Evaluation of the grating period based on laser diffraction by using a mode-locked femtosecond laser beam

Yuki SHIMIZU*, Kentaro UEHARA*, Hiraku MATSUKUMA* and Wei GAO*

*Department of Finemechanics, Tohoku University
6-6-01, Aramaki Aza Aoba, Aoba-ku, Sendai, Miyagi 980-8579, Japan
E-mail: yuki.shimizu@nano.mech.tohoku.ac.jp

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Abstract
This paper presents a newly proposed method referred to as the femtosecond laser diffraction method for measurement of the grating period. The proposed method is established in such a way that a mode-locked femtosecond laser beam is employed as a light source for the traditional laser diffraction method, in which the Littrow configuration is utilized to evaluate grating period. Since the optical modes in a mode-locked laser, which are often referred to as the optical frequency comb, have deterministic mode frequencies with equal intervals in frequency domain, multiple diffracted beams in the same diffraction order can be obtained. By utilizing the number of Littrow angles obtained from the multiple diffracted beams in the same diffraction order, highly accurate measurement of the grating period can be expected. In this paper, as the first step of research, a prototype optical setup with a mode-locked femtosecond laser source is developed, and some basic experiments are carried out to demonstrate the feasibility of proposed method.

Keywords: Diffraction grating, Grating period, Laser diffraction method, Mode-locked femtosecond laser, Littrow configuration

1. Introduction

Precision positioning is one of the key technologies utilized in many industrial fields such as the semiconductor industry or the precision machining industry (Gao et al., 2015). To carry out precision positioning, it is necessary to employ appropriate measurement technologies. Currently, optical encoders such as linear encoders (Feng et al., 2012) or surface encoders (Li et al., 2013) are often employed for the purpose due to their high resolutions over a wide measurement range. In the optical encoders, diffraction gratings having periodic fine pattern structures on its surface are employed as the scale for measurement, and are key components that determine not only the accuracy of measurement but also the measurement range of the systems (Gao, 2010). The assurance of a grating period by a reliable calibration method is therefore an important task to achieve highly accurate measurement in the optical encoders.

Many efforts have been made so far to evaluate the grating period. A method based on atomic force microscopes (AFMs) is a straightforward one in which the grating period is evaluated from three-dimensional profiles obtained by scanning of a cantilever probe on a grating surface (Dai et al., 2005, 2007). In the method, not only the grating period but also the grating depth can be evaluated simultaneously. However, due to the limited measurement range of AFMs, it takes a huge amount of time to carry out the calibration over a large area of grating surface. It is therefore not suitable for applying this method for the evaluation of grating period over a large area.

Another method for measurement of the grating period is based on the laser diffraction (Pekelsky et al., 2007), in which the grating period is calculated based on a simple theoretical equation with an angle of diffraction obtained in an experiment and a known optical frequency of the laser beam employed in the measurement (Brasil et al., 2015). In the method, an averaged value for periods in the laser-exposed area on the grating surface can be measured by a single measurement. Since non-contact optical measurement can be performed at high speed, it is suitable for the calibration of a diffraction grating over a large area. In the method, the accuracy of grating period measurement is mainly
determined by the accuracy of angles of diffraction detected in the Littrow configurations, as well as the stability of optical frequency of the laser beam employed in measurement. It is therefore preferred to obtain multiple angles of diffraction in several Littrow configurations so that an average over the values measured at a number of diffraction orders can be obtained. However, in the case of evaluating a diffraction grating having a small grating period, the number of diffracted beams that can be obtained by making a laser beam incident to the grating surface will be quite limited. In addition, the frequency of a monochromatic laser source used as a light source in the method could fluctuate during measurement, which degrades the accuracy of measurement.

In responding to the background described above, a new method referred to as the femtosecond laser diffraction method is proposed in this paper for measurement of the grating period. In the proposed method, a mode-locked femtosecond laser, which is often referred to as the optical frequency comb (Jones et al., 2000) with highly stabilized optical modes, is employed as the light source for measurement. With the employment of mode-locked femtosecond laser beam, a number of angles of diffraction corresponding to the optical frequencies of optical modes in the mode-locked femtosecond laser beam, which can be employed to improve the accuracy of grating period measurement, can be obtained. Another advantage of the proposed method is a highly stabilized optical frequency of each of the optical modes in the mode-locked femtosecond laser beam, which is expected to achieve further accurate measurement of the grating period. As the first step of research, an attention has been paid in this paper to verify the feasibility of the proposed femtosecond laser diffraction method. After a description of the principle of proposed method, the development of a prototype optical setup is described, followed by the results of grating period measurement carried out to verify the feasibility of the proposed method and the developed prototype optical setup.

2. Principle of the proposed femtosecond laser diffraction method

In the traditional laser diffraction method, a grating period can be evaluated by measuring the angle of diffraction when the Littrow configuration is established (Korpelainen et al., 2009). A schematic of the optical setup for the laser diffraction method is shown in Fig. 1. The setup is mainly composed of a laser source, a beam splitter (BS), a rotary table and a photodetector. In the setup, the angular displacement of the rotary table is measured by an angle sensor such as a rotary encoder embedded to the rotary table. A laser beam emitted from the laser source is made to passes through the BS, and is made incident to a diffraction grating, which is a target of interest, mounted on the rotary table. At first, the diffraction grating is positioned so that the zeroth-order Littrow configuration, a schematic of which is shown in Fig. 1(a), can be established. In the configuration, the zeroth-order diffracted beam is reflected back in the direction of incident laser beam, and is made incident to the detector such as a charge-coupled device (CCD) or a photodiode (PD). Since both the angle of incidence and the angle of diffraction are defined with respect to the normal of the grating surface, the Littrow configuration in the zeroth-order means that the angle of incidence is 0°. After the verification of zeroth-order Littrow configuration, rotate the grating about the Z-axis by using the rotary table to establish the first-order Littrow configuration, a schematic of which is shown in Fig. 1(b). The angular displacement given to the grating can be treated as the Littrow angle \( \theta_1 \). By using the obtained \( \theta_1 \), a grating period \( P \) can be calculated by the
following equation:

\[ P = \frac{1}{2\sin \theta_1} \frac{c}{f} \quad (1) \]

where \( f \) is the optical frequency of the light source, and \( c \) is the light speed in air. It should be noted that \( P \) is the average grating period over an area corresponding to the beam diameter. In the same manner as described above, by obtaining the Littrow angle \( \theta_m \) in the \( m \)-th order Littrow configuration, the grating period \( P \) can also be obtained based on the following equation:

\[ P = \frac{m}{2\sin \theta_m} \frac{c}{f} \quad (2) \]

As can be seen in Eqs. (1) and (2), the grating period \( P \) can be obtained not only in the first-order Littrow configuration but also in the multi-order Littrow configuration. By employing an average \( \overline{P} \) over the values measured at a number of diffraction orders, the uncertainty of grating period measurement can be improved. \( \overline{P} \) can be expressed by the following equation:

\[ \overline{P} = \sum_m \frac{m}{2\sin \theta_m} \frac{c}{f} \quad (3) \]

Meanwhile, since the maximum diffraction order by the setup is limited in theory, the number of Littrow angles \( m \) to be obtained by this method is quite limited in a practical case. Now Eq. (2) can be rewritten as follows:

\[ \sin \theta_m = \frac{m}{2\overline{P}} \frac{c}{f} \quad (4) \]

Since \( \sin \theta_m < 1 \), the maximum diffraction order \( m_{Max} \) is required to satisfy the following equation:

\[ m_{Max} < \frac{2\overline{P}}{c} \quad (5) \]

For example, in the case with a grating having a grating period \( P \) of 1000 nm and an iodine-stabilized He-Ne laser source with a wavelength of 633 nm, the number of maximum diffraction order becomes \( m_{Max} = 3 \), corresponding to a number of available Littrow angles of 6. This fact prevents the traditional laser diffraction method from achieving further precise measurement of the grating period. Another drawback of the traditional laser diffraction method is that the grating area to be irradiated by the incident laser beam changes in accordance with the change in diffraction order \( m \). In the \( m \)-th order Littrow configuration, an area of the grating to be exposed by the incident laser beam becomes \((1/\cos \theta_m)^2\) times larger than the area corresponding to the incident beam diameter. For example, in the case with a grating having a grating period \( P \) of 1000 nm and an iodine-stabilized He-Ne laser source with a wavelength of 633 nm, the grating areas to be exposed by the incident laser beam in the second- and third-order Littrow configurations become 1.5 times and 9.1 times, respectively, with respect to the area to be exposed in the first-order Littrow configuration; the change in area size to be exposed by the laser beam could affect the measurement accuracy of the grating period.

On the other hand, the proposed femtosecond laser diffraction method has a possibility of addressing the above mentioned issues of the traditional laser scattering method. In the proposed method, an innovation has been made to the above mentioned conventional laser diffraction method in such a way that a mode-locked laser is employed as the light source for the measurement, while employing a beam profiler as the photodetector to obtain a group of the first-order diffracted beams for evaluation of the grating period. A mode locked laser, which is often referred to as the optical frequency comb, has optical modes whose frequencies are arranged at equal interval in frequency domain. It is well known that the \( \text{th} \) mode frequency in the optical frequency comb can be expressed by the following equation (Jones et al., 2000):

\[ f_i = i \cdot f_{rep} + f_{CEO} \quad (6) \]

where \( f_{rep} \) and \( f_{CEO} \) are the pulse-repetition rate and the carrier envelope offset frequency, respectively. Although it is difficult to measure and control the optical frequency of a monochromatic laser on the order of several hundred THz, in
the case of femtosecond laser, \( f_{\text{rep}} \) can be synchronized with the high precision external frequency standard such as a rubidium (Rb) frequency standard, and thus all the mode frequencies can be stabilized. In the proposed femtosecond laser diffraction method, a mode-locked femtosecond laser is employed as the light source. Figure 2 shows a schematic of the optical configuration for the proposed femtosecond laser diffraction method. When a mode-locked laser beam is made incident to a grating surface, each of the optical modes in the laser beam will be diffracted with the different angle of diffraction associated with the mode frequency. It should be noted that only the positive first-order diffracted beams are shown in the figure for the sake of simplicity. Now we assume the case where the first-order diffracted beam of the \( i \)th optical mode is in the Littrow configuration as shown in Fig. 2(a). According to Eq. (2), the following equation can be obtained for the case:

\[
P_{(m,i)} = \frac{m}{2 \sin \theta_{(m,i)}} \cdot \frac{c}{f_i} = \frac{m}{2 \sin \theta_{(m,i)}} \cdot \frac{c}{i \cdot f_{\text{rep}} + f_{\text{CEO}}}
\]

(7)

where \( \theta_{(m,i)} \) is the Littrow angle of the \( m \)th-order diffracted beam of the \( i \)th optical mode with the optical frequency \( f_i \), and \( P_{(m,i)} \) is the grating period calculated by \( \theta_{(m,i)} \). Since a number of first-order diffracted beams can be obtained, a number of Littrow angles can thus be obtained by giving small angular displacements to the grating to be evaluated. By employing an average \( \bar{P} \) over the values measured at a number of Littrow configurations, which is much larger than the number of diffraction orders in the traditional laser scattering method, the uncertainty of grating period measurement can be improved in the proposed method. In the case of proposed femtosecond laser scattering method, \( \bar{P} \) can be expressed by the following equation:

\[
\bar{P} = \frac{\sum_m \sum_i P_{(m,i)}}{\sum_m \sum_i \frac{m}{2 \sin \theta_{(m,i)}} \cdot \frac{c}{f_i}}
\]

(8)

Since a number of diffracted beams in the same diffraction order from the same grating area irradiated by the incident femtosecond laser beam can be employed for the evaluation of grating period, the above mentioned drawbacks of the traditional laser scattering method are expected to be addressed in the proposed method.

3. Experiments

To verify the feasibility of proposed femtosecond laser scattering method, an optical setup was developed. Figure 3 shows a photograph of the developed optical setup for experiments. The setup was mainly composed of a femtosecond laser source (C-Fiber, Menlo Systems) having a central wavelength and a pulse repetition rate of 1550 nm and 100 MHz, respectively, a precision air-bearing spindle equipped with a rotary encoder having a resolution of 0.0038 arc-second, and a beam profiler (BP209-IR, Thorlabs) employed as the photodetector. An optical spectrum analyzer was also employed in the setup to monitor the optical spectrum of the mode-locked femtosecond laser from the laser.
source. In addition, a Fabry-Pérot cavity, which could act as an optical bandpass filter in frequency domain to periodically filter out optical modes in the mode-locked laser beam, having a free spectral range (FSR) of 100 GHz with a finesse of 3 was employed in the optical setup to modulate the angular distance between two neighboring beams in the group of first-order diffracted beams (Shimizu et al., 2017, Chen et al., 2017). Figure 4 shows the optical spectrum of the mode-locked laser beam made to pass through the Fabry-Pérot cavity obtained by the optical spectrum analyzer. In the optical setup, the Fabry-Pérot cavity was placed in the optical path of the femtosecond laser beam in such a way that the approximately half of the light power was incident to the cavity so that the pulse repetition rate can greatly be increased to about several-fold of the original FSR of the cavity (Wada et al., 2009, Shimizu et al., 2017). As can be seen in the figure, the repetition rate of the mode-locked femtosecond laser was modulated to have a repetition rate \( f'_{\text{rep}} \) of approximately 450 GHz. Denoting the frequency of \( j \)th transmitted mode with the repetition rate \( f'_{\text{rep}} \) as \( f'_{j} \), Eq. (7) can be rewritten as follows:

\[
P_{m,j} = \frac{m}{2 \sin \theta_{m,j}} \cdot \frac{c}{f'} = \frac{m}{2 \sin \theta_{m,j}} \cdot \frac{c}{f' + f'_{\text{CEO}}}
\]

Figure 5 shows a schematic of the developed optical setup shown in Fig. (3). It should be noted that a pair of mirrors (Mirrors 1 and 2 in Fig. 3) employed in the optical setup for handling the mode-locked laser beam are not indicated in the figure for the sake of simplicity. A linearly polarized mode-locked femtosecond laser beam was emitted from the light source, and was made incident to a quarter wave plate (QWP1) to obtain a circularly polarized femtosecond laser beam. The laser beam was then made to pass through the etalon to modulate its pulse repetition rate, and the modulated laser beam was split into the p-polarized and s-polarized beams. The modulated s-polarized beam was made to couple into a single-mode optical fiber, and was analyzed by the optical spectrum analyzer. Meanwhile, the modulated p-polarized beam was made to pass through another quarter wave plate (QWP2), and was then made incident to the diffraction grating, whose nominal grating period was designed to be 1052.63 nm, mounted on the
A group of first-order diffracted beams from the grating surface was then made to pass through the QWP2 again to be s-polarized beams, and were reflected by the PBS. Finally, the diffracted beams were made to focus on an active plane of the beam profiler by using an objective lens (OL1).

By using the developed optical setup, experiments were carried out to demonstrate the feasibility of the proposed femtosecond laser diffraction method. At first, the Littrow configuration in the zeroth-order was determined by using a retroreflector (Tamada et al., 2016). The retroreflector was mounted on the precision air-bearing spindle on behalf of the diffraction grating, and the femtosecond laser beam was made incident to the retroreflector. By using the femtosecond laser beam reflected from the retroreflector, a position of the beam profiler was adjusted. A schematic of the procedure for adjustment of the beam profiler is shown in Fig. 6. The angular position of the grating corresponding to the origin of the beam profiler ($x_0$) was determined by the linear approximation of five measured data points.

Following the determination of Littrow configuration in the zeroth-order described above, measurement of the grating period based on the proposed method was carried out. The grating was rotated by the precision air-bearing spindle to obtain the Littrow angles from the optical modes in the group of first-order diffracted beams. Figure 7 shows the light intensity distribution of the group of first-order diffracted beams obtained by the beam profiler. Since each diffracted optical mode had an angle of diffraction different from each other, and the mode-locked femtosecond laser beam made incident to the grating was modulated by a Fabry-Pérot cavity, multiple focused beams were observed on the beam profiler. As can be seen in the figure, the peak position of each optical mode could be detected from the light intensity distribution obtained by the beam profiler.

The change in position of the focused zeroth-order laser beam on the beam profiler due to the angular displacement of the diffraction grating.
After the verification of the group of first-order diffracted beams, an angular displacement was given to the
diffraction grating in a step of 0.02 degree over a range of 0.08 degree. Figure 8 shows the variation of light intensity
distribution of the focused first-order diffracted optical modes. As can be seen in the figure, each focused first-order
diffracted optical mode was moved along the \( X \)-direction due to the angular displacement of the grating. Figure 9
summarizes the relationship between the angular position of the grating and the peak position of the focused \( k \)th
mode.

Fig. 7  Light intensity distribution of the group of first-order diffracted beams obtained by the beam profiler at a
certain angular position

(a) A schematic of the shift of focused diffracted optical modes on the beam profiler

(b) Observed light intensity distribution

Fig. 8  Variation of the light intensity distribution of the focused first-order diffracted optical modes on the beam
profiler due to the angular displacement of the diffraction grating.

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diffracted optical mode was moved along the \( X \)-direction due to the angular displacement of the grating. Figure 9
summarizes the relationship between the angular position of the grating and the peak position of the focused \( k \)th
first-order diffracted optical mode extracted from the result in Fig. 8. As can be seen in the figure, the peak position of the focused first-order diffracted optical mode changed almost proportional to the angular displacement given to the grating. By extracting the angular position of the grating where the position of focused first-order diffracted optical mode with a mode number of $k$ became zero, the Littrow angle $\theta_{(1,k)}$ could be obtained.

Experiments were extended to rotate the grating in a step of 0.02 degree over a range of 2 degrees. Figure 10 shows the changes in position of the focused first-order diffracted optical modes as the change in angular position of the grating. It should be noted that the coefficient of determination was better than $R = 99.9\%$ in each of the linear approximations shown in Fig. 10. By using the obtained Littrow angles, grating period $P$ was evaluated based on Eq.
(9). The relationship between the calculated periods and the Littrow angles is summarized in Fig. 11. The nominal value of the grating period $P$ was evaluated to be 1052.11 nm, which was almost within the fabrication tolerance of the diffraction grating. These experimental results have demonstrated the feasibility of the proposed method. It should be noted that, as the first step of research, attentions have been paid in this paper to demonstrate the feasibility of the proposed femtosecond laser diffraction method. An optimization of the optical setup, as well as intensive analysis on the measurement uncertainty of the proposed method, will be carried out in future work.

4. Conclusion

For the evaluation of grating period, a new method referred to as the femtosecond laser diffraction method has been proposed. The method has been designed based on the conventional laser diffraction method for measurement of the grating period, while employing a mode-locked femtosecond laser as the light source having highly stable optical frequencies. Since each of the optical modes in a mode-locked femtosecond laser beam is diffracted with the different angle of diffraction associated with its mode frequency, a number of first-order diffracted beams from the same grating area irradiated by the incident femtosecond laser beam can be obtained to evaluate the grating period. To verify the feasibility of proposed femtosecond laser scattering method, an optical setup employing a mode-locked femtosecond laser beam with a central wavelength and a pulse repetition rate of 1550 nm and 100 MHz, respectively, has been developed. Since this paper has focused on the verification of the feasibility of proposed method for measurement of grating period, a Fabry-Pérot cavity, which acts as an optical bandpass filter in frequency domain to periodically filter out optical modes in the mode-locked laser beam, having a free spectral range (FSR) of 100 GHz with a finesse of 3 has been employed in the optical setup to modulate the angular distance between two neighboring beams in the group of first-order diffracted beams. A group of first-order diffracted optical modes has successfully been observed by a beam profiler, which has been employed as the photodetector in the optical setup. The Littrow angle corresponding to each of the diffracted optical modes has successfully been detected by the developed optical setup. Based on the theoretical equation with the obtained Littrow angles, the grating period has been evaluated to be 1052.11 nm, which has been almost within the fabrication tolerance of the diffraction grating. From these experimental results, the feasibility of the proposed method has been verified.

It should be noted that, as the first step of research, attentions have been paid in this paper to verify the feasibility of the proposed femtosecond laser diffraction method for measurement of the grating period. Further verification of the feasibility of proposed method by the comparison with conventional methods, a design optimization of the optical setup as well as intensive analyses on the measurement uncertainty of the proposed method are remained to be addressed, which will be carried out in future work.

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