Mathematical programming models for road repair scheduling —On aging bridges in Japan—

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Abstract

This paper proposes two mathematical programming models to deal with a scheduling problem on the renovation of several roads that need to be repaired by a certain date. A typical example of the scheduling problem is the one for aging and deteriorating bridges in Japan. Our idea behind the formulation of the models is that, if one optimizes a set of roads simultaneously under repair, then the influence on traffic can be minimized. The first model turns out to be easy to solve since it can be rewritten as a linear 0-1 integer programming problem; however it would be too simple. The second model is designed to be realistic while its computation is a difficult task in general. This paper describes how to reduce its computational cost. In addition, we compare the results for the two models to grasp their features.

Key words : Road repair scheduling, Shortest path, User equilibrium flow, Linearization, Look-up table approach, Frank-Wolfe algorithm

1. Introduction

Most of the road infrastructures in Japan was developed during the period of high economic growth from the mid-1950s through the 1960s. One of the most important road infrastructures is a bridge because of its role in a traffic network. The expected useful life of a bridge, that was built during that time, is said to be almost 50 years. Therefore, aging and deteriorating bridges are increasing in Japan and it becomes a problem in these days. Those aging bridges need to be repaired or, drastically, rebuilt. However, it is easy to imagine that restriction of traffic due to repairing or rebuilding bridges gives a significant influence on traffic.

In this paper, we consider a case that there are several roads that need to be repaired by a certain date, and we can determine the start time for repairing each of them. A typical example of this case is the renovation of almost end-of-life bridges in Japan as above. In this case, we suppose that an influence on traffic depends on a set of roads simultaneously under repair. Then, there must be a “best” schedule to repair them, which consists of an “optimized” set of roads simultaneously under repair. This study defines such a schedule with the aid of mathematical programming models. Concretely, we propose two mathematical programming (MP) models to formulate the scheduling problem, and moreover, designs a method for solving the two MP models.

One of the two MP models is called the shortest path model. This model assumes that a travel time is independent of amounts of traffic on each road and users choose a shortest (time) path to their destination. In addition, this model does not take into account traffic demands for OD pairs and supposes that repairing a road means complete closure of the road. Due to these simple assumptions, it is possible to reformulate this model as a 0-1 integer programming problem and to solve the problem efficiently with the help of a general IP solver, such as IBM ILOG CPLEX.

The other of the two MP models is called the user equilibrium flow model. Travel times depending on amounts of traffic on each road and user's route choice according to the travel times are incorporated into this model. In addition, this model considers traffic demands for OD pairs. Hence, this model would be more

realistic. However, since it is more complicated, its computation is not easy in the complexity sense. We reduce its computational cost by taking a look-up table approach.

While our models do not restrict roads needing to be repaired to bridges, we think that the restriction makes it easier for readers to understand our models, and therefore, in the rest of this paper, we address a scheduling problem for repairing **bridges**.

In the literature of a bridge maintenance, there are several models for a bridge maintenance scheduling problem, including those of Bocchini and Frangopol (2011), and Orcesi and Cremona (2011). The model of Bocchini and Frangopol is a Pareto optimization model to make an optimal schedule for preventive maintenance of several bridges under uncertainty for their reliability. Its two conflicting objectives are the minimization of the maintenance cost and maximization of the network performance. As the network performance index, they considered the inverse of a nonnegative combination of the total travel time and total travel distance. The minimization of the maintenance cost is not the objective of our models, and the network performance index they proposed is different from the objective functions of our models. Orcesi and Cremona adopted a supply and demand model to describe an optimal bridge maintenance plan, and proposed a two-step optimization process to obtain the optimal plan. In the optimization process, they considered the maintenance costs, bridge failure costs, and user costs. Our models do not assume the supply and demand mechanism. Bocchini and Frangopol, and Orcesi and Cremona both employed genetic algorithms for their optimizations, which is one of the differences from our approach.

In another literature, there is a scheduling problem combined with network design, which is called an incremental network design problem by Baxter et al. (2014). They addressed a problem on finding an optimal schedule for adding edges to a network in terms of some measures, e.g., the (cumulative) cost of shortest paths. Hence, the network is extending in a planning time horizon, which is a fundamental difference from our problem.

2. Shortest path model

2.1. Description of SP model

We first introduce a simple mathematical programming model for a bridge repair scheduling. The mathematical programming model assumes the following two situations.

- (i) A user chooses a shortest (time) path of available paths.
- (ii) Traffic restriction for repairing a bridge means the complete closure of the bridge (and, a path using the bridge is unavailable).

We call the model shortest path model, or SP model for short.

Let B be the set of bridges needing to be repaired, and let Q be the set of origin and destination (OD) pairs. As a scheduling time-span, we consider a time-span consisting of T periods, and denote it by $[T] = \{1, 2, \dots, T\}$. For each bridge $b \in B$, the deadline to repair the bridge b is denoted by $L_b \in [T]$, and the number of periods for repairing the bridge b is represented by P_b , which must be less than L_b . We assume that both L_b and P_b are designated in advance. We also suppose that, for each bridge $b \in B$ and a period $t \in [T]$, it costs $C_{b,t}$ to repair b in t , and, for each period $t \in [T]$, one has a budget of amount M_t . What we have to decide, i.e., the decision variable, is the repair start time $s_b \in [T]$ of each bridge $b \in B$. Let s be the vector $(s_b)_{b \in B}$ of all the repair start times of bridges $b \in B$. The set of bridges under repair in the period $t \in [T]$ depends on the vector s , that is, it is given by $\{b \in B \mid s_b \leq t \leq s_b + P_b - 1\}$ and denoted by $B_t(s)$.

Let B' be a subset of B . For an OD pair $q = (u, v) \in Q$, $l(q, B')$ means the shortest travel time between u and v when all the bridges in B' are under repair, that is, completely closed. In addition, we denote by $f(B')$ the sum of the shortest travel times for all OD pairs $q \in Q$, that is, $f(B') = \sum_{q \in Q} l(q, B')$. Using this function f , we can define the **delay**, denoted by $g(B')$, due to the closure of the bridges in B' to be $g(B') = f(B') - f(\emptyset)$. Then, the delay in a period $t \in [T]$ is given as $g(B_t(s))$. Now, we define the objective function of the shortest path model, which is designed to minimize the total delay, i.e.,

$$h(s) = \sum_{t \in [T]} g(B_t(s)). \quad (1)$$

Since $f(\emptyset)$ is a constant, the minimization of $h(s)$ is equivalent to the minimization of the sum of travel times for all OD pairs, i.e., $\sum_{t \in [T]} f(B_t(s))$.

In SP model, we consider the following two types of constraints.

$$(I) \quad s_b + P_b - 1 \leq L_b \quad (b \in B),$$

$$(II) \quad \sum_{b \in B_t(s)} C_{b,t} \leq M_t \quad (t \in [T]).$$

The constraint (I) means that one should complete the repair of every bridge until its deadline. The constraint (II) requires that, in each period, the cost does not exceed the budget for the period.

2.2. Properties of the objective function

We here make the following additional assumption (A): **For each OD pair, its shortest (time) path goes through exactly one of the bridges even if any (but not all) bridges are closed.** Under this assumption, SP model turns out to have some nice properties with respect to its computation. We describe the properties below.

Because of the additional assumption (A), we can, for each OD pair, specify an order of bridges depending on the lengths of shortest paths passing through each of the bridges; a bridge b_1 is prior to b_2 if the length of a shortest path through b_1 is less than (or possibly equal to) that through b_2 . By definition, this order has the following property: **If the bridges in $B'(\subseteq B)$ are closed, then an OD pair q utilizes the shortest path that passes the highest bridge in $B \setminus B'$ with respect to the order.** Hence, the order can be regarded as the priority list of bridges for each OD pair.

One can use this priority list of bridges to reduce the computational cost for the evaluation of the objective function: If, for each OD pair, we compute, as pre-processing, the travel times of shortest paths passing through each bridge and also its priority list of bridges, then it is possible to evaluate the objective function by, for each OD pair, just picking up the highest bridge in its priority list of bridges and checking the pre-computed travel time of the shortest path via the highest bridge. By using the Dijkstra method, each travel time can be calculated. If we need a lot of repeated evaluations of g (also f), then this pre-processing method drastically improves the computational performance. Furthermore, this property can be used for rewriting SP model as a linear 0-1 integer programming, which will be described in the next section.

2.3. Linearization of SP model

The property arising from the assumption (A) can be used to linearize SP model, that is, the objective function and also all constraints can be written as a linear function and linear constraints, respectively.

First, we reformulate SP model as a (nonlinear) 0-1 integer programming problem. To this end, we introduce the following 0-1 variables. For each bridge $b \in B$ and each period $t \in [T]$, 0-1 variables $x_{b,t}$ and $y_{b,t}$ are defined as follows:

$$x_{b,t} = \begin{cases} 1 & \text{if one starts to repair the bridge } b \text{ in the period } t \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

and

$$y_{b,t} = \sum_{t'=\max\{1, t-P_b+1\}}^t x_{b,t'}. \quad (3)$$

As is easily seen from the definition, a variable $y_{b,t}$ depends on variables $x_{b,t}$, and takes a value as follows:

$$y_{b,t} = \begin{cases} 1 & \text{if the bridge } b \text{ is under repair in the period } t \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

By using these variables $x_{b,t}$ and $y_{b,t}$, the constraints (I) and (II) are expressed as the following (i-1,2) and (ii), respectively:

$$(i-1) \quad \sum_{t=1}^{L_b-P_b+1} x_{b,t} = 1 \quad (b \in B),$$

$$(i-2) \quad \sum_{t=L_b-P_b+2}^T x_{b,t} = 0 \quad (b \in B),$$

$$(ii) \quad \sum_{b \in B} C_{b,t} y_{b,t} \leq M_t \quad (t \in [T]).$$

In addition, $B_t(s)$ is also replaced with $B_t(\mathbf{x}, \mathbf{y}) = \{b \in B \mid y_{b,t} = 1\}$, where $\mathbf{x}$ and $\mathbf{y}$ denote the vectors $(x_{b,t})_{b \in B, t \in [T]}$ and $(y_{b,t})_{b \in B, t \in [T]}$, respectively. Now, SP model is formulated as a 0-1 integer programming problem with the decision variables $x_{b,t}$ and $y_{b,t}$.

Next, we explain the linearization of the objective function. As is mentioned, by pre-processing, one can obtain, for each OD pair q and each bridge $b \in B$, the travel time $l_{q,b}$ when q uses a shortest path through b . For each OD pair $q \in Q$, bridge $b \in B$, and period $t \in [T]$, we introduce the following additional 0-1 variables $z_{q,b,t}$.

$$z_{q,b,t} = \begin{cases} 1 & \text{if } q \text{ uses the shortest path passing through } b \text{ in } t \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

In addition, for each OD pair $q \in Q$, let the priority list of the bridges which was introduced in Section 2.2 be denoted by

$$(b_q^1, b_q^2, \dots, b_q^{|B|}). \quad (6)$$

For each $k \in \{1, 2, \dots, |B|\}$, let B_q^k be the set of the first k bridges in the priority list above, i.e.,

$$B_q^k = \{b_q^1, b_q^2, \dots, b_q^k\}. \quad (7)$$

We impose on the variables $z_{q,b,t}$ the following constraints (iii)–(v).

$$(iii) \quad \sum_{b \in B} z_{q,b,t} = 1 \quad (q \in Q, t \in [T]),$$

$$(iv) \quad z_{q,b,t} + y_{b,t} \leq 1 \quad (q \in Q, b \in B, t \in [T]),$$

$$(v) \quad z_{q,b_q^{k+1},t} \leq \frac{\sum_{b \in B_q^k} y_{b,t}}{k} \quad (q \in Q, t \in [T], k \in \{1, 2, \dots, |B| - 1\}).$$

The constraint (iii) means that each OD pair uses exactly one bridge in each period. The constraint (iv) requires that a bridge b can be used only if the bridge b is not closed in a period t . The constraint (v) means that an OD pair q uses the $(k+1)$ th bridge b_q^{k+1} if and only if the first k bridges are all closed. Now, we show the linearized objective function:

$$\sum_{q \in Q} \sum_{b \in B} \sum_{t \in [T]} l_{q,b} z_{q,b,t}. \quad (8)$$

We call this model linearized SP model.

2.4. Computer experiment of SP model

2.4.1. Data set In computer experiments, we used a road network of 3,089 vertices, 4,629 edges (as roads), 20 ($= |Q|$) OD pairs, and 8 ($= |B|$) bridges. A part of the road network is shown in Fig. 1. The set of OD pairs is composed of all pairs of a red point and a green point in Fig. 1. Each bridge is depicted as a line sandwiched between blue lines in Fig. 1, and numbered as in Fig. 1. Those bridges are actually bridges on Shonai river in Aichi Prefecture, Japan. As a scheduling time-span, we considered a time-span of 12 ($= T$) periods. For each bridge $b \in B$, its repair deadline L_b , and the number P_b of periods needed to repair b were given by random numbers. In addition, repair costs $C_{b,t}$ and period budgets M_t were also generated by random numbers. Their value ranges are shown in Table 1. The transit time for passing through a road depends on the width of the road; it is defined to be the time needed to go through a road by 30km/h, 20km/h, and 15km/h if its width is more than 13m, from 5.5m to 13m, and from 3m to 5.5m, respectively.

2.4.2. Computation of optimal schedules Using a computer operated by (Mac) OS X 10.9.5 with an Intel Core i5 1.8GHz processor and 4GB RAM, we first compute the (original) SP model by enumerating all feasible solutions, evaluating their objective function values, and picking up an optimal solution. Without the additional assumption (A), the evaluation of the function values needs to use, for example, Dijkstra method each time to get a feasible solution. This made it take more than 6 hours to solve SP model. Subsequently, we supposed the assumption (A), and we did the function value evaluation more efficiently with the aid of the pre-processing described in Section 2.3. This reduces the computational time to approximately 7 seconds. Thus, if we adopt an enumeration method, an efficient function value evaluation is important.

Fig. 1 The road network used for the computer experiment.

Next, using a computer operated by Windows 7 with an Intel Core2 Duo 2.93GHz processor and 4GB RAM, we solved linearized SP model with the help of the IBM ILOG CPLEX 12.5.1. Owing to recent advances in general-purpose solver, the computational time was less than 1 second.

Figure 2 shows an optimal solution for SP model. In each network in Fig. 2, the bridges sandwiched between blue and orange lines mean those under repair and those available, respectively, in the periods indicated by the numbers below the network. In the optimal solution, all bridges are repaired by the 11th period. Violet lines in Fig. 3 denote the selected routes, i.e., shortest time route for each OD pair in the fifth period.

3. User equilibrium flow model

3.1. Description of UEF model

One can point out the following issues to SP model:

- (1) A user is assumed to choose a shortest (time) path of available paths.
- (2) The objective function does not take into account the traffic (demand) amount.
- (3) A travel time does not depend on the traffic volume.

These issues are typical ones in the studies on traffic. In order to deal with these issues, traffic models adopting user equilibrium flow (Wardrop, 1952) are often employed. We also incorporate the user equilibrium flow into our modeling. That is, the travel time of each path depends on the amounts of traffic on roads composing the path, and it is assumed that users choose their path in accordance with the user equilibrium traffic assignment. In addition, a new objective function is designed so that it takes traffic into account. This new model that will be described in the next section is referred to as the user equilibrium flow model, or UEF model for short.

3.2. Variables in UEF model

In UEF model, repair of bridges does not necessarily mean complete road closures, and instead we think it causes a delay. In addition to the variables used in SP model, we introduce some new variables to UEF model.

Let q be an OD pair in $\mathcal{Q}$. The traffic demand with respect to q is denoted by d_q , which is assumed to be a constant throughout all the periods. The set of available path for q is represented by R_q , and the traffic volume for q to use a path $r \in R_q$ in a period $t \in [T]$ is expressed by $\xi_{t,r}$. By definition, in each period $t \in [T]$, it holds $\sum_{r \in R_q} \xi_{t,r} = d_q$. Let R be the set of all the paths, i.e., $R = \bigcup_{q \in \mathcal{Q}} R_q$, and let ξ_t be the vector of traffic volumes $\xi_{t,r}$ on each path in a period t , i.e., $\xi_t = (\xi_{t,r})_{r \in R}$. Since the travel time of a path $r \in R$ in a period t depends on the traffic volumes ξ_t and the set $B_t(s) (= B_t(x, y))$ of bridges under repair in the period t , we write it as $\varphi_r(\xi_t, B_t(s))$. We will give later a specific form of the travel time function φ_r .

3.3. User equilibrium flow

For each OD pair $q = (u, v) \in \mathcal{Q}$ and period $t \in [T]$, let $\lambda_{q,t}$ be the shortest travel time between u and v in consideration of the traffic volumes, that is, $\lambda_{q,t} = \min_{r \in R_q} \{\varphi_r(\xi_t, B_t(s))\}$. A traffic flow ξ_t is said to be a user equilibrium flow in a period t if, for each pair OD $q \in \mathcal{Q}$ and path $r \in R_q$, ξ_t satisfies the following:

Fig. 2 An optimal schedule for SP model. A bridge sandwiched between blue lines is closed, and one between orange lines is available.

$$(a) \quad \xi_{t,r} > 0 \Rightarrow \varphi_r(\xi_t, B_t(s)) = \lambda_{q,t},$$

$$(b) \quad \xi_{t,r} = 0 \Rightarrow \varphi_r(\xi_t, B_t(s)) \geq \lambda_{q,t}.$$

It is easy to see from the definition that every path to be used in the period t has to be the shortest time path in the traffic flow ξ_t .

Furthermore, it is known that, by using additional variables $\mu_{t,r} > 0$ ($t \in [T], r \in R$), the two conditions (a) and (b) can be represented by the following inequality system:

$$\varphi_r(\xi_t, B_t(s)) - \lambda_{q,t} - \mu_{t,r} = 0 \quad (t \in [T], q \in Q, r \in R_q), \quad (9)$$

$$\xi_{t,r} \geq 0 \quad (t \in [T], r \in R), \quad (10)$$

$$\mu_{t,r} \geq 0 \quad (t \in [T], r \in R), \quad (11)$$

$$\xi_{t,r} \mu_{t,r} = 0 \quad (t \in [T], r \in R). \quad (12)$$

3.4. Objective function of UEF model

A user equilibrium flow is used to describe the traffic flow resulting from non-cooperative behavior of the

Table 1 Value ranges of variables in SP model

	Range
L_b	10, 11, or 12
P_b	3 or 4
$C_{b,j}$	Integers in $[10, 33]$
M_t	Integers in $[40, 109]$

Fig. 3 The selected routes for each OD pair in the fifth period in the optimal schedule in Fig. 2.

users. In fact, it does not necessarily minimize the following:

$$\sum_{r \in R} \xi_{t,r} \Phi_r(\xi_t, B_t(s)), \quad (13)$$

that is, the sum of the products of traffic volumes and travel times, which can be considered as “social cost”.

However, in the setting of this study, it is possible to generate a better traffic flow with respect to the social cost as a result of optimizing the set of bridges under repair at the same time. On the basis of this observation, as the objective function of UEF model we adopt the sum of the values as in Eq. (13) for all periods $t \in [T]$, that is,

$$\sum_{t \in [T]} \sum_{r \in R} \xi_{t,r} \Phi_r(\xi_t, B_t(s)). \quad (14)$$

In other words, the traffic flow assignment is determined according to the equilibrium conditions, and it is evaluated in terms of social optimality.

3.5. Objective function and equilibrium conditions

By using the equilibrium conditions, the objective function can be rewritten to be a more “explicit” form,

which is described in this section. From Eq. (9), it follows that $\mu_{t,r} = \varphi_r(\xi_t, B_t(s)) - \lambda_{q,t}$. This together with Eq. (12) gives

$$\xi_{t,r} \varphi_r(\xi_t, B_t(s)) = \xi_{t,r} \lambda_{q,t}. \quad (15)$$

Since the L.H.S. term appears in the objective function, we have

$$\sum_{t \in [T]} \sum_{r \in R} \xi_{t,r} \varphi_r(\xi_t, B_t(s)) = \sum_{t \in [T]} \sum_{r \in R} \xi_{t,r} \lambda_{q,t}. \quad (16)$$

Furthermore, since $R = \bigcup_{q \in Q} R_q$ and $\sum_{r \in R_q} \xi_{t,r} = d_q$, we obtain the following:

$$\sum_{t \in [T]} \sum_{r \in R} \xi_{t,r} \lambda_{q,t} = \sum_{t \in [T]} \sum_{q \in Q} \sum_{r \in R_q} \xi_{t,r} \lambda_{q,t} = \sum_{t \in [T]} \sum_{q \in Q} d_q \lambda_{q,t}. \quad (17)$$

Thus, in order to attain the social optimum, it needs to generate a user equilibrium flow such that its travel times are as short as possible.

We here specify the travel time function φ_r . In the context of traffic assignment, the following function ψ_e is often employed to define the function φ_r . For each road e , let η_e be the traffic volume flowing through e , and let $\psi_e(\eta_e)$ be the transit time for passing the road e which is given by

$$\psi_e(\eta_e) = \tau_e \left\{ 1 + \alpha \left(\frac{\eta_e}{\gamma_e} \right)^\beta \right\}. \quad (18)$$

This function ψ_e is known as Bureau of Public Roads (BPR) function. Then, the travel time of a path $r \in R$ is defined as the sum of transit times for all roads contained in r . Here, α and β are positive parameters, τ_e is a free-flow transit time of e , and γ_e is a positive parameter representing a capacity of e . In this paper, we also adopt a basic travel time function defined as in Eq. (18). Let $\eta_{t,e}$ denote the traffic flowing through a road e in a period t , that is,

$$\eta_{t,e} = \sum_{r \in R: e \in r} \xi_{t,r}. \quad (19)$$

For a road e , its transit function is given by $\psi_e^{(0)}$ if e is normally available, and by $\psi_e^{(1)}$ if it is under repair, where $\psi_e^{(0)}$ and $\psi_e^{(1)}$ are given by

$$\psi_e^{(0)}(\eta_{t,e}) = \tau_e^{(0)} \left\{ 1 + \alpha^{(0)} \left(\frac{\eta_{t,e}}{\gamma_e^{(0)}} \right)^{\beta^{(0)}} \right\}, \quad \psi_e^{(1)}(\eta_{t,e}) = \tau_e^{(1)} \left\{ 1 + \alpha^{(1)} \left(\frac{\eta_{t,e}}{\gamma_e^{(1)}} \right)^{\beta^{(1)}} \right\}, \quad (20)$$

and we suppose that $\psi_e^{(0)}(\eta) \leq \psi_e^{(1)}(\eta)$ for any $\eta > 0$. For a path r , the travel time function φ_r is given as follows:

$$\varphi_r(\xi_t, B_t(s)) = \sum_{e \in r \setminus B_t(s)} \psi_e^{(0)}(\eta_{t,e}) + \sum_{e \in r \cap B_t(s)} \psi_e^{(1)}(\eta_{t,e}) \quad (21)$$

Here, a path r is considered as a set of roads.

3.6. Solving UEF model

Since UEF model has an equilibrium condition, UEF model is a so-called mathematical programming with equilibrium constraints (MPEC). If we use the travel time function φ_r in Eq. (21), it is known that a solution satisfying the equilibrium condition can be represented as a **unique** optimal solution for a certain strictly convex optimization problem, and hence UEF model can be regarded as a bilevel optimization problem. As is known, in general, it is computationally difficult to solve a bilevel optimization problem as well as MPEC. However, since the user equilibrium flow is unique, the upper-level problem is trivial.

Assuming that the number of bridges is not so large, we design the following algorithm: First, as pre-processing, for all $B' \subseteq B$, we compute the user equilibrium flows and the values

$$\sum_{t \in [T]} \sum_{r \in R} \xi_{t,r} \varphi_r(\xi_t, B'), \quad (22)$$

and store them in a look-up table. Then, by enumerating all feasible solutions s , and evaluating their objective values in Eq. (14) with the help of the pre-established look-up table, we pick up an optimal solution. A look-up table approach is also employed by Bocchini and Frangopol (2011). In order to compute the user equilibrium flow, the Frank-Wolfe algorithm (Frank and Wolfe, 1956) is often utilized. We also implemented the Frank-Wolfe type algorithm.

3.7. Computer experiment of UEF model

3.7.1. Data set We applied the algorithm described in Section 3.6 to UEF model with the same road network in Fig. 1 as the one used for SP model. The variables common with SP model were set up in a similar way to SP model; namely, they are random numbers within the ranges shown in Table 2. In addition, for each OD pair q , its demand d_q was also given by random numbers from 200 to 700 as in Table 2.

For each edge e , the free-flow transit time $\tau_e^{(0)}$ was given by the transit time for passing through e defined in the same way as SP model, and the capacity $\gamma_e^{(0)}$ is defined to be 1,000, 500, and 250 if the width of e is more than 13m, from 5.5m to 13m, and from 3m to 5.5m, respectively. In addition, for all edges e in common, $\alpha_e^{(0)}$ and $\beta_e^{(0)}$ were set to be 0.15 and 4.0, respectively, which are the values used by Orcesi and Cremona (2011). In order to compare UEF model with SP model, $\tau_e^{(1)}$ for every edge e was defined to be very large number so that it means being completely closed.

Fig. 4 An optimal schedule for UEF model. A bridge sandwiched between blue lines is closed, and one between orange lines is available.

3.7.2. Computation of optimal schedules Using a computer operated by (Mac) OS X 10.9.5 with an Intel Core i5 1.8GHz processor and 4GB RAM, we computed UEF model by using the algorithm described in Section 3.6. In the case of $|B| = 8$, a look-up table approach made it possible to complete the computation in about 6 minutes, which include about 4 minutes for the establishment of a look-up table.

Figure 4 shows an optimal solution for UEF model. In each network in Fig. 4, the bridges sandwiched

Table 2 Value ranges of variables in UEF model.

	Range
L_b	10, 11, or 12
P_b	3 or 4
$C_{b,t}$	Integers in $[10, 33]$
M_t	Integers in $[40, 109]$
d_q	Integers in $[200, 700]$

between blue and orange lines mean those under repair and those available, respectively, in the periods indicated by the numbers below the network. Violet lines in Fig. 5 denote the selected routes, i.e., routes with positive flow, for each OD pair in the fifth period.

Fig. 5 The selected routes for each OD pair in the fifth period in the optimal schedule in Fig. 4.

4. Comparison of the two models

We made 50 common input data sets for the two models. The two models proposed distinct schedules for every input data set except for infeasible 7 cases. In order to grasp features of each of the two models, we counted, for every pair (i, j) of bridges i and j , the number p_{ij} of periods in which the two bridges i and j are both under repair in 43 optimal solutions of each model, where i and j are the assigned numbers to bridges as in Fig. 1. Tables 3 and 4 show the numbers p_{ij} for SP and UEF models, respectively. Table 5 consists of the differences (p_{ij} for SP model minus p_{ij} for UEF model) of the corresponding cells in Tables 3 and 4. There are remarkable differences in the pairs (4,5), (3,5), (4,7), and (3,7).

For further discussion, we introduce n -away pair of bridges: The pair of the bridges i and j is said to be n -away if $|i - j| = n$. Because of our numbering of bridges, a 1-away pair means a pair of adjacent bridges in Fig. 1,

and a 2-away pair consists of bridges two away from each other in Fig. 1, and so on. For each $n = 1, 2, \dots, 7$, we calculate the sum t_n of p_{ij} for all n -away pairs (i, j) , and their average $a_n = t_n/c_n$, where c_n is the number of n -away pairs. Tables 6 and 7 show respectively, the numbers t_n and a_n for SP and UEF models. Figure 6 is the histogram of a_n in Tables 6 and 7. From Tables 6 and 7, we note that, compared with SP model, UEF model tends to produce 2-away pairs and avoid 1- and 5-away pairs for pairs of bridges under repair at the same time. Intuitively, avoiding 1-away pairs and preferring 2-away pairs would prevent detours. About 5-away pairs, Table 5 tells that the pair (3,8) among 5-away pairs was particularly avoided in UEF model. One of the reasons of this avoidance might be that closing the bridges 3 and 8 at the same time causes traffic concentration on the bridges 4,5,6 and 7.

We also note that, for a fixed n , there can be differences of p_{ij} among the n -away pairs. For example, the pairs (4,5) and (7,8) are both 1-away pairs. The number p_{45} for SP model is much larger than p_{78} for SP model. On the other hand, the number p_{45} for UEF model is equal to 4, and, conversely, smaller than p_{78} for UEF model. The tendency of the 1-away pair (4,5) to be selected as a pair of bridges simultaneously under repair depends on the choice of the two models. Geographically, the bridges 4 and 5 can be considered as center points for all OD pairs. Since UEF model is intuitively sensitive to a traffic congestion, UEF model might avoid selecting the bridges 4 and 5 to be under repair at the same time.

Table 3 The numbers of periods in which each pair of bridges is under repair at the same time in SP model.

Bridge	1	2	3	4	5	6	7	8
1	0	56	33	34	37	60	39	29
2	56	0	46	48	41	45	41	43
3	33	46	0	0	0	59	85	45
4	34	48	0	0	117	1	1	38
5	37	41	0	117	0	0	10	43
6	60	45	59	1	0	0	0	49
7	39	41	85	1	10	0	0	44
8	29	43	45	38	43	49	44	0

Table 4 The numbers of periods in which each pair of bridges is under repair at the same time in UEF model.

Bridge	1	2	3	4	5	6	7	8
1	0	56	47	26	56	46	26	32
2	56	0	7	7	3	65	23	69
3	47	7	0	2	111	29	6	4
4	26	7	2	0	4	39	100	65
5	56	3	111	4	0	0	11	6
6	46	65	29	39	0	0	0	12
7	26	23	6	100	11	0	0	89
8	32	69	4	65	6	12	89	0

Table 5 The differences of the corresponding cells in Tables 3 and 4.

Bridge	1	2	3	4	5	6	7	8
1	0	0	-14	8	-19	14	13	-3
2	0	0	39	41	38	-20	18	-26
3	-14	39	0	-2	-111	30	79	41
4	8	41	-2	0	113	-38	-99	-27
5	-19	38	-111	113	0	0	-1	37
6	14	-20	30	-38	0	0	0	37
7	13	18	79	-99	-1	0	0	-45
8	-3	-26	41	-27	37	37	-45	0

Table 6 The numbers t_n, a_n for SP model.

n	t_n	a_n
1	263	37.6
2	141	23.5
3	178	35.6
4	205	51.2
5	146	48.7
6	82	41.0
7	29	29.0

Table 7 The numbers t_n, a_n for UEF model.

n	t_n	a_n
1	158	22.6
2	227	37.8
3	164	32.8
4	192	48.0
5	73	24.3
6	95	47.5
7	32	32.0

5. Conclusion

In this paper, we proposed two mathematical programming models to deal with a scheduling problem on the renovation of roads that need to be repaired by a certain date. A typical example of the scheduling problem is the one for almost end-of-life bridges in Japan. Our idea behind the formulation of the models is that, if one

Fig. 6 Histogram of a_n in Tables 6 and 7.

optimizes a set of roads simultaneously under repair, then the influence on traffic due to their repair can be minimized.

The first model is called SP model, and it assumes that every driver uses a shortest time path ignoring delays due to concentration of traffic, and a bridge under repair is completely closed. In addition, SP model does not consider the traffic demand for OD pairs in its objective function. Although these assumptions provide an advantage for solving SP model efficiently, these assumptions would be too simple in practical point of view.

The second model is called UEF model, whose traffic assignment is based on the user equilibrium traffic assignment, that is, UEF model assumes that drivers choose their routes according to delays due to concentration of traffic. Furthermore, the objective function of UEF model depends on the traffic demand for OD pairs. Hence, UEF model is expected to be more practical model.

We have done two computer experiments, one of which is for SP model and the other is for UEF model, and confirmed that the look-up table approach to solve UEF model is adequately fast for a set of bridges of the size equal to eight. In addition, we compared the results for the two models. As is discussed in Section 4, the two models seem to have distinct tendencies for the choice of pairs of bridges simultaneously under repair. It would be interesting to reveal the relation between the tendencies and the distribution of traffic demands or positions of bridges.

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