Evaluation of dynamic behavior of rotary axis in five-axis machining center
(Behavior around motion direction changes)

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Abstract
Several methods of evaluating the motion accuracy of the rotary axes of five-axis machining centers have been proposed in past studies. Because it is known that particular motion errors exist near motion direction changing points, it is important to evaluate the behavior of the rotary axes near these points. However, the influence of the motion error of the translational axes is included in conventional evaluation results because the translational axes reverse at the motion direction changing points about the rotary axes. In this study, a measurement system for evaluating the dynamic characteristics of the rotary axes near the motion direction changing points about these axes, excluding the influence of the translational axes, was developed. The measurement tests were also conducted using this measurement system for evaluating the dynamic characteristics of the tilt axis near the motion direction changing points about this axis. In addition, an actual machining test was conducted to evaluate the influence of the motion errors of the tilt axis near the motion direction changing points. As a result, it was confirmed that the behavior of the tilt axis near the motion direction changing points can be evaluated by using the proposed measurement method. It was also confirmed that the influence of the motion error on the machined surface is related to the size and shape of the tool and the geometric relationship between the motion error and the machined surface. It was also confirmed that the machined shape does not always contain defects when motion errors exist depending on the relationship between the motion error and the machined surface.

Key words: Five-axis machining center, Rotary axis, Dynamic behavior, Motion direction changes, Machined surface

1. Introduction
Simultaneous five-axis machining makes the machining of complex-shaped mechanical parts, molds, and dies possible; however, the accuracy of five-axis machining is typically worse than that of three-axis machining. In particular, the motion error of the rotary axes has a larger influence on the machined surface than the error of the translational axes because it is amplified by the distance between the center of the rotary axis and the machining point.

Several methods of evaluating the motion accuracy of the rotary axes using a ball-bar system, R-tests, and cone-frustum cutting tests have been proposed (Hong et al., 2011; Kato et al., 2013; NAS979, 1969; Sato and Tsutsuki, 2012; Weikert and Knapp, 2004; Yamaji et al., 2014). Most of these methods use the synchronous motion of the rotary and translational axes. Because it is known that particular motion errors exist near motion direction changing points, it is important to evaluate the behaviors near these points. This is also the case at the motion direction changing points of the rotary axes, which have a particularly large influence on the quality of machined surface. Unfortunately, however, the impact of the motion error of the rotary axes has not been studied independent of the translational axes;
that is, the influence of the motion error of the translational axes is always included in conventional evaluation results because the translational axes change the motion direction at the motion direction changing points of the rotary axes.

Therefore, in this study, a measurement method to evaluate the dynamic characteristics of the rotary axes near the motion direction changing points about these axes, excluding the influence of the translational axes, was investigated, and a measurement device that can measure the behavior of the rotary axes was developed. In addition, a machining method is proposed to confirm in what way the behavior of the rotary axes influences the machined surface. In this way, the behavior of the rotary axis at the motion direction changing point, which has a particularly large impact on the machined surface integrity of the dynamic characteristics of the rotary axes, was evaluated.

2. Motion error measurement

2.1 Machine tool structure

In this paper, a measurement method and a cutting test are proposed as techniques of evaluating the behavior of the rotary axes. Measurements and cutting tests were conducted using the proposed methods and the five-axis machining center shown in Fig. 1 to confirm the effectiveness of these methods. This five-axis machining center has a C-axis that rotates the table around the Z-axis. The C-axis is attached to the A-axis (tilt axis), which tilts the table around the X-axis. The three translational axes move the spindle in the X-, Y-, and Z-directions.

The proposed measurement method using three displacement sensors and a reference ball was employed to evaluate the motion errors. Many previous studies (Hong et al., 2011; Weikert and Knapp, 2004; Yamaji et al., 2014) have reported methods of measuring the equipment accuracy of five-axis machining center. The proposed method is to measure the displacement that occurred when the translational axes and the rotary axes move to keep between a measuring equipment and measurement target for the measurement.

Fig. 1 Configuration of the five-axis machining center (EGURO_E-32V).

2.2 Measurement device

A measurement device was developed to evaluate the dynamic characteristics of the rotary axes near the motion direction changing point. A reference ball (Renishaw plc) was adopted as the measurement target to measure the relative position of the measuring equipment and the measurement target when the posture was changed by the rotary axis motion. The sphericity of the ball is less than 1 μm, and its diameter is 19 mm.

One type of displacement sensor was chosen for the measurement device. Hong and Ibaraki (2013) previously evaluated various types of laser displacement sensors and contact-type displacement sensors commonly used in practical applications. In addition, the measurement devices are already on the market (HMS, Fidia S.p.A.; Trinity, IBS Precision Engineering). However, previous studies have not yet included comparisons between different types of displacement sensors, such as eddy current-type and laser-type sensors.
In this study, therefore, various types of displacement sensors were compared. Table 1 lists the displacement sensors compared in this study: a laser displacement sensor (IL-S025, Keyence, spot shape: elliptic, spot diameter: 25 μm [minor axis] × 1.2 mm [major axis]), a contact-type displacement sensor (GT2-H12K, Keyence), and an eddy current-type displacement sensor (EX-305, Keyence, sensor head diameter: 5.4 mm). The measurement device proposed in this study should be able to directly measure the distance to the surface of the reference ball, and the displacement sensor should be able to continuously measure the displacement to evaluate the motion error near the motion direction changing point. In addition, the sensor should be small to be compatible with the five-axis machining center used in this study, its measurement accuracy should be high, and its cost should be low.

First, the curved surface measurement tests shown in Fig. 2 were carried out with the laser and eddy current-type displacement sensors to evaluate the measurement accuracy during the curved surface measurement. In these tests, a cylinder with a diameter of 20 mm was used as the measurement target.

Figure 3 compares the measurement results. This figure demonstrates that the measurement results for the eddy current-type displacement sensor were in very poor agreement with the shape of the cylinder shown in Fig. 2, whereas the measurement results for the laser displacement sensor were in good agreement with the cylinder geometry (The error is within 1μm between the laser sensor offset ±0.2mm). One of the reasons for the poor measurement results for the eddy current-type displacement sensor may be that the magnetic flux of the eddy current tilts when it is incident on a tilted measurement surface. In addition, a possible reason the eddy current-type displacement sensor more significantly overestimated the displacement at points further from the center of the sensor is shown in Fig. 4.

<table>
<thead>
<tr>
<th>Model</th>
<th>IL-S025</th>
<th>GT2-H12K</th>
<th>EX-305</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>CMOS Laser Sensor</td>
<td>Digital Contact Sensor</td>
<td>Eddy Current-type Sensor</td>
</tr>
<tr>
<td>Measuring range [mm]</td>
<td>20~30</td>
<td>0~12</td>
<td>0~1</td>
</tr>
<tr>
<td>Accuracy [μm]</td>
<td>1.0</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Response frequency [Hz]</td>
<td>3.03×10³</td>
<td>10.0</td>
<td>18×10³</td>
</tr>
</tbody>
</table>

Fig. 2 Measurement method for comparison of sensors.

Fig. 3 Comparison of curved surface measurement results for different sensor types.

Fig. 4 Influence of surface geometry on measurement results for eddy current-type displacement sensor.
To evaluate the dynamic characteristics of the sensors, their frequency responses were also measured and compared. Figure 5 shows the method used to measure the frequency responses of the sensors. An eccentric cam was included in the measurement tests. The eccentricity of the rotational center was assumed to be the input amplitude, and the amplitude measured by the sensors was assumed to be the output amplitude.

Figure 6 shows the measured frequency responses of the laser displacement and contact-type sensors. The results confirm that the amplitude ratio (output amplitude/input amplitude) of the contact-type displacement sensor increased with increasing frequency and was larger than that of the laser displacement sensor at all frequencies. Conversely, the amplitude ratio of the laser displacement sensor was almost constant due to the higher bandwidth of the response of the displacement sensor.

The laser displacement sensor was adopted as the displacement sensor of the measurement system based on the evaluated characteristics described above. Figure 7 shows the developed measurement system, which consists of three laser displacement sensors along the X-, Y-, and Z-axes. The squareness errors of the each laser displacement sensors are approximately 50 μm / 10 mm, and the coincidences between the laser displacement sensors are adjusted within 20μm. The motion error near the motion direction changing point of the rotary axes can be found from the measurement data collected by these three directional sensors and the angles of the axes.

2.3 Movement for measurement test

In conventional methods, the influence of the translational axes is typically included in the measurement results because in typical synchronous motions of the rotary and translational axes, the translational axes reverse at the motion direction changing point of the rotary axes. Although a method of evaluating the synchronous error of the rotary and translational axes using a ball-bar system near the motion direction changing point excluding the influence of the translational axes has been reported (Sato et al., 2014), it cannot be used to evaluate the maximum influence of the motion error of the rotary axes on the synchronous error, because the measurement results are not identical with the tangential direction of the rotary axes though the maximum motion error of rotary axis appears to the tangential direction (Kato et al., 2013).
Consequently, a movement scheme was designed specifically for the measurement in this study to allow the evaluation of the behavior of only the rotary axes near the motion direction changing point by using the synchronous motion of the two rotary axes. An analysis method for the evaluation of the motion error of each rotary axis based on the measurement results of the three displacement sensors is also proposed here. The tilt axis (A-axis) was chosen as the measurement subject among the two rotary axes (A- and C-axis) because the frequent reversal of the tilt axis, which has a stroke limit, is required. In addition, it is relatively difficult to set the cutting point close to the rotational center of the tilt axis.

In typical synchronous motions of the A-axis and the translational axes, the Y- and Z-axes are reversed at the motion direction changing point of A-axis. In the proposed measurement movement scheme (Table 2), the composite velocity vector of the ball by the measurement movement about C- and A-axes remains in a same quadrant in the Y-Z plane of the machine coordinate system before and after the A-axis reversal, as shown in Fig. 8. The measurement motion of each axis is shown in Fig. 9. The dashed vertical lines in Fig. 9 indicate the motion direction changing points of the A-axis. Although the motion direction of the X-axis changes at the same time as that of the A-axis, the motion of the X-axis does not influence the present evaluation.

![Fig. 8 Schematic of measurement movement at C-axis rotational angle of 180°.](image)

Table 2 Measurement movement scheme for C- and A-axes.

<table>
<thead>
<tr>
<th>Axis</th>
<th>Motion [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>–90.0          +</td>
</tr>
<tr>
<td></td>
<td>0.0            +</td>
</tr>
<tr>
<td></td>
<td>180.0          +</td>
</tr>
<tr>
<td></td>
<td>270.0          +</td>
</tr>
<tr>
<td>A</td>
<td>–45.0          +</td>
</tr>
<tr>
<td></td>
<td>–27.0          (Reverse)</td>
</tr>
<tr>
<td></td>
<td>–63.0          (Reverse)</td>
</tr>
<tr>
<td></td>
<td>–45.0          +</td>
</tr>
</tbody>
</table>

![Fig. 9 Measurement motion of each axis.](image)
The reversal motions of the A-axis and other axes can be dissociated in the proposed movement scheme. The motion direction of the A-axis was reversed at C-axis rotational angles of 0.0° and 180.0°. The velocity vector of the ball about the C-axis in the Y-Z plane reached a maximum at these points. Figure 10 shows the measurement results of each displacement sensor near the motion direction changing points. Because the measurement directions of the sensors were set to the X-, Y-, and Z-directions in the machine coordinate system, the measurement results shown in Fig. 10 indicate the positional deviation of the reference ball in the machine coordinate system. Figure 10 shows that positional errors of almost 30 μm along the Z-axis and –10 μm along the Y-axis occurred at C-axis rotational angles of approximately 180.0° and 0.0°, respectively.

The motion error near the motion direction changing point of the A-axis can be obtained from the data measured by the three sensors. It can be considered that the center of the reference ball is located on the C-axis in the Y-Z plane because the C-axis is perpendicular to the A-axis about this five-axis machining center. From these points of view, the motion error of the A-axis can be obtained as the tangential directional error of the A-axis rotation, which corresponds to the positional error along Y'-axis, as defined in Fig. 11.

The tangential error of the A-axis can be calculated from the measurement data in the machine coordinate system (X, Y, Z) and the rotational angle $\theta_A$ of the A-axis as

$$
\begin{bmatrix}
X' \\
Y' \\
Z'
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta_A & \sin \theta_A \\
0 & -\sin \theta_A & \cos \theta_A
\end{bmatrix} 
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
$$

(1)

$$
Y' = Y \cos \theta_A + Z \sin \theta_A.
$$

(2)

The measurement results transformed into the A-axis coordinate system are shown in Fig. 12. These errors are predicted to cause defects on the machined surface. This was demonstrated by the cutting tests conducted in this study, as described in the next section. Convex errors on the machined surface are defined as being the positive direction, and errors resulting in dents (concave errors) are defined as negative.

![Fig. 10 Comparison of measured motion errors.](image)

![Fig. 11 Coordinate transformation between machine and A-axis coordinate systems.](image)
Based on the above discussion, it can be concluded that the proposed measurement system and movement scheme can be used to evaluate the dynamic behavior of the tilt axis near the motion direction changing points while excluding the influence of the other axes.

3. Cutting test

3.1 Cutting method

In this study, an actual cylindrical cutting test was carried out to inspect the measurement results for the proposed system. A cylindrical shape was adopted as the workpiece for the tests, and the movement during the machining process was set to be identical with abovementioned movement scheme to investigate the influence of the motion error of the A-axis on the machined surface.

Figure 13 illustrates the proposed cylindrical cutting motion. The tool path for the cutting test was offset from the trajectory of the functional point and could be obtained from the geometry of the workpiece and the length and radius of the tool (ball-end mill). The cutting conditions are given in Fig. 14 and Table 3. Cylindrical machining was realized using a helical machining approach from the upper side of the workpiece to its lower side. The radial depth of the cut (cross feed) was set to 0.25 mm to ensure the influence of the cusp height was sufficiently small (1 μm or less) in comparison with the motion errors. In addition, because the influence of the motion errors of the rotary axes on the machined surface were multiplied by the distance from the rotational centers of the rotary axes, the height of the workpiece was chosen based on the height of the reference ball used in the measurement test.
The surface geometry of the machined workpiece was measured by a roundness measuring machine. Figure 15 shows the measured profile and photographs of the machined surfaces. A spike-like dent with a depth of 14 μm was observed in the measured profile at C-axis rotational angles of approximately 0.0° (θ_A = –27.0°), and the error was also observed as a defect in the machined surface. This error was due to the motion error of the A-axis near 0.0°, as shown in Fig. 12(a). The influence of the feed rate and the motion direction changing point of the A-axis on the relationship between the motion error and the machined shape were investigated. The relationships between the machined shape and these two parameters are shown in Fig. 16. As shown in Fig. 16(a), the machining error increased with the A-axis feed rate. This confirms that this machining error depends on the dynamic behavior of the A-axis. In addition, Fig. 16(b) demonstrates that the machining error changed as the motion direction changing point of A-axis was varied. The experimental conditions (the movement of C- and A-axes) for the tests shown in Fig. 16(b) are listed in Table 4. Since the motion direction changing angle of A-axis is changed, it can be expected that the accuracy deterioration is come from the influence of gravity. These behaviors are evident in both the measurement and machining results.

In contrast, the machined surface had no defects at C-axis rotational angles of approximately 180.0° (θ_A = –63.0°), even though a large motion error (28 μm) existed, as shown in Fig. 12(b). In addition, the above-mentioned phenomenon did not change even when the feed rate and motion direction changing point of the A-axis were varied. These results demonstrate that although the proposed cutting test can be used to evaluate the motion error of the tilt axis, the geometric profile of the machined workpiece is not identical with the trajectory measured using the proposed method.

![Fig. 14 Cylindrical machining condition.](image)

### Table 3 Experimental conditions for machining.

<table>
<thead>
<tr>
<th>Tool</th>
<th>Type</th>
<th>Ball-end mill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of flutes</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td>16 mm</td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>A5056</td>
<td></td>
</tr>
<tr>
<td>Form</td>
<td>Column</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td>25 mm</td>
<td></td>
</tr>
<tr>
<td>Machining</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cut depth</td>
<td>2.5 mm</td>
<td></td>
</tr>
<tr>
<td>Cross feed</td>
<td>0.25 mm</td>
<td></td>
</tr>
<tr>
<td>Spindle speed</td>
<td>5000 rpm</td>
<td></td>
</tr>
</tbody>
</table>

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Table 4 Experimental conditions for the tests shown in Fig. 16(b).

<table>
<thead>
<tr>
<th>No.</th>
<th>Axis</th>
<th>Motion [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>C</td>
</tr>
<tr>
<td>(1)</td>
<td>A</td>
<td>-18.0</td>
</tr>
<tr>
<td>(2)</td>
<td>A</td>
<td>-45.0</td>
</tr>
<tr>
<td>(3)</td>
<td>A</td>
<td>-63.0</td>
</tr>
</tbody>
</table>

(a) $\theta_d = -63.0°$  
(rotational angle of C-axis: 180.0°)

(b) $\theta_d = -27.0°$  
(rotational angle of C-axis: 0.0°)

Fig. 15 Machined workpiece.

(a) Influence of feed rate of A-axis  
(rotational angle of C-axis: 0.0°,  
rotational angle of A-axis $\theta_a = -27.0°$)

(b) Influence of motion direction changing point of A-axis  
(rotational angle of C-axis: 0.0°,  
feed rate of A-axis: 50°/min)

Fig. 16 Comparison of measured motion errors.
3.2 Relationship between motion error and machined shape

The influence of the motion error profile and tool geometry on the relationship between the motion error and the surface geometry of the machined workpiece was investigated.

Figure 17 describes the relationship between the tool trajectory with a convex motion error and the surface geometry. For a tool trajectory with a convex motion error, as in the measured data shown in Fig. 12(b), it can be assumed that the tool traces points (a)–(e), as shown in Fig. 17. The maximum height \( l \) of the defects on the machined shape (height of the machining error) was smaller than the height \( H \) of the maximum motion error of the tool between points (b) and (d). The reason for this phenomenon can be described as a function of the tool radius \( R \) and the width \( W \) of the convex motion error, given by

\[
L = \sqrt{2H\left(H^2 + 4R^2\right)^{1/2}} - 2H^2.
\]

Under the condition that the height \( H \) of the tool trajectory is larger than the height \( l \) of the machining error (which is typically true). This equation suggests that the convex motion error is not reflected onto the machined surface when the tool radius is sufficiently larger than the width of the errors.

A cutting test including the simulated motion error was carried out to inspect the accuracy of this assumption, as shown in Fig. 18. In this test, height \( H \) and width \( W \) of the tool trajectory and the tool radius \( R \) were varied as follows: \( H = 0.25, 0.50, \) and \( 1.00 \) mm, \( W = 0.5–6.0 \) mm (at increments of 0.5 mm), and \( R = 1.0, 4.0, 6.0, \) and \( 7.0 \) mm. The height \( l \) of the machining error was measured using a surface roughness meter. In Figs. 19 and 20, the height \( l \) of the machining error is plotted against the width \( W \) of the tool trajectory for various heights \( H \) at a constant tool radius \( R \) of 6.0 mm and for various tool radii \( R \) at a constant height \( H \) of 0.25 mm, respectively. These plots contain the results of the cutting test and the calculation results from Eq. (3).
The characteristics of the motion error are one reason for the difference between the measured and machined results. For the measured results shown in Fig. 12(b), the height $H$ and width $W$ of the tool trajectory are approximately 28 and 157 μm, respectively. The height $l$ of the machining error was calculated to be 0.4 μm from Eq. (3). The tool radius $R$ was 8.0 mm in this case. This calculated value is negligible compared with the contributions from other error sources, such as the vibration of the tool and machine. In addition, the matching error due to the A-axis motion error was not observable on the machined surface, as shown in Fig. 15.

The results demonstrate that the convex motion error cannot be reflected onto the machined surface when the tool radius is sufficiently larger than the width of the motion error.

Fig. 19 Influence of error height $H$ on copying characteristics of motion trajectory.

Fig. 20 Influence of tool radius $R$ on copying characteristics of motion trajectory.
4. Conclusions

In this study, a measurement method and a cutting test were developed for the purpose of evaluating the behavior of the tilt axis of a five-axis machining center near the motion direction changing points about this axis. The newly developed measurement system consists of three laser displacement sensors and a reference ball. Actual measurement and cutting test were carried out to confirm the effectiveness of the proposed methods. The conclusions of this study can be summarized as follows.

1) The proposed measurement system and corresponding movement scheme can be used to evaluate the dynamic behavior of the tilt axis near the motion direction changing points about this axis while excluding the influence of the motion errors of the other axes.
2) Although the proposed cutting test can be used to evaluate the motion error of the tilt axis, the geometric profile of the workpiece machined in this study was not identical to the measured trajectory.
3) Convex motion errors cannot be reflected onto the machined surface if the tool radius is significantly larger than the width of the tool trajectory.

The proposed methods can be applied to the evaluation of the motion error of the rotary axes of five-axis machining centers. In future studies, the authors will theoretically analyze the relationship between the motion errors and the resulting machined shape.

References