform in the fixed coordinate frame can be obtained:

\[
\alpha = \theta_3, \quad \beta = -\sin^{-1}(\sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2), \quad \gamma = 90^\circ, \quad x_0 = 0, \quad y_0 = a_1 \sin \theta_1, \quad z_0 = a_1 \cos \theta_1
\]

(5)

As can be seen from Eq. (5) the platform can not move along X-axis as the value of \( x_0 \) is zero. As there are four variables \( a_1, \theta_1, \theta_2 \) and \( \theta_3 \), the 4PUS-1RPU parallel mechanism will provide 4 DOFs.

3 Workspace and Dexterity Analysis

3.1 Workspace Analysis

The workspace of the HKM is subjected to two main constraints: one is the limit of slider’s traveling length \( L_m \) and the other is the limit of the spherical-joint angle \( \theta_m \). As it can be normally avoided, the occurrence of the link interferences is not included here. The parameters of the HKM for simulation are given as follows: the angle limitation of U- and sphere-joints is set at \( \pm 45^\circ \), the spherical-joint distribution radius on the moving platform \( r \) is 200mm. The U-joint \( B_i \) on the moving platform and spherical- joint \( A_i \) on the fixed base are distributed as shown in Fig. 3; The U-joint distribution radius on the fixed base \( R \) is 480mm; The offset \( xx \) of the central points of moving platform and fixed base along X-axis is 125mm; The lengths of two sets of constant-length links are \( H_1=H_4=570mm \) and \( H_2=H_3=480mm \) respectively.

Using Matlab the position workspace of the HKM is shown in Fig. 4 (a), which indicates that the position workspace of presented HKM is foursquare in the workspace with \( Y \in [-200,200] \) and \( Z \in [500,800] \).

The orientation workspace of the proposed HKM can be described as shown in Fig. 4 (b): O-XYZ is a fixed coordinate frame, \( Z_p \) represents the normal vector of center point on
the moving platform, \( J \) refers to the angle location of \( Z_P \) with respect to \( Z \)-axis direction in the fixed frame. \( I \) refers to the roll angle of the moving platform. The orientation workspace can be expressed by angle of \( J \) that can satisfy all the constraints when the roll angle \( I \) varies from 0° to 360°. The orientation workspace of the HKM based on determined design parameters is shown in Fig.4 (c), where the cylindrical coordinate system is used to represent the orientation workspace.

![Fig. 3 Joint layout on moving platform and fixed base](image)

**Fig. 3 Joint layout on moving platform and fixed base**

**Fig. 4 The position and orientation workspace diagram and description**

### 3.2 Dexterity Analysis

In the structural scheme of this paper, the central leg is passive, and we have \( \omega_z = 0 \), \( v_z = 0 \). The first order Kinematics Influence Coefficient (KIC) matrix of input generalized coordinates with respect to the moving platform can be written as a 4 by 4 matrix as there are four actuators in structure. The central passive linkage will actually provide all constraints on the mechanism. The first order derivative KIC of input velocity with respect to platform velocity can be written as a 6 by 6 matrix:

\[
[G^p_p]_{6x6} = \begin{bmatrix}
\frac{h_1^T}{l_1b_1} & \frac{h_2^T}{l_2b_2} & \frac{h_3^T}{l_3b_3} & \frac{h_4^T}{l_4b_4} \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The velocity of the platform with respect to the input velocity of the mechanism can be written as

\[
\dot{p} = [G^p_q] \dot{q}, \quad [G^p_q] = [G^p_p]^{-1}
\]

where \([G^p_q]\) is called velocity Jacobian matrix of the mechanism.
It is verified that the force Jacobian matrix is just the transpose of the velocity Jacobian matrix, so that Eq.(8) can be obtained,

\[ J^T_p = [G^T_p] \] \hspace{1cm} (8)

A dexterity analysis is carried out with the following parameters of the HKM. The movable range of Y-axis is 300mm and the range of Z-axis is 300mm. Generally, the dexterity varies considerably for different points in the workspace and for different orientations at one given point. Based on the design parameters the orientation dexterity analysis of the HKM at three points is carried out as shown in Fig. 5, where \( \alpha \) and \( \beta \) denote the rotation angles of the platform about X- and Y-axis respectively. The dexterity analysis indicates that the variation of Z do not affect on the value of condition number, and the condition numbers over the prescribed workspace are less 20 when the maximum rotation angle of the platform is 20°.

Fig. 5: The atlases of orientation dexterity surface

4 Finite Element Modeling

It is very important to understand the dynamic characteristics for the proposed HKM. A dynamic model is required to set up before the dynamic analysis can be developed. In this paper a Finite Element (FE) model based on the solid elements for the proposed HKM is developed using MSC/Patran.

4.1 Geometric modeling

In order to model the 3D geometric solids of the HKM, one can import the model which is built by other CAD software, such as Pro/E, UG etc., into MSC/PATRAN via data exchange interface, however in this way the imported model still needs to be carefully modified as some features may miss during the model transformation. In this paper the 3D geometric model is built directly in the MSC/PATRAN environment in order to enhance the modeling efficiency. In the process of geometric modeling it is also necessary to make some simplifications for the solid element model, i.e. the small local features such as the chamfer, the fillet, etc. can be neglected in the geometric model, as these features have little influence on the results of the dynamic analysis. The modeling details for the HKM of this paper are described as follows:

The components of the proposed HKM in the geometric model include a fixed base, four supporting beams, and four vertical guiding rails with ball-screw pairs inside, the upper casting block associated with the guiding rails, a spindle, four constant-length rods, a platform and a central subchain. The configuration of the proposed HKM can be divided into two sub-structures: frame sub-structure and platform sub-structure. The frame sub-structure is composed of a fixed base, the supporting beams and the upper casting block. The fixed base and the upper casting block are made of heavy casting components in order to enhance the stability and absorb the vibrations; the spindle motor of the HKM is located in the center of the upper casting block. Four supporting beams in the fixed base are configured in a trapezoid, the precision ball screws pairs are attached on the supporting beams. The platform sub-structure includes a platform, four constant-length rods and the
central sub-chain. The platform is made in the form of a plate, but the joints connected on the platform are laid out in a trapezoid. As only the central subchain provides constraints on the platform and also bears the loading, the effective diameter of the subchain is greater than that of the constant-length rods.

For simplification purpose in this paper the universal joint and the spherical joint are treated as point contact without thermal influence. A slider connecting the universal joint and the nut of the ball screw pair is modeled with a block. The electric spindle motor is modeled with a cubic block. The weld joint and the bolted joint are simplified as rigid joint. The geometric model of the HKM is illustrated in Fig.6(a).

4.2 Finite element model

The finite element model can be obtained by discretizing the 3D solid model. In order to enhance the modeling precision the tetrahedron element is used instead of the hexahedron element. The mesh seeds are generated first so that the nodes’ locations along the edge can be explicitly specified. The locations of the mesh seeds can be determined by the setting of ‘density’ or ‘distance’. The nodes will be generated at the location where there exist seeds. The resulting shape and the meshing precision can therefore be controlled by the seeds. The tetrahedron element Tet10 with 10 vertices, each edge with 3 vertices in MSC/Patran is used. The parameter of Length in Global Edge is used to control the mesh density, which is simpler than using the Mesh Control function, the latter is used to handle more complex situation, but it is only suitable to the operations on surface. As the meshing density can directly affect the computation efficiency the proper selection of the mesh size of the components is important. In addition, there always exist errors among the nodes and elements automatically generated by the MSC/Patran, therefore, careful examination and manual modification are needed.

The procedure of meshing is based on the analysis for the HKM structures. The fixed base, upper casting block and the platform are big solid structure and the meshes for them can be coarse. As there exist many hard points on the surface the meshing operation for these surfaces should be performed first in order to avoid any singularity caused. To the components bearing loads refined meshing should be set in order to guarantee the model precision. As the constant-length rods and the central subchain exhibit more elasticity, as a result, more refined mesh should also be set, for example, we set the Global Edge Length to be 50. The finite element model of the full HKM is shown in Fig.6(b). In the FE model different young’s modulus are set for different components of the HKM due to different materials used. In the case of modal analysis it is no need to apply load on the model. However, when performing static analysis the loads have to be applied on the model. The loads can be applied either on the solid model, such as critical point, line, surface, and body,
or on the nodes and elements of the finite element model. Under the status of applied load, the platform will suffer the reaction force $F_x$ and $F_y$ in the horizontal directions of $x$ and $y$, and the reaction torque of $T_z$ in vertical direction. As the base of the HKM is fixed, the displacement constraints in Loads/BCs of MSC/NASTRAN are used to restrict its spatial location. In the finite element model of the HKM in this paper there are 10257 nodes and 4556 elements included. After pre-processing in the MSC/PATRAN, the model will be exported to MSC/NASTRAN for static analysis and modal analysis.

5 Static analysis of the HKM

As can be seen from Fig.1(b) the workpiece is located in the center of the platform, when machining the cutting load is applied to the contact points between the cutter and the workpiece, the variation of the platform of HKM will directly influence the machining precision. Based on the finite element model it is very convenient to analyze the displacement of the concerned points in the model by static analysis, say, the center point of the platform, etc.

![Fig.7  Displacement diagram of the HKM by FEA](image)

The basic structural parameters of the HKM used for the static analysis are given as follows: the dimension of each supporting beam is 60 by 80 by 1400 in millimeter. The transmission pair of ball screw, being 950 millimeters in height, is fixed to the supporting beam; the effective actuation of the ball screw is 850 millimeter. The diameter of the four constant-length rod is 20 millimeter. The lengths of the constant-length rods are divided into two groups, the length for the two short rods is 480 millimeter, while the length for the two long rods is 570 millimeter. The average diameter of the central subchain is 30 millimeter.

The simulated cutting load acting upon the platform and the upper casting block are in the same magnitude but in the opposite directions, in the paper the magnitude of the cutting load is set at 600 N and the acting directions are along X-axis and minus X-axis for the platform and the upper casting block respectively. The static analysis is performed in MSC/NASTRAN. The displacement diagram of the HKM resulted from FEA is shown in Fig.7. From the figure we can see that the displacement of the center point of the platform is 0.0072 millimeter for the parameter set of the illustrative example in the paper.

6 Modal Analysis of the HKM

The generalized dynamic equation for the HKM system can be written as Eq.(9).

$$[M]\ddot{[x]}+[C][\dot{x}]+[K][x]=[F]$$

(9)

Where $[M],[C],[K]$ refer to the mass matrix, damping matrix and stiffness matrix of the HKM system; $[x],\dot{[x]}$ and $\ddot{[x]}$ refer to the displacement vector, velocity vector and acceleration vector; $[F]$ refers to the vector of external forces. For the purpose of simplification all joints are assumed no friction or gap, then we can obtain $[C]=0$. 

The solution of the natural frequencies of the HKM system can be transformed as to solve the generalized eigenvalue of Eq. (10) as shown below.

\[
[K]\phi = \lambda[M]\phi \tag{10}
\]

where \( \lambda \) stands for the generalized eigenvalue, \( \omega = \sqrt{\lambda} \), \( \omega \) represents the angular frequency, \( \phi \) represents the vibration pattern of the \( i \)th order of frequency, the vibration frequency we need to solve can be written as Eq. (11).

\[
f = \frac{\omega}{2\pi} \tag{11}
\]

By Eq. (10) and Eq. (11) the natural characteristics of the system can be obtained, the order number of the frequencies is equal to the number of DOFs of the system.

The modal analysis is performed with MSC/NASTRAN, the results are shown in Fig. 8 (a), (b), (c) and (d) which represent the first, the second, the third, and the forth order of the natural frequency of the HKM system. The first order of vibration pattern as shown in Fig. 8(a) refers to the horizontal swing of the platform, the second order of vibration pattern as shown in Fig. 8(b) refers to the twist vibration of the platform. As the platform is connected to the constant-length rods by spherical joints, the spherical joints only constrain the translational positions, therefore the swing and the twist vibrations of the platform become the primary vibration patterns.

![Modal analysis of the HKM by FEA](image)

The third order of vibration pattern regards to the vibrations of the frame structure in the direction of Y-axis of the coordinate system, the forth order of vibration pattern refers to the vibration of the frame structure in the direction of X-axis. The frame structure includes four supporting beams and one upper casting block. The vibrations of the third and the fourth order of vibrations are due to the inherent characteristics of the frame structure.

Due to the mass of the spindle is quite big, the swing of the frame structure increases
the vibration amplitude and decreases the frequency. As the constant-length rod is connected by the spherical joints and the universal joint in its two ends, the transferring efficiency of the vibration is rather low. Therefore, the mutual influence of the vibration frequency between the frame structure and the platform is relatively small. The first six natural frequencies of the HKM model in Fig. 8 are given as follows, being 70.868Hz, 80.176Hz, 86.038Hz, 130.2Hz, 361.95Hz and 363.05Hz. From the data one can see that after the fifth order, the frequencies will exceed 350Hz, which means that first four orders of fundamental frequency have dominant effects on the dynamic characteristics of the HKM.

Table 2 Parameter setting for different cases of the HKM

<table>
<thead>
<tr>
<th>No. of HKM configuration</th>
<th>Parameter setting with different positions and orientations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Z = 600mm ; X = 0mm</td>
</tr>
<tr>
<td></td>
<td>a = 0° ; β = 0°</td>
</tr>
<tr>
<td>Case 2</td>
<td>Z = 900mm ; X = 0mm</td>
</tr>
<tr>
<td></td>
<td>a = 0° ; β = 0°</td>
</tr>
<tr>
<td>Case 3</td>
<td>Z = 700mm ; X = -80mm</td>
</tr>
<tr>
<td></td>
<td>a = 25° ; β = -25°</td>
</tr>
<tr>
<td>Case 4</td>
<td>Z = 900mm ; X = 0mm</td>
</tr>
<tr>
<td></td>
<td>a = -25° ; β = 25°</td>
</tr>
</tbody>
</table>

A comparison of the natural frequency of the platform with different setting of position and orientation parameters is developed in the paper. As listed in Table 2 the following four different cases of position and orientation parameter setting are analyzed. Case 1 and Case 2 refer to the variation of Z coordinate of the platform in the upper limit and the lower limit with the orientation angles being set at δ = θ = 0°; Case 3 and Case 4 refer to the variation of Z and X coordinates of the platform in different positions while the orientation angles being set at different values. The first order of the vibration pattern of the HKM corresponding to the four cases are shown in Fig.9(a), (b), (c) and (d) respectively.

Fig.9 indicates that when there are no orientation angles of the platform, the dominant vibration pattern is the swing of platform in Y direction; while there exists orientation angles of the platform, the dominant vibration is the swing of the platform in X direction. The reason for this phenomenon is given below: when there is no orientation angle on the platform, the only constraint of the platform in Y direction is imposed by the central sub-chain whose stiffness is much less than the stiffness in X direction, this causes the vibration of the platform in Y direction becoming influential. When there exists orientation angles to the platform, the four constant-length rods constrain the vibration of the platform in both X and Y directions. An additional constraint provided by the central subchain is
applied to the platform in Y direction. Therefore, the dominant vibration is the swing of the platform in X direction.

The first ten orders of frequencies for the HKM corresponding to the four different cases are shown in Fig.10. From the figure it is obvious that the first four orders of frequencies are dominant as there is a drastic increase after the fifth order of frequency. By looking into the first natural frequency it could be found that the frequency with the case of non-zero orientation angle of the platform is higher than that of the case with zero orientation angle of the platform.

Fig.10  Frequency comparison for different setting of position and orientation parameters

The research of the modal analysis with developed 3D solid finite element model can help to understand the dynamic characteristics of the proposed HKM, it also provides the basis for the further investigation of the parameter optimization of the HKM with consideration of dynamic performance.

7 Summary

In this paper a novel parallel mechanism with the configuration of 4PUS-1RPU is first proposed, the feature of the mechanism is that the constraints of the mechanism are provided by the central passive subchain. Based on the proposed 4-DOF parallel mechanism a 5-axis HKM scheme is developed, the workpiece is placed on the platform of the 4-DOF parallel mechanism, the spindle system is installed in an additional linear axis so that the 5-axis machining for freeform part can be performed with the developed HKM.

The kinematics for the 4-PUS parallel mechanism and the 1RPU serial central sub-chain are developed respectively. The workspace and the dexterity analysis of the HKM are carried out. The proposed HKM scheme with central subchain can help to self-calibrate the novel HKM system.

The 3D solid finite element modeling is carried out based on the MSC/PATRAN for the purpose of dynamic analysis. The proper simplifications for the finite element model is adopted, such as the universal joint and the spherical joint are treated as point contact without thermal influence, the slider and the spindle motor are modeled with a cubic block, etc. By static analysis with MSC/NASTRAN the displacement distribution diagram for the HKM is obtained, which can help to understand the stiffness of the system with loading condition. The dynamic characteristics of the novel HKM are analyzed based on the developed model. The modal analysis is performed and the natural frequencies and the vibration patterns of the HKM are obtained. The results of modal analysis show that the swing of the platform and the twist of the supporting beams and the upper casting block are influential vibration patterns of the HKM. In the modal analysis 4 different sets of the position and orientation parameters of the platform are used. The analysis results show that the first four natural frequencies are dominant, in the case of the platform with a certain
orientation angle setting the first order frequency is much higher than that of the case with zero orientation angle setting of the platform. The primary elastic deformation comes from the four constant-length rods as they have small stiffness.

The research of the dynamic analysis for the HKM can help on the improvements of the HKM structures. The work in this paper provides a basis for the parameter design of the kind of PKMs with consideration of improving the dynamic performances.

Acknowledgements

The work is supported by Natural Science Foundation of China (Project No.50505010) and Shenzhen Sci. & Tech. Scheme Fund.

References