Frequency response function measurement of a rotor system utilizing electromagnetic excitation by a built-in motor

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Received: 7 July 2019; Revised: 24 January 2020; Accepted: 9 March 2020

Abstract

In this paper, we propose a practical method for evaluating the robustness and stability of rotating machines. This entails using electromagnetic excitation and measuring the frequency response functions (FRFs) of the rotor system during rotation. The difficulty of applying excitation can be solved by utilizing a built-in motor with static eccentricity and adding a sinusoidal sweep d-axis current to the motor. A test rig with a bearingless motor (BELM) was used to verify the validity of the proposed method. A consequent-pole-type BELM is a combination of a consequent-pole-type motor and a radial magnetic bearing, in which both the motor and the suspension windings are arranged in one stator core of the BELM. The center of the motor is defined by the zero-power controller and the static eccentricity is determined by the suspension control system. The FRFs measured utilizing the electromagnetic excitation generated by the motor windings was compared with reference FRFs measured utilizing the suspension windings. The natural frequencies and damping ratios were determined from the two different FRFs. The difference between the identified natural frequencies and the difference between the damping ratios for the two different FRFs were calculated. The effectiveness of measuring the dynamic characteristics of the rotor system under various rotational conditions, such as the rotational speed and eccentricity, was clarified and the differences between the measurements obtained from the two different FRFs were generally less than 7.7%.

Keywords : Rotor dynamic characteristics, Frequency response functions (FRFs), Motor windings, Suspension windings, Static eccentricity, Electromagnetic excitation, Bearingless motor (BELM)

1. Introduction

Turbomachines driven by built-in motor, such as canned motor pumps (TEIKOKU Electric Mfg Co Ltd, 2015), compressors, and centrifugal blood pumps (Ranjit, 2008), are extremely important systems worldwide. However, the rotor dynamics of turbomachines change throughout their service life and also change with respect to internal and external conditions. Therefore, industry is investing heavily into realizing real-time health monitoring during rotation, with the aim of extending performance and maintaining the machines so that they can be operated with high stability, reliability, and durability for long periods.

With traditional inspection methods, the displacement, velocity, and acceleration of turbomachines are measured. However, the information may be insufficient for diagnosis and prognosis. Alternatively, using an excitation force to vibrate the rotor and measure the frequency response function (FRF) is a common method to monitor the dynamic characteristics, such as the natural frequency and damping ratio, of the rotor system, which can then be used to estimate the margin of stability and determine the origin of vibrations.

In FRF measurements, a few types of exciters are used. Conventionally, impact hammering and electromagnetic actuators (Aenis et al., 2002; Takahashi et al., 2012; Tsunoda et al., 2016, 2017a) are generally used, but are undesirable. Hammering a rotating body is not simple and is unrealistic because of safety. Electromagnetic exciters are able to generate the required sweep excitation force to measure the FRF from the displacement of the rotor, enabling faults to be detected

(Aenis et al., 2002), estimates of the stability margin to be made (Tsunoda et al., 2016, 2017a), and eigenvalues to be measured (Takahashi et al., 2012). However, installation of electromagnetic actuators is inconvenient and expensive, increasing the production costs. Moreover, the motor needs a larger footprint and longer rotor-shaft. Therefore, a low-cost, space-saving, safe and real-time monitoring system for turbomachines is required.

Here, the built-in motor that supplies the torque for the turbomachine is considered as an excitation source. An extra radial force, called the unbalanced magnetic force (UMF), developed as a result of the eccentricity of the built-in motor, can be utilized. In a previous study, C. I. Lee showed the characteristics of the UMF of a brushless DC motor (Lee and Jang, 2008). Factors, such as the non-uniform magnetization of a permanent magnet (PM) and the eccentricity were considered. Farhad Rezaee-Alam has calculated and analyzed the UMF in eccentric surface-mounted permanent-magnet motors under various eccentricity conditions (Rezaee-Alam et al., 2016). In this paper, the UMF is utilized as an excitation force in order to measure the dynamic characteristics of a turbomachine.

Fig. 1 Concept of proposed excitation method for rotor with a PM motor.

We propose utilizing electromagnetic excitation in order to make FRF measurements as shown in Fig. 1. Although various types of built-in motors are used to drive turbomachinery, in this method, we assume the built-in motor to be a permanent magnet (PM) synchronous motor. In general, direct-current (DC) added to the d-axis is usually used for flux-weakening control (Mwasilu et al., 2016; Nguyen et al., 2013; Liu et al., 2018). We assume the PM motor has static eccentricity. An alternating-current (AC) is injected into the d-axis to generate a sinusoidal radial excitation force, which causes the PM bias magnetic flux (Zhang and Foo, 2015) between the stator and rotor core to vary, leading to changes in the UMF in the PM motor. The varying UMF causes the rotor to vibrate in the radial direction. By measuring the displacement signal using displacement sensors, while sweeping the d-axis current, the FRFs can be obtained. This method has some advantages. First, the cost can be reduced and the mechanical components, except the non-contact displacement sensors, can be maintained. Second, the method is safe and effective. To verify our proposed method under various experimental conditions, a test rig with a bearingless motor (BELM) having motor windings and suspension windings was used. The proposed method was verified by comparing the FRFs measured using the motor winding with reference FRFs measured using the suspension windings. Rotor centricity is mainly divided into static and dynamic eccentricity. In previous research (Tsunoda et al., 2018, 2019b), we discussed measurement of the FRF of a rotor system with dynamic eccentricity. The effect of the eccentricity and the rotational speed on the FRF was not evaluated.

In the study reported in this paper, we made FRF measurements of a rotor system with static eccentricity. The rest of this paper is organized into five sections. Section 2 introduces the principle of the proposed method, and we carry out a theoretical analysis of the radial excitation force generated when using a rotor with static eccentricity. Section 3 presents the experimental setup. Section 4 describes the experimental method including simulation of the rotation of a rotor with static eccentricity and FRF measurements under various experimental conditions. Section 5 summarizes the results of the proposed method under the various test conditions and further discusses the proposed method. Section 6 gives the conclusions.

2. Nomenclature

A	Constant of fitted gain curve	B_i	Magnetic flux density in each section
c	Bearing damping coefficient	f_n	Natural frequency of rotor system
F_d	Magnetic motive force induced by the d-axis current	F_i	Attractive force induced by the motor in each section
F_{motor_x}	Attractive force in the x-direction	F_{motor_y}	Attractive force in the y-direction
F_{pm}	Magnetic motive force induced by the permanent magnet	F_x	Radial force in the x-direction
F_y	Radial force in the y-direction	g	Air gap for a rotor with static eccentricity
g_0	Nominal air gap	I_d	d-axis current
I_{d0}	Bias d-axis current	I_{d1}	d-axis current amplitude
I_{exc}	Excitation current in suspension winding	I_{sus}	Amplitude of excitation current in suspension winding
I_q	q-axis current	i_x	Current in the suspension winding in the x-direction
i_y	Current in the suspension winding in the y-direction	I_{xref}	Reference current in suspension winding in the x-direction
I_{yref}	Reference current in suspension winding in the y-direction	I_{2u}	U-phase suspension current
I_{2v}	V-phase suspension current	I_{2w}	W-phase suspension current
I_{2uref}	U-phase suspension reference current	I_{2vref}	V-phase suspension reference current
I_{2wref}	W-phase suspension reference current	I_{8v}	V-phase motor current
I_{8u}	U-phase motor current	I_{8w}	W-phase motor current
I_{8vref}	V-phase motor reference current	I_{8uref}	U-phase motor reference current
I_{8wref}	W-phase motor reference current	k	Bearing stiffness
k_d	Eccentricity-current-force coefficient of motor winding	k_i	Current-force coefficient of suspension winding
k_s	Stiffness of suspension winding	k_w	Stiffness of motor winding
L	Rotor length	L_{pm}	Thickness of the permanent magnet
m	Rotor mass	N_d	Number of turns in the d-axis winding
P_i	Permeance in each section	R	Rotor radius
R_i	Magnetic resistance of the air gap in each section	R_{pm}	Magnetic resistance of each permanent magnet
t	Time	v	Angular frequency of the excitation current
X_{ref}	Reference position in the x-direction	x	Radial displacement of the rotor in the x-direction
Y_{ref}	Reference position in the y-direction	y	Radial displacement of the rotor in the y-direction
ϵ	Static rotor eccentricity in the x-direction	ϵ_1	Rotor deformation
ϵ_0	Initial rotor eccentricity	ϵ_{st}	Static rotor eccentricity in the bearingless motor
ϵ_{amp}	Oscillation amplitude	ζ	Damping ratio of rotor system
θ	Initial angle between the centerline of one pair of PMs and x-axis	μ_0	Permeability of air gap
μ_{pm}	Permeability of the permanent magnet	ϕ	Absolute angle
ϕ_0	Rotational angle	ϕ_m	Oscillation phase

ψ_i	Magnetic flux in the air gap in each section	Ω	Rotational speed
Ω_{ref}	Reference rotational speed	ω_n	Natural angular frequency of rotor system

3. Theoretical analysis of the excitation force for a rotor with static eccentricity

3.1 Model for radial motion of a rigid spindle with static eccentricity

The equation of radial motion of a rigid rotor supported by elastic bearings can be expressed by Eq. (1):

$$\begin{cases} m\ddot{x} + c\dot{x} + kx = F_x \\ m\ddot{y} + c\dot{y} + ky = F_y \end{cases} \quad (1)$$

where m , k , and c are the mass, stiffness and damping coefficient, respectively, x and y are the radial displacements of the rotor in the x- and y-directions, respectively. The supporting stiffness and damping is isotropic in the radial direction. F_x and F_y are the radial forces. The rigid rotor is assumed to rotate with static eccentricity.

3.2 Generation of radial excitation force

In the proposed method, an AC current is injected into the d-axis of the motor to generate a radial excitation force. The bias magnetic flux generated by the PMs is changed by the DC d-axis current to extend the speed range. A consequent-pole-type motor with four PMs is used in the rotor system. Figure 2 shows a cross-sectional view of the rotor, which has static eccentricity ϵ in the x-direction. The black lines show the magnetic flux from the PMs and the red lines show the magnetic flux from the motor windings. As shown in Fig. 3, the line between the centers of one pair of PMs has an initial angle θ with respect to the x-axis. The rotational angle ϕ_0 is obtained from the product of the rotational speed Ω and time t . The absolute angle ϕ is obtained by adding these together, as in the following equation:

$$\phi = \phi_0 + \theta \quad (2)$$

Fig. 2 Cross-sectional view of a consequent-pole-type motor with static eccentricity and the magnetic flux path from the PMs and the motor windings.

Fig. 3 The angular relationship between the absolute, rotational, and initial angles.

As shown in Fig. 4, the motor model is divided into eight 45-degree sections, and the sections are labeled in turn in the counterclockwise direction. We assume that the magnetic flux is uniformly distributed within each section. In each section, the magnetic resistance of the air gap between the stator and rotor core is R_i ($i = 1, 2, \dots, 8$), the magnetic flux in the air gap is ψ_i , and the attractive force induced by the motor is F_i . The magnetic resistance of each permanent magnet can be calculated from the following Eq. (3):

$$R_{pm} = 4L_{pm}/(\mu_{pm}RL\pi) \quad (3)$$

where L_{pm} and μ_{pm} are the thickness and permeability of the permanent magnet, respectively. R and L are the radius

and length of the rotor, respectively.

Fig. 4 The magnetic resistance and unbalanced magnetic force divided into eight sections.

The air gap for a rotor with static eccentricity is given by Eq. (4):

$$g = g_0 \left\{ 1 - \frac{\epsilon}{g_0} \cos(\theta + \phi_0) \right\} \quad (4)$$

where ϵ is much smaller than the nominal air gap g_0 ($1 \gg \epsilon/g_0$). The permeance P_i ($i = 1, 2, \dots, 8$) in each section can be obtained from Eq. (5):

$$P_i = \frac{\mu_0 RL}{g_0} \int_{(-\pi/8)+(i-1)\pi/4}^{(\pi/8)+(i-1)\pi/4} \left\{ 1 - \frac{\epsilon}{g_0} \cos(\theta + \phi_0) \right\} d\theta \quad (i = 1, 2, \dots, 8) \quad (5)$$

Based on the magnetic flux path in Fig. 2, the equivalent magnetic circuit is obtained as shown in Fig. 5. F_{pm} and $F_d (= N_d I_d)$ represent the magnetic motive force induced by the permanent magnet and the d-axis current, respectively. The magnetic flux ψ_i ($i = 1, 2, \dots, 8$) in the air gaps in the respective sections is given by Eq. (6):

Fig. 5 The equivalent circuit for the magnetic flux path.

$$\left\{ \begin{array}{l} (R_1 + R_{pm})\psi_1 = (R_3 + R_{pm})\psi_3 \\ (R_3 + R_{pm})\psi_3 = (R_5 + R_{pm})\psi_5 \\ (R_5 + R_{pm})\psi_5 = (R_7 + R_{pm})\psi_7 \\ R_2\psi_2 = R_4\psi_4 \\ R_4\psi_4 = R_6\psi_6 \\ R_6\psi_6 = R_8\psi_8 \\ F_{pm} + F_d - (R_1 + R_{pm})\psi_1 = R_2\psi_2 \\ \psi_1 + \psi_3 + \psi_5 + \psi_7 = \psi_2 + \psi_4 + \psi_6 + \psi_8 \end{array} \right. \quad (6)$$

The magnetic flux is divided by the area to obtain the magnetic flux density $B_i = \psi_i / (RL\pi/4)$. The attractive force $F_i = (RL\pi/4)B_i^2/\mu_0$ can be calculated utilizing the magnetic flux density. Finally, the attractive forces in the x- and y-directions (F_{motor_x}, F_{motor_y}) can be obtained from the following equations:

$$\left\{ \begin{array}{l} F_{motor_x} = \sum_{i=1}^8 F_i \cos \left\{ \frac{\pi}{4}(i-1) + \phi_0 \right\} \\ F_{motor_y} = \sum_{i=1}^8 F_i \sin \left\{ \frac{\pi}{4}(i-1) + \phi_0 \right\} \end{array} \right. \quad (7)$$

The above calculation is performed using Mathematica[®] and an analytical solution for the attractive forces in the x- and y-directions can be obtained using Eq. (8):

$$\left\{ \begin{array}{l} F_{motor_x} = -\frac{4(-1)^{5/8}\{1 + (-1)^{3/4}\}(F_{pm} + F_d)^2 RL\mu_0}{(g_0 + L_{pm})(2g_0 + L_{pm})} \frac{\epsilon}{g_0} + O\left(\frac{\epsilon}{g_0}\right)^3 \approx k_w\epsilon + k_d I_d \epsilon \\ F_{motor_y} = 0 \end{array} \right. \quad (8)$$

where k_w and k_d are the initial negative stiffness and the eccentricity-current-force coefficient, respectively. $O\left(\frac{\epsilon}{g_0}\right)^3$ can be ignored. In this case, since ϕ_0 is not in the above equation, the attractive force is independent of the rotational angle of the rotor. Since $F_{pm} \gg F_d = N_d I_d$, the formula can be simplified into a linear relationship.

3.3 Motor radial force model

With a static eccentricity ϵ in the x-direction, the attractive force in the y-direction is zero and the equation of motion in the x-direction is given by Eq. (9)

$$m\ddot{x} + c\dot{x} + kx = F_{motor_x} \quad (9)$$

The center of the stator is set as the origin of the coordinates. When the initial eccentricity of the rotor in the x-direction is ϵ_0 , the rotor has a small deformation ϵ_1 that generates a restoring force that balances the attractive force induced by the motor, which is given by Eq. (10):

$$k\epsilon_1 = k_w\epsilon + k_d I_d \epsilon \quad (10)$$

where ϵ is the static eccentricity, which is equal to the sum of the initial eccentricity ϵ_0 and the deformation of the rotor ϵ_1 as shown in Fig. 6.

Fig. 6 Eccentricities of the rotor supported by two bearings

We assumed that the displacement of the rotor from the initial eccentricity ϵ_0 is $x = \epsilon_1 + \Delta x$ and the d-axis current is $I_d = I_{d0} + I_{d1}\cos vt$, in which the AC component excites the vibration. The displacement, attractive force, and d-axis current can be combined with Eqs. (9) and (10) resulting in the following equation of motion:

$$m(\epsilon_1 + \Delta x) + c(\epsilon_1 + \Delta x) + k(\epsilon_1 + \Delta x) = k_w(\epsilon + \Delta x) + k_d I_{d0}(\epsilon + \Delta x) + k_d I_{d1}(\epsilon + \Delta x)\cos vt \quad (11)$$

Since $k_w \gg k_d I_{d0}$ and $\epsilon \gg \Delta x$, the motor radial force model can be simplified to Eq. (12) by substituting Eq. (10) into Eq. (11):

$$m\ddot{\Delta x} + c\dot{\Delta x} + (k - k_w)\Delta x = k_d I_{d1}\epsilon \cos vt \quad (12)$$

This equation describes the dynamic model of the rigid rotor and we can define the excitation force as $F_{exc} = k_d I_{d1}\epsilon \cos vt$. The excitation force can be controlled by the d-axis current.

4. Experimental test rig

4.1 Test rig with a bearingless motor

A compact and simple test rig with a BELM was used as shown in Fig. 7 (Tsunoda et al., 2019a, 2019b). The rotor is supported by two circular oil-film bearings and rotated by a consequent-pole-type BELM (Chiba et al. 2005) placed at the midpoint of the rotor-shaft. The oil viscosity is 49 mPa·s and the 1st bending frequency of the rotor-shaft is 42 Hz. Table 1 shows the dimensions of the test rig. Displacement sensors (PU-05, Applied electronics Crop.) are mounted symmetrically on both sides of the BELM in the x- and y-directions. A non-contact encoder is placed at the end of the rotor-shaft to detect the rotational speed and angle. To avoid any rotor-stator contact during magnetic suspension, two touchdown bearings made of polyacetal are located on both sides of the BELM. Figure 8 shows a cross-sectional view of the consequent-pole-type BELM, which has motor windings and suspension windings in one stator. Table 2 summarizes the dimensions of the BELM. The motor windings, which are three phase and have eight poles, are used for rotation. The suspension windings are three phase and have two poles, and are used for positioning the rotor. Table 3 shows the configurations and characteristics of the two types of windings.

Fig. 7 Photograph and configuration of the test rig.

Fig. 8 Cross-sectional view of the consequent pole-type BELM (The U-phase suspension windings and motor windings are illustrated. The suspension flux is indicated by red lines).

As illustrated in Fig. 8, The radial force can be generated by the flux from the suspension windings, which can strengthen or weaken the bias magnetic fluxes on both the right and left hand sides. Therefore, the position of the rotor can be controlled by the current in the suspension windings. In previous simulations and experiments (Tsunoda et al., 2017b), the radial forces in the x- and y-directions were measured. These can be expressed by the following equations:

$$\begin{cases} F_x = k_i i_x - k_s x \\ F_y = k_i i_y - k_s y \end{cases} \quad (13)$$

where $k_i = 27.0 \text{ N/A}$ and $k_s = -971 \text{ N/mm}$ are the current-force coefficient and the negative stiffness, respectively. i_x and i_y are the currents in the suspension winding in the x- and y-directions. x and y are the displacements of the rotor in the x- and y-directions.

Table 1 Dimensions of test rig.

Rotor		Oil-film bearing	
Mass	2.6 [kg]	Diameter	25 [mm]
Diameter	15 [mm]	Clearance	0.1 [mm]
Length	720 [mm]	Length	25 [mm]

Table 2 Dimensions of consequent-pole-type BELM.

Rotor diameter	70 [mm]	PM outer diameter	69 [mm]
Stator diameter	160 [mm]	PM thickness	5 [mm]
Clearance	1 [mm]	PM arc	45 [°]
Length	50 [mm]	Length of teeth	29 [mm]
Slot number	24	Teeth width	4.8 [mm]

4.2 Suspension and rotation control systems

Proportional-integral-derivative (PID) controllers were used in the suspension control system. The values of the PID parameters were tuned experimentally. Figure 9 shows the suspension control system. The reference currents I_{xref} and

I_{yref} are outputted from the PID controller. These are transformed from two-phase into three-phase currents I_{2uref} , I_{2vref} , and I_{2wref} , and then inputted into three current amplifiers to generate the radial force on the BELM. These controls are carried out using a DSP system (MicroLabBox, dSPACE, GmbH.) with a sampling frequency of 20 kHz. Three PWM single-phase amplifiers (Junus JSP-180-20, Copley Controls Corp.) are used for the suspension control system and the bandwidth of current feedback system I_{2u}/I_{2uref} is about 600 Hz.

Table 3 Configurations of windings and characteristics of BELM.

The design of coils	Copper, $\varnothing 0.8$ mm, Max. current 5 A	
Motor windings	Parallel wires	2
	Series turns per slot	25
	Average eccentricity-current-force coefficient k_d	18.5 [N/A · mm]
Suspension windings	Single wire	1
	Series turns per slot	10
	Current-force coefficient k_i	27.0 [N/A]
	Negative stiffness k_s	-971 [N/mm]

Figure 10 shows the rotational control system. A PI controller is used and the coordinates are transformed from direct-quadrature to three-phase to control the rotational speed. The absolute rotor angle is measured by the non-contact encoder and the rotational speed Ω is calculated using a frequency-to-voltage (FV) converter through the sawtooth wave of the rotor angle. The differences between the reference rotational speed Ω_{ref} and the measured rotational speed Ω is inputted into the PI controller and the amplitude of the q-axis current I_q is calculated by the PI controller, whose parameters are 0.5 A/rps for the proportional gain and 0.5 A/rps · s for the integral gain. I_q and I_d are transformed to three-phase reference currents and the motor currents I_{8u} , I_{8v} and I_{8w} act on the motor windings to generate the motor torque through the amplifiers. The same DSP system used for the suspension control system is applied. Current amplifiers (7425 AC, Copley Controls Corp.) are used and the bandwidth of the current feedback system I_{8u}/I_{8uref} is approximately 1000 Hz.

Fig. 9 Block diagram for the suspension control system and configuration of the FRF measurement using the suspension windings.

Fig. 10 Block diagram for the rotational control system and configuration of the FRF measurement using the motor windings.

5. Experimental method

5.1 Realization of rotation with static eccentricity

5.1.1 Motor center

In this experiment, the center of the motor is defined when the DC current through the suspension windings is zero. The static eccentricity of the rotor is defined utilizing the distance and direction from the motor center. To find the center of the motor, the zero-power controller was used by first setting the reference position X_{ref} and Y_{ref} to (0 μ m, 0 μ m).

Due to the roundness error of the rotor, the position of the motor center depends on the rotational angle. Therefore, the average position ($-20 \mu\text{m}$, $70 \mu\text{m}$) during the rotation is defined as the motor center.

5.1.2 Elimination of whirling

When the rotor rotates at a speed of 40 rps, the amplitude of whirling is approximately $95 \mu\text{m}$. To eliminate this, the reference positions in the suspension control system were sinusoidally changed synchronously with the rotation to generate a radial force to counteract the unbalanced force (Tsunoda et al., 2017b, 2019a). Furthermore, the static eccentricity ϵ_{st} was added to the reference positions in the suspension control system as follows:

$$\begin{cases} X_{ref} = \epsilon_{amp} \cos(\Omega t + \phi_m) + \epsilon_{st} \\ Y_{ref} = \epsilon_{amp} \sin(\Omega t + \phi_m) \end{cases} \quad (14)$$

Fig. 11 Simulation of the static eccentricity rotation.

Note that ϵ_{amp} is the amplitude of the oscillation and Ω is the rotational speed, and ϕ_m is the phase, which depends on the position of the center of mass. Figure 11 shows the amplitude of whirling is reduced to $20 \mu\text{m}$ and the static eccentricity rotation is fixed at $100 \mu\text{m}$.

5.2 FRF measurement using the suspension windings and the motor windings

5.2.1 FRFs measured utilizing the suspension windings

To verify the proposed method, FRF measurements utilizing the radial excitation force generated by the suspension windings of the BELM were used as a reference. Figure 9 shows the configuration of the FRF measurements using the suspension windings. The excitation current $I_{exc} = I_{sus} \cos vt$ from a frequency response (FR) analyzer (FRA5022, NF Corp.) was inputted into the reference current in the x-direction, where I_{sus} is the amplitude of the excitation current in the suspension windings and v is the angular frequency of the excitation current. The amplitude of excitation current I_{sus} was set to 0.5 A. The FRFs from the excitation current I_{sus} to the displacement of the rotor were measured by a FR analyzer from 1 Hz to 300 Hz during rotation.

5.2.2 FRFs measured utilizing motor windings

Figure 10 shows the configuration of the FRF measurement using the motor windings. An AC current signal of $I_{d1} \cos vt$ from the FR analyzer was added to the d-axis current I_d . During rotation, the d-axis current was as follows:

$$I_d = I_{d0} + I_{d1} \cos vt \quad (15)$$

Note that I_{d0} is the bias d-axis current for flux control during rotation and the value of the bias d-axis current depends on the rotational speed due to the flux-weakening control. I_{d1} is the amplitude of the excitation current, which was set to 2.5 A. The FRFs from the current command I_{d1} to the displacement of the rotor were also measured by the FR analyzer.

5.3 Test conditions

To more completely verify the feasibility of the proposed method, some of the rotational conditions, such as the

eccentricity and rotational speed, were varied. Furthermore, the rotor dynamics were also changed by tuning the PID parameters of the BELM controller. In these experiments, three sets of the PID controller parameters were implemented as shown in Table 4. The controller parameters were different in the x- and y-directions to ensure the same closed-loop in both directions due to the position of the levitation having a non-uniform air gap, with the zero-power controller used to balance the force due to gravity. In the PID controller, parameters 1, 2, and 3 had the highest, middle and lowest derivative gains, respectively, to simulate the changes in the dynamic characteristics of the rotor system. The proportional and integral gains remain constant to set the natural frequency response of the first shaft bending mode at approximately 50-70 Hz.

Table 4 PID controller parameters.

		Parameter 1 (High)	Parameter 2 (Medium)	Parameter 3 (Low)
X-direction	K_p [A/m]	40,000		
	K_i [A/m·s]	13,100		
	K_d [A·s/m]	52	36	20
Y-direction	K_p [A/m]	53,000		
	K_i [A/m·s]	17,700		
	K_d [A·s/m]	60	43	25

Table 5 lists the experimental eccentricities and rotational speeds. As shown in Eq. (12), the excitation force is proportional to the eccentricity and the eccentricities of rotational machines are different due to imprecise nature of assembly. Thus, the effect of eccentricity on the measurement accuracy of the FRF was tested. The values of the static eccentricities were set to 100 μm , 50 μm , and 25 μm in the negative x-direction, respectively.

The dynamic characteristics of the rotor system also depend on the rotational speed due to the stiffness and damping variation of the oil-film bearings. The effect of the rotational speed on the proposed method needs to be considered. The rotational speed in our experiments was set to 40 rps, 30 rps, and 20 rps and the I_{d0} bias d-axis current was set to -4.5 A, -2.5 A, and -2.0 A, respectively.

Table 5 Experimental eccentricities and rotational speeds.

Eccentricities [μm]	100
	50
	25
Rotational speeds [rps]	40
	30
	20

6. Experimental results

6.1 Electromagnetic excitation

Various rotational speeds, eccentricities, and rotor dynamics were changed in the experiments. Figure 12 shows the waveform of a typical experiment utilizing the motor windings. The rotational speed was set to 40 rps and the static eccentricity ϵ_{st} was fixed as 100 μm in the negative x-direction. The offset of the d-axis current I_{d0} and the amplitude of the d-axis current I_{d1} were set to -4.5 A and 2.5 A, respectively. Parameter 3, having a low damping ratio, was chosen due to the obvious peak of the natural frequency. With an eccentricity of 100 μm in the negative x-direction and excitation signal at 1 Hz, the amplitude of the rotor calculated utilizing a FFT at 1 Hz was 9.7 μm . The average q-axis current used to generate the torque was 0.68 A. The measured rotational speed had an error of 1 rps and 0.1 rps with and without excitation, respectively.

Fig. 12 (a) Waveform of displacement and FFT analysis of displacement in the negative x-direction at 1 Hz excitation in a typical experiment. (b) Motor current at 1 Hz excitation in a typical experiment. (c) Rotational speed with and without 1 Hz excitation in a typical experiment.

6.2 FRF measurement

6.2.1 Effect of rotor dynamics

Figure 13 shows the measured gain plots of the FRFs with different PID parameters and fitted lines from 1 Hz to 300 Hz under typical experimental conditions. The unit of gain is mm/A. The red and black lines were measured utilizing excitation forces generated by the motor windings and the suspension windings, respectively, and the three pairs of red and black gain curves are similar, except for an offset of 20 dB owing to the different current-force coefficients. The gain curves, shown by the blue lines, have been fitted utilizing the least squares method to the measured curves using Eq. (16):

$$G_p(s) = \frac{A}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (16)$$

Note that A is a constant, ζ is the damping ratio, and ω_n is the natural angular frequency, respectively. The natural frequency f_n is obtained as $f_n = \omega_n/2\pi$.

Table 6 lists the identified damping ratio, natural frequency, and the percentage differences in the values obtained using the motor windings compared to those obtained using the suspension windings. The difference between the identified natural frequencies and the difference between the damping ratios for the two different FRFs are less than 5.2%, proving the validity of the proposed method.

Fig. 13 Measured FRFs with various rotor dynamics.

Table 6 Identified parameters and differences with various rotor dynamics.

		Suspension windings (standard)	Motor windings	Differences (%)
Parameter 1	f_n [Hz]	99	95	4.0
	ζ	1.14	1.18	3.5
Parameter 2	f_n [Hz]	80	77	3.8
	ζ	0.58	0.61	5.2
Parameter 3	f_n [Hz]	69	66	4.4
	ζ	0.20	0.19	5.0

6.2.2 Effect of eccentricity

Figure 14 illustrates the frequency responses from 1 Hz to 300 Hz and the results of the displacement in the x-direction with static eccentricities of 25 μm , 50 μm , and 100 μm . The rotational speed was 40 rps, the bias d-axis current was -4.5 A and the amplitude of the d-axis current was 2.5 A. In Fig. 14 (a), the different offsets in the gain plots measured by the motor windings are a result of the different excitation forces caused by the different eccentricities. In Fig 14 (b), the amplitudes of displacement at 1 Hz are 4.3 μm , 6.1 μm , and 9.7 μm with the static eccentricities fixed at 25 μm , 50 μm , and 100 μm , respectively, in which the eccentricity is positively correlated with the amplitude of the displacement. The peak at 40 Hz is due to the unbalanced vibration of the rotor at 40 rps. Table 7 summaries the identified natural frequencies, damping ratios, and percentage differences. The differences are less than 6.3%. Therefore, even when the eccentricity is 25 μm , the FRF measurement can be achieved using the proposed method.

Fig. 14 (a) Frequency responses with different eccentricities at 25 μm , 50 μm and 100 μm at a speed of 40 rps. (b) FFT analysis of rotor displacement with different eccentricities applying 1 Hz excitation at a speed of 40 rps.

Table 7 Identified parameters and differences with different eccentricities.

		Suspension windings (standard)	Motor windings	Differences (%)
25 μm	f_n [Hz]	69	66	4.4
	ζ	0.20	0.19	5.0
50 μm	f_n [Hz]	68	66	2.9
	ζ	0.16	0.15	6.3
100 μm	f_n [Hz]	68	67	1.5
	ζ	0.15	0.15	0

6.2.3 Effect of rotational speed

Table 8 lists the identified natural frequencies, damping ratios, and the differences at different rotational speeds. The eccentricities are 100 μm and the amplitudes of the d-axis currents are 2.5 A. The natural frequencies and damping ratios were changed slightly depending on the rotational speed due to the variation in stiffness and damping of the oil-film bearings. However, the differences between the identified parameters measured using the suspension windings and the motor windings are less than 7.7%. Therefore, the FRF can be measured using the proposed method.

Table 8 Identified parameters and differences at various rotational speeds.

		Suspension windings (standard)	Motor windings	Differences (%)
20 rps	f_n [Hz]	66	67	1.5
	ζ	0.13	0.14	7.7
30 rps	f_n [Hz]	67	66	1.5
	ζ	0.15	0.16	6.8
40 rps	f_n [Hz]	69	66	4.4
	ζ	0.20	0.19	5.0

6.2.4 Discussion

In the experimental results, many of the natural frequencies measured by the suspension winding excitation are higher than those measured by the motor winding excitation. The natural frequency of the rotor system is affected by the bias d-axis currents, as shown in Eq. (11). However, the same bias d-axis currents were set for the rotational control of the motor both in the suspension and motor winding excitations. Another possibility of having a difference in the measured natural frequencies is that the excitation force is not the same in both excitation methods. It may cause the difference in frequency responses due to nonlinear dynamic of the rotor system. For more accurate analysis, further research is required about the effect of the rotor non-linearity.

7. Conclusion

This paper proposes a practical method of electromagnetic excitation utilizing the motor windings of a built-in motor to evaluate the dynamic characteristics of turbomachinery. The principle of generating an excitation force utilizing an AC d-axis current in a rotor with static eccentricity of the rotor is presented. The theoretical equations show that the radial excitation force is not affected by the rotational angle but is proportional to the eccentricity and the d-axis current. A test rig with a BELM was used to verify the feasibility of the proposed method. The FRFs measured utilizing the suspension windings and the motor windings of BELM were compared. The natural frequencies and damping ratios were identified from the two types of FRF. The effectiveness of the proposed method with different rotor dynamics and rotational conditions, such as the eccentricity and rotational speed, was studied. FRF measurements with three different sets of parameters for the rotor dynamics were made and the percentage differences between measurements made with the two different FRFs were less than 5.2%. The proposed method was verified for different rotational conditions. Even when the

eccentricity was 25 μm and the rotor dynamics were changed by varying the rotational speed, the differences between the natural frequencies and the differences between the damping ratios obtained from the two different FRFs were less than 7.7%.

Future work will focus on extending the proposed electromagnetic excitation, including evaluations using canned motor pumps or artificial blood pumps. Furthermore, sensorless measurement for the displacement will be considered.

Acknowledgement

This research is partly supported by NSK Foundation for Advancement of Mechatronic.

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