Geometry generation principle and meshing properties of a new gear drive

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Received: 29 September 2017; Revised: 26 November 2017; Accepted: 3 January 2018

Abstract

The generation principle of composite cycloid is proposed for the tooth profile of external drive. Firstly, a two-link mechanism is developed as the equivalent mechanism to describe the geometric principle of cycloid by introducing the motion transforming method. Then the path curve of \( n \)-order cycloid motion is generalized using \( n+1 \) link mechanism. The new second-order, third-order and fourth-order composite cycloid equations of tooth profiles, including the corresponding link mechanisms, are derived and compared. It is found that the fourth-order composite cycloid is more suitable and potential for the gear design. Secondly, based on differential geometry and meshing theory, the mathematical models, including original composite cycloid profile, meshing equation, conjugate profile and meshing line are established. Subsequently, the meshing properties, such as contact ratio, sliding ratio and mechanics property analysis are conducted and compared with involute gear drive. Transmission efficiencies under different operating conditions are performed on the FZG gear test rig. Theoretical and experimental results demonstrate that greater contact ratio, smaller sliding ratio, superior bending and contact stress, and high efficiency of the new gear are represented in comparison with the involute gear.

Keywords: Gear drive, Composite cycloid gear, Meshing characteristics, Mathematical model

1. Introduction

As one of key component, gears are widely used in various industrial applications for power speed and torque conversion. At present, the main types of tooth profiles are involute, circular-arc and cycloid. Especially, considering the production and assembly in practical application, the involute profile is the most commonly tooth profile. However, some shortcomings of the three tooth profiles are pointed out as follows: The involute profile suffers from relatively low load capacity, poor lubrication and proneness to interference. The inevitable axial load of helical profile of circular arc is a big shortcoming. Because of the geometric principle and conjugate theory of cycloid drive, the gears such as pump and watch gears cannot be applied in external drive for power transmission except to motion transmission (Luo, et al., 2008), (Liang, et al., 2015).

Theoretically, the meshing tooth profiles are the fundamental element to determine transmission performance of a gear drive. Due to the demands of high performance such as high power density, high speed, high precision and high reliability in recent decades, the gear drive, which has higher bending strength, contact strength and transmission efficiency, is needed increasingly and urgently. The further enhancement of gear performance is limited by the current gear design technique. Therefore, new kinds of tooth profiles should be proposed and developed to fulfill the high performance demand (Qin, 2014). Cycloid drive, which has big transmission ratio, compact size and large load capacity including high bending and contact strength, has been the popular gear reducers of small teeth difference planetary applied in precision transmission such as robot and aerospace areas (Li, 2014). A lot of researches about cycloid drives, including conjugate theory (Chang, 2003), meshing characteristics (Hwang, 2007) and mechanical properties (Biernecki, 2015), have been reported. It should be a valuable research topic that a new type of cycloid profile which have these disadvantages in the external drive can be developed. The main studies dealing with some new types of
cycloid profiles are conducted by XUE (2005), LI (2010), and CHEN (2012). XUE (2005) proposed a new cycloid drive with cycloid rack and pin roller gear. LI (2010) derived a new type of universal cycloid gear with relatively small sliding ratio. CHEN (2012) developed a combined profiles of curtate epicycloid and prolate hypocycloid that can realize the minimum teeth number of six teeth without undercutting. In general, the researches on the new type of cycloid profile are less relatively, and it is well worth studying.

To create a new type of cycloid profile with large contact ratio, small sliding ratio and high load capacity, a motion transforming method of equivalent two-link mechanism is introduced to describe the generation principle of traditional cycloid, then \( n \)-orders cycloid motion can be generalized using \( n+1 \) link mechanism. The fourth-order composite cycloid which is more suitable for gear drive can be deduced. Meshing characteristics and mechanics properties of this new gear drive are conducted in comparison with involute gear. The transmission efficiencies of gear samples are also performed. Finally, some valuable conclusive summaries of this study are drawn.

2. Geometric principle of composite cycloid

2.1 Geometric principle of cycloid

As shown in Fig. 1 (a) and Fig. 1 (b), when the rolling circle of radius \( r_g \) rotates counterclockwise around the circumference of the base circle of radius \( r_0 \) in a pure rolling motion, point \( M \) is attached to or fixed on the rolling circle. So the trajectory of point \( M \) will trace a cycloid path (LI, 2010), (CHEN, 2012). When the point \( M \) is locating \((\lambda_1=1)\), inside \((0<\lambda_1<1)\) and outside \((\lambda_1>1)\) the rolling circle respectively, point \( M \) will generate the equal-amplitude cycloid, curtate cycloid and prolate cycloid. Accordingly, the cycloids in Fig. 1(a) and Fig. 1(b) are epicycloid and hypocycloid. As a simplification, these six cycloids can be uniformly called generalized cycloid in this paper. Respectively, the equations of cycloid in Fig. 1(a) and Fig. 1(b) can be expressed as Eq. (1) and Eq. (2).

\[
\begin{align*}
    x(\alpha) &= e_0 \cos \alpha \pm e_1 \cos(z+1)\alpha \\
    y(\alpha) &= e_0 \sin \alpha \pm e_1 \sin(z+1)\alpha \\
    e_0 &= r_0 + r_g \\
    x(\alpha) &= e_0 \cos \alpha \mp e_1 \cos(z-1)\alpha \\
    y(\alpha) &= e_0 \sin \alpha \mp e_1 \sin(z-1)\alpha \\
    e_0 &= r_0 - r_g 
\end{align*}
\]

where \( z \) represents the periodicities of cycloid curve \( \Sigma \), \( z = r_0/r_g \), \( \lambda_1 \) is the prolate coefficient, \( e_1 \) is the distance from point \( M \) to the center of rolling circle and can be defined as \( e_1 = \lambda_1 r_g \). As for the symbols “±” and “∓”, the upper symbols are adopted when point \( M \) is located to the left of center \( O_1 \), otherwise the lower symbols are adopted when point \( M \) is located to the right of center \( O_1 \).

![Fig. 1 Geometric principle of cycloid](image-url)

2.2 Equivalent two-link mechanism of cycloid motion

As shown in Fig. 2, the geometric principle of cycloid can be considered as the two-link mechanism in which link \( O_1O \) is the side link and link \( O_1O_M \) is the follower link. Then the geometric principle of cycloid can be described as the movement trajectory of point \( M \) which is locating on the follower \( O_1O_M \). The geometric property of cycloid is mainly controlled by parameters \( r_0, r_g \) and \( \lambda_1 \). Obviously, the control parameters are too small to limit the application range of cycloid profile, and cycloid profile is only applied in some specific fields such as pin-cycloid planetary reducer, clock gear and rotor pump. Therefore, in order to improve the geometric property of cycloid profile for external drive, some
innovations should be worked out in the next sections.

![Equivalent two-link mechanism of cycloid](image)

**Fig. 2 Equivalent two-link mechanism of cycloid**

### 2.3 New Composite Cycloid

Based on the motion transforming method of equivalent two-link mechanism of cycloid principle, the path curve of \( n \)-order cycloid motion can be expanded and generalized by using the expanding \( n+1 \) link mechanism, as shown in Fig. 3. In the initial position, the link endpoint \( A_{n-1} \) is located on the left side of the point \( A_n \). The moving direction of each link is counterclockwise, and the polar equation of point \( P_m \) can be expressed as

\[
I(\alpha) = x + I = e_{n-1}^{+1 \alpha} e^{i\omega_{n-1} t}, \quad \alpha_{n-1} = \omega_{n-1} t
\]

where \( e_{n-1}, \omega_i (i = 1 \sim n + 1) \) and \( \alpha_n \) donate the angular velocity, rotation angle and length of each link respectively, the trajectory curve of endpoint is defined as \( n \)-orders cycloid.

![\( n \)-order cycloid generated by \( n+1 \)-link mechanism](image)

**Fig. 3 \( n \)-order cycloid generated by \( n+1 \)-link mechanism**

Then the point \( P_m \) in cartesian coordinate system can be represented as

\[
\begin{align*}
x(\alpha_n) &= e_1 \cos \alpha_n + e_2 \cos \alpha_1 + \ldots + e_n \cos \alpha_n \\
y(\alpha_n) &= e_1 \sin \alpha_n + e_2 \sin \alpha_1 + \ldots + e_n \sin \alpha_n
\end{align*}
\]

(4)

Based on the geometric characteristic of link mechanism, the only certain trajectory of endpoint can be determined when the length \( e_i \) and rotation angle \( \alpha_n \) of each link are defined. Consequently, different initial positions of each link, which have only relationship with \( \alpha_n \), can not change the whole trajectory. So the Eq. (4) can be defined as generalized \( n \)-order cycloid equation, and the generalized cycloid of equivalent two-link mechanism can be regarded as the first-order cycloid.

If the cycloid profile can be applied effectively for power transmission of parallel axles with external engagement engage, the control parameters of cycloid tooth should be added to increase contact ratio to change the geometric characteristics of traditional cycloid fundamentally. Based on the transforming method on link mechanism of cycloid motion, the generalized \( n \)-orders cycloid is referred, then the two-link cycloid motion can be expand to three-link, four-link and five-link mechanism. However, the most important difference is the joint pattern of each circles which is making rolling motion inside and outside the front circle in turn. Accordingly, the new composite cycloid of second-order, third-order and fourth-order can be generated. The equation of these three new cycloid can be written as

\[
\begin{align*}
x(\alpha) &= r_1 \cos \alpha - c_1 \cos(z-1) \alpha - c_2 \cos(z+1) \alpha + c_3 \cos(2z-1) \alpha + c_4 \cos(2z+1) \alpha \\
y(\alpha) &= r_1 \sin \alpha + c_1 \sin(z-1) \alpha - c_2 \sin(z+1) \alpha + c_3 \sin(2z-1) \alpha + c_4 \sin(2z+1) \alpha
\end{align*}
\]

(5)
where $\alpha$ is the link angle in initial position, $c_1$, $c_1'$, $c_2$ and $c_2'$ are the length of each link. From the relationship among each length listed in Table 1, the curves can be classified into second-order, third-order and fourth-order composite cycloid respectively, the gear parameters including $r$, $r_f$, $r_s$, $h_r$ and $h_a$ of the three tooth profiles of composite cycloid can be deduced without immediate considering of tip clearance coefficient $c^*$. Where $r$ is the radius of reference circle, $m$ is the modulus of tooth profile, and $r = 0.5mz$.

It can be known from Table 1 that a certain composite cycloid is decided by the teeth $z$, module $m$, addendum coefficient $h_r'$ and tip clearance factor $c^*$, notice that in this case the involute gear still has the variable pressure angle under different working conditions. In addition, the gear parameters of third-order cycloid profile are against both the gear parametrization design and the follow-up study of transmission characteristics.

In a similar motion mode of $n+1$-link mechanism, the $n$-order composite cycloid can be obtained and deduced. The overall process is similar to the above composite cycloid, so we do not describe them again here. When $n \geq 5$, $c_s$ can be deduced as $c_s = r_l \frac{\prod \lambda_i}{z''}$, the expression of $c_n$ shows that the end-link length follows an exponential decreasing. Then the high curve sensitivity, the intractable machining and the practical application will be resulted. Therefore, the five or bigger orders composite cycloid will not be researched in this article.

To sum up the above study of composite cycloid, it can be known that the fourth-order composite cycloid has a potential to the application of tooth profile.

### Table 1 Parameters of composite cycloid

<table>
<thead>
<tr>
<th>Profile</th>
<th>Second-order</th>
<th>Third-order</th>
<th>Fourth-order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length relation of each link</td>
<td>$c_1 = c_1' \neq 0$, $c_2 = c_2' = 0$</td>
<td>$c_1 = c_1' \neq 0$, $c_2 = c_2' = 0$, $c_3 = 0$</td>
<td>$c_1 = c_1' \neq 0$, $c_2 = c_2'$, $c_3 = 0$</td>
</tr>
<tr>
<td>Reference circle $r$</td>
<td>$r_1$</td>
<td>$r_1 + c_2$</td>
<td>$r_1$</td>
</tr>
<tr>
<td>Root radius $r_f$</td>
<td>$r_1 - 2c_1$</td>
<td>$r_1 - 2c_1 - c_2$</td>
<td>$r_1 - 2c_1$</td>
</tr>
<tr>
<td>Tip radius $r_s$</td>
<td>$r_1 + 2c_1$</td>
<td>$r_1 + 2c_1 - c_2$</td>
<td>$r_1 + 2c_1$</td>
</tr>
<tr>
<td>Dedendum $h_r$</td>
<td>$2c_1$</td>
<td>$2(c_1 + c_2)$</td>
<td>$2c_1$</td>
</tr>
<tr>
<td>Addendum $h_a$</td>
<td>$2c_1$</td>
<td>$2(c_1 - c_2)$</td>
<td>$2c_1$</td>
</tr>
</tbody>
</table>

### 3. Meshing principle of conjugate curves

As shown in Fig. 4, the coordinate systems are established according to the right-hand rule (Hwang, 2007), (Li, 2014), $S_{O_1}(O_{1}x_{1}, y_{1}, z_{1})$ and $S_{O_2}(O_{2}x_{2}, y_{2}, z_{2})$ are the fixed coordinate systems. Respectively, conjugate curves $\Gamma_1$ and $\Gamma_2$ are defined by using the two movable coordinate systems $S_1(O_{1}x_{1}, y_{1}, z_{1})$ and $S_2(O_{2}x_{2}, y_{2}, z_{2})$ which are connected to the pinion 1 and wheel 2. $S_{O_3}(O_{3}x_{3}, y_{3}, z_{3})$ is the fixed coordinate system to describe meshing line. The two gears rotate in opposite direction with instantaneous angular velocities $\omega_1$ and $\omega_2$. Respectively $\phi_1$ and $\phi_2$ are the angular displacements. $a$ is the center distance between the mated gears, $M$ is the contact point, point $P$ is the intersection of pitch circles. According to the transmission ratio $i$ of the transform mechanism, the rotation speeds and rotation angles have the following relationship equation

$$i = \frac{1}{l_{z_1} = \omega_1 / \omega_2} = \frac{z_2}{z_1} = \frac{\phi_1}{\phi_2} = \frac{r_2}{r_1}$$

(6)

![Fig. 4 Coordinate systems of composite cycloid gear](image-url)
\( M_i \) is the point in the composite cycloid profile \( \Gamma_i \), its position vector and parameter are \( r_i(\alpha) \) and \( \alpha \). The equation of the original composite cycloid profile \( \Gamma_1 \) in the coordinate system \( S_1 \) can be described as

\[
r_i(\alpha) = \begin{bmatrix} x_i(\alpha) \\ y_i(\alpha) \\ 1 \end{bmatrix}^T = \begin{bmatrix} r_i \cos \alpha - 2c_i \cos(z\alpha) \cos\alpha - 2c_2 \sin(2\zeta\alpha) \sin\alpha \\ r_i \sin \alpha - 2c_i \cos(z\alpha) \sin \alpha + 2c_2 \sin(2\zeta\alpha) \cos \alpha \\ 1 \end{bmatrix}
\]  

(7)

where \( c_i \) is the adjusting parameter of tooth height, \( c_2 \) is the control parameter of profile, and \( c_2 = f_i c_1 \), \( f_i \) is the adjusting factor of profile.

The meshing equation of enveloping motion between conjugate curves \( \Gamma_1 \) and \( \Gamma_2 \) can be expressed as

\[
f(\alpha, \varphi_1) = N_i V^{(12)}_i = 0
\]

(8)

The normal vector \( N_i \) and relative velocity \( V^{(12)}_i \) are represented as

\[
N_i = -\frac{dy_i(\alpha)}{d\alpha} i + \frac{dx_i(\alpha)}{d\alpha} j
\]

(9)

\[
V^{(12)}_i = [a i_i \cos \varphi_1 - y_i(\alpha)(1 + i_2)] i + [x_i(\alpha)(1 + i_2) - a i_2 \sin \varphi_1] j
\]

(10)

where \( i_i \) and \( j_i \) are the unit vectors of coordinate system \( S_i \).

Substituting Eq.(9) and Eq.(10) into Eq. (8) yields the following meshing equation

\[
f(\alpha, \varphi_1) = r_i \left[ r_i - 2c_z \cos(\zeta\alpha) + 4f_i c_1 z_i \cos(2\zeta\alpha) \right] \cos(\alpha + \varphi_1) + 2c_2 r_i \left[ z_i - 2f_i \cos(\zeta\alpha) \right] \sin(\zeta\alpha) \sin(\alpha + \varphi_1) + 2c_i z_i \left[ -r_i \sin(\zeta\alpha) + c_1 \sin(2\zeta\alpha) - 2f_i c_1 \sin(4\zeta\alpha) \right] = 0
\]

(11)

The equation of conjugate tooth profile \( \Gamma_2 \) can be determined by the following coordinate transformation in coordinate system \( S_2 \).

\[
r_2(\alpha, \varphi_1) = [x_2(\alpha) \quad y_2(\alpha) \quad 1]^T = M_{21}(\varphi_1) r_i(\alpha)
\]

(12)

where

\[
M_{21} = \begin{bmatrix} \cos(\varphi_1 + \varphi_2) & -\sin(\varphi_1 + \varphi_2) & a \cos \varphi_1 \\ \sin(\varphi_1 + \varphi_2) & \cos(\varphi_1 + \varphi_2) & -a \sin \varphi_1 \\ 0 & 0 & 1 \end{bmatrix}
\]

(13)

Therefore, the \( x \)-component and \( y \)-component in Eq.(12) can be yielded as the following expression.

\[
\begin{align*}
x_2(\alpha) &= -2f_i c_1 \sin(2\zeta\alpha) \sin[\alpha + (1 + i_1) \varphi_1] + [r_i - 2c_z \cos(\zeta\alpha)] \cos[\alpha + (1 + i_2) \varphi_1] + r_i \left( 1 + i_2 \right) \sin(2\iota_2 \varphi_1) \\
y_2(\alpha) &= [r_i - 2c_z \cos(\zeta\alpha)] \sin[\alpha + (1 + i_2) \varphi_1] + 2f_i c_1 \sin(2\zeta\alpha) \cos[\alpha + (1 + i_2) \varphi_1] - r_i \left( 1 + i_2 \right) \cos(2\iota_2 \varphi_1)
\end{align*}
\]

(14)

The equation of meshing line between the conjugate tooth profiles \( \Gamma_1 \) and \( \Gamma_2 \) can be expressed as

\[
r_{ml}(\alpha, \varphi_1) = M_{ml}(\varphi_1) r_i(\alpha) = [x_{ml}(\alpha) \quad y_{ml}(\alpha) \quad 1]^T
\]

(15)

where

\[
M_{ml} = \begin{bmatrix} \cos \varphi_1 & -\sin \varphi_1 & 0 \\ \sin \varphi_1 & \cos \varphi_1 & -r_i \\ 0 & 0 & 1 \end{bmatrix}
\]

(16)
Substituting Eq.(7) and Eq.(16) into Eq.(15), the equation of meshing line in the coordinate system $S_{ml}$ can be expressed as

$$
\begin{align*}
{x_{ml}(\alpha)} &= -2fz_1\sin(2z_1\alpha)\sin(\alpha+\varphi) + \left[ r_1 - 2z_1\cos(z_1\alpha) \right] \cos(\alpha+\varphi) \\
y_{ml}(\alpha) &= 2fz_1\sin(2z_1\alpha)\cos(\alpha+\varphi) + \left[ r_1 - 2z_1\cos(z_1\alpha) \right] \sin(\alpha+\varphi) - r_1
\end{align*}
$$

(17)

4. Geometrical modeling of gear pair

A series of theoretical calculations are carried out based on the designated parameters case 1 and case 2 of composite cycloid gears in Table 2. A simplified meshing model of gear pair is developed by means of MATLAB software. As displayed in Fig.5 (a) and Fig.5 (b), the precise three-dimensional models of the case 1 and case 2 are established using SolidWorks software, respectively. For the case 2 in SolidWorks, the pinion speed $v_p$ is set as 2275 r/min, the exact contact function is adopt to handle the cation way in the gear rotation process. Then a computerized kinematics simulation of the meshing process is also carried out by using SolidWorks. In the simulation process, no meshing interference is found. Fig.5 (c) shows that the mating tooth surfaces in kinematics process are meshing in line which is similar to the involute gear drive. As shown in Fig.5 (d), the simulation result of wheel speed $v_w$ in stable process is 1499.43 r/min, so a constant gear ratio $i_{c2}$ that $i_{c2}=44/29=2275/1499.43=1.517$ can be concluded in the power transmission. Moreover, the stable gear ratio can also demonstrate of no meshing interference in the process.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central distance $a$ (mm)</td>
<td>91.5</td>
<td>91.5</td>
</tr>
<tr>
<td>Module $m$ (mm)</td>
<td>4.575</td>
<td>2.507</td>
</tr>
<tr>
<td>Pressure angle $\alpha_p$ (°)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Transmission ratio $i$</td>
<td>0.667</td>
<td>0.659</td>
</tr>
<tr>
<td>Tooth number of pinion $z_1$</td>
<td>16</td>
<td>29</td>
</tr>
<tr>
<td>Tooth number of wheel $z_2$</td>
<td>24</td>
<td>44</td>
</tr>
<tr>
<td>Addendum coefficient $h_1^a$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Tip clearance factor $c^*$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Tooth height factor $h$</td>
<td>0.625</td>
<td>0.625</td>
</tr>
<tr>
<td>Adjusting factor of profile $f_1$</td>
<td>0.018</td>
<td>0.018</td>
</tr>
<tr>
<td>Adjusting parameter of tooth height $c_1$ (mm)</td>
<td>2.8594</td>
<td>2.8594</td>
</tr>
<tr>
<td>Control parameter of profile $c_2$ (mm)</td>
<td>0.05147</td>
<td>0.05640</td>
</tr>
<tr>
<td>Tooth width $b$ (mm)</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

![Fig. 5 Three-dimensional solid models of new gear drive](image)

(a) Case 1

(b) Case 2

(c) Line contact in meshing

(d) Kinematics simulation
Meanwhile, the original composite cycloid profile can be calculated, as shown in Fig. 6. By the same parameters of teeth \( z_1/z_2 \), module \( m \), addendum coefficient \( h'_* \) and tip clearance factor \( c' \), the involute profile can be also depicted in Fig. 6. Comparisons show that the composite tooth profile in the upper part appears very close to the involute tooth profile. However, for the section area of dedendum, the tooth thickness of composite profile is greater than that of involute gear. Therefore the new gear drive should have a higher bending strength theoretically.

5. Meshing characteristics

5.1 Contact Ratio

The contact ratio of a gear drive is defined as the average number of teeth being in mesh simultaneously. Also, it can be described as the gear rotating angle from running-in to running-out divided by the angle between every two teeth (Pedrero, et al., 1996). As shown in Fig. 6, the contact ratio of composite gear drive can be expressed as

\[
\varepsilon_a = (\alpha_s - \alpha_i)/(2\pi/z_1)
\]  

(18)

where \( \alpha_s \) and \( \alpha_i \) are the rotation angle of meshing point \( M \) in the position of running-in and running-out.

Based on the geometrical relationship \( O_1O_{wa} + O_{wa}M_q = O_1M_{wp} \), the following equation can be deduced as

\[
y_{wu}(\alpha_s) + r_i = r_i \sin(\pi - \alpha_s)
\]  

(19)

With triangle cosine theorem of \( \Delta O_1M_qO_{wa} \), geometric equation can be expressed as

\[
a^2 + \frac{x_{wu}^2(\alpha_s)}{\cos^2 \alpha_s} - r_{s2}^2 = 2a \frac{x_{wu}(\alpha_s) \cos(\pi/2 - \alpha_s)}{\cos \alpha_s}
\]  

(20)

Respectively, \( \alpha_s \) and \( \alpha_i \) can be calculated by the above Eq. (19) and Eq. (20). Then the contact ratio can be carried out by the Eq. (18). As shown in Table 3, the contact ratios of the five sets of gear parameters, including composite cycloid and involute gear, are carried out by Matlab.

According to Table 3, the contact ratio \( \varepsilon_{wa} \) of composite cycloid gear drive is bigger than that contact ratio \( \varepsilon_{wa} \) of
involute gear drive. So the high drive stability and load capacity can be realized by the new gear drive.

### Table 3  Contact ratio

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module $m$</td>
<td>1 3 5 7 9</td>
</tr>
<tr>
<td>Pinion $z_1$</td>
<td>24 17 18 22 23</td>
</tr>
<tr>
<td>Wheel $z_2$</td>
<td>29 33 55 90 119</td>
</tr>
<tr>
<td>Transmission ratio $i$</td>
<td>1.21 2.06 3.06 4.09 5.17</td>
</tr>
<tr>
<td>$e_{at}$</td>
<td>1.64 1.62 1.68 1.73 1.75</td>
</tr>
<tr>
<td>$e_{at}$</td>
<td>1.62 1.59 1.65 1.71 1.73</td>
</tr>
</tbody>
</table>

### 5.2 Sliding ratio

In this chapter, the incredibly small sliding ratio in composite cycloid drive can be verified by the following calculation. The sliding ratio is a measure to describe the sliding characteristic during the meshing process. It is defined as a ratio of the length of sliding arc relative to the length of the corresponding arc in meshing profiles (Wang, et al., 2010). A lower ratio will have greater transmission efficiency due to the less friction.

![Sliding principle of composite cycloid gear](image)

Supposing a driving gear with original curve $\Gamma_1$ transmits movement to a driven gear with its conjugated curve $\Gamma_2$. They contact at point $M_s$ as displayed in Fig.8, in an extremely tiny pieces of time $\Delta t$ which approaches to zero during the meshing process, respectively point $M_s$ will travel the arc length $M_sM_{s1}$ and $M_sM_{s2}$ in the conjugate curves. Accordingly, $\Delta s_1$ and $\Delta s_2$ represents the arc length $M_sM_{s1}$ and $M_sM_{s2}$. Assuming the relative sliding exists, the arc length $\Delta s_1$ is not equal to that of arc $\Delta s_2$, and the difference between $\Delta s_1$ and $\Delta s_2$ is defined as the sliding arc. According to the meshing theory, the sliding ratio $\sigma_1$ and $\sigma_2$ of pinion and wheel of this gear drive can be expressed as

\[
\sigma_1 = \lim_{\Delta s_1 \to 0} \frac{\Delta s_1 - \Delta s_2}{\Delta s_1} = \left( \frac{dr_1}{dt} - \frac{dr_2}{dt} \right) \frac{ds_1}{ds_1} = \frac{ds_1 - ds_2}{ds_1} \tag{21}
\]

\[
\sigma_2 = \lim_{\Delta s_2 \to 0} \frac{\Delta s_2 - \Delta s_1}{\Delta s_2} = \left( \frac{dr_2}{dt} - \frac{dr_1}{dt} \right) \frac{ds_2}{ds_2} = \frac{ds_2 - ds_1}{ds_2} \tag{22}
\]

Then the whole and detail expression of sliding ratios of the new gear pair can be obtained as Eq. (23) and Eq. (24). Similarly, this sliding model is suitable for the arbitrary conjugate tooth profiles that contact along line paralleling to axis direction, and it does not need to derive complicated geometrical relationship.

\[
\sigma_1 = (1 + i_1) \frac{B_1 B_2}{B_1 B_1} \tag{23}
\]

\[
\sigma_2 = (1 + i_2) \frac{B_1 B_2}{B_1 B_1} \tag{24}
\]

where the expressions of symbols are introduced as a simplification, their expressions are the followings
Given parameters are as follows: \( m = 3 \) mm, \( z_1/z_2 = 30/73 \), \( a = 154.5 \) mm, \( \alpha_p = 20^\circ \). And the calculation results of sliding ratio of composite cycloid gears are obtained as shown in Fig. 9. Meanwhile the sliding ratio of involute gears, as shown in Fig. 10, is also received by the way proposed in the literature (Yan, et al., 2001). The following conclusions can be drawn from these numerical results: (1) the sliding ratio of composite cycloid gear drive is much less than that of involute gear drive; (2) the biggest value of sliding ratio are presented in the meshing point of running-in and running-out, however, the absolute values are still lower than 0.1 and are reduced by about 10 times in comparison with involute gear. Meanwhile, the sliding ratio is very close to 0 in pitch point. So the superior sliding ratio can be achieved in the composite cycloid gear drive.

### 6. Mechanics properties analysis

In this section, mechanics properties for the new gear pair such as contact and bending stresses are analyzed. For this study, the FEA program ABAQUS has been used (Miryam, et al., 2016). The solid model of Case 1 established in Fig. 5 is applied and processed into FEA model. The material is 40 CrNi with the properties of Young's modulus \( E = 206 \) GPa and Poisson's ratio \( \nu = 0.3 \). The torque loaded in the pinion is 270 N·m. For calculating efficiently and accurately, the solid model with five teeth is separated. Fig. 11 is the FE model established including the pinion and wheel in contact. The network is plotted into six blocks of each tooth and is constructed low hexahedron unit. Generally, the working tooth surfaces should be chosen as master, correspondingly the meshing tooth surface of wheel are defined as slave surfaces. The rigid surface \( R_{s1} \) of pinion is coupling with the reference point \( C_{p1} \) in the center axis \( R_{x1} \) of pinion, the rotation angle \( R_{v1} \) is given to this point. Respectively, the rigid surface \( R_{s2} \) of wheel is coupling with the reference point \( C_{p2} \) in the center axis \( R_{x2} \) of wheel, the putting off load \( T_{z2} = 270 \times 24/16 \) N·m = 405 N·m is imposed to \( C_{p2} \). Under the same parameters of teeth \( z_1/z_2 \), module \( m \), addendum coefficient \( h^* \) and tip clearance factor \( e^* \), stress distribution of involute gear drive is also analyzed. Furthermore, to validate the FE model presented above, the contact stress and bending stress are calculated with the module of contact analysis in KISSsoft, the calculation principle is mainly based on the method B of international standard ISO 6336 published in 2006.
Fig. 11 Finite element model for the contact analysis

Fig. 12 Analysis of the contact stress

(a) Bending stress of pinion

(b) Bending stress of wheel

Table 4 Maximum contact and bending stress

<table>
<thead>
<tr>
<th>Gear drive</th>
<th>Max contact stress (MPa)</th>
<th>Max bending stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>New gear drive</td>
<td>1185.2</td>
<td>302.6</td>
</tr>
<tr>
<td>Involute gear drive</td>
<td>1358.6</td>
<td>332.3</td>
</tr>
</tbody>
</table>

Fig.12 and Fig.13 show that the stress curves of contact stress and bending stress of the new gear drive in comparison with traditional involute gear are changeable with the pinion rotation angle, respectively. It can be observed that contact stress and bending stress of the involute drive calculated with ABAQUS are very close with the according results in KISSsoft. The good agreements are present in both Fig.12 and Fig.13. It indicates that the calculation method of FE model for Case 1 with ABAQUS is reasonable and convinced. Meanwhile, for the new drive, the better performances on both contact strength and bending strength in one meshing cycle have expressed. The Von-Mises stresses, including maximum contact stress and bending stress in Table 4, are obtained in the numerical results as shown in Fig.12 and Fig.13. These values, as listed in the Table 4, show that the maximum contact stress of the composite cycloid gear is reduced by about 12.75% accordingly in comparison with the involute gear. Meanwhile the bending stresses are 26.35% and 8.9% less than that of involute gear. Therefore, the application of the composite cycloid profile can reduce both contact and bending stresses.

7. Performance experiment

As shown in Fig.14, the gear samples with the composite cycloid gear parameter of case 1 in Table 2 are processed
using low-speed wire-cutting electrical discharge machining, the involute gears are in the process of gear hobbing and grinding, as shown in Fig.14. FZG gear testing rig is applied, as described in Fig.15. The transmission efficiencies under different operating conditions are performed with the rig, as shown in Fig.16. The torque \( T_1 \) is loaded on the pinion by using the loader set and can be tested by the sensors, and the variable rotational speed of the gear is controlled by the electric motor. The average gear mesh mechanical efficiency can be calculated by the following equation

\[
\eta = \frac{T_1 z_2}{T_2 z_1}
\]

(29)

where \( T_1 \) and \( T_2 \) are the average torques on the pinion and wheel, respectively.

---

Fig. 14 Gear samples

Fig. 15 Schematic diagram of the gear test rig

1 Test gearbox; 2 Test wheel; 3 Shaft I; 4 Elastic coupling I; 5 Torque sensor II; 6 Slave gearbox; 7 Slave wheel; 8 Drive motor; 9 Slave pinion; 10 Shaft II; 11 Load clutch; 12 Elastic coupling II; 13 Torque sensor I; 14 Test pinion

Fig. 16 Testing process

For this testing, five kinds of torque \( T_1 \) are loaded as 14.1 N·m, 35.3 N·m, 60.8 N·m, 94.8 N·m and 135.2 N·m,
each load is proceeded under four different speeds of 1000 r/min, 1500 r/min, 2000 r/min and 2500 r/min. The testing results of transmission efficiencies are shown in Fig.17. It can be concluded that the transmission efficiency will increase if the rotational speed increases and the torque keeps constant. Similarly, it will also increase if increasing torque and keeping rotational speed constant. The maximum efficiency of composite cycloid gear samples may be up to 98.96% at the load of 135.2 N·m and it is tending towards stability. The efficiencies of composite cycloid gears, which is in the range of 97.20 % to 98.96 %, are positively correlating with the loading torque. Meanwhile, the machining precision of involute gears are higher than the new gears, but the composite cycloid gears still have advantages in high efficiency. Therefore, the efficiency superiority will be more obvious in future if the machining precision is improving.

Fig. 17 Transmission efficiencies tested in FZG test rig

8. Conclusions

In this paper, the geometric principle of a new composite cycloid is put forward by proposing a motion transforming method. Conjugate theory, meshing characteristics, mechanics property and efficiency testing of the new composite cycloid gear drive are conducted and discussed in comparison with the involute gear. The theoretical and experimental results show that the excellent meshing characteristics and high transmission efficiency of the proposed profile can be obtained and are available for gear design of the new type drive. The main conclusions can be drawn as follows: design of new types of gear drive

(1) The motion transforming method of equivalent two-link mechanism is developed to describe traditional cycloid. The new second-order, third-order and fourth-order composite cycloid equations including the corresponding link mechanisms are derived and compared. The fourth-order composite cycloid is more suitable for the tooth profile of gear drive.

(2) The mathematical models of meshing equation, conjugate profile and meshing line of the new gear drive are established. Through the three-dimensional solid model and motion simulation analysis, this new gear pair can mesh in line contact with a constant gear ratio and no meshing interference.

(3) Meshing properties and mechanics properties, including greater contact ratio, smaller sliding ratio and superior both bending stress and contact stress of the new gear, are represented.

(4) The experimental tests demonstrate that the transmission efficiencies of new drive range from 97.20 % to 98.96 % positively correlating with the loading torque. Importantly, under the same operating conditions they are higher compared with the involute gear, and it is also indicated that the new gear drive show a commendable value in application to the external gear drive of practical engineering.

References

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