Analysis of the effect on gear accuracy of workpiece/tool positioning accuracy in the hobbing process

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Abstract
Accompanying the improvement of vehicle interior quietness in recent years, there has been a need to manufacture low-noise transmission gears at low cost. In response to this need, we adopted a gear honing process as the finishing method following heat treatment in order to reduce the source of gear noise. Previously, gear manufacturing processes generally proceeded in the order of hobbing, gear shaving, heat treatment and gear honing. For the purpose of reducing manufacturing costs, however, there has been a trend in recent years to eliminate gear shaving, resulting in just the processes of hobbing, heat treatment and gear honing. However, with the manufacturing processes of hobbing, heat treatment and gear honing, there are several factors that make it necessary to improve the machining accuracy of the hobbing process. Therefore, this paper describes a hobbing simulation that has been conceived as a first step toward improving hobbing accuracy. This simulation was devised to make clear the effect of the positional relationship between the workpiece and the tool on gear accuracy, including workpiece pitch error and tooth runout. It has been verified on the basis of machining tests. The following results were confirmed by the simulation and machining tests. (1) Work piece positioning accuracy during hobbing has a large effect on pitch error and tooth runout. (2) Tool positioning accuracy greatly affects the tooth profile and helix undulation and also has a large effect on imparting periodic error to pitch error and tooth runout. These results confirmed the effectiveness of the hobbing simulation in determining hobbing process control values.

Keywords : Hobbing simulation, Workpiece/tool positioning accuracy, Gear accuracy, Hobbing process

1. Introduction
As a result of the improvement of vehicle interior quietness in recent years, there is need for low-noise automotive transmission gears that can be manufactured at low cost. In response to this need, we adopted a gear honing process as the gear finishing method following heat treatment as a measure for addressing the sources of gear noise.

Gears have traditionally been manufactured in a series of processes consisting of hobbing, gear shaving, heat treatment and gear honing. However, for the purpose of reducing manufacturing costs, there has been a trend in recent years toward eliminating gear shaving, resulting in just the processes of hobbing, heat treatment and gear honing. This simplified manufacturing method is referred to here as hobbing plus honing.

However, this hobbing plus honing method has certain tendencies. (1) The amount of stock that must be removed from the tooth surface by gear honing increases. (2) Because less cutting force is applied in gear honing than in gear grinding, it is more difficult to eliminate the pitch error that occurs in the hobbing process (Fig. 1). In other words, an
essential requirement for the hobbing plus honing method is that the machining accuracy of the hobbing process must be improved.

A hobbing simulation was developed in this study as a first step toward improving hobbing accuracy. Methods of simulating the gear hobbing process have long been researched (Ariura et al., 1986), (Umezaki et al., 1986), (Chiu et al., 1987), (Chiu et al., 1988), (Chiu et al., 1989), (Chiu et al., 1992), (Gravel. 2013). All of them focused on tool mounting error, rotational error of the hobbing machine itself, or tool manufacturing error, but workpiece mounting error was not included in the factors analyzed. Meanwhile, in recent years, tool manufacturing accuracy has been improved and the structure of hobbing machines has been changed such as by the switch to direct drive through the elimination of the master worm gears of the table.

As a result, the positional relationship between the workpiece and the tool can be presumed to have a greater effect now on gear accuracy.

In view of this situation, we developed a hobbing simulation to make clear the effect of workpiece/tool positioning error on gear accuracy, including workpiece pitch error and tooth space run-out. This hobbing simulation takes into account workpiece mounting error in consideration of actual production machining and has been validated on the basis of machining experiments.

2. Hobbing simulation

In order to make clear the accuracy required in the hobbing process of the hobbing plus honing method, the hobbing simulation can also simulate pitch error and tooth space run-out, in addition to the tooth profile and tooth trace geometry. It also takes into account the hob shift by which the hob is shifted axially so that each workpiece is machined with a new and separate cutting edge in mass production.

2.1 Simulation procedure

This section explains the procedure of the hobbing simulation.
A) The coordinate points of each cutting blade are found from the hob specifications (Fig. 2).
B) A virtual hob like that shown in Fig. 3 is calculated based on the coordinate points of each cutting blade.
C) The coordinates of the shape generating motions determined by the hobbing conditions are converted to the virtual hob. The hobbing conditions include the workpiece and hob specifications, hob setting angle, center distance and hob feedrate.
D) The coordinate point group and envelope of all the cutting blades passing through an arbitrary cross section of the virtual workpiece are found by numerical analysis, and the tooth shape profile around the entire circumference of the workpiece is calculated (Fig. 4).
E) The normal error (fHα, fFα, fHβ, and fFβ) of all the teeth, pitch error and tooth space runout are calculated from the tooth shape profile on the workpiece circumference (Fig. 5).
It will be noted that calculating the tooth shape profile on the workpiece circumference makes it possible to simulate as far as the tooth root fillet shape.

2.2. Definition of coordinate systems

The coordinate systems of the workpiece and the hob are shown in Fig. 6. In hob cutting, for one rotation of the hob on its axis $\psi$, the hob revolves around the workpiece $\mathcal{P}$ and simultaneously moves in the $Z_g$ direction by an amount equal to $f$.

\[ \begin{bmatrix} x_s' \\ y_s' \\ z_s' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Gamma & 0 & -\sin \Gamma & 0 \\ 0 & -1 & 0 & a \\ \sin \Gamma & 0 & \cos \Gamma & -f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix} \quad (1) \]

\[ \begin{bmatrix} x_g' \\ y_g' \\ z_g' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos (-\Psi) & -\sin (-\Psi) & 0 & 0 \\ \sin (-\Psi) & \cos (-\Psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_g \\ y_g \\ z_g \\ 1 \end{bmatrix} \quad (2) \]

In addition, hob revolution $\Psi$ and amount of movement $f$ are given by the following equations.

\[ \Psi = \pm \left\{ \Psi \cdot (1 \pm f \phi) \cdot \frac{\sin \beta}{m_n \cdot \pi \cdot z_n} \cdot z_h \frac{z_u}{z_u} \right\} \quad (3) \]

2.3 Conversion of hob coordinate system to workpiece coordinate system

The following equations hold true for converting the hob coordinate system to the workpiece coordinate system.

Fig. 4 Analysis of tooth profile around entire workpiece circumference in the hobbing

Fig. 5 Gear accuracy vs. pitch error and tooth space run-out
where $x_t$-$y_t$-$z_t$ are the cutting blade coordinate points, $x_g$-$y_g$-$z_g$ are the coordinate points after converting the cutting blade coordinate points to the workpiece coordinate system, $\Gamma$ is the hob setting angle (the direction shown in Fig. 6 is positive), $a$ is the center distance, $f_t$ is the feedrate (mm/rev) (Conv. is positive and climb is negative), $f$ is the amount of hob movement (mm), $\beta$ is the helix angle (right-helix is positive and left-helix is negative), $m_n$ is the normal module, $z_w$ is the number of workpiece teeth, $z_h$ is the number of hob threads, $\psi$ is the angle of hob rotation, $\Psi$ is the angle of hob revolution (clockwise direction around the $Z_g$ axis is positive), and the ±notations in the equation indicate positive for a right-helix hob and negative for a left-helix hob.

2.4. Workpiece mounting error and tool mounting error

One of the major features of the hobbing simulation is that it takes into account outer diameter run-out and end face run-out of the workpiece, which are measured at the positions shown in Fig. 7.

Figure 8 shows the positions where right end run-out and left end run-out of the hob are measured.

\[
f = \frac{\pm f_t}{(2\pi \pm \frac{2 \cdot f_t \cdot \sin \beta}{m_n \cdot z_w}) \cdot \Psi}
\]

2.4.1 Effect of outer diameter run-out $E_o$ of the workpiece

The effect of outer diameter run-out of the workpiece is found by transforming $x_g'$ and $y_g'$ in Eq. (1) as shown in the following matrix.

\[
\begin{bmatrix}
  x_g' \\
  y_g' \\
  z_g' \\
  1
\end{bmatrix} =
\begin{bmatrix}
  \cos \Gamma & 0 & -\sin \Gamma & E_o/2 \cdot \sin(P_o + \Psi) \\
  0 & -1 & 0 & a - E_o/2 \cdot \cos(P_o + \Psi) \\
  \sin \Gamma & 0 & \cos \Gamma & -f \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_t \\
  y_t \\
  z_t \\
  1
\end{bmatrix}
\]

(5)
2.4.2 Effect of end face run-out \( E_f \) of the workpiece

The effect of end face run-out of the workpiece is found by transforming Eq. (2) as shown in the following matrix.

\[
\begin{bmatrix}
  x_s \\
y_s \\
z_s \\
1
\end{bmatrix} =
\begin{bmatrix}
  \cos(-\Psi) & -\sin(-\Psi) & 0 & 0 \\
  \cos \varepsilon \sin(-\Psi) & \cos \varepsilon \cos(-\Psi) & \sin \varepsilon & 0 \\
  -\sin \varepsilon \sin(-\Psi) & -\sin \varepsilon \cos(-\Psi) & \cos \varepsilon & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_e' \\
y_e' \\
z_e' \\
1
\end{bmatrix}
\]

\( \varepsilon = \sin^{-1} \left( \frac{E_f}{d_a} \right) \)  

(7)

2.4.3 Effect of left end run-out \( E_{il} \) and right end run-out \( E_{ir} \) of the hob

The effect of left end and right end run-out of the hob is calculated with the equations below.

\[
\begin{align*}
y_{icr} &= \frac{E_{ir}}{2} \cos(P_{ir} + \psi) \\
z_{icr} &= -\frac{E_{ir}}{2} \sin(P_{ir} + \psi) \\
y_{iel} &= \frac{E_{il}}{2} \cos(P_{il} + \psi) \\
z_{iel} &= -\frac{E_{il}}{2} \sin(P_{il} + \psi) \\
x_{iCR} &= \sqrt{L_n^2 - (y_{iCR} - y_{iCL})^2 - (z_{iCR} - z_{iCL})^2} / 2 \\
x_{iCL} &= -x_{iCR}
\end{align*}
\]

(8)

\[
\begin{align*}
\theta_{xy} &= \tan^{-1} \left( \frac{y_{iCR} - y_{iCL}}{x_{iCR} - x_{iCL}} \right) \\
\theta_{xz} &= \tan^{-1} \left( \frac{z_{iCR} - z_{iCL}}{x_{iCR} - x_{iCL}} \right)
\end{align*}
\]

(9)

\[
\begin{align*}
T_x &= H_s \\
T_y &= (y_{iCR} + y_{iCL}) / 2 \\
T_z &= (z_{iCR} + z_{iCL}) / 2
\end{align*}
\]

(10)

\[
\begin{bmatrix}
  x_i \\
y_i \\
z_i \\
1
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta_{xz} \cos \theta_{xy} & \cos \theta_{xz} \sin \theta_{xy} & -\sin \theta_{xz} & -T_s \cos \theta_{xz} \cos \theta_{xy} + T_s \\
  -\sin \theta_{xy} & \cos \theta_{xy} & 0 & T_s \sin \theta_{xy} + T_y \\
  \sin \theta_{xz} \cos \theta_{xy} & \sin \theta_{xz} \sin \theta_{xy} & \cos \theta_{xz} & -T_s \sin \theta_{xz} \cos \theta_{xy} + T_z \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_r0 + H_s \\
y_r0 \\
z_r0 \\
1
\end{bmatrix}
\]

(11)

The cutting blade coordinates are calculated with Eqs. (8) to (11) in cases where hob run-out occurs. The notation \( L_0 \) is the overall hob length, \( P_0 \) is the initial phase where outer diameter run-out of the workpiece is maximum, \( P_1 \) is the initial phase where hob run-out is maximum, \( x_{CR}-Y_{0C}-Z_{0C} \) are the central coordinates of the left and right ends of the hob axis, \( \theta_{xy} \) is the hob axis deflection angle on the \( X_r-Y_r \) plane and \( \theta_{xz} \) is that on the \( X_r-Z_r \) plane, \( H_s \) is the amount of hob shift, \( T_r-T_y-T_z \) indicate the movement of the hob axis center point to \( X_r-Y_r-Z_r \), and \( x_{r0}-y_{r0}-z_{r0} \) are theoretical cutting blade coordinates without any error. The suffixes \( R \) and \( L \) indicate right and left.
2.5 Analysis of normal error

Figure 9 shows the method of calculating normal error based on the tooth profile coordinate values. The specific procedure is explained below.

A) Calculate the tangent between point $P$ and the base circle $r_b$ and the coordinates of point $B$.

B) Calculate the normal length error $L'$ from point $P$ and point $B$.

C) Find the theoretical normal length $L$ using the following equation.

$$
\alpha = \tan^{-1}(\xi + \eta) \\
L = (\alpha + \text{inv} \alpha) \cdot r_b
$$

D) Calculate the normal error, Error, as $L' - L$.

E) Calculate the normal error by repeating steps A) to D) for the tooth profile coordinate points of the right and left flanks of all the teeth.

It will be noted that $\eta$ is an arbitrary angle that is given near the involute origin of each tooth.

3. Validation of hobbing simulation based on actual machining experiments

Machining experiments were conducted to verify the validity of the hobbing simulation. The target gear used in the experiments was the primary reduction gear (i.e., idler gear) of the CVT, which is designed for use on midsize vehicles (Mizuochi et al., 2013). A cross-sectional view of the CVT is shown in Fig. 10 along with the target gear. The gear specifications are listed in Table 1 and the hob specifications in Table 2. It will be noted that this hob has standard specifications that are used in production machining.

<table>
<thead>
<tr>
<th>Table 1 Gear specifications</th>
<th>Target gear (Idler gear)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module</td>
<td>mm 1.95</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>deg. 17.2</td>
</tr>
<tr>
<td>Helix angle</td>
<td>deg. 30.5(LH)</td>
</tr>
<tr>
<td>Number of teeth</td>
<td>- 52</td>
</tr>
<tr>
<td>Tip diameter</td>
<td>mm 123.5</td>
</tr>
<tr>
<td>Root diameter</td>
<td>mm 112.038</td>
</tr>
<tr>
<td>Face width</td>
<td>mm 28.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2 Hob specifications</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Module</td>
<td>mm 1.95</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>deg. 17.2</td>
</tr>
<tr>
<td>Number of gashes</td>
<td>16</td>
</tr>
<tr>
<td>Number of starts</td>
<td>- 3(LH)</td>
</tr>
<tr>
<td>Tip diameter</td>
<td>mm 80</td>
</tr>
<tr>
<td>Total length</td>
<td>mm 180</td>
</tr>
</tbody>
</table>

Fig. 10 CVT cross-sectional view & target gear

The effects of the tool rotational speed, tool feedrate, workpiece run-out and hob cutter run-out were investigated in order to verify the validity of the hobbing simulation. The tool feedrate was used to examine the tool marks (undulations in the tooth profile and trace directions) that appear on the workpiece tooth surface following the hobbing process. Workpiece run-out was used to facilitate an investigation of this parameter, which is one of the distinctive capabilities of the hobbing simulation. Table 3 shows the six types of machining experiments that were conducted using different combinations of four factors: tool rotational speed (rpm), feedrate (mm/rev.) workpiece run-out (μm) and hob cutter run-out (μm).

In experiment 1, the tool feedrate was set at 1.5 mm/rev while suppressing workpiece run-out and hob run-out. The condition given in experiment 2 was a tool feedrate of 0.5 mm/rev. The condition given in experiment 3 was workpiece outer diameter run-out of 50 μm. The condition given in experiment 4 was workpiece outer diameter run-out of 100 μm. The condition given in experiment 5 was workpiece end face run-out of 50 μm. The condition given in experiment 6 was workpiece end face run-out of 100 μm.
Table 3 Experimental conditions

<table>
<thead>
<tr>
<th>Exp</th>
<th>Speed (m/min)</th>
<th>Feed (mm/rev)</th>
<th>Workpiece run-out</th>
<th>Hob run-out</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Outer (μm)</td>
<td>Face (μm)</td>
</tr>
<tr>
<td>1</td>
<td>150</td>
<td>1.5 Climb</td>
<td>0 target</td>
<td>0 target</td>
</tr>
<tr>
<td>2</td>
<td>↑</td>
<td>0.5 Climb</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>3</td>
<td>↑</td>
<td>1.5 Climb</td>
<td>50 target</td>
<td>↑</td>
</tr>
<tr>
<td>4</td>
<td>↑</td>
<td>↑</td>
<td>100 target</td>
<td>↑</td>
</tr>
<tr>
<td>5</td>
<td>↑</td>
<td>↑</td>
<td>0 target</td>
<td>50 target</td>
</tr>
<tr>
<td>6</td>
<td>↑</td>
<td>↑</td>
<td>0 target</td>
<td>100 target</td>
</tr>
</tbody>
</table>

Five workpieces (n = 5) were machined in each experiment with the hob shift position set in the center. Workpiece run-out and hob run-out were measured before each workpiece was machined to confirm that the targeted run-out values shown in Table 3 were obtained with the machining settings. The workpiece run-out values measured in each experiment (n = 5) are plotted in Fig. 11. The measured data confirmed that workpiece run-out was consistent with the targeted values.

Table 4 shows the measured left end and right end run-out of the hob and the corresponding phase values. These results confirmed that the run-out values obtained with the settings were within the targets for hob run-out in Table 3. Hob run-out values did not change in all six experiments.

Table 4 Measured values of hob run-out in each experiment

<table>
<thead>
<tr>
<th>Hob run-out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left (μm)</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

Based on the foregoing measured results for experiments 1-6, the following seven parameters were presumed to be the most important items for obtaining the required gear accuracy in the hobbing process: (1) tooth profiles, (2) tooth traces, (3) cumulative pitch error, (4) tooth run-out, (5) periodic error, (6) fFα average tooth trace undulation of all the teeth, and (7) fHα variation in helix angle of all the teeth. A comparison of the measured data and simulated results for these seven items verified the validity of the hobbing simulation. Periodic error will be explained in detail in section 3.3.

### 3.1 Comparison of tooth profiles

Figure 12 compares the simulated and measured results for tooth profiles, which are representative of hobbing accuracy. The left side shows the tooth profiles for experiment 1 and the right side the tooth profiles for experiment 4. The upper graphs are for the right tooth flank and the lower graphs are for the left tooth flank. The upper blue lines are the measured tooth profiles and the lower black lines are the simulated tooth profiles. The results confirmed that the measured and simulated tooth profiles agreed well. The results obtained in the other experiments also showed good agreement between the measured and simulated profiles.
3.2 Comparison of tooth traces

A comparison of measured and simulated tooth traces is shown next in Fig. 13. The tooth trace data are shown for comparison in the same way as the tooth profile data. It was found that the measured and simulated tooth traces also showed good agreement with regard to the number of undulations and their shapes. The measured and simulated tooth traces also agreed well in the other experiments.

Because the measured and simulated results showed good agreement for both the tooth profiles and tooth traces, the validity of the hobbing simulation was verified on the basis of the tooth surface shape data.

3.3 Gear accuracy verification results

Having confirmed that the measured and simulated tooth profiles and tooth traces agreed well, the simulated and measured values for gear pitch accuracy were compared to verify the validity of the simulation.

When there is hob run-out, a machined gear displays periodic error originating in pitch error components, as shown in the left-hand graph in Fig. 14. Periodic error has the characteristic of the periodic components of an integer order (= number of teeth divided by the number of starts). In these experiments, the gears had 52 teeth and there were three starts, so periodic error appeared in the 17th order component. If periodic error is included in the pitch error after the hobbing process, it cannot be removed by gear honing and may become the cause of abnormal noise. Therefore, a frequency analysis was performed on the workpiece cumulative pitch error data using the Fourier transform in order to determine whether periodic error caused by hob run-out could be evaluated by the simulation. The evaluation results are shown in the right-hand graph in Fig. 14. The results for the cumulative pitch error, tooth space run-out and periodic error (17th order) are shown in Figs. 15, 16 and 17, respectively. In all three figures, the vertical axis shows the measured values and the horizontal axis the simulated values.
These results confirmed that the measured and simulated values also agreed well for the cumulative pitch error, tooth space run-out and periodic error of the machined workpieces, thereby verifying the validity of the simulation.

The simulated results were then verified for the workpiece gear accuracy \( f_{F\alpha} \) average and \( f_{H\beta} \) variation in the hobbing process, which affect the amount of tooth surface stock removal in the gear honing process. \( f_{F\alpha} \) average is found by calculating the tooth trace undulation for all the teeth and taking the average value. \( f_{H\beta} \) variation shows the difference between the maximum and minimum helix angle error of all the teeth.

Figures 18 and 19 show the relationship between the simulated and measured values for \( f_{F\alpha} \) average and \( f_{H\beta} \) variation, respectively. The measured and simulated results for \( f_{F\alpha} \) average and \( f_{H\beta} \) variation also show good agreement, thereby verifying the validity of the simulation.

The validation results presented in sections 3.1, 3.2 and 3.3 above thus confirmed the validity of the newly developed hobbing simulation.

4. Identification of important control factors in the hobbing plus honing method

The hobbing simulation was then used to identify the important control factors, and to determine their control values, in the hobbing process that are the keys to improving the gear accuracy obtained with the hobbing plus honing method. Various run-out values were applied to the workpiece and the hob, having the specifications listed respectively in Tables 1 and 2, and simulations were conducted to investigate the effect on cumulative pitch error and periodic error.
The tool feedrate used in the simulations was set at 1.5 mm/rev climb, which is a standard value for mass production, and the following six factors were analyzed: (1) effect of workpiece outer diameter run-out, (2) effect of workpiece end face run-out, (3) effect of the interaction between workpiece outer diameter run-out and end face run-out when both were present, (4) effect of the initial phase of hob left end and right end run-out, (5) effect of hob left end and right end run-out in opposite phase, and (6) effect of the hob shift.

4.1 Effect of workpiece run-out

The effect of workpiece run-out on cumulative pitch error and periodic error (17th order) was analyzed for three types of run-out conditions: for workpiece outer diameter run-out alone, for workpiece end face run-out alone, and for both workpiece outer diameter and end face run-out together. In order to clarify the effect of workpiece run-out alone, hob run-out was set at 0 μm.

4.1.1 Effect of workpiece outer diameter run-out

The effect of workpiece outer diameter run-out on cumulative pitch error and periodic error is shown in Fig. 20. In the left-hand graph, the vertical axis shows the cumulative pitch error in relation to the outer diameter run-out along the horizontal axis. In the right-hand graph, the vertical axis shows the periodic error (17th order) in relation to the outer diameter run-out along the horizontal axis. The blue and orange bars show the simulated results for the left and right tooth flanks, respectively. The workpiece outer diameter run-out was varied from 0 to 200 μm in 50 μm increments in the simulation. The results in the left-hand graph show that the cumulative pitch error of the left and right tooth flanks nearly coincides with the amount of outer diameter run-out, indicating that this run-out has a very large effect on cumulative pitch accuracy.

In contrast, the results in the right-hand graph show that even for large outer diameter run-out of 200 μm, periodic error is only 0.3 μm, indicating that the effect is very small.

4.1.2 Effect of workpiece end face run-out

Figure 21 shows the effect of workpiece end face run-out on cumulative pitch error and periodic error. In the left-hand graph, the vertical axis shows the cumulative pitch error in relation to the end face run-out along the horizontal axis. In the right-hand graph, the vertical axis shows the periodic error (17th order) in relation to the end face run-out along the horizontal axis. The workpiece outer diameter run-out was varied from 0 to 200 μm in 50 μm increments in the simulation. The results in the left-hand graph show that the cumulative pitch error of the left and right tooth flanks is about one-half of the workpiece end face run-out, indicating that the effect is about one-half as much as that seen for the workpiece outer diameter run-out. The results in the right-hand graph show that the effect on periodic error is very small, similar to that seen for the outer diameter run-out.
4.1.3 Effect of interaction between workpiece outer diameter and end face run-out

Figure 22 shows the effect of workpiece outer diameter run-out and end face run-out together as well as their phase difference on cumulative pitch error and periodic error. In the left-hand graph, the vertical axis shows the cumulative pitch error in relation to the phase difference between the outer diameter run-out and end face run-out along the horizontal axis. In the right-hand graph, the vertical axis shows the periodic error (17th order) in relation to the phase difference between the outer diameter and end face run-out. In the simulation, both types of run-out were set at 100 μm and their phase difference was varied from 0° to 300° in 60° increments.

The results show that, depending on the phase difference, there are cases where the cumulative pitch error is added and becomes maximum and other cases where it is cancelled out and becomes minimum. These results made it clear that the phase difference between workpiece outer diameter and end face run-out must be taken into account when analyzing the effect of workpiece run-out on gear accuracy. It will be noted that even in the case of both workpiece outer diameter run-out and end face run-out together, the effect on periodic error is very small.

4.2 Effect of hob run-out

A simulation was then conducted to analyze the effect of hob run-out on cumulative pitch error and periodic error (17th order). Table 5 shows typical examples of hob run-out that can occur during gear production. Example 1 for hob left end and right end run-out in the same phase rarely occurs in mass production. In most cases, the left end run-out and right end run-out of the hob are in different phases. In example 3 in particular where the left end run-out and right end run-out of the hob are in opposite phase (= 180°), hob end face run-out becomes maximum. It is easy to imagine that it has the largest effect on the accuracy of machined gears in this case. Therefore, the simulation was conducted under a condition where the left end run-out and right end run-out of the hob were in opposite phase (= 180°). Workpiece run-out was set at 0 μm in order to identify the effect of hob run-out alone.
### 4.2.1 Effect of the initial phase of hob left end and right end run-out

Figure 23 shows the effect of the initial phase of hob left end and right end run-out on cumulative pitch error and periodic error. In the left-hand graph, the vertical axis shows the cumulative pitch error in relation to the initial phase of the hob left end and right end run-out. In the right-hand graph, the vertical axis shows the periodic error (17th order) in relation to the initial phase along the horizontal axis. It will be noted that the hob left end run-out and right end run-out were in opposite phase (= 180°) as in example 3 in Table 5. The initial phase of hob right end run-out was varied from 0° to 315° in 45° increments while varying the initial phase of hob left end run-out from 180° to 135° in 45° increments. Hob run-out at both ends was set at 100 μm in the simulation.

The results of this simulation revealed that the effect on the left and right flanks of the workpiece differed depending on the initial phase of the hob left end and right end run-out. In other words, when simulating hob run-out, it is necessary to consider the initial phase of the run-out and also that the left end run-out and right end run-out should be in opposite phase. Notably, the results revealed that the effect on cumulative pitch error was smaller compared with that of workpiece run-out, but, on the other hand, the effect on periodic error was very large.

![Fig. 23 Effect of initial phase of hob left end and right end run-out](image)

### 4.2.2 Effect of left end and right end run-out in opposite phase

A simulation was then conducted to analyze the effect of the amount of hob run-out. Figure 24 shows the effect of the amount of hob run-out on cumulative pitch error and periodic error. In the left-hand graph, the vertical axis shows the cumulative pitch error in relation to the amount of hob run-out along the horizontal axis. In the right-hand graph, the vertical axis shows the periodic error (17th order) in relation to the amount of hob run-out along the horizontal axis. Both the hob left end run-out and right end run-out were varied from 0 to 200 μm in 20 μm increments in the simulation. The right end run-out and left end run-out were initially in opposite phase (= 180°), with the initial phase of the left end run-out set at 0° and that of the right end run-out at 180°. Because of the effect of the initial phase of hob run-out, the impact on the right flank of the workpiece was larger.

A characteristic of the results is that cumulative pitch error and periodic error display high sensitivity to hob run-out until approximately 80 μm, indicating that hob run-out has a large effect on gear accuracy, but sensitivity
decreases at larger run-out values. In other words, it has a large effect on cumulative pitch error and periodic error in the region of small hob run-out, making it clear that special attention should be paid to controlling hob run-out.

4.2.3 Effect of the hob shift

The hob is shifted in the axial direction in actual production so that each workpiece is machined with a new and separate cutting edge. When there is hob cutter run-out, presumably the effect of that run-out increases closer to the end of the hob because of the hob shift. Therefore, a simulation was run to make clear the effect of the hob shift on the workpiece, and the results are presented in Fig. 25. In the left-hand graph, the vertical axis shows the cumulative pitch error in relation to the hob shift amount along the horizontal axis. In the right-hand graph, the vertical axis shows the periodic error (17th order) in relation to the hob shift amount along the horizontal axis. Based on the hob and workpiece specifications, the hob shift amount was determined in the range where generative machining was viable. Hob left end run-out and right end run-out were in opposite phase (= 180°) and hob run-out at both ends was set at 100 μm in the simulation. In addition, because it was made clear in section 4.2.1 that the initial phase of hob run-out must be considered, the initial phase of hob right end run-out was varied from 0° to 315° in 45° increments. The initial phase of hob left end run-out was varied from 180° to 135° in 45° increments. The maximum values were taken as the simulation values.

The results confirmed that the effect of hob run-out increased closer to the hob end. The effect of the hob shift on the left tooth flank was the greatest when the amount of hob shift was +73.7 mm for machining with the left end of the hob. The effect of the hob shift on the right tooth flank was the greatest when the amount of hob shift was -73.7 mm for machining with the right end of the hob. These results signify that simulations should be conducted at the maximum ends of the hob shift range.

The results of the investigations described in sections 4.1 and 4.2 above made clear the simulation conditions under which the workpiece run-out and hob cutter run-out that might occur in gear production would have the largest effect on gear accuracy.
5. Hobbing process control values

Run-out control values were then determined for the hobbing process, assuming the use of the hobbing plus honing method in gear production, and a simulation was conducted using the above-mentioned conditions where workpiece run-out and hob run-out would have the largest effect on gear accuracy. The purpose was to determine whether the required gear accuracy could be satisfied. Table 6 shows the hobbing process control values, and Table 7 shows the simulated values for workpiece cumulative pitch error and periodic error. The control value for hob left end run-out was set larger than that for right end run-out in order to take into account the difficulty of accurately clamping the hob arbor to the sub-spindle at the left end of the hob. The results confirmed that the hobbing process control values were sufficiently feasible for mass production and that the required gear accuracy was satisfied.

<table>
<thead>
<tr>
<th>Table 6 Process control values</th>
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<tbody>
<tr>
<td>Workpiece run-out</td>
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<td>Outer (μm)</td>
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<td>10</td>
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<tr>
<th>Table 7 Simulated values for cumulative pitch error and periodic error from process control values</th>
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<tr>
<td>Pitch error</td>
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<tr>
<td>Left flank (μm)</td>
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<td>23.2</td>
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</table>

6. Conclusions

A hobbing simulation was developed for analyzing the effect of workpiece/tool positioning accuracy in the hobbing process on gear accuracy, including cumulative pitch error and tooth space run-out. As a result of conducting machining experiments to verify the validity of the simulation, the following results were made clear.

(1) Workpiece positioning accuracy has an especially large effect on cumulative pitch error. Tool positioning accuracy has a large effect on the tooth profile and tooth trace undulations. In addition, it also has a large effect on imparting periodic error to cumulative pitch error. These effects were confirmed by the results of simulations and actual machining experiments.

(2) The effects of workpiece run-out and hob run-out that might occur in mass production were found to be the largest under the following simulation conditions. When simulating workpiece run-out, the phase difference between workpiece outer diameter run-out and end face run-out must be taken into account. When simulating hob run-out, hob left end run-out and right end run-out in opposite phase (= 180°) and also their initial phase must be considered. When simulating the hob shift, the very end values of the hob shift range should be used in the simulation.

(3) Based on the use of this hobbing simulation, a procedure was explained for determining the process control values for workpiece run-out and hob run-out in the hobbing process of the hobbing plus honing method.

It should be noted that simulations must match the actual conditions involved because the effect of each type of run-out on gear accuracy will vary if the workpiece and hob specifications and the feedrate are changed.

References


Chiu, H., Ariura, Y., Ueno, K., Ozaki, H., Sato, E., Improvement of Gear Accuracy in Gear Hobbing 6th Report,