Determination of the zero-position for an optical angle sensor

Jun TAMADA*, Yukitoshi KUDO*, Yuan-Liu CHEN*, Yuki SHIMIZU* and Wei GAO*
*Department of Nanomechanics, Tohoku University
6-6-01, Aramaki Azab Aoba, Aoba-ku, Sendai, Miyagi 980-8579, Japan
E-mail: yuki.shimizu@nano.mech.tohoku.ac.jp

Received 26 February 2016

Abstract
This paper proposes a new optical angle sensor, in which a mode-locked femtosecond laser referred to as the optical frequency comb is employed as the light source for the sensor. By using the optical frequency comb, whose carrier frequency is well stabilized by using an external frequency standard with the uncertainty of $10^{-11}$, both the sensor stability and the sensor sensitivity are expected to be improved. In this paper, the angle error caused by the frequency fluctuation of the laser beam is considered in the case of both the mode-locked femtosecond laser and a laser diode, which is a conventional light source for the optical angle sensor. A prototype optical sensor head with the mode-locked femtosecond laser is then fabricated for the angle sensor. Since the light wavelength of the mode-locked femtosecond laser used in this paper is out of the visible range, an alignment method based on laser autocollimation with a retroreflector is introduced to determine the zero-position for the angle sensor. This method is also effective in suppressing the influence of the alignment error such as a cosine error. Some basic experiments are carried out to verify the feasibility of the developed angle sensor with the optical frequency comb.

Key words: Optical frequency comb, Optical angle sensor, Angular displacement, Laser autocollimation, Retroreflector

1. Introduction

Fine optical parts and highly integrated semiconductor devices are produced by precision machine tools and semiconductor manufacturing instruments, whose machining accuracy is limited to the positioning accuracy of positioning systems employed in such instruments (Buice et al., 1999). Measurement of the positioning accuracy of such positioning system is therefore essential to fabricate products with further higher accuracy and to guarantee the quality of products. A positioning error of a precision linear stage along its motion axis is measured by displacement sensors such as linear encoders or laser interferometers so that closed-loop control can be carried out. However, on the other hand, angular error motion components of the linear stage are rarely measured in most of the cases. Measurement of angular error motion components of a stage table is desired since the Abbe error, which is caused by the combination of the angular error motion components and an offset between the motion axis of the stage table and the measurement axis of the position sensor, can result in micrometric positioning error (Bryan et al., 1979). Therefore, the closed-loop control with the high-precision angle sensor is necessary for further precise positioning of the stage table (Steinmetz et al., 1990).

In order to detect angular error motions of such stage systems with high accuracy, an autocollimator is often used. The autocollimator is based on autocollimation method (Jones et al., 1961), which can detect the angular displacement of a measurement target by detecting the displacement of a light spot focused on a photodetector with the employment of a collimator objective. Meanwhile, the autocollimators used for measurement of the stage angular error motion components require a high sensitivity, a high resolution and a high response speed. In addition, the size of the optical sensor head for the autocollimator is required to be small enough to be applied for measurement of the stage system. Although the conventional autocollimators have the high sensitivity and high resolution, their response speed is limited.
since their position detector is a two-dimensional Charge-Coupled Device (CCD), whose response speed is not enough to carry out feedback control of the stage table, in most of the cases. In addition, a focal length of the collimator objective of the angle sensor for the higher sensor sensitivity prevents miniaturization of the sensor size. To overcome the problems, an optical angle sensor based on laser autocollimation, which employs a laser beam and photodiodes as the light source and the position detectors, respectively, has therefore been developed (Ennos et al., 1982). Further detailed investigation has revealed that the sensitivity of the laser autocollimator, which is based on the laser autocollimation and employs a four-cell photodiode, is independent from the focal length of the collimator objective (Saito et al., 2007). An employment of the collimator objective with a shorter focal length has allowed the optical sensor head of the laser autocollimator to be designed in a compact size, while keeping its high measurement sensitivity. Furthermore, the laser autocollimator employing single-cell photodiodes (SPDs) as the position detector has also been proposed, and the measurement resolution of 0.001 arc-second has successfully been demonstrated (Shimizu et al., 2016). Meanwhile, it is also known that the sensitivity of the angle sensor based on the laser autocollimation is a function of the laser beam frequency (Saito et al., 2007). Although a laser diode (LD) is often employed as a light source for the angle sensors, it has a problem on its frequency fluctuation due to the mode hop caused by temperature variation (Koechner et al., 1999). The frequency fluctuation of the laser beam directly affects the diameter of the focused laser beam on PDs, resulting in the sensitivity deviation of the angle sensor. Although the precise control of the LD temperature can stabilize the optical frequency (Corwin et al., 1998) to a certain level, further stabilization of the optical frequency is desired for further stabilized sensor sensitivity.

In responding to the background described above, in this paper, we have employed a mode-locked femtosecond laser, which is often referred to as the optical frequency comb, as a highly-stabilized laser beam for the angle sensor. The frequency of the mode-locked femtosecond laser can be stabilized by an external frequency standard (Haus et al., 2000). By applying the optical frequency comb to the optical angle sensor, both the sensor stability and sensitivity are expected to be improved. A new optical configuration has designed for the optical sensor head of the angle sensor. In the designed optical configuration, the SPDs are employed to detect the positions of the focused laser beams. One of the drawbacks of the employment of the mode-locked femtosecond laser is that its optical alignment becomes difficult due to the light wavelength of the emitted laser beam, which is out of the visible range. We have therefore proposed an alignment method based on laser autocollimation to determine the zero-position for the angle sensor by employing a retroreflector. The proposed method is useful for the precision positioning of a laser beam and can be used to easily determine the zero position of the angle sensor even if the laser wavelength is out of the visible range as the mode-locked femtosecond laser used in this paper.

2. Measurement principle of angular displacement

Optical angle sensors based on the laser autocollimation are widely used in many applications to measure the small angular displacements of measurement targets. Figure 1(a) shows a schematic of the optical setup for conventional laser autocollimator employing a two-dimensional CCD as a photodetector. In terms of the size of the optical setup, a LD has usually been employed as the light source for the laser autocollimators. A collimated laser beam from the LD is reflected by a target reflector, and the reflected light is focused onto a position sensor by a collimator objective (CO). The light spot moves on the sensor surface according to the angular displacement $\Delta \theta$ of the target reflector. The light spot displacement on the photodetector $\Delta d$ can be described by the following equation (Virdee et al., 1988):

$$\Delta d = 2F \Delta \theta$$

(a) CCD type

![CCD type](image1)

(b) Single-cell Type

![Single-cell Type](image2)

Fig. 1 Principle of the laser autocollimation for the detection of the small angular displacement
\[ \Delta d = 2F \Delta \theta \]  

(1)

where \( F \) is a focal length of the CO. The sensitivity of the conventional autocollimator is high enough to measure the small angular displacement of the precision linear stage. However, the size of the optical sensor head for the angle sensor tends to become large because CO with a long focal length \( F \), which contributes to magnify the light spot displacement \( \Delta d \), is required to achieve the high sensitivity. In addition, the response speed of the angle sensor is not so high because the photodetector employed in the optical sensor head is the two-dimensional CCD, whose response speed is low in principle due to the limited readout rate and frame rate for reading out the millions of CCD pixels one by one in a repetitive shift-and-read process (Möller-Werdel optical GmbH data sheet). Although a linear image sensor (line CCD) can achieve a high readout speed, it is not suitable as the photodetector of the precision laser autocollimator in terms of the resolution of position detection. In addition, when the angular motion occurs in another axis of the rotation, the light spot will easily be moved out from the active cells of the linear image sensor, resulting in the loss of the sensor output.

To solve the problems, an optical sensor head employing quadrant photodiodes or SPDs, which have also been employed in this paper, has been proposed (Gao, 2010). Figure 1(b) shows the optical setup for the angle sensor employing a SPD as the photodetector. In the proposed optical setup, the spot size on the photodetector is one of the most important factors that affect the sensor sensitivity. The size of the light spot focused on the SPDs can be described by the following equation (Srivastava et al., 2014):

\[ 2w = \frac{4F c}{\pi D \nu} \]  

(2)

where \( w \) is the spot radius at which the intensity is \( 1/e^2 \) of its maximum value, \( D \) is the diameter of the collimated laser beam made incident to the collimator objective, \( F \) is the focal length of the collimator objective, \( c \) is the speed of the light in air, and \( \nu \) is the typical frequency of the light source. As shown in the equation, the instability of \( \nu \) directly affects the spot size. However, little attention has been paid to the stability of the light frequency so far. Assuming that \( \Delta \nu \) is the fluctuation of the light frequency, the deviation of the light spot radius \( \Delta w \) is calculated as follows based on Eq. (2):

\[ \Delta w = \frac{2}{\pi} \frac{F c}{D} \frac{1}{\nu} \left( 2F c \left( \frac{\Delta \nu}{\nu} \right) \right) \]  

(3)

Table 1 summarizes the angle fluctuation calculated based on the equation. In the calculation, \( F \) and \( D \) are set to be 50 mm and 0.9 mm, respectively. The frequency fluctuation of the LD of 1 THz results in the instability of the optical spot diameter of approximately 0.1 \( \mu \)m. Although the estimated deviation of the optical spot diameter is not so large, this can be a problem in the case of highly-sensitive angular displacement measurement. With the precise control of the temperature, LD can provide the stable frequency with the uncertainty of \( 10^{-9} \) (Corwin et al., 1998). For further reduction of the influence of laser frequency fluctuation on the sensor sensitivity, in this paper, a mode-locked femtosecond laser is employed as a light source for the optical angle sensor. Figure 2 shows a schematic of the spectrum of the mode-locked laser, which consists of a series of discrete, equally spaced laser modes. The mode frequency of a specific comb can be expressed as follows (Jones et al., 2000):

\[ \nu_i = i \nu_{\text{rep}} + \nu_{\text{CEO}} \]  

(4)

where \( \nu_i \) is the frequency of the \( i \)-th mode laser, \( \nu_{\text{rep}} \) is a pulse repetition rate, \( \nu_{\text{CEO}} \) is a carrier envelope offset frequency, and \( i \) is an integer. The repetition rate can be stabilized by the rubidium (Rb) frequency standard with the uncertainty of \( 10^{-11} \). Since the fluctuation of the \( \nu_{\text{CEO}} \) (less than 50 MHz) is much less than that of the laser diode, the sensor stability

| Table 1  Deviation of the focused light spot diameter due to the laser frequency fluctuation |
|-----------------|-----------------|-----------------|
| Frequency Fluctuation [MHz] | Laser diode | Femtosecond laser |
| Typical Frequency [THz] | 450 | 190 |
| Typical spot diameter [\( \mu \)m] | 94 | 223 |
| Difference of spot radius [nm] | 104 | 0.029 |
and the sensor sensitivity are expected to be improved by the employment of the optical frequency comb. In the case of the mode-locked femtosecond laser having the frequency fluctuation of 50 MHz, the instability of the optical spot diameter is estimated to be 29 pm, which is negligibly small.

Regarding the high stability of the mode-locked femtosecond laser, the optical angle sensor, in which the SPDs and the femtosecond laser are employed as the position sensors and the light source, respectively, is proposed in this paper. Figure 3 shows a schematic of the part of the optical configuration for the optical angle sensor employing the SPDs as the position detector. In the method, the optical configuration consists of two units; a reference unit with SPD2, and a measurement unit with SPD1. The SPD1 is employed for measurement of the angular displacement based on the laser autocollimation, while the SPD2 is for compensating the fluctuation of the light intensity, which can influence the sensor sensitivity. The laser beam from the light source is divided into a reference beam and a measurement beam by the beam splitter (BS) in the reference unit. The reference beam is captured by the SPD2. The light spot focused by the CO is positioned in the mid of the active area on the SPD2 so that all the light spot can be received. The measurement beam, on the other hand, is made incident to a target reflector. The beam reflected from the target reflector is bent by a polarized beam splitter (PBS), and is focused on the SPD1 by another CO. The light spot focused on the SPD1 is aligned to be on the edge of the active area of the SPD1 when the normal of the target reflector is parallel with the optical axis of the measurement beam.

The intensity fluctuation of the laser beam can be suppressed by dividing the photocurrent output from the SPD1 ($I_1$) by that from the SPD2 ($I_2$). The normalized output $I$, which is referred to as the sensor output in this paper, can be written as follows (Murata et al., 2013):

$$I = \frac{I_1}{I_2} \times 100 \%$$  \hspace{1cm} (5)

With the angular displacement $\Delta \theta$ of the target reflector, the light spot on the SPD1 will have a displacement $\Delta d$. Then the SPD1 detects the change in the total intensity introduced by $\Delta d$. On the assumption that $\Delta \theta$ is small, the sensor output can be written as follows (Murata et al., 2013):

$$I = \frac{2w\Delta d}{\pi w^2} = \frac{2\Delta d}{\pi w}$$  \hspace{1cm} (6)
Here $2w$ corresponds to the diameter of the light spot focused on the SPDs. From Eqs. (1), (3) and (6), the sensitivity $S$ of the angle sensor can therefore be calculated as follows (Saito et al., 2007):

$$S = \frac{2Dv}{c}$$

This equation is valid until the angular displacement of the measurement target is small. The sensor sensitivity is a function of $D$, $c$ and $v$. Since $D$ and $c$ are constant, the stability of sensor sensitivity $S$ is mainly determined by $v$. According to Eq. (7), the sensitivity is easily influenced by the fluctuation of the light source. In this paper, we have therefore employed a mode-locked femtosecond laser as a highly-stabilized laser beam so that the sensitivity of the angle sensor can be stabilized.

Meanwhile, a new alignment method using a retroreflector, which can reflect the incident light to the source with high accuracy regardless of the angle of incidence, is introduced to determine the zero-position for the angle sensor. This method can be used even in the case of employing invisible light such as the femtosecond laser employed as the light source for the angle sensor. In addition, the proposed method is also effective in reducing the influence of the cosine error caused by the misalignments. Figure 4 shows a schematic of the setup for determining the zero-position for the angle sensor. At first, the position of the SPD is adjusted in such a way that the focused light spot is positioned in the mid of the active area of the SPD, while employing the retroreflector as the target reflector (Fig. 4(a)). After that, a plane reflector is replaced with the retroreflector, and is mounted on a rotary stage as shown in Fig. 4(b). With the replacement, the focused light spot moves out from the mid of the SPD active area. Therefore, in the next step, the turntable of the rotary stage is moved in such a way that the focused light spot will be placed in the mid of the SPD active area. During the operation, the photocurrent from the SPD is monitored so that the plane mirror can be adjusted to the initial angular position as shown in Fig. 4(c). Throughout the procedure, the zero-position for the angle sensor, where the axis of laser source is perpendicular to the mirror, can be determined. In the following section, some experiments are carried out to verify the feasibility of the proposed method.

![Diagram of experimental setup](image_url)
3. Experiments

An experimental setup was constructed to verify the feasibility of the proposed method of determining the zero-position for the angle sensor, and to evaluate its sensitivity. Figure 5 shows a schematic of the experimental setup. The setup mainly consists of an optical sensor head developed for this study, a mirror reflector mounted on a rotary stage and a laser autocollimator, which is employed as a reference in this paper. In the optical configuration of the sensor head, the collimated laser beam from the femtosecond laser source (Menlo Systems C-Fiber) with a pulse repetition rate of 100 MHz and a pulse width of approximately 150 fs is employed as the measurement beam. The measurement beam is to pass through a polarized beam splitter (PBS) and a quarter wave plate (QWP), and is then made incident to a flat mirror reflector mounted on a digital swivel stage (Beldex Co./DMS06-09R) having the measurement resolution and range of 1 arc-second and ±18000 arc-seconds, respectively. The measurement beam reflected from the mirror reflector passes through the QWP again, and is bent at the PBS. After that, the beam is focused onto a single-cell photodiode (SPD) by a collimator objective (CO). A photocurrent generated by the SPD is then converted to the voltage output by using an I-V circuit, and the variation of the voltage output is observed by a digital oscilloscope. The design parameters for the optical sensor head are summarized in Table 2.

As a first step of the experiments, sensitivity of the angle sensor was evaluated. The focused light spot on the SPD was at first positioned to be on the edge of the SPD active area. The flat mirror reflector was then made to rotate about the X-axis with a travel range of ±20 arc-seconds. Figure 6(a) shows the variation of the angle sensor output during the stage rotation. The angular displacement of the stage was measured by the laser autocollimator. From the result, the sensitivity was confirmed to be 0.0062 V/arc-second. The noise component in the SPD output was also evaluated as shown in Fig. 6(b). The peak to peak value of the noise component was evaluated to be 12.7 mV, which corresponds to the measurement resolution of approximately 2.0 arc-seconds. It should be noted that this study is a first step of the angle sensor employing the femtosecond laser; due to the large spot size of approximately 100 µm on the SPD surface,
the acquired sensor sensitivity is relatively low compared with the state-of-the-art optical angle sensors. Meanwhile, on the other hand, the size of spot cannot be made to be smaller because the spot size is determined by the wavelength as shown in Eq. (2). The optimization of the spot diameter on the SPD plane has a possibility of improving the sensor sensitivity for further higher measurement resolution. Besides, the measurement resolution can be improved by suppressing the noise component in the SPD output.

In the proposed alignment procedure shown in Fig. 4, the accuracy of determining the zero-position depends on the accuracy of the retroreflectivity of the retroreflector employed in the setup. The retroreflectivity of the retroreflector was therefore evaluated by using the developed optical setup. Figure 7(a) shows a schematic of the experimental setup prepared for the evaluation of the retroreflector. In the setup, the retroreflector was mounted on the swivel stage instead of the mirror reflector. The measurement beam reflected by the retroreflector was captured by the SPD, which were aligned in such a way that the light spot focused on the SPD plane by the CO was positioned to be on the edge of the SPD active area for highly-sensitive retroreflectivity evaluation. Figure 7(b) shows the deviation of the SPD output during the rotation of the retroreflector. The deviation of the SPD output in the travel range of 8000 arc-seconds was found to be 1.6 mV peak-to-peak, corresponding to the angle error of 0.26 arc-second. Since this value was small compared with the measurement resolution of the developed angle sensor, the retroreflectivity of the retroreflector employed in the experiment was evaluated to be enough.

In the proposed alignment procedure shown in Fig. 4, the accuracy of determining the zero-position for the angle sensor was then carried out by using the developed system. In the experiment, mirror tilt angle about the X-axis was changed within the range of ±300 arc-seconds by the digital swivel stage. Figure 8 shows the variation of the angle sensor output due to the angular displacement applied to the mirror reflector. The zero-position for the flat mirror reflector was identified from a normal distribution curve, which was fit to the measured SPD output as shown in the figure. The resolution of determining the zero position agrees with that of the angle sensor. To further verify the feasibility of the proposed method, computer simulation was carried out. In the simulation, a circular light spot having Gaussian distribution was estimated to go across the active area of the SPD. The spot diameter was set to be 105 µm, which was calculated from Eq. (2). In the calculation, the parameters shown in Table 2 were employed. Figure 9 shows the simulation results. In the figure, the curve acquired in the experiment is also plotted for comparison. As can be seen in the figure, the experimental result showed a fairly good agreement with the simulation results. The SPD output was
almost constant while the whole optical spot was on the SPD active area. A slight difference between the simulation result and the experimental result can be explained as the influences of the misalignments of the optics in the experimental setup and the aberration of each optical component. These results mean that the zero-position for the optical angle sensor employing the laser beam emitted from the femtosecond laser with the wavelength of 1560 nm, which is out of the visible range of the light wavelength, was successfully determined by using the proposed method. It should be noted that the optimization of the size of the focused spot on the SPD surface with respect to the size of the SPD active area is necessary to determine the zero-position for the angle sensor with higher accuracy, which will be carried out as future work.

![Diagram](a) Experimental setup

![Diagram](b) Variation of the sensor output due to the angular displacement applied to the retroreflector

Fig. 7 Evaluation of the retroreflectivity of the retroreflector employed in the setup

![Diagram](Fig. 8 Variation of the angle sensor output due to the applied angular displacement)
4. Conclusion

An optical angle sensor employing a mode-locked femtosecond laser with the typical wavelength of 1560 nm as the light source has been proposed. A prototype of the optical sensor head based on the laser autocollimation method has been developed. The sensitivity and the resolution of the angle sensor have confirmed to be 0.0062 V/arc-second and 2.0 arc-seconds, respectively. To determine the zero-position for the developed optical angle sensor, whose optical alignment is difficult and time-consuming task due to the invisible laser beam employed in the optical setup, a unique alignment technique based on laser autocollimation method employing a retroreflector has been proposed. Some experiments have been carried out to verify the feasibility of the proposed method. It has been verified from both the simulation and experimental results that the zero-position for the optical angle sensor can successfully be determined by the proposed method.

Acknowledgement

This project was supported by Japan Society for the Promotion of Science (JSPS) Grant Number 15H05759.

References

Gao, W., Precision Nanometerology, Springer (2010).