Optimization and a reliability analysis of a cam-roller follower mechanism

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Abstract
The aim of this work is to synthesize a cam mechanism with translating roller follower based on optimization approaches and reliability analysis. The study consists of two parts. At first, this study performs preliminary deterministic optimization to find the optimum size of a cam system and to ensure its high operating performance. For this, an objective function is defined and that takes into account the three major design parameters typically influence the design of this type of mechanism: the base radius of the cam, the radius of the roller and the eccentricity. Also, constraints on performance and resistance indicators such as the acceptable pressure angles, size radius of curvature of directories curves, the efficiency of the transmission and the specific contact pressure between the cam and follower are taken into account in this work. The second part is devoted to a reliability-mechanism study whose system failure probability is estimated by two complementary methods: the approximation methods FORM / SORM and Monte Carlo simulation using the Phimeca reliability engineer Software. Moreover, inverse FORM was used to provide an elasticity factors evaluation in order to carry out possibilities of the cam mechanism design and reliability improvement. The study has shown that the reliability is greatly improved.

Keywords : Cam mechanism, Optimization, Reliability based design, FORM, SORM methods, Monte Carlo simulation

1. Introduction

For their advantage of obtaining different types of movements, cam follower mechanisms find applications in a wide variety of mechanical means, such as printing presses, machine tools, textile machines, automobile engines and so on. This type of mechanism consists of two essential mobile elements, the cam is a rotating element which gives reciprocating or oscillating motions to another element known as follower as well as used to transform rotary motion into a translating motion.

The requirement of high performance machinery demands efficient methods for the design of cams. In a cam follower system, the cam size minimization is usually the first condition in the designing process of a cam mechanism by its base circle radius. However, it is also necessary to take into consideration other parameters to improve the performance of the cam system while optimizing the robustness of the mechanism. These design parameters can be considered as optimization criteria.

In the area of the optimum design of cam mechanisms, several research studies are developed in regard to analysis and deterministic design optimization of cam mechanisms. Chan et al (1996) have presented an approach to design and optimize disk cams with different follower configurations. This approach is based on an exploratory search method known as the Monte Carlo method. Yu and Lee (1997) analyzed the problem of size optimization of a cam mechanism...
with a translating roller follower using non-linear programming techniques and a family of parametric polynomials for describing the motion curve. In the process of optimization, they used the kinematic characteristics, the curvature of the cam profile and the amount of offset of the follower. However, Ananthasuresh (2001) presented the design of a cam-roller mechanism using the kinematic design equations wherein roller-crank drives the cam. Mitsi et al. (2001) determined the design parameters by the minimization of the maximum compressive stress at the contact area of a cam-disk mechanism with translating roller follower. Later, Flores (2010) presented a computational approach for optimizing the design of disk cam mechanisms with eccentric translator rollers. For this, he considered an objective function with seven constraints. Recently, Mahesh et al. (2012) used finite element approach to optimize the shape of a flat face of existing follower into a curved face of modified follower. Moreover, a genetic algorithm is employed for the optimization of a cam mechanism with translating flat face follower from a multi objective point of view (Tsiafis et al. 2013). More recently, the design of a cam profile is presented by Laxmi et al. 2016 using B-Spline. B-Spline is applied to approximate the basic cycloidal velocity curve which has better motion characteristics. In their work, a computer-aided design and computer-aided manufacturing (CAD/CAM) system is developed for follower motion, i.e. displacement, velocity, acceleration, jerk and cam profiles.

The major disadvantage of these mechanisms consists in the occurrence of contact pressures that subjects the materials to important stresses in the immediate vicinity of the contact (Alaci et al. 2011). In addition, the direct contact between cam and follower induces a load torque on the cam due to friction while causing a loss of energy dissipated as heat in the two parts in contact. However, using a roller improves the efficiency of the transmission system by reducing friction, but increases the size of the mechanism. An attempt is made by Mahesh et al. (2012) to change the flat face of follower to a curved face follower of cam-follower mechanism of a four stroke internal combustion engine.

In contrast, very little work has been done in the field of probabilistic optimization design of cam mechanisms, although reliability based design methods are becoming quite well mastered and are much applied to obtain robust designs of certain applications in engineering.

In reality, uncertainties are inherent in mechanical systems (Du et al. 2009), such as those in random dimensions (tolerances), random clearances at joints, random deformations of structural components, mechanical parts materials, working conditions, etc... The ignorance or mistreatment of the uncertainties during mechanism synthesis may result in significant kinematic and dynamic errors and are therefore found to have critical effect on the final rating of machines.

Taking account of uncertainty in the mechanical analysis is an indispensable condition for optimal and robust design of structures. However, a Monte Carlo simulation approach is applied for the reliability analysis of a four-bar straight-line mechanism for which the effect of the probability distribution of the random parameters on the overall reliability is investigated (Crawford et al. 1987). A study on kinematics accuracy reliability for high speed press mechanisms is proposed (Jia et al. 2009). In this study, a universal accuracy analytical model of planar linkage mechanism is established, and the calculation model for kinematic accuracy reliability index of linkage mechanisms is built in terms of the state function of reliability. From these results, it is shown that the value of the failure probability of movement and reliability changes periodically giving advice to the design and early production. Moreover, in a recent work (Jianing et al. 2016), an evaluation of the reliability of a crank-slider mechanism with joint clearances is presented where the displacement of the slider is used to measure output accuracy. An indirect probability model (IPM) combined with the Kaplan–Meier estimator is proposed to express the mechanism reliability as a function of time. Results show that relative errors between the theoretical and experimental results of mechanism reliability are less than 5%, demonstrating the effectiveness of the proposed method. However, only a single mode of failure is considered in this work. Extensions to deal with non-homogenous data and multiple failure mechanisms should be investigated in order to get a more precise and realistic reliability of the system.

The present paper, prepared in this context, aims to perform a synthesis of a first type of cam-follower mechanism based on deterministic optimization approaches and reliability analysis. The study consists of two parts. The first part performs a preliminary deterministic optimization to find the optimal size of a cam system and to ensure its high operating performance. For this, an objective function is defined that takes into account the three major parameters typically influence the design of this type of mechanism: the base radius of the cam, the radius of the roller and the eccentricity. In addition to geometrical restrictions, constraints on performance and resistance indicators such as the pressure angle, the minimum radius of curvature of the cam, the transmission efficiency and the Hertzian pressure are taken into account in this work. The second part consists of reliability-mechanism analysis whose system failure probability is estimated by approximated methods: FORM/SORM and Monte Carlo simulation method, using the...
Phimeca reliability engineer Software. Moreover, inverse FORM is used to provide an elasticity factors evaluation in order to carry out possibilities of the cam mechanism reliability improvement.

2. Objective function and constraints

First, it is necessary to clearly define the objective function, which must take into account the basic parameters that influence the design and operation of the cam mechanism to be optimized. Since the objective function defined in this study aims to minimize the size and to ensure a good performance of this mechanism. However, decreasing the size means diminishing the dimensions of the mechanism and thus reducing the masses of the cam and the roller follower while maintaining adequate performance of the system. Thus the resisting forces will decrease such as: weight and the centrifugal force applied to the cam and the forces applied to the follower which are weight and inertia force. This will lead to minimizing the driving force of the electric motor and thus gaining energy during the period of the permanent movement of the mechanism where the work of all the engines forces is equal to the work of all resistant forces.

In the case of this work, this leads to minimizing the base radius of the cam \( R_b \), the maximum pressure angle reached during the rise phase \( \varphi_{\text{max-rise}} \) and the maximum pressure angle reached during phase of descent \( \varphi_{\text{max-return}} \). The first compound affects the mass of the cam via the minimization of the size of the cam and the two other terms associated with the pressure angle are related to the system performance.

The main variables in the design of this mechanism are therefore, the base radius \( R_b \), the radius of the roller \( R_g \) and the eccentricity \( e \) as shown in Fig. 1.

![Fig. 1 Kinematic scheme of the disc cam with translating follower (Flores 2013)](image)

However, this problem can be considered as an optimization problem, where the objective function \( f \) takes into account the size of the cam \( R_b \) and the pressure angle \( \varphi \). In this case, the pressure angle \( \varphi \), is formed (at any point) between the normal to the pitch curve and the direction of the follower motion.

The objective function to be optimized can be formulated as: (Flores 2013)

\[
f(R_b, R_g, e) = R_b + \varphi_{\text{max-rise}} + \varphi_{\text{max-return}}
\]

(1)

This optimization is subject to the constraints mentioned below.
- The first two constraints are imposed on \( \varphi_{\text{max-rise}} \) and \( \varphi_{\text{max-return}} \) and are as follows (Hajji 2003):

\[
g_1 = \varphi_{\text{max-rise}} \leq 30^\circ
\]

(2)

\[
g_2 = \varphi_{\text{max-return}} \leq 45^\circ
\]

(3)

- The third constraint arises from Fig. 1
\[ g_3 = R_b + R_g \geq e \]  

(4)

The radius of curvature of the cam profile is another factor that also affects the performance of the mechanism. It determines the radius of the roller used to ensure contact between the cam and the follower. To avoid undercutting as well as to ensure correct desired movement, it is not acceptable to have only a value of the cam curvature radius within the range \( 0 \leq \rho \leq R_g \) as mentioned by Flores (2013). This means if the cam is on the convex part, it is necessary that \( \rho_{\min} > R_g \). In fact, if the cam is on the concave part, the absolute value of the minimal curvature radius \( |\rho_{\min}| \) should be at least 2 to 3 times above to the radius of the roller (Norton 1999). Consequently, the fourth constraint, oriented to the radius of curvature, can be written as follows:

\[ g_4 = \begin{cases} 
\text{For } \rho \geq 0 & \rightarrow \rho_{\min} > R_g \\
\text{For } \rho < 0 & \rightarrow |\rho_{\min}| \geq 3R_g 
\end{cases} \]  

(5)

- The fifth and sixth constraints are specified by the following constructive inequalities to ensure that the mechanism can easily be assembled (Flores 2013):

\[ g_5 = R_g \leq e \]  

(6)

\[ g_6 = e \leq R_b \]  

(7)

- The next constraint is the Hertzian pressure. The contact pressure studied here refers to the stress developed at the contact surfaces between the roller and the cam due to tangential and normal loads (Chablat et al. 2007). Thus, the seventh is based on a criterion of mechanical strength, namely that the applied Hertzian stress to be minimized must not exceed the permissible stress. This constraint is determined by:

\[ g_7 = \sigma_{\max} - \sigma_{adm} < 0 \]  

(8)

The maximal applied Hertzian stress is given by: (Wang 1989)

\[ \sigma_{\max} = 0.564 \sqrt{\frac{F_n((1/(\rho-R_g)+1/R_g)}{[1-(\mu_1^2+1)/E_1+(1-\mu_2^2)/E_2)]} \]  

Where:

\[ F_n = \frac{P}{[1-(\mu(2\xi+1)\tan \varphi)]\cos \varphi} \]  

P is the total external load acting on the follower. This load is the algebraic sum of the external applied load, the follower weight, the inertia force and the spring force; \( t \) is the cam thickness; \( \mu_1, \mu_2 \) are Poisson’s ratio of the cam and follower, respectively and \( E_1, E_2 \) are the moduli of elasticity of the cam and follower, respectively; \( \mu \) is the coefficient of friction between the follower stem and the guide bearing;

\[ \xi = \left( q - \left( (R_b+R_g)^2 - e^2 + s \right) \right)/b \]  

q is the distance between the cam centre and the guide; \( s \) is the displacement; \( b \) is the length of the follower guide.

- The eighth constraint taken into account is the mechanical efficiency. The transmission of the power of the mechanism must be done with a minimum loss of energy due mainly to friction and pression angle. Thereby, the constraint on the minimal limit of energy loss (efficiency) is established as follows:

\[ g_8 = \eta_{\min} \geq 0.90 \]  

(9)

Where: \( \eta = 1 - (\mu(2\xi+1)\tan \varphi) \) (Artobolevsky 1997).

It is worth to note that in friction transmission mechanical systems, the efficiency is generally low comparatively to gear transmissions. For this reason, the minimum yield is fixed at 0.9.
Finally, lower and upper bands of the variables must be defined:

\[
R_{b}^{\text{inf}} \leq R_{b} \leq R_{b}^{\sup} \tag{10}
\]
\[
R_{g}^{\text{inf}} \leq R_{g} \leq R_{g}^{\sup} \tag{11}
\]
\[
e^{\text{inf}} \leq e \leq e^{\sup} \tag{12}
\]

### 3. Results and discussion

In this section, a case study is presented. The follower pursues a cosinusoidal law of the type Rise-Dwell-Return-Dwell. The input parameters are as given in Table 1.

Considering kinematic requirements the displacement, velocity and acceleration of the follower are determined (Fig. 2). The follower acceleration is critically parameter for design considerations where its values are related to the system inertia forces in which an infinite acceleration leads to an infinite inertia forces. This occurs when there is a discontinuity (two values at the same angle) in the velocity curve. In the present work, this problem was avoided by the choice of a motion law which offered an acceleration curve with finite values.

![Fig. 2 The follower motion diagrams](image)

The optimization problem formulated previously is to find the optimum size of the cam-roller follower mechanism under the eight constraints named above while considering the limit values of the base radius $R_b$, the radius of the roller $R_g$ and the eccentricity $e$. Thus the different combinations of values of these three parameters give different results. To resolve this problem, a “Differential Evolution DE” algorithm, developed previously (Hamoudi, Djeddou, et al. 2015 and 2016), is used. Furthermore, the results obtained are validated using the « Matlab fmincon » function.
which is a component of the MATLAB version 7.1 optimization toolbox (mathworks.com). This algorithm, which implements a quasi-Newtonian minimization routine with constraints, generally exhibits good results in solving technology problems (Xiaoping et al. 2008). Table 2 shows the results obtained by the two methods.

The DE control parameters considered for this case study are as follows:

- The population size NP = 50
- The maximum number of generations G\text{max} = 200
- The mutation factors F_1 = 0.8, F_2 = 1
- The crossing probability CR = 0.9

The three design variables R_b, R_g and e are taken into account within the intervals (15, 55), (4, 20) and (5, 20) respectively.

Table 1  Input parameters of the case study

<table>
<thead>
<tr>
<th>Materials properties (Tsiafis et al. 2013)</th>
<th>Functional requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cam Poisson’s ratio : ( \mu_1 = 0.3 )</td>
<td>Rod rise: ( h = 15 ) mm</td>
</tr>
<tr>
<td>Follower Poisson’s ratio : ( \mu_2 = 0.26 )</td>
<td>Rise angle: ( \theta \text{Rise} = 120^\circ )</td>
</tr>
<tr>
<td>Cam modulus of elasticity : ( E_1 = 2.1 \times 10^5 \frac{N}{mm^2} )</td>
<td>Dwell angle ( \theta \text{Dwell-up} = 60^\circ )</td>
</tr>
<tr>
<td>Follower modulus of elasticity : ( E_2 = 1.15 \times 10^5 \frac{N}{mm^2} )</td>
<td>Return angle: ( \theta \text{Return} = 60^\circ )</td>
</tr>
<tr>
<td>Cam permitted contact stress : ( \sigma_{adm} = 1750 \frac{N}{mm^2} )</td>
<td>Dwell angle: ( \theta \text{Dwell-down} = 120^\circ )</td>
</tr>
<tr>
<td>Spring constant of elasticity : 3.004 N/mm</td>
<td>External load : 100 N</td>
</tr>
<tr>
<td>Friction coefficient of follower/guide : ( \mu = 0.1 )</td>
<td>Rod mass : 0.3 kg</td>
</tr>
<tr>
<td>The spring preset : ( \delta = 0 ) mm</td>
<td>Cam rotation speed : 120 rev/min</td>
</tr>
<tr>
<td>Distance between cam /guide : ( q = 60 ) mm</td>
<td>Cam thickness : 8 mm</td>
</tr>
<tr>
<td>Guide length: ( b = 60 ) mm</td>
<td>Guide length: ( b = 60 ) mm</td>
</tr>
</tbody>
</table>

By analyzing the above Table, it can be clearly seen that the two methods give identical results with a total satisfaction of the constraints and a relatively short execution time (0.458938 seconds). Finally, as an example, Fig. 3 shows the two profiles of the cam obtained before and after optimization for \( R_b=36 \) mm, \( R_g=15 \) mm and \( e=8 \) mm.

Fig. 4 and 5 show respectively the evolution of the pressure angle and the radius of curvature as a function of the rotation angle of the cam. After observation of Fig. 4, it can clearly be seen that the maximum pressure angle in the rise and the return phases is smaller than that of permissible values of Eq. (2) and Eq. (3), which proves that these constraints are respected and therefore the mechanism does work correctly. In Fig. 5, the radius of curvature \( \rho \) of the cam is plotted as a function of the angle of rotation. The transition from convex (\( \rho > 0 \)) to concave regions (\( \rho < 0 \)) can be observed. In particular, the absolute minimum radius \( \rho_{\text{min}} \) in the concave region is 1773.5 mm, while the minimum radius \( \rho_{\text{min}} \) on the convex region is 22.962 mm. Both values prove that this mechanism is working well and undercutting will not occur in the concave region (where \( \rho < 0 \)). Also, contact stresses will not be high since Eq. (5) \( (g_4 = \left| \rho_{\text{min}} \right| \geq 3R_g ) \) is satisfied.

The addition of constraints \( g_4 \) and \( g_5 \) has improved significantly results of this optimization relatively to earlier research works. In fact, the satisfaction of the constraint of the radius of curvature has a positive effect on the Hertzian pressure. Also the same for the efficiency constraint which has a positive effect on the pressure angle ie a good transmission of energy.
Table 2  Global results obtained with the numerical approaches

<table>
<thead>
<tr>
<th>Optimization method</th>
<th>DEA</th>
<th>Matlab fmincon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function</td>
<td>72.4426</td>
<td>72.3977</td>
</tr>
<tr>
<td>Base radius $R_b$ (mm)</td>
<td>32.2500</td>
<td>32.1718</td>
</tr>
<tr>
<td>Roller radius $R_g$ (mm)</td>
<td>5.1000</td>
<td>5.1281</td>
</tr>
<tr>
<td>Exentricity $e$ (mm)</td>
<td>5.1000</td>
<td>5.1281</td>
</tr>
<tr>
<td>$</td>
<td>\varphi_{\text{max}-\text{Rise}}</td>
<td>$ ($^\circ$)</td>
</tr>
<tr>
<td>$</td>
<td>\varphi_{\text{max}-\text{Return}}</td>
<td>$ ($^\circ$)</td>
</tr>
<tr>
<td>$</td>
<td>\rho_{\text{min}}</td>
<td>$ (mm)</td>
</tr>
<tr>
<td>$\eta_{\text{min}}$</td>
<td>0.9004</td>
<td>0.9001</td>
</tr>
<tr>
<td>$g_1$</td>
<td>-22.0714</td>
<td>-22.0991</td>
</tr>
<tr>
<td>$g_2$</td>
<td>-12.7360</td>
<td>-12.6750</td>
</tr>
<tr>
<td>$g_3$</td>
<td>-32.2500</td>
<td>-32.1718</td>
</tr>
<tr>
<td>$g_4$ (for $\rho &lt; 0$)</td>
<td>-1758.2000</td>
<td>-1761.3000</td>
</tr>
<tr>
<td>$g_5$</td>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>$g_6$</td>
<td>-27.1500</td>
<td>-27.0437</td>
</tr>
<tr>
<td>$g_7$</td>
<td>-569.2580</td>
<td>-570.1000</td>
</tr>
<tr>
<td>$g_8$</td>
<td>-0.0004</td>
<td>-0.0001</td>
</tr>
</tbody>
</table>

Fig. 3  Standard cam profile and optimized cam profile obtained with the numerical optimization approach
4. Reliability analysis

The purpose of this section is to predict the probability (reliability) of failure of the system considered, followed by a sensitivity and a parametric analysis to assess the effect of the different design variables on the performance of the mechanism to highlight ways to improve the design. The probability of failure (reliability) in this work is estimated by calculating the reliability index $\beta$ (Lemaire 2005). Two complementary techniques are generally used: the first is the Monte Carlo simulation (Rubinstein 1981) and the second is based on first order and second order approximation methods called First Order Reliability Method (FORM) and Second Order Reliability Method (SORM) (Lemaire 2009). Both methods are implemented in the reliability software called Phimeca Software (phimeca.com).

4.1 Performance state function

To perform a reliability analysis, a performance limit state function must be defined that should take into account the
fundamental parameters that influence the design and operation of the cam follower mechanism. The performance function, taken here as a failure criterion, is defined by the descent of the minimum instantaneous efficiency $\eta_{\text{min}}$ to the fixed value of 0.9. The term resistance is therefore $\eta_{\text{min}}$ and it depends on several random variables. On the other hand, the term solicitation is the constant value 0.9. The performance function can be written as follows:

$$G = \eta_{\text{min}} - 0.9$$  \hspace{1cm} (13)

The failure is observed when $G < 0$.

The three design variables of this mechanism are taken first as variables following normal distributions and whose mean values are those obtained from the above optimization, and their variation coefficients (Table 3) values are according to bibliographic sources (Zhang et al. 2003).

Table 3  Statistics of random design variables

<table>
<thead>
<tr>
<th>Design random variable</th>
<th>Distribution law</th>
<th>Mean</th>
<th>Variation coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_b$ (mm)</td>
<td>Normal</td>
<td>32.25</td>
<td>1%</td>
</tr>
<tr>
<td>$R_g$ (mm)</td>
<td>Normal</td>
<td>5.1</td>
<td>1%</td>
</tr>
<tr>
<td>$e$ (mm)</td>
<td>Normal</td>
<td>5.1</td>
<td>1%</td>
</tr>
</tbody>
</table>

4.2 Reliability results

After obtaining results from the above deterministic optimization, the reliability of the cam mechanism of fig. 1 can be calculated and a sensitivity study may be then carried out.

As it is an usual, in similar studies, the simulation of Monte Carlo is taken here as a reference solution and from which the reliability indices, the failure probability and its covariance results are obtained as:

$$P_{\text{f, MCS}} = 0.3823, \quad \beta_{\text{MCS}} = 0.299446 \quad \text{with} \quad N=1000000 \text{ simulations and } CV(P_{\text{f}}) = 0.127\%.$$ 

Both FORM and SORM approximation methods, implemented in Phimeca software, are used to search for the reliability index and the failure probability. The limit state of Eq. (13) is approximated by a hyper plane based on the Abdo-Rackwitz algorithm (Abdo et al. 1990) for FORM and by a quadratic hyper surface for SORM (Hohenbichler et al. 1989, Tvedt 1990).

By analyzing results of Table 4, it can be said that the SORM method gives results comparable to those obtained by MC simulation while being more efficient in terms of calls to the limit state. However a small gap between the FORM and SORM results is revealed. This can be explained by the nonlinear limit state around the critical point. Moreover, the gap between SORM and Monte Carlo results probably means that the SORM is not very sufficient to cover the nonlinearity around the critical point. The calculation times are very short and are of the order of a few seconds. Convergence was achieved in a maximum of 172 iterations.

It should be also noted that the reliability indices above, obtained for the case study of this work, are very low ($\beta = 0.2976$ corresponding to a reliability of 61.70%). This is due in fact to the optimization results where the optimal efficiency value ($\eta_{\text{min}} = 0.9004$, table 2) is very close to the permissible limit of the constraint $g_8$. This clearly justifies why parametric, elasticity and inverse reliability methods are really necessary in such problems in order to increase the reliability value of the mechanism by making minor modifications to design variables.

Table 4  Reliability approximation results

<table>
<thead>
<tr>
<th>Approximation method</th>
<th>Nb alls</th>
<th>$P_f$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORM (Abdo-Rackwitz)</td>
<td>18</td>
<td>0.3830</td>
<td>0.2976</td>
</tr>
<tr>
<td>SORM (Hohenbichler / Breitung)</td>
<td>22</td>
<td>0.3830</td>
<td>0.2977</td>
</tr>
<tr>
<td>SORM (Tvedt, SQP)</td>
<td>172</td>
<td>0.3830</td>
<td>0.2977</td>
</tr>
</tbody>
</table>
5. Reliability products
5.1 Sensitivity and elasticity analysis

The sensitivity calculation with respect to probabilistic parameters, allows evaluating the sensitivities and the elasticities of the reliability index (the failure probability) with respect to all design distribution parameters. The elasticity of a parameter defines the impact of this parameter variation on the evolution of the reliability index (respectively the failure probability). It is calculated by differentiation of the index (respectively the probability) with respect to the first parameter (mean) or the second parameter (standard deviation). These derivatives are multiplied by the parameter/index (respectively parameter/probability) ratio to reach a dimensionless value.

Fig. 6 and 7 show respectively the histograms of the reliability index sensitivity and elasticity with respect to physical deterministic variables. The sensitivity diagram gives in % the magnitude and the sign of contribution of each design variable on the reliability (the failure probability) of the cam system. Whereas, the elasticity diagram may be very useful to point out ways of design improvements. Both figures indicate that failure probability level is largely influenced by the cam mean and standard deviation of the base circle. It can also be said that the failure probability level is quite sensitive to the roller and eccentricity mean values but not to their standard deviations.

![Reliability Index Histogram](image1)

**Fig. 6** Reliability index Histogram

![Sensitivities/Elasticities Histogram](image2)

**Fig. 7** Sensitivities/Elasticities histogram
Table 5 Sensitivities/Elasticities recapitulation results

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>e [ET]</td>
<td>-0.1163</td>
<td>-0.0200</td>
<td>0.0446</td>
<td>5.94e-03</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>( p_b ) [Moy]</td>
<td>3.0312</td>
<td>32.4551</td>
<td>-1.1569</td>
<td>-97.4166</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>( p_b ) [ET]</td>
<td>-0.5792</td>
<td>-0.9527</td>
<td>0.3356</td>
<td>0.2826</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>( p_g ) [Moy]</td>
<td>3.0332</td>
<td>51.9756</td>
<td>-1.1577</td>
<td>-15.4156</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>( p_g ) [ET]</td>
<td>-0.1370</td>
<td>-0.0235</td>
<td>0.0523</td>
<td>6.95e-03</td>
<td>NC</td>
<td>NC</td>
</tr>
</tbody>
</table>

5.2 Parametric analysis

As it can be concluded from the sensitivity and elasticity studies (Fig. 6 and 7), the cam base radius is the most influential parameter in the design of the cam mechanism. Fig. 8 allows quantifying, in detail, the decrease of the failure probability \( P_f \) or the reliability index \( \beta \) as a function of the cam base circle. Furthermore, this shows that the probability of failure decreases very rapidly (exponentially) with the increase in the average of the base radius of the cam until very low values of the order of 10^-12 are reached. As a result, the designer can choose the appropriate base radius for the value of the reliability he wishes to have.

![Fig. 8 Evolution of \( P_f \) and \( \beta \) as function of \( R_b \)](image)

5.3 Proposed reliability based design method

The aim of this problem analysis is to give the value of a deterministic parameter and to calculate the failure probability (or the reliability index) is known. During this problem analysis, PhimecaSoft computes the different reliability results that can be evaluated through a reliability calculation using an approximation method. However, In the case of this work, the study is focused on the cam base radius \( R_b \) due to its great effect on the reliability index \( \beta \) relatively to \( R_g \) and \( e \) (see Fig. 6 and 7). Therefore, the problem is to give mean values of \( R_b \) and to obtain the desired failure probability of \( P_f \), while \( R_g \) and \( e \) are fixed to their mean values \( (R_g\text{mean} = e\text{mean} = 5.1 \text{ mm}) \). The obtained results using this trial and error method are illustrated in Table 6. It is worth to notice that small changes in the mean value of the cam base radius results in a significant change in the reliability (failure probability) while value of the objective function vary slightly. By this way, the mechanical designer can analyze closely the optimal design for the comparison and the choice.
Table 6 Inverse FORM results

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$P_f$</th>
<th>Mean $R_b$</th>
<th>Efficiency $\eta_{min}$</th>
<th>Objective function $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2977</td>
<td>0.3830</td>
<td>32.25</td>
<td>0.9004</td>
<td>72.4426</td>
</tr>
<tr>
<td>0.7491</td>
<td>0.2269</td>
<td>32.4</td>
<td>0.9011</td>
<td>72.4746</td>
</tr>
<tr>
<td>1.0478</td>
<td>0.1474</td>
<td>32.5</td>
<td>0.9016</td>
<td>72.4963</td>
</tr>
<tr>
<td>1.6399</td>
<td>0.0505</td>
<td>32.7</td>
<td>0.9024</td>
<td>72.5406</td>
</tr>
<tr>
<td>2.5152</td>
<td>0.00595</td>
<td>33</td>
<td>0.9037</td>
<td>72.6093</td>
</tr>
<tr>
<td>3.0904</td>
<td>9.99 $\cdot 10^{-4}$</td>
<td>33.2</td>
<td>0.9046</td>
<td>72.6566</td>
</tr>
<tr>
<td>3.2333</td>
<td>6.12 $\cdot 10^{-4}$</td>
<td>33.25</td>
<td>0.9048</td>
<td>72.6685</td>
</tr>
<tr>
<td>3.3755</td>
<td>3.63 $\cdot 10^{-4}$</td>
<td>33.3</td>
<td>0.9050</td>
<td>72.6806</td>
</tr>
<tr>
<td>3.9408</td>
<td>4.06 $\cdot 10^{-5}$</td>
<td>33.5</td>
<td>0.9059</td>
<td>72.7295</td>
</tr>
<tr>
<td>4.7769</td>
<td>8.90 $\cdot 10^{-7}$</td>
<td>33.8</td>
<td>0.9071</td>
<td>72.8048</td>
</tr>
<tr>
<td>5.3264</td>
<td>5.01 $\cdot 10^{-8}$</td>
<td>34</td>
<td>0.9080</td>
<td>72.8564</td>
</tr>
</tbody>
</table>

Finally, as an example, Fig. 9 presents the cam profile of a robust solution corresponding to a base radius equal to 33.5 mm giving a failure probability of $4.06 \cdot 10^{-5}$, an efficiency of an order of 0.9059 and a value of 72.7295 for the objective function.

![An example of the obtained cam profile using reliability analysis](image)

6. Conclusions

The deterministic optimization of a cam size may enable the mechanical designer to select the appropriate geometric parameters of the mechanism elements to make them of a minimum size while respecting the constraints on performance indicators, such as the pressure angle and radius of curvature of the cam profile. However, according to the results obtained from this work, the weakness of deterministic optimization lies in the fact that its solutions are not robust, and therefore a design based on reliability is very necessary. Thereafter, the calculation of the reliability of the system is performed with very promising results. Furthermore, the search of the failure probability of the system, in relation to the dreaded event, was done by different ways, namely: Monte Carlo simulations and FORM/SORM approximation methods. Using approximation methods, we were able to obtain measurements of sensitivities and...
elasticity’s of this probability with respect to the mean and the standard deviations of each random variable. The latter measurements can really be a design support tool to choose the variables underlying the design of any mechanical system.

Finally, it appears convenient to conclude this study with reference to three points:
- Designing by deterministic optimization tools may lead to solutions which are not robust.
- The reliability analysis of a system may be considered like decision-making aid.
- The reliability approximation methods may be very useful to point out ways of system design improvements.

Acknowledgment

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