Diagnosis of gear damage based on coefficient of variation method by analyzing vibration accelerations on one gear tooth

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Abstract
Since minor gear damage may cause serious failures of the entire equipment, early detection of gear damage is one of the important measures to prevent the machine system from malfunction. This paper proposes a method of diagnosing early gear damage by analyzing the vibration accelerations of one gear tooth. Gears manufactured with three different kinds of methods are tested in this study. Their tooth profile error is different from each other. Therefore, the influence of tooth profile error to the damage diagnosis is also discussed in this study. The vibration acceleration on the gear box was acquired as the original signal. In order to eliminate the interference components from the original signal, the time synchronous averaging method is adopted to process the original signal. Thus, the time synchronous averaging signal of driving gear is obtained as the analytical signal. Because the abnormal wave is not obvious in the whole signal, the vibration accelerations of one gear tooth was extracted from the time synchronous averaging signal for studying in detail. For illustrating the representative character of the vibration signal, statistical parameters are calculated from the vibration acceleration of one gear tooth to evaluate the conditions of gear tooth. In addition, the coefficient of variation method is employed to evaluate the contribution ratio of these parameters to gear damage and the weight coefficient of each parameter is obtained. The statistical parameters are synthesized into a unified feature based on the weight coefficient. The results show that the unified feature distance of damaged tooth is distinctive from the normal teeth, and the gear damage can be detected by the present method in this study.

Key words : Gear, Damage diagnosis, Vibration acceleration, Time synchronous averaging, Coefficient of variation, Unified feature

1. Introduction

Gear is one of the most important and commonly used parts in the machine system, the gear damage may cause serious failure of the entire equipment. Therefore, it is crucial to detect the gear damage as early as possible. In the published reports, researchers have done countless effort to study the characteristics of gear failures and have developed many methods to detect gear failures, such as techniques of acoustic emission (Al-Balushi, et al., 2002, Toutountzakis and Samanta, 2005), stress wave analysis (Board, 2003), laser scattering (Tanaka, et al., 2011), vibration signal analysis (Feng, et al., 2001) and so on. According to the prior studies, analyzing the vibration signal has been proved to be one of the effective methods to diagnose gear conditions. Moreover, because the vibration signal can be acquired without disassembling machines, this method has been widely applied in the field of damage detection in industrial production because of its convenient and high efficiency.

Because the vibration signal on gear box is always inevitably affected by the vibration of other components in the system or the environment disturbance, the abnormal symptoms caused by gear failure are not obvious in the signal,
especially when the gear failures are small. Thus, how to emphasize the failure characteristics of the vibration signal and extract suitable failure features from the collected data is crucial to improve the diagnostic accuracy of gear failures. To address this problem, researchers have developed many techniques in time domain, frequency domain and time-frequency domain of the signal. It is well known that techniques of Fast Fourier transform, cepstrum analysis, high-resolution spectral analysis and so on are commonly used to analyze the vibration signal in frequency domain. Gear damage can be detected by analyzing the frequency spectrum of the vibration signal because of the modulation phenomenon. Nevertheless, the spectra analysis may be incapable of detecting gear failures when they are small. Many techniques of time-frequency domain analysis, including wavelet transform, wavelet packet transform, Hilbert transform and so on, have been developed and applied to the non-stationary vibration signal processing and failure diagnosis (Mcfadden, 1994, Peng and Chu, 2004, Staszewski and Tomlinson, 1994). However, it is difficult to reduce the influence of other components to the vibration signal of interested gears using these methods. The vibration signal of test driving gear is unable to be clearly extracted from the original data. In the field of time-domain analysis, time synchronous averaging has been proved to be an effective technique to reduce noise and environment disturbances by removing uninterested non-periodic events from the original signal (Matsumura and Houjoh, 2004, Christian, 2007). It has been reported in many literatures that some processing techniques based on time-synchronous averaging have been developed for the detection of gear failures. For example, Mcfadden (1986, 1988) and Dalpiaz (1990) adopted time-synchronous averaging method to acquire residual signal by subtracting the regular tooth meshing harmonics from the original vibration signal, and estimated the location of damage on tooth surface by calculating the amplitude and phase angle of the vibration function of residual signal. Therefore, in order to separate the vibration signal of test driving gear from the signal of other machine parts and strengthen the characteristics of abnormal wave, we try to adopt the time synchronous averaging method to reduce the interference components from the original signal in this study. In the past studies, most of them focused on researching the whole vibration signal of the gear. Because the abnormal wave of early gear damage is weak in the whole signal, we intend to analyze the vibration signal on one gear tooth to study the vibration characteristics in detail.

This paper proposes a method of diagnosing early gear damage by analyzing the vibration accelerations on one gear tooth. Test gears with different tooth profile errors are tested in this study. Therefore, the influence of tooth profile error to the damage diagnosis is also discussed in this study. The time synchronous averaging method is adopted to process the original signal acquired in the experiments. Moreover, because the statistic method has been confirmed to be an effective technique to extract distinctive failure features from the vibration signal for damage detection in several studies (Pachaud, et al., 1997, Wang, et al., 2010), statistical parameters are also calculated to represent the characteristics of the signal. In addition, the coefficient of variation method is employed to evaluate the sensitivity degree of statistical parameters to gear damage and the weight coefficient of each parameter is obtained. The statistical parameters are synthesized into one unified feature based on the weight coefficient. Finally, the gear damage is diagnosed with the unified feature.

2. Time synchronous averaging method

Time synchronous averaging method is an effective signal processing method to extract the interested signal of some particular gear from the original signal. The computation of synchronous averaging process can be explained as follows. Supposing a time signal \( s(t) \), which consists of periodic signal \( f(t) \) and Gaussian white noise \( n(t) \), defined as (McFadden, 1987):

\[
s(t) = f(t) + n(t)
\]

Then, the signal \( s(t) \) is truncated with the cycle period \( T \) of signal \( f(t) \), and \( N \) segments of signal are obtained. Adding up each segment of the signal from their beginning points. Because of the independency of the Gaussian white noise, it can be acquired that:

\[
s(t_i) = Nf(t_i) + \sqrt{N}n(t_i)
\]

Averaging to the signal \( s(t_i) \), the output signal \( y(t_i) \) is obtained as:

\[
y(t_i) = f(t_i) + \frac{n(t_i)}{\sqrt{N}}
\]
It can be seen that the periodic signal \( f(t) \) does not change, while the Gaussian white noise of the output signal \( y(t) \) is reduced to \( 1/\sqrt{N} \) the noise of the original input signal \( s(t) \). If the cycle number \( N \) is large enough, the white noise will be eliminated clearly from the signal. Thus, the periodic signal \( f(t) \) can be emphasized.

Actually, the computation of time synchronous averaging can be regarded as a signal filtering process using a comb filter. The synchronous average of \( s(t) \) is equal to a convolution using a trigger signal as the follow equation \( ^{(12)} \):

\[
y(t) = c(t) * s(t)
\]  

where, \( y(t) \) is the output signal, \( c(t) \) is the trigger signal with a frequency \( f_o \), it is defined as:

\[
c(t) = \frac{1}{N} \sum_{r=0}^{N-1} s(t + rT_o)
\]

where, \( T_o = 1/f_o \) \( N \) is the number of averages. The convolution equals to the following multiplication in frequency domain:

\[
Y(f) = C(f) * S(f)
\]

where, \( S(f) \) is the Fourier transform of the signal \( s(t) \), \( C(f) \) is the Fourier transform of \( c(t) \). \( C(f) \) is calculated as:

\[
C(f) = \frac{1}{N} \frac{\sin(\pi N T_o f)}{\sin(\pi T_o f)}
\]

If the number of averages \( N \) is large enough, only the integer multiples of frequency \( f_o \) can be passed. Thus, if we set the frequency \( f_o \) equals to the rotational frequency of shaft having the driving gear in this study, the integer multiples of the driving shaft rotational frequency is kept, and other signal will be eliminated.

In this study, in order to reduce the interference components from the original signal and to strengthen the characteristics of abnormal signal generated by gear damage, the time synchronous averaging technique is adopted to process the original signal. During the experiments, the vibration signal on gear box was collected. Meanwhile, the pitch signal was also obtained by an index disc mounted on the driving gear shaft and a photo sensor. There is a hole drilled in the index disc, which corresponds to the location of the damaged tooth. When the driving gear making one revolution, the light would pass through the hole and the voltage of the photo sensor becomes larger. Therefore, the pitch signal records the exact revolution cycles, the length of the vibration data in one cycle and the location of damaged tooth of the driving gear. We divided the original vibration signal into many segments according to the pitch signal. Each segment of the signal having the same length contains the vibration accelerations acquired when the driving gear rotating a circle. Then, the divided signal segments were averaged. Because the meshing signal of driving gear is periodic and its frequency is the integer multiples of the shaft rotational frequency, after the process of time synchronous averaging, the meshing signal of the driving gear will be emphasized, but the other components of the original signal will be reduced conversely.

---

**Fig. 1** A scheme of the power circulating type gear testing machine
3. Experimental set up and test gears
3.1 Experimental apparatus

The experiments were accomplished on a power circulating type gear testing machine. Figure 1 shows a scheme of the machine, which mainly consists of a driving motor, a slave gear pair, loading device, an index disc, an photo sensor, gear box and bearing stand. The test gears and bearings were mounted in the gear box and bearing stand separately. The gear box and bearing stand were installed on the same baseplate. Therefore, the vibration of test gears could be transmitted to the gearbox through gear shaft, bearing, bearing stand and the baseplate. An accelerometer was set at the center of the gear box cover. The vibration accelerations were measured by the accelerometer and were recorded by PC through the amplifier and AD converter. Meanwhile, to extract the signal of the driving gear one revolution cycle, the pitch signal is measured by both index disc mounted on the gear shaft and a photo sensor. The lubrication method was oil bath and turbine oil ISO VG32 was selected as the lubrication oil. To stabilize the viscosity of lubrication oil, the oil temperature was kept at $313 \pm 3$ K using the electric heater and refrigeration pipe. A thermometer was used to monitor the temperature of the lubricating oil. Test gears were driven by the motor with speed of 1800 rpm, and the torque was 70N-m which was loaded on the test gears by the loading device. The sampling frequency is 50 KHz, and the sampling time is 2s.

3.2 Test gears

The specification of test gears is shown in Table 1. There are three kinds of test gears in this study. The test gears are respectively manufactured by three different surface finishing methods, which are hobbing, wire electrical discharge machining and grinding. Therefore, the test gears are called gear H, gear W and gear G in this study. The material of test gears is JIS S45C Thermal refining steel. The other geometry of the test gears are the same as shown in Table 1. All the test gears are the involute spur gears whose module is 4mm, and pressure angle is $20^\circ$. The number of driving gear teeth is 27, and the number of driven gear teeth is 31.

For the reference test, test gears with two different conditions are investigated in this study, which are named normal gear and spot damaged gear. The normal gear has no failure on tooth surface. For simulating the slight failure and its growing on tooth surface, we advisedly drilled some spots damage on the tooth surface of test gears, called spot damage gear. The spots damage are all drilled around the pitch point of the tooth, at an extremal angle to the tooth surface. They are similar to ellipse halls. We defined the damage degree according to the damage area ratio. When the maximum damage area ratio of one tooth $\leq 5\%$ and the damage area ratio of the gear $\leq 0.2\%$, it is considered as the slight failure. The maximum damage area ratio of one tooth is defined as a ratio of the failure area on one tooth surface to the contact area of a tooth, damage area ratio of the gear is defined as a ratio of all the failure area to all the contact area of a gear. Especially, according to the number of spots damage on tooth surface and the surface finishing method, we named the damaged gears as spot damaged gear H-I and H-II, W-I and W-II, G-I and G-II respectively. There is one spot damage on the No.14 tooth surface of spot damaged gear I, two spots damage on the No.14 tooth surface of spot damaged gear II. For spot damaged gear I and spot damaged gear II, the damage area ratio of one tooth and the damage area ratio of the gear are 4.5% and 0.16%, 10.8% and 0.4% respectively.

<table>
<thead>
<tr>
<th>Table 1 Specification of test gears</th>
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<tr>
<td>Module $m [\text{mm}]$</td>
</tr>
<tr>
<td>Pressure angle $a_0 [\text{deg}]$</td>
</tr>
<tr>
<td>Number of teeth $z_1/z_2$</td>
</tr>
<tr>
<td>Pitch circle diameter $d [\text{mm}]$</td>
</tr>
<tr>
<td>Tip circle diameter $d_a [\text{mm}]$</td>
</tr>
<tr>
<td>Face width $b [\text{mm}]$</td>
</tr>
<tr>
<td>Contact ratio $\varepsilon$</td>
</tr>
<tr>
<td>Tooth profile</td>
</tr>
<tr>
<td>Material</td>
</tr>
<tr>
<td>Surface finishing</td>
</tr>
</tbody>
</table>

Figure 3 shows the tooth profile error of test driving gears. All the gears have spot damage round the pitch point, which is displayed by the dot-dash line. The depth of all the spot is about 0.3 mm. Figures 3(a)-(b) present the tooth profile error of spot damaged gear H-I and H-II. Except for the spot damage, the tooth profile error of spot damaged gear H is approximately within 30 μm. Figures 3 (c)-(f) show the tooth profile error of spot damaged gears W and G respectively. The tooth profile of spot damaged gear W and G changes more slightly over the whole tooth than gear H and the fluctuation is about within 5 μm. Because the test gears are manufactured by different methods, the tooth profile error of each type gear is distinct from each other. Especially, the tooth profile error of spot damaged gear H is the largest of all. In addition, the tooth profile error of gear G is a little smaller than the tooth profile error of gear W.
4. Experimental results and discussions
4.1 Time synchronous averaging signal

The vibration accelerations of gear box were obtained in the experiment. The spot damage is drilled on the tooth surface of driving gear, thus, the vibration of driving gear would be the most significantly affected by the damage. In order to reduce the influence of the vibration from other components in the machine system, we intend to analyze the vibration accelerations of the driving gear only. The method of time synchronous averaging (TSA) is utilized to process the original signal. Moreover, the period of abnormal signal caused by damage is as same as the period of driving gear shaft. We have to average the original signal according to the cycle period of driving gear shaft to emphasize the characteristics of driving gear signal and abnormal signal. The teeth number of driving gear and driven gear are 27 and 31 respectively, as a result, the vibration accelerations of the driven gear and the other interference signal will be reduced. The signal obtained after time synchronous averaging is called TSA signal in this paper.

In order to compare the difference between the original signal and the TSA signal, we adopted the vibration signals of spot damaged gear H-I as the example to illustrate. As shown in Fig. 4, in the original signal, there are some small interference signals in the major signal. These interference signals are caused by the vibration of the motor or the other parts in the testing machine, the environmental electromagnetic disturbances and the white noise generated by the accelerometer. They would impact the diagnosis of the useful signal. In the figure of TSA signal, the peak of the wave becomes clearer and its number is 27 which equals to the teeth number of driving gear. Furthermore, the waveform shape of TSA signal is much smoother than that of the original signal. The peak value of No. 14 tooth is a little larger, however, the difference is not obvious. It can be concluded that the time synchronous averaging method could effectively extract the vibration signal of interested gear, and enable to eliminate the interference components from the original signal. However, the abnormal signal caused by damaged tooth could not be detected in TSA signal.

![Fig. 4 Original signal and the TSA signal in one revolution of driving gear (spot damaged gear H-I)](image-url)
4.2 Vibration accelerations of one gear tooth

Because the spot damage is small, the waveform fluctuation caused by the spot damage is not obvious as shown in Fig. 4. It is still difficult to observe the abnormal wave in the whole TSA signal. Therefore, we try to extract the signal of one gear tooth from the TSA signal to study the vibration characteristics in detail. The TSA signal in one revolution is divided into several segments according to the product of the driving gear tooth number and the contact ratio. Each segment of the signal represents the TSA signal on one tooth. We averaged all the segmented signals of one test gear including signals on both normal teeth and damaged teeth, and acquired the average signal. As shown in Fig. 4 (b), although some waveforms of the normal teeth are also a little larger or smaller, most of them are stable and similar. Therefore, the average signal is approximate with most normal teeth waveforms and can be used as the representative signal for waveforms of normal teeth. Because it is complicated and confused to compare all the teeth waveforms of the test gear in a figure, the signal of No.14 damaged tooth, signal of one normal tooth and average signal are adopted as representative waveforms for comparison. As shown in Fig. 5, the red curve is the TSA signal of damaged tooth, the blue one represents the TSA signal of one normal tooth and the green curve shows the average signal.

![Graphs of TSA signal on one gear tooth](image-url)
Figures 5 (a) and (b) present the TSA signal of spot damaged gears H-I and II. The TSA signal of spot damaged gear H-I in Fig. 5 (a) is extracted from the signal shown in Fig. 4 (b). In both of the TSA signal for gears H-I and II, the vibration accelerations on damaged tooth is larger than those on normal teeth and average signal. The average signal is similar with the signal of normal tooth. Figures 5 (c) and (d) show the TSA signal of spot damaged gear W-I and II. In Fig. 5(c), the signal of normal tooth is nearly as same as the average signal. The maximum vibration acceleration of damaged tooth is a little larger than that of normal tooth and average signal. Fig. 5(d) shows the signal of spot damaged gear w-II. The signal of normal tooth is also similar with the average signal. The vibration acceleration of damaged tooth is larger than that of normal tooth. Figures 5 (e) and (f) show the TSA signal of spot damaged gear G. The TSA signal on damaged tooth is larger than the TSA signal on normal tooth and the average signal. The difference of the signal between the damaged tooth and the normal tooth is clear. Moreover, the vibration acceleration of gear G-II is larger than that of gear G-I. The results indicate that the spot damage could increase the vibration accelerations of the damaged tooth. We also compared the signal of damaged tooth No. 14 with all the normal teeth waveforms of the test gear, it is verified that the waveform of No. 14 tooth is different from most of the waveforms of normal teeth.

Because most of the teeth condition is normal, the average signal is approximate with the signal on normal teeth and is different from the signal on damaged tooth. The difference from the average signal can be employed to detect damage initially. It also could be observed that the signal fluctuation of damaged tooth in spot damaged gear II is larger than that in spot damaged gear I. The reason can be considered as the damage area of spot damaged gear II is larger, which can intensify the vibration of the damaged teeth or the adjacent teeth. Moreover, the vibration acceleration of spot damaged gears H, W and G gradually become smaller. However, the evidence of the difference between the damaged tooth and the normal tooth does not change, it even becomes more obvious conversely. According to Fig. 3 ‘Tooth profile error of test driving gears’, the tooth profile error of spot damaged gear H is the largest of all, and the tooth profile error of spot damaged gear G is smaller than gear W. Therefore, the result demonstrates that the larger tooth profile error would intensify the vibration of gears, which will decrease the gear performance and the fatigue life of gears.

5. Extract feature parameters and diagnose gear damage using coefficient of variation method

5.1 Extract feature parameters from the vibration accelerations

As discussed in section 4.2, the difference of vibration accelerations between the normal tooth and the damaged tooth can be observed in the TSA signal. However, it would be complicated for us to compare the TSA signal of all the gear teeth one by one. Besides, the result will depend on the artificial experience and is unstable. Therefore, we try to adopt some statistical parameters to quantitatively present these differences. Statistical parameters have been proved to be effective in explaining the characteristics of the signal in many prior studies. Moreover, it has been reported that parameters such as standard deviation, kurtosis, skewness, clearance factor, kurtosis factor and so on are more sensitive to the fluctuation of the signal. The standard deviation stands for the dispersion degree of data distribution, which focuses on the trend of the vibration. Kurtosis is a measurement of how the data probability distribution is peaked around the mean value. If some peak is sharp in a vibration signal because of the higher value, then its data probability distribution is sharper around the mean value, which would be caused by gear damage. Skewness is a measure of asymmetry in a statistical distribution and can be quantified to define the extent to which distribution differs from a normal distribution. Clearance factor is defined as the peak value of the signal divided by the square root value of the signal. When there is slight failure on tooth surface, the square root value of the signal changes little, while the peak value increases. Therefore, the value of clearance factor increases. This parameter is appropriate for indicating slight failure, (Pachaud, et al., 1997). Consequently, statistical parameters of standard deviation ($\sigma$), maximum value ($X_{\text{max}}$), kurtosis ($Ku$), skewness ($Sk$) and clearance factor ($CL_v$) are calculated from the TSA signal of each gear tooth in this study. Supposing the vibration signal is $X$, these parameters are defined as the following formulas (Takeyasu, 2004):

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} \quad (8)$$
\[
K_u = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^4}{n\sigma^4}
\]  

(9)

\[
Sk = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left( \frac{x_i - \bar{x}}{\sigma} \right)^3
\]  

(10)

\[
CL_f = \frac{X_{\text{max}}}{X_r}
\]  

(11)

where, \(x_i\) is the data of the signal, \(\bar{x}\) is the mean value of the signal, \(n\) is the data number of the signal, \(X_{\text{max}}\) is the maximum value of the signal, \(X_r = (\sum_{i=1}^{n} \sqrt{|x_i|})^2\) is the square root value of the signal.

Because the computation process of statistical parameters is the same, we employed the normal gear and damaged gear with one spot to typically illustrate the characteristics of each parameter. Figure 6 shows the statistical parameters extracted from the TSA signal on each gear tooth. In the figures, the abscissa shows the tooth number. There are 27 teeth in the test driving gear, and the No. 14 tooth has the spot damage. The non-dimensional parameters of kurtosis, skewness and clearance factor are drawn in the same figure, and the dimensional parameters of standard deviation, maximum value are drawn in the other figure.

Figure 6(a) shows the statistical parameters of normal gear H (manufactured by hobbing). All the curves of statistical parameters are undulation from No. 1 tooth to No. 27 tooth. Especially, the clearance factor of No. 3 tooth is a little larger than the other teeth. However, there is no uniform specific regularity about the fluctuation of all the parameters. The fluctuation of clearance factor and maximum value seems stronger than the other parameters.

Figure 6(b) presents the statistical parameters of spot damaged gear H-I. The statistical parameters of each gear tooth are different from each other. From parameters of kurtosis, clearance factor and maximum value, it can be observed that the parameters value at No. 14 tooth is a little larger than the other teeth, though the difference is small. However, it is difficult to find the difference between No. 14 tooth and the other teeth from parameters of skewness and standard deviation.

Figure 6(c) shows the statistical parameters calculated from the TSA signal of spot damaged gear W-I. All the statistical parameters value changes slightly from tooth No. 1to No. 27. Especially, the fluctuation range of maximum value is larger than other parameters. Besides, all the parameters value around No. 14 tooth is a little larger than those of the other teeth. However, the difference is not obvious in parameters of skewness and clearance factor.

Figures 6 (d) shows the statistical parameters calculated from the TSA signal of spot damaged gear G-I. The parameters of maximum value and clearance factor at No. 14 tooth are apparently larger than the other teeth. It also can be seen that the parameters of kurtosis, clearance factor and standard deviation at No. 14 tooth are larger than the other teeth, but the difference is not obvious. However, it is hard to detect the abnormal value of No. 14 tooth based on the skewness.

According to the results, the statistical parameters value of damaged tooth is larger than that of the normal tooth, which can effectively reflect the difference between the damaged tooth and the normal tooth. Therefore, the statistical parameters can be employed as the effective indicators to evaluate the conditions of gear tooth. Especially, parameters of maximum value, clearance factor and kurtosis are more sensitive to the variation of the signal and can be employed to detect the early gear damage. In addition, comparing statistical parameters of spot damaged gear H-I, W-I and G-I, the difference of parameters value between No. 14 tooth and the other teeth is more obvious in figures of spot damaged gear W-I and G-I. Because the tooth profile error of spot damaged gear H is larger than gears W and G, the vibration acceleration of spot damaged gear H is more largely influenced by the tooth profile error. It demonstrates that the larger tooth profile error can reduce the features of the failure and decrease the diagnosis accuracy of damaged tooth. Besides, statistical parameters fluctuate among each tooth of both normal gear and damaged gear. The fluctuation of normal gear is slighter than the damaged gear, and the change has no regularity and uniformity. Though the difference between normal tooth and damaged tooth can be observed, there are still also instability in some parameters. It is difficult to diagnose gear damage based on any single statistical parameter. Therefore, it is necessary to develop some method to unify the characteristics of all the statistical parameters into one feature parameter.
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Fig. 6 Statistical parameters calculated from the TSA signal on each gear tooth
5.2 Coefficient of variation method

As discussed above, almost all of the extracted statistical parameters can express the characteristic features of the damaged tooth, and we try to composite all the parameters into one feature parameter. However, the sensitivity of these parameters to the changes of damage is different. For example, the difference between the normal teeth and the damaged teeth is seemed to be more obviously shown by parameters of standard deviation and clearance factor. In order to evaluate the sensitivity degree of each parameter, we try to calculate the weight coefficient of each statistical parameter. Weight coefficient is defined as a proportionality factor to describe the importance of a parameter in the whole parameters. Therefore, these parameters can be unified into a composite indicator based on the weight coefficient to diagnose the tooth damage more easily. It is known that the larger the dispersion of the parameter’s value, the more obvious difference can be reflected with this parameter, in other words, the more important the parameter is. Thus, the coefficient of variation method was adopted to calculate the variance of the parameter. Coefficient of variation (CV) is a standardized criterion for the dispersion of a data set with regards to the mean value. It is effective to describe the variation of the data series and is useful to compare the degree of variation from one data set to another. The coefficient of variation is defined as a ratio of the standard deviation to the mean value. Supposing the statistical parameters is a matrix as follows:

\[
P = \begin{bmatrix}
  p_{i1} & \ldots & p_{ij} \\
  \vdots & \ddots & \vdots \\
  p_{i6} & \ldots & p_{ij}
\end{bmatrix} \quad (i = 1, 2, \ldots, 27; j = 1, 2, \ldots, 6)
\]  

where \( p_{ij} \) is the \( j \)-th statistical parameter of \( i \)-th tooth, \( i = 1, \ldots, 27 \) represents the number of teeth, \( j \) is the kind of statistical parameters.

The coefficient of variation can be calculated by the following formula (Faber and Korn, 1991):

\[
V_j = \frac{\sigma_j}{\bar{x}_j} \quad (j = 1, 2, \ldots, 6)
\]  

(13)

where \( V_j \) is the coefficient of variation of \( j \)-th parameter, \( \sigma_j \) is the standard deviation of \( j \)-th parameter, \( \bar{x}_j \) is mean value of \( j \)-th parameter.

The weight coefficient is defined as the proportion of each coefficient of variation to the total variations of all the parameters. It is acquired as follows:

\[
W_j = \frac{V_j}{\sum_{j=1}^{6} V_j}
\]  

(14)

where, \( W_j \) is the weight coefficient of \( j \)-th parameter. Then, we unified the statistical parameters into one indicator according to the weight coefficient. In order to eliminate the effect of dimensions, we normalize the statistical parameter at first. The computation of unification is shown as follows:

\[
z_{ij} = \frac{(p_{ij} - \bar{x}_j)}{\sigma_j} \quad (i = 1, 2, \ldots, 27; j = 1, 2, \ldots, 6)
\]  

(15)

\[
F_i = \sum_{j=1}^{6} z_{ij} \times W_j \quad (i = 1, 2, \ldots, 27)
\]  

(16)

where, \( z_{ij} \) is the normalized parameter, \( F_i \) is the unified feature of \( i \)-th tooth.

We acquired the weight coefficient of all the statistical parameters according to the coefficient of variation method. As shown in Table 2, the calculation process and weight coefficients of spot damaged gear W-I are adopted as the example for instruction. It is observed that the weight coefficients of kurtosis, clearance factor and maximum value are larger, while the weight coefficients of standard deviation and skewness are smaller. According to the results, parameters of clearance factor, maximum factor and kurtosis are more sensitive to the variation of tooth damage, which agrees with the results of the statistical parameters discussed in the last section 5.1.
Table 2 Weight coefficient of statistical parameters for spot damaged gear W-I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Normal gear H</th>
<th>Spot damaged gear H-I</th>
<th>Spot damaged gear H-II</th>
<th>Normal gear W</th>
<th>Spot damaged gear W-I</th>
<th>Spot damaged gear W-II</th>
<th>Normal gear G</th>
<th>Spot damaged gear G-I</th>
<th>Spot damaged gear G-II</th>
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<tr>
<td>Maximum value</td>
<td>23.875</td>
<td>23.875</td>
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<td>Kurtosis</td>
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Fig. 7 Distance of unified feature

5.3 Diagnose damaged tooth by distance of unified feature

Because the damaged teeth number is few, and the damage is small, the vibration acceleration and statistical parameters of normal tooth is similar with the average signal. Besides, the difference between the signal of damaged tooth and the average signal is a little larger. Therefore, we diagnose the damaged tooth by the unified feature distance between each tooth and the average signal. The distance of unified feature for each kind of gears is shown in Fig. 7. As shown in Fig. 7, the abscissa shows the tooth number, and the ordinate is the distance of unified feature. The feature distance of No. 14 tooth with spot damage is painted by darker color. As shown in Figs. 7 (a), (d) and (g), the feature distance of all the teeth on normal gear is within 1, and there is no obviously large data among them. Figures 7(b) - (c) show the feature distance of spot damaged gear H. It is clearly observed that the feature distance of No. 14 tooth is larger than the other normal teeth. Especially, for spot damaged gear H-I, the difference is more obvious than that shown in the statistical parameters. In Figs. 7 (e) - (f), the feature distance of No. 14 tooth is also significantly different with other teeth in spot damaged gear W. Figures. 7 (h) - (i) present the unified feature distance of spot damaged gear G. The feature distance of No. 14 tooth is still larger than the other teeth. However, several feature distances of normal teeth behind the No. 14 tooth still have larger value, especially in spot damaged gear G-II. It is because the strong
vibration of the damaged tooth could not be reduced immediately, the impact still influence the adjacent teeth behind which would also affect the diagnosis accuracy of damaged tooth. Therefore, it is important to reduce this influence during the signal analysis in the future study. Anyway, the feature distance of No. 14 tooth is significantly different from the other teeth, by which the damaged tooth can be diagnosed.

6. Conclusions

This paper proposed a method of diagnosing gear damage based on the unified feature distance by analyzing the vibration acceleration on one gear tooth, and investigated the influence of different tooth profile error to the diagnosis of damaged tooth. Gears manufactured with three different kinds of methods were tested in the experiments and their tooth profile error is different from each other. The vibration acceleration on gear box was acquired as the original data. Then, the method of time synchronous averaging method was adopted to extract the vibration signal of driving gear from the original data. The result indicated that the time synchronous averaging method could effectively eliminate the interference components from the original signal. Because the abnormal wave is not obvious in the whole signal, the vibration accelerations of one gear tooth was extracted from the time synchronous averaging signal for analyzing in detail. The results show that the vibration acceleration on damaged tooth is larger than the normal teeth. Moreover, larger tooth profile error would intensify the vibration of gears, which will decrease the gear performance and the fatigue life of gears. In order to illustrate the character of the signal quantitatively, statistical parameters were calculated from the vibration signal of one gear tooth. According to the results, the statistical parameters can effectively reflect the difference between the damaged tooth and the normal tooth, and varied with the change of the damage. Therefore, the statistical parameters can be employed as the effective indicators to evaluate the conditions of gear tooth. In addition, the coefficient of variation method was employed to evaluate the sensitivity degree of these parameters to gear damage and the weight coefficient of each parameter was obtained. The statistical parameters were synthesized into one unified feature based on the weight coefficient. The unified feature distance of damaged tooth is obviously different from the normal teeth, by which the gear damage can be detected. It demonstrates that the proposed method is effective for detecting the early gear damage.

References


