Nonlinear estimation of the transmission-input-shaft torque in a full-scale automotive vehicle

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Abstract

A nonlinear torque estimator for accurately estimating non-measurable transmission-input-shaft torque in a full-scale automotive vehicle with continuously variable transmission is proposed. The estimator is based on a single-degree-of-freedom, nonlinear drivetrain model; it describes the clearance-induced torsional dynamics of the drivetrain and simulates the non-measurable torque arising from measurable engine-output-shaft torque and drive-shaft torque. Parameters of the estimator are identified from time series obtained from the full-scale vehicle via particle swarm optimization. The estimated transmission-input-shaft torque agrees with the physical torque, suggesting that the estimation quality is above 94.5%. Our estimator extrapolates the transmission-input-shaft torque outside the identified time interval and estimates an unknown sample of the torque measured from a separate trial of the experiment. Our estimator is further improved by delaying the time series of input torques.

Keywords : Drivetrain, Vibration, Clearance, Full-scale vehicle, Nonlinear estimation, System identification, Particle swarm optimization

1. Introduction

Suppression control of clearance-induced torsional shocks and vibration in automotive drivetrains is of concern to vehicle manufactures, who aim to improve driving performance and fuel efficiency (Ebisu et al., 2016; Inoue, 1993; Morimura, 2003; Okabe and Murayama, 1989; Tomiyama and Kojima, 2009; Yoshikawa and Nagamatsu, 2000a,b). In recent years, such control has mostly been achieved via electronic control units (ECUs) (Kamaga et al., 2010; Tsuchida et al., 2014).

To make this control more accurate, the state variables of the drivetrain should be measured or estimated as accurately as possible; however, this is not an easy task because there are many non-measurable drivetrain variables due to the limited number of sensors, especially in production vehicles. To make matters worse, such variables are not easy to estimate due to the strong nonlinearity caused by the clearance and friction of the drivetrain, as has often been reported by vehicle manufacturers investigating acceleration/deceleration shocks (Inoue, 1993; Yoshikawa and Nagamatsu, 2000b).

In this study, we propose a novel nonlinear estimator that allows us to accurately estimate a non-measurable drivetrain torque, i.e., the transmission-input-shaft torque, which is usually hidden within the gearbox casing and is hardly measurable in production vehicles. We design our estimator as a double-input single-output (DISO) estimator that receives the measured engine-output-shaft torque T_{eg} and drive-shaft torque T_{ds} and outputs the estimated transmission-input-shaft torque T_{tm}^* , as shown in Fig. 1. In our estimator, the nonlinear dynamics of the drivetrain are described by a single degree-of-freedom (DOF) piecewise linear vibration model, which comprises a torque transfer function successfully used in our previous study (Yoshida et al., 2016) wherein another type of drivetrain model of single-input single-output (SISO) was built to predict the measurable drive-shaft torque.

Our single DOF approach can contribute to a low computational cost suitable for ECU implementation, in sharp contrast to any other studies based on multiple DOF models (Crowther and Zhang, 2005; Crowther et al., 2007a,b; Li

Fig. 3 Mechanical model representation of experimental torques.

During this period, the test driver received large fore-aft shocks from the vehicle body. Therefore, our proposed estimator should reproduce the dynamics of $T_{tm}(t)$ especially during this period, so as to suppress shocks and vibration.

2.2. Vibration model

To mechanically represent the dynamics of $T_{eg}(t)$, $T_{tm}(t)$, and $T_{ds}(t)$, we first consider a two-DOF vibration model with a viscoelastic support having a clearance shown in Fig. 3. In this model, it is assumed that the total inertia of the vehicle body is sufficiently larger than the other moments of inertia. Initially, we also ignore the external resistances for simplicity. In this case, the equations of motion of this model become

$$\ddot{\theta}_1 = \rho_1(T_{tm} - T_{ds}), \quad \ddot{\theta}_2 = \rho_1 T_{ds} - (\rho_1 + \rho_2) T_{tm} + \rho_2 T_{eg}, \quad \rho_i := I_i^{-1} \quad (i = 1, 2), \quad (2)$$

where θ_1 and I_1 respectively represent the relative angular displacement and the equivalent total inertia between the transmission-input-shaft and the drive-shaft, and θ_2 and I_2 represent those between the engine-output-shaft and the transmission-input-shaft.

Next, we modify the second equation of (2) to make it suitable for developing our estimator, which receives $T_{eg}(t)$ and $T_{ds}(t)$ and outputs $T_{tm}(t)$. As successfully demonstrated in our previous study (Yoshida et al., 2016), the effects of the external resistances can be modeled by the equivalent constant components of torques. Thus, we assume

$$T_{eg} := S_{eg} + \Delta_{eg}, \quad T_{tm} := S_{tm} + \Delta_{tm}, \quad T_{ds} := S_{ds} + \Delta_{ds}, \quad (3)$$

where S_{eg} , S_{tm} , and S_{ds} are the vibrational components of T_{eg} , T_{tm} , and T_{ds} , respectively, and Δ_{eg} , Δ_{tm} , and Δ_{ds} are their respective constant components. By substituting them into the second equation of (2), we obtain

$$\ddot{\theta}_2 = \rho_1 S_{ds} - (\rho_1 + \rho_2) S_{tm} + \rho_2 S_{eg} + \Delta, \quad (4)$$

where $\Delta := \rho_1 \Delta_{ds} - (\rho_1 + \rho_2) \Delta_{tm} + \rho_2 \Delta_{eg}$ is a constant. Then, we rewrite (4) around the equilibrium point $\bar{\theta}_2$ by introducing a new state variable $\phi := \theta_2 - \bar{\theta}_2$ and obtain

$$\ddot{\phi} = \rho_1 S_{ds} - (\rho_1 + \rho_2) S_{tm} + \rho_2 S_{eg}. \quad (5)$$

The resulting single-DOF vibration model (5) describes the dynamics of the vibrational component, ϕ , of θ_2 .

3. Construction of torque estimator

In this section, we propose a novel torque estimator that receives measurements of the engine-output-shaft torque T_{eg} and drive-shaft torque T_{ds} , and outputs an estimate of T_{tm} .

3.1. Torque transfer model

In our estimator design, we assume that the inputs T_{eg} and T_{ds} are known time-series, i.e.,

$$T_{eg} := T_{eg}(t) = S_{eg}(t) + \Delta_{eg}, \quad T_{ds} := T_{ds}(t) = S_{ds}(t) + \Delta_{ds}, \quad (6)$$

and the vibrational component of the output T_{tm} is a function of ϕ and $\dot{\phi}$, i.e.,

$$T_{tm} - \Delta_{tm} = S_{tm} := F(\phi, \dot{\phi}; \mathbf{q}), \quad (7)$$

where F is a torque transfer function that models forward and backward contacts between the clearance or backlash of a drivetrain; this was already given in our previous study (Yoshida et al., 2016) as

$$F(\phi, \dot{\phi}; \mathbf{q}) := F_k(\phi; \mathbf{q}) + F_c(\phi; \mathbf{q}) \dot{\phi}, \quad \mathbf{q} = (\mu_i, k^-, k^+, c^-, c^+, \sigma), \quad (8)$$

Fig. 4 Samples of functions (9) for $\sigma = 40$ and 10^3 .

Fig. 5 Proposed torque estimator that estimates $T_{tm}(t)$ from measured torques, $T_{ds}(t)$ and $T_{eg}(t)$.

where $\mathbf{q}$ is a parameter vector, μ [rad] is the width of the clearance, c^- [Ns/rad] and k^- [N/rad] are viscoelastic coefficients during the backward contact between the clearance, and c^+ and k^+ are those during the forward contact. Here, F_k and F_c are smoothed piecewise functions of steepness σ , as shown in Fig. 4 and given by

$$\begin{aligned} F_k(\phi; \mathbf{q}) &:= -k^- \text{ramp}(-\phi - \mu; \sigma) + k^+ \text{ramp}(\phi - \mu; \sigma), \\ F_c(\phi; \mathbf{q}) &:= c^- \text{step}(-\phi - \mu; \sigma) + c^+ \text{step}(\phi - \mu; \sigma), \end{aligned} \quad (9)$$

where $\text{ramp}(\cdot)$ and $\text{step}(\cdot)$ are smoothed unit ramp and unit step functions of the forms:

$$\text{ramp}(x; \sigma) := \frac{\ln\{1 + \exp(\sigma x)\}}{\sigma}, \quad \text{step}(x; \sigma) := \frac{1 + \tanh(\sigma x)}{2}. \quad (10)$$

3.2. Torque estimator

Substituting (6) and (7) into (5), we propose a torque estimator of the following form:

$$\ddot{\phi}(t) = \rho_1 \{T_{ds}(t) - \Delta_{ds}\} - (\rho_1 + \rho_2) S_{tm}^*(t) + \rho_2 \{T_{eg}(t) - \Delta_{eg}\}, \quad (\phi(0), \dot{\phi}(0)) = \phi_0, \quad (11a)$$

$$T_{tm}^*(t) := S_{tm}^*(t) + \Delta_{tm}, \quad S_{tm}^*(t) := F(\phi(t), \dot{\phi}(t); \mathbf{q}), \quad (11b)$$

where $T_{eg}(t)$ and $T_{ds}(t)$ are the measured torques already described in Section 2.1, $T_{tm}^*(t)$ is the estimated T_{tm} , ϕ_0 is the initial condition, and $F(\phi(t), \dot{\phi}(t); \mathbf{q})$ is the torque transfer function provided in (8).

Figure 5 shows a block diagram of our proposed torque estimator (11), which receives the measured engine-output-shaft torque $T_{eg}(t)$ and drive-shaft torque $T_{ds}(t)$ and outputs the estimated transmission-input-shaft torque $T_{tm}^*(t)$. The dashed rectangle in the left part of the diagram represents (11a), which receives $T_{eg}(t)$, $T_{ds}(t)$, and $S_{tm}^*(t)$ and outputs $\phi(t)$ and $\dot{\phi}(t)$. The right dashed rectangle indicates (11b), which receives $\phi(t)$ and $\dot{\phi}(t)$ and outputs $T_{tm}^*(t)$.

4. Identification of unknown parameters

The estimator (11) has eleven unknown system parameters:

$$\mathbf{p} = (p_1, p_2, \dots, p_{11}) := (\rho_1, \rho_2, \Delta_{eg}, \Delta_{ds}, k^-, k^+, c^-, c^+, \mu, \sigma, \Delta_{tm}), \quad (12)$$

as listed in Table 1. These parameters are identified based on the measured time series (1) as follows.

Table 1 Parameters of the estimator.

Parameter		Unit	Equation
ρ_1	Inverse of inertia moment of the first rotor	$(\text{kg m}^2)^{-1}$	(11a)
ρ_2	Inverse of inertia moment of the second rotor	$(\text{kg m}^2)^{-1}$	
Δ_{eg}	Constant component of engine-output-shaft torque	Nm	
Δ_{ds}	Constant component of drive-shaft torque	Nm	
k^-	Stiffness during backward contact	N/rad	(11b)
k^+	Stiffness during forward contact	N/rad	
c^-	Viscous damping during backward contact	Ns/rad	
c^+	Viscous damping during forward contact	Ns/rad	
μ	Width of clearance	Rad	(11b)
σ	Steepness of smoothing functions		
Δ_{tm}	Constant component of transmission-input-shaft torque	Nm	

Table 2 Feasible ranges of the parameters p_j in (12), where $\mathbb{R}$ is the set of real numbers and $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} | x \geq 0\}$ is the set of non-negative real numbers.

Parameter	Feasible range	Parameter	Feasible range	Parameter	Feasible range
$p_1 (= \rho_1)$	$\mathbb{S}_1 := \mathbb{R}_{\geq 0}$	$p_5 (= k^-)$	$\mathbb{S}_5 := \mathbb{R}_{\geq 0}$	$p_9 (= \mu)$	$\mathbb{S}_9 := \mathbb{R}_{\geq 0}$
$p_2 (= \rho_2)$	$\mathbb{S}_2 := \mathbb{R}_{\geq 0}$	$p_6 (= k^+)$	$\mathbb{S}_6 := \mathbb{R}_{\geq 0}$	$p_{10} (= \sigma)$	$\mathbb{S}_{10} := \mathbb{R}_{\geq 0}$
$p_3 (= \Delta_{\text{eg}})$	$\mathbb{S}_3 := \mathbb{R}$	$p_7 (= c^-)$	$\mathbb{S}_7 := \mathbb{R}_{\geq 0}$	$p_{11} (= \Delta_{\text{tm}})$	$\mathbb{S}_{11} := \mathbb{R}$
$p_4 (= \Delta_{\text{ds}})$	$\mathbb{S}_4 := \mathbb{R}$	$p_8 (= c^+)$	$\mathbb{S}_8 := \mathbb{R}_{\geq 0}$		

4.1. Formulation of identification problem

We introduce a cost function of $\mathbf{p}$ in the form:

$$g(\mathbf{p}) := \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |T_{\text{tm}}(\tau) - T_{\text{tm}}^*(\tau; \mathbf{p})| d\tau, \quad (13)$$

where $T_{\text{tm}}(t)$ is the measured transmission-input-shaft torque in (1), $T_{\text{tm}}^*(t; \mathbf{p})$ is the torque estimated with (11) using $\mathbf{p}$, and $t_1 \leq t \leq t_2$ is the time interval of interest, which is selected as $[t_1, t_2] = [1.5, 3]$ so that it covers the initial stationary state during $t < 1.6$ s, the large-amplitude vibration during $1.7 < t < 2.2$ s, and the final stationary state during $t > 2.8$ s of the measured data in Fig. 2.

The cost function (13) is numerically calculated as follows. Using $\mathbf{p}$, we first numerically solve the estimator (11) for the constant input $T_{\text{ds}}(t) = T_{\text{ds}}(t_1)$ and $T_{\text{eg}}(t) = T_{\text{eg}}(t_1)$ to determine the initial equilibrium $\bar{\phi}(t_1)$ at $t = t_1$. Then, we obtain the numerical solution of (11), starting from $\phi_0 = (\bar{\phi}(t_1), 0)$ and calculate

$$g(\mathbf{p}) = \frac{1}{m} \sum_{n=1}^m |T_{\text{tm}}(t^n) - T_{\text{tm}}^*(t^n; \mathbf{p})|, \quad t^n := t_1 + n\delta_t, \quad t^m := t_2, \quad (14)$$

where n and m are integers and $\delta_t := 10^{-3}$ s is the step size of a numerical solution that is identical to the sampling interval of the measured data, $T(t)$, in (1).

In this study, the unknown parameters $p_j, j = 1, \dots, 11$ in (12) have feasible physical ranges $\mathbb{S}_j$ that are specified in Table 2, where $\mathbb{R}$ is the set of real numbers and $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} | x \geq 0\}$ is the set of non-negative real numbers. Here, the inversed inertia moments $p_1, p_2 (= \rho_1, \rho_2)$, stiffnesses $p_5, p_6 (= k^-, k^+)$, viscous dampings $p_7, p_8 (= c^-, c^+)$, and width of clearance $p_9 (= \mu)$ are physically given as non-negative real numbers. The steepness of the smoothing functions $\sigma (= p_{10})$ is also assumed to be a non-negative real number. On the contrary, $\Delta_{\text{eg}}, \Delta_{\text{ds}}$, and $\Delta_{\text{tm}} (= p_3, p_4, \text{ and } p_{11})$ are assigned arbitrary real values.

Using the cost function (13) and $\mathbb{S}_j$ in Table 2, we define a constrained optimization problem as follows:

$$\underset{\mathbf{p}}{\text{Minimize}} \quad g(\mathbf{p}) \quad \text{subject to} \quad \mathbf{p} \in \mathbb{S}_1 \times \dots \times \mathbb{S}_{11}. \quad (15)$$

Hereafter, $\mathbf{p}^*$ denotes the optimal solution of (15). The constrained problem (15) can be rewritten as an equivalent unconstrained problem (Parsopoulos and Vrahatis, 2002) of the following form:

$$\underset{\mathbf{p}}{\text{Minimize}} \quad G(\mathbf{p}) := \begin{cases} V & \text{for } \mathbf{p} \notin \mathbb{S}_1 \times \dots \times \mathbb{S}_{11}, \\ g(\mathbf{p}) & \text{otherwise,} \end{cases} \quad \text{subject to} \quad \mathbf{p} \in \mathbb{R}^{11}, \quad (16)$$

where V is a penalty value. If $V > g(\mathbf{p}), \forall \mathbf{p} \in \mathbb{S}_1 \times \dots \times \mathbb{S}_{11}$, the optimal solution of (16) equals $\mathbf{p}^*$; therefore, we can solve (16) using a sufficiently large V .

Fig. 6 Cost $G(\mathbf{gb}(k))$ as a function of PSO iteration k .

Table 3 Identified parameter vector $\mathbf{p}_o$ from measured $\mathbf{T}(t)$.

Component	Value	Unit	Component	Value	Unit
$p_1(= \rho_1)$	1.113	$(\text{kg m}^2)^{-1}$	$p_7(= c^-)$	1.976×10^2	Ns/rad
$p_2(= \rho_2)$	2.085×10^{-1}	$(\text{kg m}^2)^{-1}$	$p_8(= c^+)$	6.520×10	Ns/rad
$p_3(= \Delta_{eg})$	2.818×10	Nm	$p_9(= \mu)$	1.212×10^{-2}	Rad
$p_4(= \Delta_{ds})$	-4.395	Nm	$p_{10}(= \sigma)$	6.397×10	
$p_5(= k^-)$	2.000×10^3	N/rad	$p_{11}(= \Delta_{tm})$	1.504×10	Nm
$p_6(= k^+)$	1.743×10^{-2}	N/rad			

4.2. Particle swarm optimization

We employ PSO (Shi and Eberhart, 1998) to solve (16). We consider a swarm of N candidate solutions:

$$\{\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_i, \dots, \mathbf{p}_N\}, \quad \mathbf{p}_i \in \mathbb{R}^{11}, \quad (17)$$

which are called particle positions. Each component of $\mathbf{p}_i$ is recursively updated by

$$\begin{cases} v_{ij}(k+1) = \omega v_{ij}(k) + c_1 \text{rand}_{1ij}(k) (p_{bij}(k) - p_{ij}(k)) + c_2 \text{rand}_{2ij}(k) (g_{bj}(k) - p_{ij}(k)), \\ p_{ij}(k+1) = p_{ij}(k) + v_{ij}(k+1), \quad (k = 0, 1, \dots, k_{\max}), \end{cases} \quad (18)$$

where $p_{ij}(k)$ is the j -th component of $\mathbf{p}_i$ at iteration k ; $v_{ij}(k)$ is the corresponding velocity; ω , c_1 , and c_2 are system parameters of PSO; $\text{rand}_{1ij}(k)$ and $\text{rand}_{2ij}(k)$ are random values independently generated for each i , j , and k with a uniform distribution in $[0, 1]$; and $p_{bij}(k)$ and $g_{bj}(k)$ are the j -th components of the vectors $\mathbf{pb}_i(k)$ and $\mathbf{gb}(k) \in \mathbb{R}^{11}$, respectively. $\mathbf{pb}_i(k)$ is the position of the particle taking the lowest cost among those at $\mathbf{p}_i(0), \dots, \mathbf{p}_i(k)$; this is called the personal best. $\mathbf{gb}(k)$ is the position of the particle with the lowest cost among all particles for all iterations up to k ; this is called the global best. For sufficiently large $k_{\max}$, $\mathbf{gb}(k_{\max})$ is expected to be close to the optimal solution $\mathbf{p}^*$. The PSO parameters are set to $\omega = 0.729$ and $c_1 = c_2 = 1.49445$ (Clerc and Kennedy, 2002; Eberhart and Shi, 2000) in this study.

5. Estimation results

5.1. Identified parameters of the estimator

The identification conditions are empirically selected as follows. The penalty value of the unconstrained problem (16) is set to $V = 10^4$. All initial particle velocities $\mathbf{v}_i(0) = (v_{i1}(0), \dots, v_{i11}(0))$ are set to $(0, \dots, 0)$. The initial particle positions $\mathbf{p}_i(0)$ are distributed at 2^{11} uniform grid points on an 11-dimensional subspace

$$\begin{aligned} D := & [5, 6] \times [0, 1] \times [25, 30] \times [-10, -5] \times [10^3, 10^5] \times [5 \times 10^2, 10^3] \\ & \times [90, 95] \times [85, 90] \times [7 \times 10^{-3}, 8 \times 10^{-3}] \times [10^3, 5 \times 10^3] \times [10, 15]. \end{aligned} \quad (19)$$

Thus, the number of particles becomes $N = 2^{11} = 2048$.

Applying the method in Section 4 and the above conditions, we identify the parameter vector $\mathbf{p}$ in (12). Figure 6 shows the cost $G(\mathbf{gb}(k))$ as a function of the PSO iteration k ; the cost rapidly decreases and mostly converges after $k = 150$. Table 3 shows the identified parameter vector $\mathbf{p}_o := \mathbf{gb}(k = 250)$ with $G(\mathbf{p}_o) \approx 2.71707$.

5.2. Torque estimation result

Figure 7 shows the time series of the measured and estimated torques. In the upper graph, the blue and green lines indicate the measured time series of the engine-output-shaft torque $T_{eg}(t)$ and the drive-shaft torque $T_{ds}(t)$, respectively, which are already shown in Fig. 2. The lower graph shows the time series of the transmission-input-shaft torques. The black line indicates the measured torque $T_{tm}(t)$, which is already shown in Fig. 2. The red line indicates the estimated

Fig. 7 Measured and estimated torques during the time interval $1.5 < t < 3$ s, corresponding to those of parameter identification.

Fig. 8 Extrapolation of transmission-input-shaft torque.

torque $T_{tm}^*(t; \mathbf{p}_o)$ using our estimator (11) with the identified parameter vector $\mathbf{p}_o$ in Table 3, subjected to $T_{eg}(t)$ and $T_{ds}(t)$ as presented in the upper graph. The labels P_i and R_i ($i = 1, 2, 3$) denote the i -th peak and the ravine of the torque, respectively. The estimated torque $T_{tm}^*(t; \mathbf{p}_o)$ mostly agrees with the measured torque $T_{tm}(t)$. That is, $T_{tm}^*(t; \mathbf{p}_o)$ almost reproduces the waveform and the presence of all peaks P_i and ravines R_i . In particular, $T_{tm}^*(t; \mathbf{p}_o)$ accurately reproduces the peak value of P_1 and also the first rising timing at $t \approx 1.7$ s.

We evaluate the accuracy of the estimated torque as the quality of the following form (Brown and Grande-Moran, 1997; Yoshida et al., 2016):

$$Q(\mathbf{p}, t_1, t_2) := \left(1 - \frac{e(\mathbf{p}, t_1, t_2)}{e_0(t_1, t_2)} \right) \times 100 [\%], \quad (20)$$

where $e(\mathbf{p}, t_1, t_2) := (t_2 - t_1)^{-1} \int_{t_1}^{t_2} |T_{tm}(\tau) - T_{tm}^*(\tau; \mathbf{p})| d\tau$ and $e_0(t_1, t_2) := (t_2 - t_1)^{-1} \int_{t_1}^{t_2} |T_{tm}(\tau)| d\tau$. In the case of Fig. 7, we have $Q(\mathbf{p}_o, 1.5, 3) \approx 95.4\%$.

From the result shown in Fig. 7, we find that our single-DOF estimator with the identified parameters can successfully estimate the physically measured transmission-input-shaft torque in a full-scale vehicle.

5.3. Estimation capability for unknown data

5.3.1. Extrapolation of response outside the identified time interval

In Fig. 8, the blue and green lines in the upper graph and the black line in the lower graph indicate the measured torques $T_{eg}(t)$, $T_{ds}(t)$, and $T_{tm}(t)$, respectively, which are the same data presented in Fig. 7 with the plotting range extended

Fig. 9 Additional example of experimental data from a separate trial as a function of time.

Fig. 10 Measured and estimated transmission-input-shaft torque of the second sample.

to $t = 5.5$ s. The red line in the lower graph indicates the estimated torque $T_{tm}^*(t; \mathbf{p}_o)$ according to our estimator (11) using the parameter vector $\mathbf{p}_o$ mentioned in Table 3. The estimated $T_{tm}^*(t; \mathbf{p}_o)$ during $1.5 \leq t \leq 3$ s provides the same result as in Fig. 7. However, after $t = 3$ s, $T_{tm}^*(t; \mathbf{p}_o)$ is extrapolated based on the parameter vector $\mathbf{p}_o$ identified between $1.5 \leq t \leq 3$ s. In the extrapolated time interval, good agreement between $T_{tm}^*(t; \mathbf{p}_o)$ and $T_{tm}(t)$ is maintained and $Q(\mathbf{p}_o, 3, 5.5)$ is found to be $\approx 98.6\%$.

Therefore, it is clear that our estimator can extrapolate the transmission-input-shaft torque outside of the identified time interval.

5.3.2. Estimation of an unknown data sample

Figure 9 shows a further sample of the experimental data. This was taken from a separate trial of the same experiment (with the same vehicle, environment, and experimental conditions) as the sample already shown in Fig. 2. The measured torques are labeled $T_{eg2}(t)$, $T_{tm2}(t)$, and $T_{ds2}(t)$, indicating that they are related to the second sample. The axes and legends are all similar to those of the first sample. The lower graph shows that a second transient vibration began near $t \approx 4.2$ s. This second transient was due to an external disturbance in the vehicle due to the environment.

Figure 10 shows the estimation result. The black and red lines indicate the measured torque $T_{tm2}(t)$ of the second sample shown in Fig. 9 and the estimated torque $T_{tm2}^*(t; \mathbf{p}_o)$, respectively. Here, the manner of obtaining $T_{tm2}^*(t; \mathbf{p}_o)$ is different from the estimation cases that yield Figs. 7 and 8. The parameter vector $\mathbf{p}_o$ is the same as that shown in Table 3, which is identified from the first sample of data in Fig. 2. The input torques received by the estimator are replaced with $T_{eg2}(t)$ and $T_{ds2}(t)$. Thus, based on the estimator identified from the first sample, $T_{tm2}^*(t; \mathbf{p}_o)$ estimates the $T_{tm2}(t)$ value of the second sample.

In Fig. 10, it is clear that the estimated torque $T_{tm2}^*(t; \mathbf{p}_o)$ mostly agrees with the measured torque $T_{tm2}(t)$ including the second transient vibration beginning at $t \approx 4.2$ s. In this case, $Q(\mathbf{p}_o, 1.5, 3) \approx 94.5\%$ and $Q(\mathbf{p}_o, 3, 5.5) \approx 98.3\%$; these

Fig. 11 Cost $G(\mathbf{gb}(k))$ as a function of Δ_t^{eg} with $\Delta_t^{\text{ds}} = 0$ s.

Fig. 12 Cost $G(\mathbf{gb}(k))$ as a function of Δ_t^{ds} with $\Delta_t^{\text{eg}} = 0$ s.

Table 4 Identified parameter vector $\mathbf{p}_{o2}$ via time-shifted $T'_{\text{eg}}(t)$ and $T'_{\text{ds}}(t)$ for $(\Delta_t^{\text{eg}}, \Delta_t^{\text{ds}}) = (0, 15 \times 10^{-3})$ s at $k = 250$.

Component	Value	Unit	Component	Value	Unit
$p_1(= \rho_1)$	3.727×10^{-1}	$(\text{kg m}^2)^{-1}$	$p_7(= c^-)$	9.234×10	Ns/rad
$p_2(= \rho_2)$	1.214×10^{-2}	$(\text{kg m}^2)^{-1}$	$p_8(= c^+)$	1.070×10^2	Ns/rad
$p_3(= \Delta_{\text{eg}})$	2.276×10	Nm	$p_9(= \mu)$	8.742×10^{-3}	Rad
$p_4(= \Delta_{\text{ds}})$	-2.867×10^{-1}	Nm	$p_{10}(= \sigma)$	1.898×10^2	
$p_5(= k^-)$	6.902×10^2	N/rad	$p_{11}(= \Delta_{\text{tm}})$	1.847×10	Nm
$p_6(= k^+)$	2.304×10^2	N/rad			

are comparable to the values of 95.4% for the estimation case in Fig. 7 and 98.6% for the extrapolation case in Fig. 8.

Therefore, it is apparent that our estimator can accurately estimate the unknown sample of data, even if it contains a second transient vibration due to an external disturbance.

6. Improvement of the estimator with time shift

In the above torque estimation results shown in Figs. 7, 8, and 10, we can find some minor amplitude discrepancies between the estimated and measured torques near the first ravine R_1 and the second peak P_2 . In this section, we improve these discrepancies by modifying our estimator.

6.1. Estimator modification with time shift

We modify the estimator (11) to receive

$$T'_{\text{eg}}(t) := T_{\text{eg}}(t + \Delta_t^{\text{eg}}), \quad T'_{\text{ds}}(t) := T_{\text{ds}}(t + \Delta_t^{\text{ds}}), \quad (21)$$

in place of $T_{\text{eg}}(t)$ and $T_{\text{ds}}(t)$, respectively, where Δ_t^{eg} and Δ_t^{ds} [s] are time shifts.

To examine the effects of Δ_t^{eg} and Δ_t^{ds} , Fig. 11 shows the cost $G(\mathbf{gb}(k = 250))$ for $(\Delta_t^{\text{eg}}, \Delta_t^{\text{ds}}) = (i \times 10^{-3}, 0)$ [s], and Fig. 12 shows those for $(\Delta_t^{\text{eg}}, \Delta_t^{\text{ds}}) = (0, i \times 10^{-3})$ [s], $i = -15, -14, \dots, 30$, indicating that adding time shifts can reduce the cost and improve the estimation accuracy. The maximal accuracy is found at $(\Delta_t^{\text{eg}}, \Delta_t^{\text{ds}}) = (0, 15 \times 10^{-3})$ [s]; the corresponding parameter vector $\mathbf{p} = \mathbf{p}_{o2}$ is shown in Table 4. In this case, the cost improves to 1.71755 (63.2% of the value without time shifting).

6.2. Torque results from the modified estimator

Figure 13 shows the estimation result. The black line indicates the measured torque $T_{\text{tm}}(t)$, which is already shown in Fig. 8. The red line indicates the estimated torque $T_{\text{tm}}^{**}(t; \mathbf{p}_{o2})$ using the modified estimator utilizing the parameter vector $\mathbf{p}_{o2}$ mentioned in Table 4 subjected to $T'_{\text{eg}}(t)$ and $T'_{\text{ds}}(t)$ with $(\Delta_t^{\text{eg}}, \Delta_t^{\text{ds}}) = (0, 15 \times 10^{-3})$ [s]. We use the time interval $1.5 \leq t \leq 3$ s for parameter identification.

The result shows that the waveform difference is reduced at the first ravine R_1 and the second peak P_2 , whereas it shows a relatively significant increase at the first peak P_1 and during the extrapolated interval in comparison with our first results shown in Figs. 7 and 8. In the parameter identification range $1.5 \leq t \leq 3$ s, the estimation quality of the result shown in Fig. 13 takes $Q(\mathbf{p}_{o2}, 1.5, 3) \approx 97.1\%$, which is about 1.7 % better than that of the first result in Fig. 7 (95.4 %). In the extrapolated range, the estimation quality given by the modified estimator takes $Q(\mathbf{p}_{o2}, 3, 5.5) \approx 97.6\%$, which is comparable to that of the first result in Fig. 8 (98.6%).

Fig. 13 Measured and estimated transmission-input-shaft torque of first sample with time shifting of $(\Delta_t^{\text{eg}}, \Delta_t^{\text{ds}}) = (0, 15 \times 10^{-3})$ [s].

Fig. 14 Measured and estimated transmission-input-shaft torque of the second sample with time shifting of $(\Delta_t^{\text{eg}}, \Delta_t^{\text{ds}}) = (0, 15 \times 10^{-3})$ [s].

Figure 14 shows the estimation result for the second sample's measured data. The black line indicates the measured torque $T_{\text{tm}2}(t)$, which is already shown in Fig. 9. The red line indicates the torque $T_{\text{tm}2}^*(t; \mathbf{p}_{o2})$ estimated by the modified estimator using the parameter vector $\mathbf{p}_{o2}$ mentioned in Table 4 and receiving time-shifted inputs $T'_{\text{eg}2}(t) := T_{\text{eg}2}(t + \Delta_t^{\text{eg}})$ and $T'_{\text{ds}2}(t) := T_{\text{ds}2}(t + \Delta_t^{\text{ds}})$ with $(\Delta_t^{\text{eg}}, \Delta_t^{\text{ds}}) = (0, 15 \times 10^{-3})$ s.

The result in Fig. 14 shows that the waveform difference is reduced at the first ravine R_1 and the second peak P_2 , whereas a relatively significant increase is shown around the second transient vibration near $t \approx 4.2$ s in comparison with the first estimation for the second sample in Fig. 10. The estimation quality becomes $Q(\mathbf{p}_{o2}, 1.5, 3) \approx 96.1\%$ and $Q(\mathbf{p}_{o2}, 3, 5.5) \approx 97.5\%$ for the result in Fig. 14; the former is about 1.6% better than its counterpart in Fig. 10 (94.5%) whereas the latter is comparable to its counterpart (98.3%).

In practical use of the time-shifted estimator described above, the estimation time delay $t_d = \max(\Delta_t^{\text{eg}}, \Delta_t^{\text{ds}}) = 15 \times 10^{-3}$ s arises due to the delayed inputs in (21). Nonetheless, we find that if the delayed estimation of 15×10^{-3} s is acceptable, superior performance of our estimator can be achieved by our proposed time shifting.

7. Conclusion

We proposed a nonlinear torque estimator to accurately estimate the non-measurable transmission-input-shaft torque T_{tm} in a production CVT vehicle. First, by developing a single-DOF vibration model with a torque transfer function, we designed an estimator capable of receiving the measurable engine-output-shaft torque T_{eg} and drive-shaft torque T_{ds} and then outputting an estimate of T_{tm} . Next, we identified the parameters of the estimator using PSO to minimize the estimation error between the estimated and physical T_{tm} and obtained the following results.

- The estimated transmission-input-shaft torque T_{tm} was in good agreement with the physical T_{tm} , marking an estimation quality of more than 94.5%.
- The estimator can extrapolate the transmission-input-shaft torque outside the identified time interval, thus maintaining the estimation quality.
- The estimator can also estimate an unknown transmission-input-shaft torque measured from a separate trial of the experiment with an accuracy comparable to the estimation of identified sample data, even if the unknown data contains a second transient vibration due to an external disturbance.
- The estimator's performance was further improved by time shifting the input data.

From these results, we conclude that our single-DOF estimator can accurately estimate the non-measurable transmission-input-shaft torque in a full-scale automotive vehicle.

In future work, we plan to investigate our estimator's capability for other types of vehicles and driving conditions. We also plan to design another estimator to estimate the drive-shaft torque T_{ds} from the engine-output-shaft torque T_{eg} , based on techniques presented in this paper.

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