Fatigue residual life prediction of casting crane under track defect model

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Abstract
For the safety problems caused by abnormal changes of stress-time history at fatigue risk points of casting crane metal structure due to the vertical impact effect of track defects, the forecast method of fatigue residual life of casting crane under track defect model is proposed. Guided by the kinematics and dynamics theory, the kinematic-dynamic model of dynamic load resulted from crane travelling on defective track including the high-low dislocation and horizontal gap is established. The influence of track defects on the vertical impact effect of operation crane is analyzed. Furthermore, combining with the collection of technological process and usage of casting crane, the characteristic parameters needed in the process of stress spectrum acquisition are determined. The allowable stress method combined with finite element simulation is used to get the fatigue risk cross-sections and points of crane metal structure and the first principal stress-time histories of dangerous points are obtained. The double parameter stress spectrum of dangerous points is extracted by rain flow technology. Based on fracture mechanics and damage tolerance design, the fatigue residual life of dangerous points is estimated by Paris formula. Taking the 100/40t-28.5m casting crane as an example, the feasibility of the above model and method is verified.

Keywords: Casting crane metal structure, High-low dislocation of rail joint, Fatigue residual life prediction, First principal stress-time history, Paris formula

1. Introduction

The casting crane is one of the main equipments for steel-making and concasting process. Because of the poor working environment, high working level and complex structure, the event will pose a great threat to the lives of the operator and the surrounding people. Some will even lead to crane crash and casualties (Wu X et al., 2011). The crane track plays the role of bearing and guiding function to the casting crane. The installation quality of track seriously affects the operation and service life of the whole machine. In recent years, the accidents of casting cranes caused by track defects have emerged in an endless stream. Due to the high-low dislocation and horizontal gap of track, the fatigue failure for crane track girder is very easy to occur at welding joint (Rykaluk K et al., 2018), the bearing damage of 450t crane wheel occurs frequently for ‘Bayuquan Iron and Steel Branch’ (Guanghui L., 2016) and the weld seam of the parent material at the corner is cracked for main beam of 480/80t casting crane of ‘Shougang Jingtang United Iron & Steel Co., Ltd’, which greatly reduces the service life of the main beam (Jianping P., 2013).

At present, in view of the above phenomena, scholars at home and abroad have carried out extensive research and achieved some results. The crane track damage and the remaining life of workshop rail crossbeam during crane operation are introduced and the necessary measures is proposed in (Kulka J et al., 2015). Through the measurement of track geometric deviations and the determination of skew motion caused by eccentric load, the failure problem of 200t casting crane caused by the rail wear is discussed in (Kulka J et al., 2016). Veronika (Vašková V et al. 2017) analyses the crane operation failure attributed to the deformation of the track caused by the insufficient rigidity of crane track girder.
Rettenmeier (Rettenmeier P, et al., 2016) puts forward the fatigue residual life evaluation method of crane rail girder under multi-axial stress state for combining wheel load with and welding residual stress. The fatigue residual life of track girder of crane running for 30 years is described in (Ávila G et al., 2017) and the results show that the fatigue failure is most likely to occur at the welded joint. Under the condition of load spectrum generated by crane historical operation record data, the fatigue residual life of the rail girder is given in (Caglayan O et al., 2010) by the standard static load test combined with the established finite element model. For the abnormal phenomenon of rail gnawing and rail breaking of 450t bridge crane, the finite element analysis method for the influence of the high-low dislocation of crane track on the track girder is proposed in (Zhiqiang L et al., 2017). Through field testing and calculation, the cause of rail biting is diagnosed in (Baoyu, Z et al., 2003), because crane wheels of bridge crane of ‘Wuhan Iron and Steel Co., Ltd’ need to be replaced less than one month.

The above studies mainly discuss the causes for both the rail gnawing phenomenon and fatigue failure of welded joint of crane track girder. There is less analysis of weld seam cracking of the crane metal structure caused by track defects.

Because of crane track defect, the continuity of track is destroyed, the carrying capacity of track is reduced and the running resistance of crane and the vertical impact effect are increased. Thus, abnormal changes of first principal stress-time history of fatigue risk points are produced. Furthermore the fatigue life of the crane metal structure is greatly reduced.

For the above problems, this paper establishes the kinematic-dynamic model of dynamic load resulted from crane travelling on defective track including the high-low dislocation and horizontal gap and analyzes the influence of track defects on the vertical impact effect of operation crane. Besides, the fatigue residual life prediction method of the casting crane under track defect mode is put forward.

2. Impact Coefficient for Crane Travelling on Defective Track

The dynamic load and impact coefficient caused by crane travelling on the defective track joint can be estimated by using an appropriate kinematics-dynamics model. For the high-low dislocation of track joint, the kinematics-dynamics model is as shown in Fig.1. For the horizontal gap of track joint, the model is similar.

![Kinematics-dynamics model of dynamic load caused by crane travelling on high-low dislocation of track joint](image)

\begin{align}
m\ddot{x}(t) + kx(t) &= kh(t) \quad (1) \\
\end{align}

As shown in Fig.1 c), the track roughness function \( h(t) \) is established as Eq.(2) based on the cosine function, about which the center position of each wheel passes through the high-low dislocation of rail joint.

\begin{align}
h(t) &= A + B\cos\Omega t \quad (2) \\
\end{align}

Where, \( A, B \) are coefficients, calculated as follows.

Given in Fig.1 b) and c), for \( t = 0 \), \( h(0) = 0 \) and for \( t = t_s, h(t_s) = h_s \). Therefore, Eq.(2) can be transformed into Eq.(3).
\[ \begin{align*}
A &= B = \frac{h_s}{2} \\
h(t) &= \frac{h_s}{2} (1 - \cos \Omega t)
\end{align*} \tag{3} \]

Eq. (1) is converted to Eq. (4) by substituting Eq. (3) into Eq. (1).

\[ m \ddot{x}(t) + kx(t) = k \frac{h_s}{2} (1 - \cos \Omega t) \tag{4} \]

According to the superposition principle, the steady-state solution of Eq. (4) is the sum of solutions of the following equations.

\[ m \ddot{x}(t) + kx(t) = k \frac{h_s}{2} \cos \Omega t \tag{5} \]

\[ m \ddot{x}(t) + kx(t) = -k \frac{h_s}{2} \sin \Omega t \tag{6} \]

The solution of Eq. (5) is obtained as:

\[ x_p(t) = \frac{h_s}{2} \]

The general solution of Eq. (6) is designed as:

\[ x_h(t) = C_1 \cos \omega t + C_2 \sin \omega t \tag{8} \]

Where \( \omega = \left( \frac{k}{m} \right)^{1/2} \) is the system natural frequency. The special solution of Eq. (6) is assumed to \( x_p(t) = -x \cos \Omega t \) because of the excitation \( k \cdot h(t) \) being the simple harmonic form, which is substituted into Eq. (6) and the result is as follows:

\[ X = \frac{h_s}{k - m \omega^2} = \frac{h_s}{2[1 - (\Omega/\omega)^2]} \tag{9} \]

Thus, the solution of Eq. (6) is determined as:

\[ x_1(t) = x_h(t) - X \cos \Omega t = C_1 \cos \omega t + C_2 \sin \omega t - \frac{h_s}{2[1 - (\Omega/\omega)^2]} \cos \Omega t \tag{10} \]

The whole solution of Eq. (4) is confirmed as:

\[ x(t) = x_p(t) + x_1(t) = \frac{h_s}{2} + C_1 \cos \omega t + C_2 \sin \omega t - \frac{h_s}{2[1 - (\Omega/\omega)^2]} \cos \Omega t \tag{11} \]

The initial conditions \( x(t = 0) = x_0 \) and \( \dot{x}(t = 0) = \dot{x}_0 \) are used and coefficients \( C_1 \) and \( C_2 \) are obtained as follows:

\[ \begin{align*}
C_1 &= x_0 + \frac{h_s}{2[1 - (\Omega/\omega)^2]} + \frac{h_s}{2} \\
C_2 &= \frac{\dot{x}_0}{\omega}
\end{align*} \tag{12} \]

The response of the system is

\[ x(t) = \frac{h_s}{2} + \left( x_0 + \frac{h_s}{2[1 - (\Omega/\omega)^2]} - \frac{h_s}{2} \right) \cos \omega t + \frac{x_0}{\omega} \sin \omega t - \frac{h_s}{2[1 - (\Omega/\omega)^2]} \cos \Omega t \tag{13} \]

The vertical velocity of the crane mass \( m \) across the high-low dislocation of rail joint is obtained as follows:

\[ \dot{x}(t) = -\left( x_0 + \frac{h_s}{2[1 - (\Omega/\omega)^2]} - \frac{h_s}{2} \right) \omega \sin \omega t + \frac{x_0}{\omega} \omega \cos \omega t + \frac{h_s}{2[1 - (\Omega/\omega)^2]} \Omega \sin \Omega t \tag{14} \]

The vertical acceleration of the crane mass \( m \) is design as follows:

\[ \ddot{x}(t) = -\left( x_0 + \frac{h_s}{2[1 - (\Omega/\omega)^2]} - \frac{h_s}{2} \right) \omega^2 \cos \omega t - \frac{x_0}{\omega} \omega^2 \sin \omega t + \frac{h_s}{2[1 - (\Omega/\omega)^2]} \Omega^2 \cos \Omega t \tag{15} \]

The initial vertical displacement \( x_0 \) and the initial velocity \( \dot{x}_0 \) are all equal to zero when the crane mass \( m \) goes through the high-low dislocation of rail joint.

\[ \dot{x}(t) = -\left( \frac{h_s}{2[1 - (\Omega/\omega)^2]} - \frac{h_s}{2} \right) \omega \sin \omega t + \frac{h_s}{2[1 - (\Omega/\omega)^2]} \Omega \sin \Omega t \tag{16} \]

\[ \ddot{x}(t) = \frac{h_s}{2} \frac{\Omega^2}{[(\Omega/\omega)^2 - 1]} \left( \cos \omega t - \cos \Omega t \right) \tag{17} \]

Let \( \alpha_f = \omega / \Omega \).

\[ \dddot{x}(t) = \frac{h_s}{2(\alpha_f^2 - 1)} \omega \sin \omega t + \frac{h_s}{2(\alpha_f^2 - 1)} \Omega \sin \Omega t \tag{18} \]
The energy conservation theorem is applied and the kinetic energy at time $t_s$ is converted into the potential energy in $\Delta t$ time.

$$\frac{1}{2} mx^2(t_s) = \frac{1}{2} k \Delta x^2$$

$$\Delta x = \sqrt{\frac{m}{k}} \dot{x}(t_s)$$

Where, $\Delta x$ is the spring variation from time $t_s$ to $t_s + \Delta t$.

Within $\Delta t$ time, the spring elasticity is $k\Delta x$ and the acceleration is expressed as:

$$a_{\Delta t} = \frac{k}{m} \frac{\Delta x}{\Delta t} = \frac{k}{m} \dot{x}(t_s)$$

At time $t_s + \Delta t$, the maximum acceleration of the system is determined as:

$$\ddot{x}_{\text{max}} = \ddot{x}(t_s) + a_{\Delta t} = \frac{h_s}{2} \Omega^2 \frac{a_f^2}{1-a_f^2} (1 + \cos \omega t_s) + \omega \dot{x}(t_s) = \frac{h_s}{2} \Omega^2 \frac{a_f^2}{1-a_f^2} (1 + \cos \omega t_s) + \omega^2 \frac{h_s}{2(1-a_f^2)} \sin \omega t_s$$

$$= \frac{h_s}{2} \Omega^2 \frac{a_f^2}{1-a_f^2} (1 + \cos \omega t_s + \sin \omega t_s) = \frac{h_s}{2} \Omega^2 \frac{a_f^2}{1-a_f^2} (1 + \cos \pi a_f + \sin \pi a_f)$$

Let $\xi'(a_f) = \frac{a_f^2}{2(1-a_f^2)} (1 + \cos \pi a_f + \sin \pi a_f)$, Eq. (25) can be changed as:

$$\ddot{x}_{\text{max}} = \frac{h_s}{2} \Omega^2 \frac{a_f^2}{1-a_f^2} \xi'(a_f) = \frac{h_s}{2} \frac{a_f^2}{1-a_f^2} \xi'(a_f) \approx \frac{h_s}{2} \frac{a_f^2}{1-a_f^2} \frac{\pi^2}{2} \xi'(a_f)$$

$$a_f = \frac{\omega}{\Omega} = \frac{\varepsilon}{\nu} = \frac{\varepsilon}{\nu} = \frac{\omega}{\nu} \frac{R^2 - (R - h_s)^2}{\nu} \approx \frac{\omega}{\nu} \frac{2R}{h_s} = \frac{2R}{h_s}$$

The impact coefficient of the crane travelling on the low-high dislocation of crane track joint is defined as:

$$\phi_4 = \frac{m c_{\text{max}} + mg}{mg} = \frac{\dot{x}_{\text{max}} + \dot{g}}{g} = 1 + \left(\frac{\pi}{2}\right)^2 \frac{\nu^2}{Rg} \xi'(a_f)$$

3 Fatigue Residual Life Prediction of Casting crane

3.1 Characteristic parameters influencing of stress spectrum of fatigue risk points

In the light of GB/T 3811-2008 'crane design specification' (Technical Committees., 2008), the normal stress of fatigue risk points of casting crane metal structure is expressed as:

$$\sigma = 1.15 \phi_1 \left( \frac{M_y}{W_x} + \frac{M_x}{W_y} \right)$$

Where, $\sigma$ is the normal stress at fatigue risk points, in MPa. For the movement of hoisting trolley, $\phi_1 = 1$, while for the movement of crane, $\phi_1 = \phi_4$, $M_x$ and $M_y$ are bending moments of cross-sections of fatigue danger points to x and y axis, respectively, in N×mm. $W_x$ and $W_y$ are bending section modulus of cross-sections, in mm$^3$.

The shear stress of fatigue danger points on the web of casting crane metal structure is designed as:

$$\tau = \frac{F_{Sx}}{1.6}$$

Where, $\tau$ is shear stress for fatigue risk points, in MPa. $F$ is the shear force of cross-section of fatigue risk points, in N. $S_{x0}$ is the maximum static moment of gross section, in mm$^3$.

Under multiaxial complex stress state, fatigue cracks usually are produced at the maximum direction of stress triaxiality. The crack producing plane is the plane with the largest shear stress amplitude and the crack propagation
direction is the vertical direction of the maximum tensile stress (i.e. the first principal stress). Therefore, it is necessary to convert the normal stress and shear stress of fatigue risk point into the first principal stress.

From the two stress state transformation formula, the first principal stress of fatigue danger point is given as:

\[
\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}
\]  

(31)

Where, \(\sigma_1\) is the first principal stress of fatigue risk points, in MPa.

The bending moments \(M_y\), \(M_x\) and shear force \(F\) are deducted under three cases at fatigue danger point based on structural mechanics, as given in Fig.2.

1) The right wheel of hoisting trolley is located on the left side of the fatigue danger point of the main girder, presented in Fig.2 a), namely \(L-(x+b)>0\).

\[
\begin{align*}
M_y^2 & = \frac{[P_1Jx+P_2J(x+b)+FqS^2/2](S-L)-Fq(S-L)^2}{2} - FqL^2, \\
M_x^2 & = \frac{[P_1JHx+P_2JH(x+b)+FqH(S^2/2+M_S)](S-L)-M_S-FqH(S-L)^2}{2} - FqL, \\
F & = \frac{[P_1Jx+P_2J(x+b)+FqS^2/2]}{2} - Fq(S-L) \\
\end{align*}
\]  

(32)

Where, \(P_{1J}\) and \(P_{2J}\) are wheel pressures of left and right wheels of hoisting trolley, in N. \(F_q\) is the uniform load of main girder, in N/mm. \(S\) is the span of main beam, in mm. \(x\) is the distance from the fatigue danger point to the left end of main girder, in mm. \((x+b)\) is the distance from left wheel of hoisting trolley to left end of main girder, in mm. \(b\) is the wheelbase of hoisting trolley, in mm. \(u\) is the distance between the dangerous point and the hoisting trolley wheel, in mm. \(M_y\) is the bending moment caused by deflection lateral force, in N-mm.

2) The left wheel of hoisting trolley is located on the right side of the fatigue danger point of the main girder, introduced in Fig.2 b), namely \(L-x<0\).

\[
\begin{align*}
M_y & = \frac{[P_1J(S-x)+P_2J(S-x-b)+FqS^2/2]L-FqL^2}{2}, \\
M_x & = \frac{[P_1JH(S-x)+P_2JH(S-x-b)+FqH(S^2/2+M_S)]L-M_S-FqH(L-x)^2}{2} - P_{1J}H(L-x), \\
F & = \frac{[P_1J(S-x)+P_2J(S-x-b)+FqS^2/2]}{2} - FqL - P_{1J} \\
\end{align*}
\]  

(33)

3) The left and right wheels are located on both sides of fatigue danger point of main girder, presented in Fig.2 c).

\[
\begin{align*}
M_y & = \frac{[P_1J(S-x)+P_2J(S-x-b)+FqS^2/2]L-FqL^2}{2} - P_{1J}(L-x), \\
M_x & = \frac{[P_1JH(S-x)+P_2JH(S-x-b)+FqH(S^2/2+M_S)]L-M_S-FqH(L-x)^2}{2} - P_{1JH}(L-x), \\
F & = \frac{[P_1J(S-x)+P_2J(S-x-b)+FqS^2/2]}{2} - FqL - P_{1J} \\
\end{align*}
\]  

(34)

![a) Force diagram of right wheel on the left side of fatigue danger point of main girder](image)

![b) Force diagram of left wheel on the right side of fatigue danger point of main girder](image)
A working cycle of the casting crane is a complete process of lifting an object from the ground to starting the next item, including lifting goods, traversing and normal rest. The stress spectrum of fatigue risk points of crane metal structure is extracted from the first principal stress-time history of all the working cycles. In a working cycle, from Eqs. (29)-(34), it can be known that characteristic parameters of fatigue risk points of crane metal structure are composed of lifting load, location of hoisting trolley wheel and relative position of fatigue risk sections.

### 3.2 Acquisition and processing of characteristic parameters

From the technological process of casting crane given in Fig. 3 (originating from the extensive investigation finished by experts from Taiyuan Heavy Machinery Group Co., Ltd), the number of working cycles is determined and the characteristic parameters of fatigue risk points of crane metal structure are counted. The related results are shown in Table 1.

<table>
<thead>
<tr>
<th>Working cycle</th>
<th>Lifting load (t)</th>
<th>The number of trolley left wheel passing through dangerous cross-section f-f</th>
<th>The number of trolley left wheel passing through each section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working cycle 1</td>
<td>m_{q1}</td>
<td>3</td>
<td>B-C 3  C-D 3  D-E 3  E-F 1  F-G 1  G-H 1</td>
</tr>
<tr>
<td>Working cycle 2</td>
<td>m_{q2}</td>
<td>1</td>
<td>1 1 1 1 1 1</td>
</tr>
</tbody>
</table>

According to the casting crane usage, the average daily number of transportation is obtained, including average hoisting times \( N_{Q1} \) of lifting full casting furnace \( m_{Q1} \) and \( N_{Q2} \) of lifting empty casting furnace \( m_{Q2} \). Which is combined with working days per year \( t_d \) and working years \( T \), total hoisting times of lifting full and empty casting furnace are calculated, respectively:

\[
\begin{align*}
N_1 &= N_{Q1} t_d T \\
N_2 &= N_{Q2} t_d T
\end{align*}
\]

Where \( N_1 \) and \( N_2 \) are the total number of hoisting times under full and empty package operation, respectively.

### 3.3 Fatigue dangerous position

The fatigue failure of casting crane metal structure occurs mostly in the weak fatigue performance, the maximum stress or the stress concentration. Through theoretical calculation, finite element analysis or model beam test (Jialin L., 2011), the fatigue dangerous position of main beam for periodic inspection are determined, shown in Fig. 4.

1) The crack is formed at the weld toe of the weld joint of large baffle and lower flange plate (see Fig. 4 a), point 3 and point 5 of A-A section) and the crack propagation direction is as given in Fig. 4 b), which eventually lead to the failure of main beam.

2) The crack source is formed on the weld joint of main web and lower flange plate (see Fig. 4 a), point 2 and point 4 of A-A section) and the crack is extended to the web and the flange plate at the same time and the crack
propagation direction is presented in Fig. 4 b), thus the main beam is invalid.

3) There is a crack at the weld joint between the upper flange plate and the accessory web plate (see Fig. 4 a), point 1 of the A-A section) and the crack stays in situ, causing no failure of the main beam.

4) The crack is generated at the weld joint between the main web plate and T-type steel (see Fig. 4 a), point 6 of the A-A section) and the crack stays in situ, causing no failure of the main beam.

3.4 Stress spectrum acquisition of fatigue risk point

With GB/T 3811-2008 ‘crane design specification’ and ‘mechanical equipment metal structure design’ being as the theoretical guidance, the first principal stress-time history of each fatigue risk point of crane metal structure is determined by Eqs. (29) - (34). The first principal stress-time history of fatigue risk point 2 for one technological process is given in Fig. 5.

![Diagram showing stress spectrum acquisition of fatigue risk point](image)

As shown in Fig. 5, the a-b-c-d-e section is the first principal stress variation curve for wheelbase center line of full package hoisting trolley moving from H point of main girder to B point. The e-f-g section is that moving from B point of...
main girder to E point. The g-h-k section is that moving from E point of main girder to B point. For lifting empty casting furnace, the l-o-p-q-r section is the first principal stress variation curve for wheelbase center line of empty package hoisting trolley moving from B point of main girder to H point. The abnormal change of first principal stress-history is caused by the impact response of crane travelling on high-low dislocation of track joint.

For the particular casting crane, the technological process is fixed and the first principal stress-time histories of one fatigue risk point under technological processes are the same, thus forming the typical stress-time history segment. On this basis, the two-dimensional stress spectrum that is the double parameter stress spectrum of fatigue risk points of crane metal structure is obtained by the rain flow technology, including the mean stress spectrum and the stress amplitude spectrum.

3.5 Residual fatigue life prediction

For fatigue failure of casting crane metal structure, with fatigue dangerous points on dangerous cross-sections being as the research object, the above method is applied to obtain the stress spectrum of fatigue risk points under the characteristic parameters. Based on Miner’s linear cumulative damage theory and linear elastic fracture mechanics, the formula calculating fatigue remaining life is deduced by using the Paris equation and the fatigue residual life estimation of crane metal structure is realized.

Before estimating fatigue crack growth life, the critical crack size $a_t$ for structure being subjected to fatigue fracture under a given load should be determined by the criterion of linear elastic fracture (Cheng, R., 2015).

$$a_t = \frac{1}{\pi} \left( \frac{K_c}{Y\sigma_{max}} \right)^2$$  \hspace{1cm} (36)

Where, $\sigma_{max}$ is the maximum cyclic stress. $K_c$ is the fracture toughness of materials. $Y$ is the stress intensity correction factor, namely shape parameter. The shape parameter $Y$ generally refers to the function of crack size $a$ and plate width $W$, namely $Y(a,W)$.

In fracture mechanics, cracks are divided into 3 basic types, including open-type, sliding-type and tearing-type. The cracks causing the failure of casting crane main beam are mainly located at the welds connected to the large partition plate, the web plate and the lower flange plate in the tensile area. In the light of stress distribution of cross-section of the main beam (see Fig. 6 a)), these cracks are all open-type cracks (see Fig. 6 b) and c) and belong to the penetrating thickness crack for the plate thickness rather than the surface crack. Besides, the crack propagation direction is given in Fig.4 b), namely expanding along the direction of flange width and web width.

Therefore, for the casting crane main beam, either the flange width or the web width is far greater than the crack size, so the cracked flange or web is considered as the center or unilateral crack infinite plate. For the center crack infinite plate, $Y$ is equal to 1. For the unilateral crack infinite plate, $Y$ is 1.12.

In the middle speed expansion stage, the fatigue crack propagation rate $da/dN$ and the stress intensity factor amplitude $\Delta K$ satisfy the Paris model.

$$da / dN = C(\Delta K)^m$$  \hspace{1cm} (37)

Where, $a$ is the crack size. $N$ is the stress cycles. $C$ and $m$ are parameters of fatigue crack propagation rate.
For the open-type crack, the crack extension occurs. The crack will be in the closed state under the action of the compression load. Therefore, the negative stress part of stress cycling has no contribution to crack propagation and the stress intensity factor can be defined as:

\[
\Delta K = \begin{cases} 
\Delta K_{\max} - K_{\max} = Y\Delta \sigma \sqrt{a} & R > 0 \\
\Delta K_{\max} = Y\sigma_{\max} \sqrt{a} & R \leq 0 
\end{cases}
\]  

\( (38) \)

Where, \( \Delta \sigma \) is the stress variation range, \( \Delta \sigma = \sigma_{\max} - \sigma_{\min} \cdot \sigma_{\max} \) is the maximum stress, \( \sigma_{\min} \) is the minimum stress, \( R \) is the stress ratio.

After the initial crack \( a_0 \) and critical crack \( a_l \) are given and Eq. (38) is integrated into Eq. (37), the fatigue life is obtained as:

\[
N_f = \begin{cases} 
\frac{1}{C(Y\Delta \sigma \sqrt{\pi})^n(0.5m - 1)} \left(1/ a_0^{0.5m-1} - 1/ a_l^{0.5m-1}\right) & m \neq 2, R > 0 \\
\frac{1}{C(Y\Delta \sigma \sqrt{\pi})^n} \ln(a_c / a_0) & m = 2, R \leq 0 \\
\frac{1}{C(Y\sigma_{\max} \sqrt{\pi})^n(0.5m - 1)} \left(1/ a_0^{0.5m-1} - 1/ a_l^{0.5m-1}\right) & m \neq 2, R > 0 \\
\frac{1}{C(Y\sigma_{\max} \sqrt{\pi})^n} \ln(a_c / a_0) & m = 2, R \leq 0 
\end{cases}
\]  

\( (39) \)

In order to eliminate the influence of average stress, based on the principle of equal life, the Goodman formula (37) is used to transform all amplitude stress into the stress range \( \Delta \sigma \) under cycling performance \( R=0 \).

\[
\frac{\sigma_{\text{eq}}}{\sigma_b} + \frac{\sigma_{\text{eq}}}{\sigma_b} = 1
\]

\( (40) \)

\[
\Delta \sigma = \frac{2 (\sigma_{\text{eq}} + \sigma_{\text{eq}})}{1 + \sigma_{\text{eq}}}
\]

\( (41) \)

Where, \( \sigma_{\text{eq}} \) is the material yield limit for \( R=-1 \). \( \sigma_b \) is the material tensile strength. \( \sigma_{\text{eq}} \) is the mean stress for cycling performance \( R \). \( \sigma_{\text{eq}} \) is the stress amplitude for cycling performance \( R \). \( \Delta \sigma \) is stress variation range under \( R=0 \), namely, \( \Delta \sigma = \sigma_{\max} - \sigma_{\min} = \sigma_{\max} \).

For the variable amplitude load, it is converted to the equal amplitude load through the Miner stress amplitude equivalent method, according to the principle of equal life.

\[
\sigma = \sqrt{\sum \alpha_i \sigma_i^m}
\]

\( (42) \)

Where, \( \alpha_i \) is the ratio of stress amplitude cycles at different levels and \( N_f \). \( \sigma_i \) is the stress amplitude at all levels.

4 Engineering Example

Taking the 100/40t-28.5m eccentric rail casting crane (Liangbing H., 2017) as an example, the fatigue residual life of the metal structure is determined.

4.1 Parameters determination

Basic parameters, overall design parameters and main beam design parameters are introduced in (Liangbing H., 2017). Through the analysis of static strength and fatigue strength of main beam, it is found that the fatigue risk section is the middle span section and the distribution of fatigue risk points is shown in Fig.3. The property of middle span dangerous section is given in Table 2. With the same as the daily lifting times for hoisting empty casting furnace, those for lifting full casting furnace are 60, namely \( N_{Q1} = N_{Q2} = 60 \) and the working days per year are 365, namely \( t_d = 365 \).

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Property of middle span dangerous section</th>
</tr>
</thead>
</table>

Cross-sectional areas $A$ (mm$^2$) | $x$ (mm) | $y$ (mm) | $I_x$ (mm$^4$) | $I_y$ (mm$^4$)  
--- | --- | --- | --- | ---  
93048 | 889 | 1236 | 56904510397.888 | 93322539316.186  
Static moment to X axis (mm$^3$) | $S_x : 6.1107e+007$ | Static moment to Y axis (mm$^3$) | $S_y : 4.4429e+007$  

According to the damage tolerance design theory (Munstermann S., 2012), the maximum size of existing crack measured by nondestructive testing (Mandal N R., 2017) should be taken as the initial crack. The minimum crack length can be measured from 0.05 to 0.50mm at present. When the crack size cannot be obtained, the maximum size of initial crack is 0.5mm and the life calculation result is in favor of a security. A large number of tests (Aihong W., 2012) show that the initial crack of metal structure obeys the log-normal distributed and the mean value is 0.526mm and the standard deviation is equal to 0.504. Based on the above analysis, it is reasonable to take 0.5mm as the conservative value of initial crack in the paper.

4.2 Fatigue life prediction

The impact coefficient for crane travelling on the high-low dislocation of rail joint is calculated by Eqs. (1)-(28), as given in Table 3. Then, the first principal stress-time histories of fatigue danger points are obtained by Eqs. (29)-(34). Next, the stress amplitude spectrum and mean spectrum of fatigue risk points are extracted by the rain flow counting method from the results of first principal stress-time history of fatigue risk points. On these basis, the critical crack size and the fatigue residual life of each point are determined by Eqs. (36)-(42) combined with the fracture toughness of Q345 steel (see Table 4 (The International Organization for Standardization, 2012)), as presented in Table 5~Table 7.

Table 3  Impact coefficient for crane travelling on high-low dislocation of rail joint

<table>
<thead>
<tr>
<th>High-low dislocation of rail joint $h_i$(mm)</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact coefficient for crane travelling $\varnothing_4$</td>
<td>1.0787</td>
<td>1.2541</td>
<td>1.4328</td>
<td>1.6065</td>
<td>1.6902</td>
<td>1.7714</td>
<td>1.8499</td>
</tr>
</tbody>
</table>

Table 4  Q345 fracture toughness of Q345 steel

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield strength</th>
<th>Tensile strength</th>
<th>Threshold value of fatigue crack propagation $\Delta K_{th}$ (R=0) (MPa $\sqrt{m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q345</td>
<td>345</td>
<td>460</td>
<td>6.04</td>
</tr>
</tbody>
</table>

| Material | Fracture toughness $\Delta K_1$ (MPa $\sqrt{m}$) | Parameters of fatigue crack propagation rate $C \times 10^{-13}$ | $m$ |
|---|---|---|
| Q345 | 92.7 | 1.06 | 4.66 |

Table 5  Critical crack size of every dangerous point for different high-low dislocation of rail joint (mm)

<table>
<thead>
<tr>
<th>No.</th>
<th>High-low dislocation of rail joint $h_i$(mm)</th>
<th>Point 1</th>
<th>Point 2</th>
<th>Point 3</th>
<th>Point 4</th>
<th>Point 5</th>
<th>Point 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>54934</td>
<td>129</td>
<td>172</td>
<td>133</td>
<td>180</td>
<td>309000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>465762</td>
<td>112</td>
<td>151</td>
<td>117</td>
<td>159</td>
<td>260699</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>329947</td>
<td>86</td>
<td>117</td>
<td>89</td>
<td>121</td>
<td>184440</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>245910</td>
<td>69</td>
<td>92</td>
<td>70</td>
<td>96</td>
<td>135870</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>190323</td>
<td>558</td>
<td>75</td>
<td>57</td>
<td>79</td>
<td>105714</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>170362</td>
<td>50</td>
<td>69</td>
<td>52</td>
<td>71</td>
<td>94270</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>154259</td>
<td>46</td>
<td>63</td>
<td>47</td>
<td>65</td>
<td>84946</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>140336</td>
<td>43</td>
<td>59</td>
<td>44</td>
<td>60</td>
<td>77250</td>
</tr>
</tbody>
</table>

From Table 5, the results of critical crack size of fatigue risk point 1 and point 6 are distorted. This is due to the initial crack sizes of point 1 and point 6 are 0.5mm and for the various high-low dislocation of rail joint, the stress intensity factor $\Delta K$ values (see Table 7) are all less than the fatigue crack propagation threshold $\Delta K_{th}$ (See Table 4). In this case, the crack extension will not be appeared, which is accordance with the results of the finite element analysis or model beam test (Jialin L., 2011).
Table 6  Calculation results of fatigue residual life

<table>
<thead>
<tr>
<th>No.</th>
<th>High-low dislocation of rail joint ( h_s (\text{mm}) )</th>
<th>Fatigue residual life (\text{years})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\text{Point 1}</td>
<td>\text{Point 2}</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2.61E+10</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1.74E+10</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>7.90E+09</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>3.97E+09</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>2.20E+09</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>1.70E+09</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>1.34E+09</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>1.08E+09</td>
</tr>
</tbody>
</table>

Table 7  Stress intensity factors of each dangerous point \( \Delta K (\text{MPa} \sqrt{\text{m}}) \)

<table>
<thead>
<tr>
<th>No.</th>
<th>High-low dislocation of rail joint ( h_s (\text{mm}) )</th>
<th>\text{Point 1}</th>
<th>\text{Point 2}</th>
<th>\text{Point 3}</th>
<th>\text{Point 4}</th>
<th>\text{Point 5}</th>
<th>\text{Point 6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.26</td>
<td>9.66</td>
<td>8.37</td>
<td>9.46</td>
<td>8.18</td>
<td>0.35</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.28</td>
<td>10.23</td>
<td>8.86</td>
<td>10.04</td>
<td>8.67</td>
<td>0.37</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.29</td>
<td>10.66</td>
<td>9.22</td>
<td>10.46</td>
<td>9.03</td>
<td>0.40</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.31</td>
<td>10.94</td>
<td>9.47</td>
<td>10.76</td>
<td>9.27</td>
<td>0.41</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>0.32</td>
<td>11.19</td>
<td>9.67</td>
<td>11.00</td>
<td>9.47</td>
<td>0.42</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>0.32</td>
<td>11.29</td>
<td>9.76</td>
<td>11.11</td>
<td>9.56</td>
<td>0.43</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>0.32</td>
<td>11.39</td>
<td>9.83</td>
<td>11.20</td>
<td>9.65</td>
<td>0.43</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>0.33</td>
<td>11.49</td>
<td>9.92</td>
<td>11.29</td>
<td>9.73</td>
<td>0.44</td>
</tr>
</tbody>
</table>

As given in Table 6, the result of fatigue residual life of point 1 is far greater than the design life that is generally 30~50 years. The reason for this is that point 1 is in the compressive stress region (see Fig.7) which bears bending normal stress \( \sigma \) \((\sigma > 0)\) is compression) and shear stress \( \tau \). The principal stress can be expressed as

\[
\sigma = \frac{-\sigma_1}{2} \left( \frac{\sigma_1}{2} + \tau_x \right) \leq \sigma_1
\]

It shows that the shear stress has great influence on the first principal stress. For point 1, the shear stress is smaller and its first principal stress is smaller, which leads to the larger critical crack size and longer fatigue residual life. Therefore, the weld joint at the pressure stress area and the smaller shear stress are not considered for the calculation of fatigue residual life. Calculation results of fatigue residual life of point 4 and point 5 on the main web are greater than those of point 2 and point 3 on the secondary web. The reason is that the thickness of main web is greater than that of secondary web. Hence, the fatigue residual life of danger point is smaller in tensile zone of secondary web.

Fig. 7 Distribution of stress zone in dangerous cross-section

From Table 6 and Fig.7, similar to point 1, the fatigue risk point 6 is in the compressive stress region. But when the hoisting trolley runs to fatigue risk position, point 6 will be subjected to the local stress. By the two stresses being synthesized, the first principal stress of point 6 is larger than that of point 1. So, the fatigue residual life of point 6 is less than the result of point1. However, compared with the fatigue risk points in the tensile stress region, results of first principal stress of point 6 and point 1 are all smaller and results of fatigue residual life are greater. Thus, the fatigue residual life of casting crane metal structure depend on the first principal stresses of fatigue risk points in the tension stress region.
5 Discussion

In order to verify the scientificity, rationality and adaptability of the method proposed in this paper, taking the engineering example (100/40t-28.5m eccentric rail casting crane originating from the extensive texts finished by testing personnel from Taiyuan Heavy Machinery Group Co., Ltd) as the research object, three aspects in the calculation process are analyzed and discussed as follows:

1) In engineering practice, the formation of high-low dislocation is often accompanied by the horizontal gap of rail joint (see Fig. 8). The coupling effect between high-low dislocation and horizontal gap of rail joint is discussed and the critical horizontal gap considering coupling effect under diverse high-low dislocation is given.

2) To demonstrate the rationality of the impact coefficient calculated by the kinematics-dynamics model of dynamic load caused by crane travelling on the high-low dislocation of track joint, the formula (9) of GB3811-2008 (Technical Committees., 2008) and formula (D.10) of ISO 8686-1:2012 (Xiaopeng L., 2012) are analyzed.

3) The Influence of high-low dislocation of track joint on fatigue residual life based on impact coefficients of different international standards is described, thus the completeness of above theory in the paper being proved.

Fig. 8 Track defect of casting crane with partial rail

5.1 Analysis of coupling effect between high-low dislocation and horizontal gap of rail joint

For the wheel crossing the track defect with coupling between high-low dislocation and horizontal gap of rail joint, the centroid trajectory of wheel is shown in Fig. 9. $e$ is the horizontal gap length of rail joint and $e_{s} = \sqrt{e^{2} - R^{2}}$.

For wheel radius being 400 mm, the critical horizontal gap considering the coupling effect under diverse high-low dislocation is calculated in Table 8. For diverse wheel radius and high-low dislocation, the critical horizontal gap taking into account the coupling effect is introduced in Fig. 10.
Table 8  Critical horizontal gap considering coupling effect (wheel radius being 400 mm)

<table>
<thead>
<tr>
<th>High-low dislocation $h_s$ (mm)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical gap $e_s$ (mm)</td>
<td>29.267</td>
<td>39.95</td>
<td>49.898</td>
<td>56.427</td>
<td>63.048</td>
</tr>
<tr>
<td>High-low dislocation $h_s$ (mm)</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Critical gap $e_s$ (mm)</td>
<td>69.022</td>
<td>74.505</td>
<td>79.599</td>
<td>84.374</td>
<td>89.882</td>
</tr>
</tbody>
</table>

As given in Table 8, for the wheel radius being 400mm and the high-low dislocation $h_s$ being 1mm, the corresponding critical horizontal gap $e_s$ is equal to 29.267mm. The analysis of the rest of high-low dislocation is similar to that of $h_s$ being 1mm.

From Fig.10, under the wheel radius being 100mm, the results of critical horizontal gap $e_s$ are equal to 4.4710 mm and 43.589mm, corresponding with the high-low dislocation $h_s$ being 0.1mm and 10mm. For the wheel radius being 500mm, the results of critical horizontal gap $e_s$ are equal to 9.9995 mm and 99.4987mm, which is corresponding with the high-low dislocation $h_s$ being 0.1mm an10mm. It can be seen that when the wheel radius is constant, the critical horizontal gap increases with the increase of the high-low dislocation. When the high-low dislocation is constant, the critical horizontal gap decreases with the decrease of wheel radius.

Fig. 10 Critical horizontal gap considering coupling effect under diverse wheel radius and high-low dislocation

Through the above analysis, if $e > e_s$, the coupling effect should be considered. But for $e \leq e_s$, the high-low dislocation should only be covered instead of the horizontal gap of track joint. Because the wheel radius R is far larger than the critical gap $e_s$, the coupling effect between the high-low dislocation and horizontal gap of rail joint generally can be replaced by the analysis process and results for the high-low dislocation of track joint.

5.2 Difference of impact coefficients based on variety international standards

The impact coefficients of variety international standards are shown in Fig.11, thus demonstrating the limitations of formula (9) of GB3811-2008 and the incompleteness of formula (D.10) of ISO 8686-1:2012.
As presented in Fig.11, for the high-low dislocation of rail joint being less than 2.7mm (4mm), the impact coefficient calculated by the formula (9) of GB3811-2008 is greater than that determined by the above theory in the paper (the formula (D.10) of ISO 8686-1:2012). The reason for this is that the formula (9) of GB3811-2008 is an empirical formula which only considers the running speed of casting crane and the minor high-low dislocation of rail joint and is suitable for the design stage. For a given arbitrary high-low dislocation of rail joint $h_s$, the impact coefficient obtained by the above theory in the paper is larger than that confirmed by the formula (D.10) of ISO 8686-1:2012. The reason is that the partial potential energy which the kinetic energy at time $t_s$ is converted into the potential energy in $\Delta t$ time is ignored in the formula (D.10) of ISO 8686-1:2012.

### 5.3 Influence of high-low dislocation of track joint on fatigue residual life

In order to study the influence of high-low dislocation of track joint on the fatigue residual life of casting crane metal structure, the influence degree is calculated by Eq.(42). The related calculations are shown in the Table 9 and Fig.12.

$$D = 1 - \frac{N_f}{N_{f0}}$$  \hspace{1cm} (43)

Where, $D$ is the influence of high-low dislocation of track joint on the fatigue residual life. $N_{f0}$ is the fatigue residual life for track without imperfection, namely the high-low dislocation being 0mm. $N_{fi}$ is the fatigue residual life for the high-low dislocation being $i$ mm.

<table>
<thead>
<tr>
<th>High-low dislocation of rail joint (h_s (mm))</th>
<th>Fatigue residual life (years)</th>
<th>Influence degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Based on above theory in the paper</td>
<td>Based on (\varnothing_4) calculated by ISO 8686-1:2012</td>
<td>Based on (\varnothing_4) calculated by GB3811-2008</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>-------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>0</td>
<td>59.9962</td>
<td>59.9962</td>
</tr>
<tr>
<td>1</td>
<td>43.5509</td>
<td>46.0604</td>
</tr>
<tr>
<td>3</td>
<td>23.5438</td>
<td>29.3239</td>
</tr>
<tr>
<td>5</td>
<td>13.5298</td>
<td>19.586</td>
</tr>
<tr>
<td>7</td>
<td>9.3745</td>
<td>13.6516</td>
</tr>
<tr>
<td>8</td>
<td>6.7593</td>
<td>11.5612</td>
</tr>
<tr>
<td>9</td>
<td>5.5343</td>
<td>9.8771</td>
</tr>
<tr>
<td>10</td>
<td>4.5859</td>
<td>8.4854</td>
</tr>
</tbody>
</table>
As described in Table 8 and Fig.12, the Influence of fatigue residual life increases with the increase of high-low dislocation of track joint regardless of $\Phi_4$ calculated by anyone of ISO 8686-1:2012, GB3811-2008 and above theory in the paper. Compared with GB/T3811, the influence degree curve obtained by either ISO 8686-1:2012 or the above theory is higher. The main cause is the different impact coefficients obtained by the formula (D.10) of ISO 8686-1:2012, the formula (9) of GB3811-2008 and the above method in the paper.

6 Conclusion

1) Aiming at the vertical impact dynamic response of casting crane mental structure caused by the defective track, the kinematics-dynamics model of dynamic load caused by crane travelling on defective track is proposed and the calculation formula of impact coefficient $\Phi_4$ is deduced. The coupling effect between high-low dislocation and gap of rail joint is discussed and the critical horizontal gap considering coupling effect under diverse high-low dislocation is given.

2) In view of security problems caused by performance degradation of the casting crane metal structure, the prediction method of fatigue residual life based on defective track is put forward and the influence of high-low dislocation of track joint on fatigue residual life is described on the basis of impact coefficients of different international standards, thus demonstrating the completeness of proposed method in the paper.

Acknowledgment

This paper is based on the extensive investigation and testing results finished by experts and testers from Taiyuan Heavy Machinery Group Co., Ltd. The author cordially thanks experts and testers for on-site survey and real-time detection. At the same time, this paper is sponsored by the National Science-technology Support Projects for the 13th Five-year Plan (2017YFC0805703-4) and Safety Assessment and Residual Life Assessment Method for Hoisting Machinery (2017JD01) and the Fund for Shanxi ‘1331Project’ key Subjects Construction.

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