Analyzing just-in-time purchasing strategy in supply chains using an evolutionary game approach

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Abstract

Many researchers have focused on the comparison between the JIT model and the EOQ model. However, few of them studied this problem from an evolutionary perspective. In this paper, a JIT purchasing with the single-setup-multi-delivery model is introduced to compare the total costs of the JIT model and the EOQ model. Also, we extend the classical JIT-EOQ models to a two-echelon supply chain which consists of one manufacturer and one supplier. Considering the bounded rationality of players and the quickly changing market, an evolutionary game model is proposed to discuss how these factors impact the strategy selection of the companies. And the evolutionarily stable strategy of the proposed model is analyzed. Based on the analysis, we derive the conditions when the supply chain system will choose the JIT strategy and propose a contract method to ensure that the system converges to the JIT strategy. Several numerical experiments are provided to observe the JIT and EOQ purchasing strategy selection of the manufacturer and the supplier. The results suggest that, in most situations, the JIT strategy is preferred. However, the EOQ strategy remains competitive when the supplier's inventory cost level is high or the demand is low. Supply chain members can choose the EOQ strategy even when the JIT strategy is more profitable. In some situations, strategy selection also depends on the market situation. The JIT policy with low investment costs and high supply chain performance is preferred for the companies.

Keywords : Just-in-time, Economic order quantity, Purchasing, Supply chain management, Evolutionarily stable strategy

1. Introduction

JIT purchasing refers to those practices that aimed at eliminating all forms of waste from the purchasing process (Bond et al. 2019), most notably the frequent deliveries of small lot sizes (Dong et al., 2001). By adopting just-in-time (JIT) purchasing policy, firms can reduce their inventory costs and lead times, improve product quality and achieve higher productivity (Dong et al., 2001). JIT purchasing is a vital component of JIT implementation (Fazel, 1997). The purchasing system is a relationship that exists between suppliers and manufacturers. By considering the supplier and the manufacturer as a whole, the JIT purchasing system will eventually reduce costs for the all supply chain (Kim and Ha, 2003).

The goal of JIT purchasing is to reduce waste from a company's operations, and eventually, to reach zero inventories (Ha and Kim, 1997), and to improve the flow of materials (Waters, 1995). JIT purchasing policy is different from conventional policies. Typically, JIT purchasing results in small lot sizes, elimination of inventories and frequent deliveries (Handfield, 1993). To implement JIT purchasing, usually, there are two key phases. First, manufacturers need to select suppliers who are willing to cooperate and build the JIT purchasing system with them. Second, both manufacturers and suppliers need to change their purchasing system from conventional purchasing to the JIT purchasing system.

The only way to achieve JIT purchasing system is to build a long-term relationship between supplier and manufacturer (Ha and Kim, 1997). Therefore, the important challenges of JIT purchasing systems are the tasks of encouraging firms to change the current purchasing to JIT purchasing, finding suitable suppliers who are willing to cooperate, building and keeping a stable relationship with them. The bounded rationality of suppliers and manufacturers is also an important reason that may finish the long-term relationship. Although there are many papers investigated JIT purchasing, few of them touched on when will suppliers and manufacturers adopt the JIT purchasing system as a strategy

by a game theoretical approach.

Economic order quantity (EOQ) is one of the most classic models in inventory management. This model was originally proposed by Harris (1913). In the classical EOQ model, the total cost consists of three items: holding cost, ordering cost and purchase cost. The key to implementing this model is to find the optimal order quantity that minimizes the total cost. The EOQ model provides a method to mathematically analyze the order quantity in management science. Although it has been proposed for a long time, the EOQ model is still widely accepted by many industries today (Andriolo et al., 2014).

In the past decades, the comparison between JIT and EOQ has been addressed widely in the literature. Fazel (1997) established a mathematical model to evaluate and compare the total costs of the JIT system and EOQ system and found the cost indifference point. A basic assumption in their work is that, in the JIT purchasing system, the ordering and carrying costs are transferred from the buyer to the seller and the transferred costs are indirectly charged to the manufacturer as part of the JIT purchase price. Based on Fazel's (1997) model, many researchers developed a new model to compare EOQ purchasing and JIT purchasing systems. Later, Fazel et al. (1998) developed a JIT-EOQ model considering a price discount in the EOQ model. Based on the former works of Fazel (1998), Schniederjans and Cao (2001) proposed an alternative model that included relevant physical distribution cost savings in their total JIT cost function. Wu and Low (2006) improved Fazel et al.'s (1998) work. They extended the classical EOQ with a price discount model that includes inventory operating costs which were left out before. Continually, Wu et al. (2013) proposed a new JIT-EOQ model by considering the stockout risk in the JIT model. Wang and Ye (2018) studied the JIT and EOQ systems with carbon emissions in a two-echelon supply chain with one manufacturer and multiple retailers.

However, most studies in the field of comparison between JIT and EOQ purchasing models have only focus on the profit of the manufacturer. A successful JIT purchasing system is based on the long-term mutual relationship between manufacturer and supplier (Ha and Kim, 1997). Therefore, the supplier is also should be considered in the study of the JIT-EOQ model.

Some academic literature on JIT policy with game theory are summarized. Transchel and Minner (2011) investigated a dynamic quantity competition game between two retailers. The first retailer adopts the EOQ policy and the second retailer places order in JIT policy. To solve this dynamic problem in continuous time, they proposed a differential game. Bylka (2011) proposed a non-cooperative game with one vendor and multiple retailers. The vendor delivers the goods in JIT shipments to each retailer. They proved the existence of Nash equilibria in several types of sub-games of their proposed game model. Elyasi et al. (2014) studied a two-echelon supply chain model with imperfect quality by game theory. The manufacturer and the supplier in the supply chain use JIT purchasing policy to manage their inventory. Nash equilibrium, Stackelberg equilibrium and the results derived from the centralized model were compared. They found the centralized model has the best solution.

To compare JIT and EOQ policy, Transchel and Minner (2011) assumed two competing retailers that adopt JIT and EOQ policy respectively. However, the successful implementation of the JIT system depends on the cooperation relationship between (Ha and Kim, 1997). A two-echelon supply chain model that consists of one supplier and one manufacturer is discussed in this paper. Researchers also study the JIT systems in the non-cooperative and cooperative relationship between the supplier and the buyer and indicate that the centralized model has a higher profit. In this paper, we consider a cooperative relationship in the supply chain. To the best of the author's knowledge, no research was found in the context of the comparison between JIT and EOQ purchasing models using evolutionary game models.

Some researchers have focused on the study of the single-setup-multi-delivery (SSMD) model which is an integrated JIT lot-splitting model (Ha and Kim, 1997). In the SSMD model, the supplier can produce the manufacturer's order quantity at one setup and deliver those items by multiple deliveries (Kim and Ha, 2003). Continually, Cao et al. (2007) and Sarkar (2018) extend the SSMD model to a more complex JIT environment.

JIT purchasing refers to those practices that have frequent deliveries of small lot sizes (Dong et al. 2001). Liu and Nishi (2019a) extended the classical JIT-EOQ model to a two-echelon supply chain and developed an evolutionary game model to analyze the stability of the JIT strategy and the EOQ strategy. However, their study is still based on a common assumption that has been used in the previous studies to compare EOQ and JIT purchasing policy, that is the buyer does not hold any inventory, which is not a practical assumption. On the other hand, in the SSMD model, the manufacturer keeps a small size of inventory. In this paper, we introduce the SSMD model into the JIT-EOQ comparison model.

The evolutionary game model can be applied to analyze various social systems, such as beliefs, culture forms, practices, and techniques. In evolutionary game theory, when the strategy has an above-average payoff, the frequency of it increases. That indicates the evolution dynamic is not the best-reply dynamic. The players have limited and localized knowledge concerning the dynamic system as a whole. It is known as the bounded rationality of players (Gintis, 2000). Wang and Ye (2018) studied the JIT-EOQ model with multiple retailers. In one of their examples, they indicate that part of the retailers prefer the JIT strategy and the other part of retailers prefer the EOQ strategy. As a result, the manufacturer and the retailers may sign contracts separately and make separate transportations with different strategies. However, if the supply chain switches from the EOQ model to the JIT model, it is possible to save money for the whole supply chain. They suggest the strategy transformation may improve the performance of the whole supply chain. But they did not

discuss how to make the transformation in the JIT-EOQ model. In this study, we try to adopt the evolutionary game approach to analyze the JIT-EOQ model to derive the stability conditions for each equilibrium.

Much research in supply chain management has emphasized the use of evolutionary game theory. Yu et al. (2009) examined the effects of vendor-managed inventory (VMI) strategy in an integrated supply chain by the evolutionary game approach. Their study suggests that the short-term VMI implementation will result in loss of profit for the upstream company. However, long-term implementation will increase the profit for the integrated supply chain. Yi and Yang (2017) discussed the profit maximization and revenue maximization problem in a market with network externality through the evolutionary game approach. The evolutionary game has been used to analyze the complex evolutionary process and long-term behaviors in economic systems when players under bounded rationality (Esmaeili et al. 2016). Recently, Liu and Nishi (2019b) developed a three-stage game model to investigate the subsidy policy and pricing strategy in a closed-loop supply chain under government intervention.

Different from the extant literature, this study applies an evolutionary game approach to study the JIT-EOQ model in a supply chain. In this paper, to tackle the problems of the transition from conventional purchasing system to JIT purchasing system and the bounded rationality of suppliers and manufacturers, we propose a JIT-EOQ purchasing model that uses evolutionary game theory to investigate when the purchasing system is moving towards JIT policy. By considering the SSMD model, we extended the JIT-EOQ model to a two-echelon supply chain model. Then an evolutionary game model is developed to investigate the evolutionary tendency of the supplier and manufacturer. Previous studies of comparison between JIT and EOQ purchasing models have not dealt with the bounded rationality of suppliers and manufacturers. In the classical model, they assume that players have fully rationality and always choose the optimal strategy. However, in practical, players normally have bounded rationality, which is the basic assumption in evolutionary game theory (Xiao and Yu, 2006). Finally, the previous papers do not attempt to consider the potential impact of the existence of multiple manufacturers and suppliers in the market. A field study conducted by Low and Wu (2005) shows that the ready-mixed concrete industries have many manufacturers and suppliers. By adopting an evolutionary game model, we analyze how decision-makers' decisions impact each other. This study offers some new insights into the JIT-EOQ model by adopting the evolutionary game approach.

In this paper, we use an evolutionary game approach and simulation method to find the most stable purchasing strategy, JIT or EOQ in supply chains. Based on the proposed model, the following questions are mainly discussed:

1. Under what conditions the supply chain system will converge to JIT purchasing strategy?
2. How the bounded rationality impacts the strategy selection of the manufacturers and the suppliers?
3. How different parameters affect strategy selection?

The remaining part of the paper has been organized in the following way. In section 2, the basic expressions of the JIT-EOQ purchasing model are derived and the evolutionary game model is proposed, and the revised game model with penalty costs is presented. In section 3, several numerical experiments are conducted. Section 4 presents managerial insights. Section 5 gives a summary.

2. An evolutionary game model for JIT-EOQ purchasing model

In this section, cost functions for both JIT and EOQ purchasing systems are formulated. Based on those functions, the payoff matrix is presented and an evolutionary game model is developed to analyze the evolutionarily stable strategy of the JIT-EOQ model.

2.1 Notations

Indices:

$i \in \{s, m\}$, index of supply chain members, which denotes the supplier s and the manufacturer m .

$k \in \{J, E\}$, index of strategies, which denotes the JIT strategy J and the EOQ strategy E .

Parameters:

S_i : order setup cost per order for supply chain member i .

H_i : inventory holding cost per unit for supply chain member i .

D : annual demand rate of products.

P : annual production rate.

C_i : investment cost to implement the JIT system for supply chain member i .

F : aggregate cost per shipment.

Variables:

Q_k : contract quantity in purchasing system k .

N : number of shipments per contract.

q : lot size in the JIT model, $q = Q_J/N$.

TC_i^{JIT} : cost function for supply chain member i in JIT purchasing system.

TC_i^{EOQ} : cost function for supply chain member i in EOQ purchasing system.

2.2 Basic model

Consider a two-echelon supply chain that consists of one supplier and one manufacturer. The supplier produces raw materials and sells those items to the manufacturer. Then the manufacturer produces goods from the raw materials and sales of those goods to the customers.

S_i , H_i , D , P , C_i , and F are the given parameters. Q_k , N , and q are the decision variables. Several assumptions are made to propose our model. First, we assume there are two possible purchasing systems in a supply chain: JIT purchasing system or EOQ purchasing system. The comparison between JIT and EOQ purchasing system is widely discussed in both academic and practice (Fazel et al. 1998; Wu and Low 2006; Wang and Ye, 2018).

Second, both JIT and EOQ models are considered to be a centralized supply chain decision system where the supplier and manufacturer cooperate with each other to determine the best purchasing policy. In the EOQ model, S_i , H_i , D , C_i , F are assumed to be common knowledge in the integrated supply chain. This information is used to decide the optimal order quantity. The JIT model requires a higher level of cooperation that the supplier's production rate should also be shared with the manufacturer. By sharing this information, the optimal order quantity and the number of shipments are derived.

Third, to successfully implement JIT purchasing, it is assumed that the production rate should be larger than the demand rate. This is a basic assumption of the SSMD model (Ha and Kim, 1997; Kim and Ha, 2003).

2.2.1 JIT purchasing system

Previous papers that compare JIT and EOQ models (Fazel, 1997; Fazel et al. 1998; Schniederjans and Cao, 2001; Wu and Low 2006; Wu et al. 2013) only considered the single manufacturer and assume that the manufacturer does not hold any inventory. However, the JIT purchasing system depends on the cooperation relationship between manufacturer and supplier (Ha and Kim, 1997). So, it is necessary to consider a supplier in the JIT model. Also, in practical, JIT policy often associates with frequent deliveries of small lot sizes (Dong et al. 2001). It is also important to describe the small lot sizes of JIT policy.

Therefore, in this section, the SSMD model is considered. This model consists of one manufacturer and one supplier, and the order quantity in this model is divided into equal delivery sizes and the manufacturer can produce at a single setup. This model is the same as the SSMD model proposed by Ha and Kim (1997).

Figure 1 shows the inventory levels of the manufacturer and the supplier. This is an example of the SSMD model that the supplier produces the manufacturer's order quantity at one setup and deliveries the small lot-sizes in six times. The supplier produces all the order quantity at production time Q/P , and the inventory is increased with a rate of P . The aggregation of production time and non-production time of one order quantity is Q/D . We also assume that each delivery arrives at the time when the manufacturer used all the inventory. The manufacturer's depletion time of each delivery is q/D and the inventory is decreased with a rate of $-D$.

Fig. 1 An example of inventory level for SSMD by Kim and Ha (2003).

Under the SSMD scenario, an order quantity is produced at one setup, but divided into N deliveries. And it is assumed the delivered items arrive at the time when the manufacturer used all the inventory. As a result, the productions are setup D/Q_j times, and the manufacturer only needs to keep the average quantity $Q_j/2N$ as inventory because the order quantity is divided into N shipments, the number of shipments per order quantity is derived as DN/Q_j . In the

SSMD model, the manufacturer need to pay the transportation cost as a way that the manufacturer compensates the supplier for managing the inventory (Kim and Ha, 2013). Consequently, the total cost of the manufacturer in a JIT purchasing system is given as follows:

$$TC_m^{JIT} = \frac{D}{Q_J} S_m + \frac{Q_J}{2N} H_m + \frac{D}{Q_J} NF + C_m \quad (1)$$

where the first item is the setup cost. The second item is inventory cost. The third item is the transportation cost. The last item is the investment cost. According to Frazier et al. (1988), to implement the JIT strategy, moderate levels of investments are necessary for both the supplier and the manufacturer. The manufacturer's production plans and schedules should be shared with the supplier at frequent intervals. The supplier may also need to invest in physical assets such as a new factory near to the manufacturer's plant, new warehouses and so on. Specifically, this JIT investment can enhance production efficiency and logistics performance (Buvik and Halskau, 2001). JIT investments include employee training, restructuring, and maintaining the new production, purchasing, and inventory system. In our model, C_s and C_m are the JIT investment costs that made by supplier and manufacturer to enhance the supply chain performance.

In the SSMD model, the supplier produces the order quantity Q_J at one setup. Then the supplier splits Q_J into several equal lot sizes q , and delivers these items over N times to the manufacturer. Hence, the productions are setup D/Q_J times. The inventory cost is obtained from the total area of the shape of one order quantity (the sum of the triangle area (I), saw-toothed area (II), rectangular area (III), and step area (IV)) in Fig. 1 (b). This expression for the inventory was derived by Ha and Kim (1997). Figure 1 is an example that $3q/D < Q/P < 4q/D$. The SSMD model also can be used in other cases by assuming that $(i-1)q/D < Q/P < iq/D$.

Each area of the shape of one order quantity of SSMD can be derived by:

$$\begin{aligned} I &= q^2/2P \\ II &= \frac{(Q-q)^2}{2P} - \frac{(i-2)(i-3)q^2}{2D} - (i-2)q \left[\frac{Q-q}{P} - \frac{(i-2)q}{D} \right] \\ III &= [N - (i-1)]q \left[\frac{(i-1)q}{D} - \frac{Q-q}{P} \right] \\ IV &= \frac{(N-i)[N - (i-1)]q^2}{2D} \end{aligned}$$

The order quantity is produced D/Q_J times and the inventory cost is H_s . Then, the total inventory is obtained by

$$HC = \frac{D}{Q_J} H_s (I + II + III + IV)$$

The total cost for the supplier is composed of the setup cost, the inventory cost, and the JIT investment cost. Hence, the supplier's total cost in a JIT purchasing system is given as follows:

$$TC_s^{JIT} = \frac{D}{Q_J} S_s + \frac{Q_J H_s}{2N} \left[(2-N) \frac{D}{P} + N - 1 \right] + C_s \quad (2)$$

The aggregation of total costs for manufacturer and supplier is as follows,

$$TC^{JIT} = \frac{D}{Q_J} (S_m + S_s + NF) + \frac{Q_J}{2N} H_m + \frac{Q_J H_s}{2N} \left[(2-N) \frac{D}{P} + N - 1 \right] + C_s + C_m \quad (3)$$

As there is one decision-maker in JIT purchasing, the ordering quantity is derived according to the total cost function of the integrated supply chain. By calculating the first derivatives of TC^{JIT} , the optimal number of deliveries and the optimal order quantity can be gained:

$$N^* = \sqrt{\frac{(S_m + S_s)[(H_m - H_s)P + 2DH_s]}{F(P-D)H_s}} \quad (4)$$

and,

$$Q_J^* = \sqrt{\frac{2D(S_s + S_m)}{H_s(1 - D/P)}} \quad (5)$$

The number of shipments is an integer that larger than or equal to one. If N^* is not an integer, we choose the nearest integer that larger or smaller than N^* (Kim and Ha, 2003).

2.2.2 EOQ purchasing system

Many researchers have studied the EOQ model with two-echelon supply chains (Dong and Xu, 2002; Hsiao and Lin, 2005; Esmacili et al., 2009; Elyasi et al., 2014). To investigate the performance of the Vendor Managed Inventory (VMI) system, Dong and Xu (2002) proposed an EOQ model between supplier and buyer and compare it with the VMI system. In their EOQ model, the manufacturer is the supply chain leader, and it determines the order quantity based on its profit function. Hsiao and Lin (2005) discussed an EOQ model based on the Stackelberg game in the supply chain. In their paper, they discussed the situation when the supplier is the leader. Esmacili et al. (2009) discussed the seller-buyer supply chains based on the EOQ model. Stackelberg equilibriums are obtained by assuming the seller is the leader and the buyer is the leader. Recently, Chernonog (2020) investigate a two-echelon supply chain by considering an EOQ framework. Manufacturer-leader and retailer-leader are analyzed and the equilibriums are obtained.

From the previous works, we found that the EOQ model with two-echelon supply chains has been widely studied by assuming the leader-follower relationship in a two-echelon supply chain. Many researchers study the supply chain models by assuming the manufacturer-leader situation (Dong and Xu, 2002; Esmacili et al., 2009; Chernonog, 2020). See Tsuboi et al. (2018) for a detailed discussion of leadership structures in supply chains.

However, in this paper, we consider a centralized EOQ purchasing strategy. Because the JIT purchasing system and EOQ purchasing system are compared, and the JIT model is considered to be a centralized model, it is reasonable to propose a centralized EOQ model. Hence, in this paper, we assume that an integrated EOQ purchasing model. The inventory is managed by both the manufacturer and the supplier.

The total cost of the manufacturer in an EOQ purchasing system is given as follows:

$$TC_m^{EOQ} = \frac{D}{Q_E} S_m + \frac{Q_E}{2} H_m + \frac{D}{Q_E} F \quad (6)$$

The first item is the setup cost. The second item is holding cost and the third item is the transportation cost. The cost function of the supplier in an EOQ purchasing system is given as follows:

$$TC_s^{EOQ} = \frac{D}{Q_E} S_s + \frac{Q_E}{2} H_s \quad (7)$$

The integrated profit under EOQ policy is given as:

$$TC^{EOQ} = \frac{D}{Q_E} (S_m + S_s + F) + \frac{Q_E}{2} (H_m + H_s) \quad (8)$$

In an integrated two-echelon supply chain with EOQ purchasing policy, the manufacturer and supplier decide the optimal order quantity. Economic order quantity can be obtained from the first-order condition:

$$Q_E^* = \sqrt{\frac{2D(S_m + S_s + F)}{H_m + H_s}} \quad (9)$$

The cost difference between JIT and EOQ purchasing model can be calculated as:

$$Z = TC^{JIT} - TC^{EOQ} \quad (10)$$

2.3 The evolutionary game model

In this game model, we assume that there are many markets that sell homogeneous products and the manufacturers use similar raw materials to produce those products. A homogeneous product can easily be substituted by other products. Therefore, suppliers can change their downstream manufacturers, manufacturers also can change their suppliers.

Each market has a supply chain that consists of one supplier and one manufacturer. In a supply chain, a supplier supplies raw materials to a manufacturer. And the manufacturer uses those raw materials to produce products. We

randomly let a supplier match with a manufacturer in a market. This assumption is frequently used in evolutionary game models (Xiao and Yu, 2006; Li and Yang, 2017; Liu and Nishi, 2019b). Both the supplier and manufacturer have two possible purchasing strategies, JIT purchasing strategy or EOQ purchasing strategy (Wang and Ye, 2018).

Fig. 2 Market environment based on evolutionary game model

In practice, Wu and Low (2005) investigated the implementation of JIT purchasing and EOQ purchasing of 15 ready-mixed concrete companies in Singapore and 20 companies in Chongqing, China. Because of the quick changes in the ready-mixed concrete market, the sole sourcing vendor strategy was not implemented in both Singapore and Chongqing. That results in the manufacturer in ready-mixed concrete industrial frequently changing their suppliers. Their results also indicate that both the JIT purchasing and the EOQ purchasing strategy can be adopted to manage the purchasing process of raw materials in the ready-mixed concrete industry. Also, some companies that use the EOQ strategy are wondering whether they should change to JIT purchasing strategy (Wu and Low, 2006). From this example of field study, we find the assumptions in our evolutionary game model have been met in this case. Various life forms can be seen as the product of evolutionary dynamics, such as beliefs, practices, and techniques (Gintis, 2000). Evolutionary game theory provides the theoretical basis for our model. In this study, two important concepts in evolutionary game theory, replicator dynamics and evolutionarily stable strategy, are used to analyze the decision-making process of suppliers and manufacturers.

Based on the profit functions, the payoff matrix is gained as shown in Table 1.

Table 1 Payoff matrix of the JIT-EOQ model.				
Supplier	Manufacturer			
	JIT		EOQ	
JIT	$-TC_s^{JIT}$	$-TC_m^{JIT}$	$-TC_s^{EOQ} - C_s$	$-TC_m^{EOQ}$
EOQ	$-TC_s^{EOQ}$	$-TC_m^{EOQ} - C_m$	$-TC_s^{EOQ}$	$-TC_m^{EOQ}$

Because the payoffs are analyzed in the game model, all the cost functions in the matrix become negative. When both the supplier and the manufacturer adopt the JIT (EOQ) policy, the payoffs should be the negative cost functions of the JIT (EOQ) model that we derived before. Also, in our game model, we assume that the JIT investment should be required when JIT purchasing policy is adopted. Also, according to Buvik and Halskau (2001), If either one or both of the supplier and the manufacturer perform inadequately, the JIT purchasing system is severely interrupted. Therefore, we assume that when the JIT relationship is failed (one or both companies adopt the EOQ strategy), the profit should be calculated by the EOQ model. As shown in the matrix, when one player adopts the JIT policy and the other adopts the EOQ policy, the payoff should be calculated by the EOQ model and the JIT investment is also added to the player who adopts the JIT policy.

Let x represents the proportion of suppliers who select the JIT purchasing policy among all of them. Then, $(1 - x)$

is the fraction of the suppliers who adopt the EOQ strategy. Similarly, we use y to represent the proportion of the manufacturers who will adopt JIT purchasing policy, and use $(1 - y)$ to represent who will not. Accordingly, $(x, y) \in [0, 1] \times [0, 1]$. During this game process, companies adjust their strategies, and x and y will change until this system achieves a steady state.

Let U_{1J} be the fitness payoff of the suppliers when they adopt the JIT purchasing strategy. The fitness payoffs can be expressed as

$$U_{1J} = y(-TC_s^{JIT}) + (1 - y)(-TC_s^{EOQ} - C_s) \quad (11)$$

Furthermore, the fitness payoff of the suppliers who choose the EOQ strategy is given by

$$U_{1E} = y(-TC_s^{EOQ}) + (1 - y)(-TC_s^{EOQ} - C_s) \quad (12)$$

When JIT with population x and EOQ with the fraction $(1 - x)$, the average fitness payoff of the supplier is obtained:

$$\bar{U}_1 = xU_{1J} + (1 - x)U_{1E} \quad (13)$$

Similarly, the fitness payoffs of the manufacturers with two different strategies are U_{2J} and U_{2E} , where

$$U_{2J} = x(-TC_m^{JIT}) + (1 - x)(-TC_m^{EOQ} - C_m) \quad (14)$$

$$U_{2E} = x(-TC_m^{EOQ}) + (1 - x)(-TC_m^{EOQ} - C_m) \quad (15)$$

Similarly, when JIT with population y and EOQ with the fraction $(1 - y)$, the average fitness payoff of the manufacturers can be defined as

$$\bar{U}_2 = yU_{2J} + (1 - y)U_{2E} \quad (16)$$

According to the analysis above, the replicator dynamics for this game model are as follows:

$$\dot{x} = \frac{dx}{dt} = x(U_{1J} - \bar{U}_1) = x(1 - x)[y(TC_s^{EOQ} + C_s - TC_s^{JIT}) - C_s] \quad (17)$$

$$\dot{y} = \frac{dy}{dt} = y(U_{2J} - \bar{U}_2) = y(1 - y)[x(TC_m^{EOQ} + C_m - TC_m^{JIT}) - C_m] \quad (18)$$

Equations (17) and (18) respectively illustrate the rate of proportion change of suppliers and manufacturers.

2.4 Model analysis

An evolutionarily stable strategy (ESS) is an important concept in evolutionary game theory. Consider a random pairwise matching in a large population to play a game. Assume that all the individuals are “programmed” to play a pure or mixed strategy. Then, a small population share of individuals who are programmed to play another pure or mixed strategy participate in this game. The incumbent strategy is said to be evolutionarily stable strategy if the strategy can resist the invasion by mutant strategies (Weibull, 1997).

Definition (Maynard Smith, 1982). A strategy $x \in \Delta$ is an ESS if and only if either of (1) or (2) holds:

$$(1) u(x, x) > u(y, x) \quad \forall y,$$

$$(2) u(x, x) = U(y, x), \text{ then } u(x, y) > u(y, y).$$

The first condition means that x played against x is better than any y played against x . The second condition means that if x played against x is as much any y played against x , then x played against any y should be better than y played against y (Barron, 2013).

The evolutionarily stable strategy (ESS) of the JIT-EOQ game model is analyzed in this section.

Proposition 1.

(1) The replicator dynamic system's steady-states are (0,0), (0,1), (1,0), (1,1).

(2) If $C_m < \frac{H_m(NQ_E - Q_J)}{2N} - \frac{D(FNQ_E - FQ_J + Q_ES_m - Q_JS_m)}{Q_EQ_J}$ and $C_s < \frac{H_sQ_E}{2} - \frac{H_sQ_J}{2N} \left(n - 1 - \frac{D(N-2)}{P} \right) + \frac{DS_s}{Q_E} - \frac{DS_s}{Q_J}$. Then $(x_s =$

$$\frac{C_m}{TC_m^{EOQ} + C_m - TC_m^{JIT}}, y_s = \frac{C_s}{TC_s^{EOQ} + C_s - TC_s^{JIT}}) \text{ is also a steady-state.}$$

Proof.

In order to obtain the equilibrium points of this system, let values of (18) and (19) equal to 0.

$$\frac{dy}{dt} = 0, \frac{dx}{dt} = 0$$

Obviously, (0,0), (0,1), (1,0), (1,1) are equilibrium points of this system.

When $C_m < \frac{H_m(NQ_E - Q_J)}{2N} - \frac{D(FNQ_E - FQ_J + Q_ES_m - Q_J S_m)}{Q_E Q_J}$ and $C_s < \frac{H_s Q_E}{2} - \frac{H_s Q_J}{2N} \left(n - 1 - \frac{D(N-2)}{P} \right) + \frac{DS_s}{Q_E} - \frac{DS_s}{Q_J}$, the point (x_s, y_s) is inside the square $(x, y) \in [0,1] \times [0,1]$. Thus, (x_s, y_s) is a steady-state.

Proposition 2.

- (1) The steady-state (0,0) is ESS.
- (2) The steady-states (0,1) and (1,0) are unstable.
- (3) The steady-state (1,1) is ESS if (x_s, y_s) is a steady-state. Otherwise, it is unstable.

Proof.

The stability of a replicator dynamic system can be analyzed by a Jacobian matrix. Based on equations in (17) and (18), the Jacobian matrix is as follows:

$$J = \begin{bmatrix} (1-2x)[y(TC_s^{JIT} - TC_s^{EOQ} - C_s) + C_s] & x(1-x)(TC_s^{JIT} - TC_s^{EOQ} - C_s) \\ y(1-y)(TC_m^{JIT} - TC_m^{EOQ} - C_m) & (1-2y)[x(TC_m^{JIT} - TC_m^{EOQ} - C_m) + C_m] \end{bmatrix}$$

Then, the determinant and the trace of J can be derived.

$$\det J = (1-2x)[y(TC_s^{JIT} - TC_s^{EOQ} - C_s) + C_s] \cdot (1-2y)[x(TC_m^{JIT} - TC_m^{EOQ} - C_m) + C_m] - x(1-x)(TC_s^{JIT} - TC_s^{EOQ} - C_s) \cdot y(1-y)(TC_m^{JIT} - TC_m^{EOQ} - C_m)$$

and

$$\text{tr} J = (1-2x)[y(TC_s^{JIT} - TC_s^{EOQ} - C_s) + C_s] + (1-2y)[x(TC_m^{JIT} - TC_m^{EOQ} - C_m) + C_m]$$

A steady-state solution (x^*, y^*) of the dynamic system is ESS if $\text{tr} J < 0$ and $\det J > 0$. If either $\text{tr} J > 0$ or $\det J > 0$, the steady-state solution (x^*, y^*) is unstable (Barron, 2013).

The stability analysis at each steady-state is shown in Table 2.

Table 2 The stability analysis of the JIT-EOQ model.

Steady-state	$\det(J)$	$\text{tr}(J)$	Stability
(0,0)	$C_s C_m$	$-C_s - C_m$	ESS
(0,1)	$(TC_s^{EOQ} - TC_s^{JIT})C_m$	$TC_s^{EOQ} - TC_s^{JIT} + C_m$	Unstable
(1,0)	$(TC_m^{EOQ} - TC_m^{JIT})C_s$	$TC_m^{EOQ} - TC_m^{JIT} + C_s$	Unstable
(1,1)	$(TC_s^{JIT} - TC_s^{EOQ})(TC_m^{JIT} - TC_m^{EOQ})$	$(TC_s^{JIT} - TC_s^{EOQ} + TC_m^{JIT} - TC_m^{EOQ})$	ESS or not
(x_s, y_s)		0	Unstable

At the steady-state (0, 0), we always have $\det(J) > 0$ and $\text{tr}(J) < 0$, because of $C_s > 0$, and $C_m > 0$. Therefore, (0, 0) is an ESS.

The steady-states (0,1) and (1,0) cannot satisfy both $\det(J) > 0$ and $\text{tr}(J) < 0$. Hence, (0,1) and (1,0) are not stable.

At the steady-state (1, 1), if (x_s, y_s) is a steady-state, we can get

$$C_m < \frac{H_m(NQ_E - Q_J)}{2N} - \frac{D(FNQ_E - FQ_J + Q_ES_m - Q_J S_m)}{Q_E Q_J}$$

and

$$C_s < \frac{H_s Q_E}{2} - \frac{H_s Q_J}{2N} \left(N - 1 - \frac{D(N-2)}{P} \right) + \frac{DS_s}{Q_E} - \frac{DS_s}{Q_J}$$

These two conditions ensure $\det(J) > 0$ and $\text{tr}(J) < 0$ at steady-state (1, 1). Thus (1, 1) is an ESS. Otherwise, it is unstable.

In this evolutionary game model, when (x_s, y_s) is a steady-state, both (0,0) and (1,1) are ESSs. Otherwise, the steady-state (0,0) is the only ESS. It is interesting that even the total cost of the JIT system is lower than that of the EOQ system, the EOQ purchasing strategy is still a stable strategy. That means when most of the suppliers and manufacturers adopt the EOQ policy, EOQ policy is an attractive strategy for all the companies even though suppliers and manufacturers can improve their profit if all companies adopt JIT policy.

Researchers compare JIT and EOQ models (Fazel, 1997; Fazel et al. 1998; Schniederjans and Cao, 2001; Wu and Low 2006; Wu et al. 2013) by comparing the manufacturer's cost in these two systems, namely, the difference between TC_m^{JIT} and TC_m^{EOQ} . However, when the JIT-EOQ model is extended to a two-echelon supply chain that consists of

multiple suppliers and manufacturers, the analytical results are different from the previous studies. First, the strategy selection is not only affected by the manufacturer, but both of the manufacturer and the supplier. Second, because of the existence of JIT investments, we found that the EOQ policy is always a stable strategy. That indicates that if there are considerable companies adopt the EOQ policy at the beginning of the game, the EOQ policy will be adopted by all the companies eventually. Third, we found the condition, $TC_m^{JIT} < TC_m^{EOQ}$, no longer guarantees the adoption of JIT policy. Even when the JIT policy is more cost-efficient ($TC_m^{JIT} < TC_m^{EOQ}$ and $TC_s^{JIT} < TC_s^{EOQ}$), the JIT and EOQ policy are both stable strategies. In this situation, the result depends on the initial distribution of the population.

Because both of the JIT and EOQ policies are ESS, we are interested in the long-term behavior of the system as $t \rightarrow \infty$. Our model is described by the differential equations in (17) and (18) with two unknown variables x and y . Hence we need to find the solution (x^*, y^*) when $t \rightarrow \infty$. Instead of discussing the closed-form, researchers often analysis the system by the phase diagram. In section 3, several phase diagrams will be provided to illustrate examples.

2.5 Game model with penalty costs

Because of the JIT investments, the EOQ strategy is always a stable strategy even when the JIT strategy is more cost-efficient. To promote the manufacturer and the supplier adopt the JIT strategy when the JIT policy could improve the supply chain performance, in this model, we consider a contract between the supplier and the manufacturer. A contract is broken by the one who adopts the EOQ policy unilaterally, while the other player adopts the JIT policy. The one who adopted the EOQ policy will be punished for breaking the contract. The penalty will be transferred from the company that breaks the contract to the company that follows the contract. The payoff matrix of this game model with penalty costs is shown in Table 3.

Table 3 Payoff matrix with penalty costs.

Supplier	Manufacturer	
	JIT	EOQ
JIT	$-TC_s^{JIT}, -TC_m^{JIT}$	$-TC_s^{EOQ} - C_s + G_m, -TC_m^{EOQ} - G_m$
EOQ	$-TC_s^{EOQ} - G_s, -TC_m^{EOQ} - C_m + G_s$	$-TC_s^{EOQ}, -TC_m^{EOQ}$

By the same method in section 2.3, the replicator dynamics equations with penalty costs are as follows:

$$\begin{aligned}\dot{x} &= \frac{dx}{dt} = x(u_{11} - \bar{U}_1) = x(1-x)[y(TC_s^{EOQ} + C_s - TC_s^{JIT} + G_s - G_m) + G_m - C_s] \\ \dot{y} &= \frac{dy}{dt} = y(u_{21} - \bar{U}_2) = y(1-y)[x(TC_m^{EOQ} + C_m - TC_m^{JIT} + G_m - G_s) + G_s - C_m]\end{aligned}$$

The Jacobian matrix is as follows:

$$J = \begin{bmatrix} (1-2x)[y(TC_s^{EOQ} + C_s - TC_s^{JIT} + G_s - G_m) + G_m - C_s] & x(1-x)(TC_s^{EOQ} + C_s - TC_s^{JIT} + G_s - G_m) \\ y(1-y)(TC_m^{EOQ} + C_m - TC_m^{JIT} + G_m - G_s) & (1-2y)[x(TC_m^{EOQ} + C_m - TC_m^{JIT} + G_m - G_s) + G_s - C_m] \end{bmatrix}$$

Then, the determinant and the trace of J are derived:

$$\det J = (1-2x)[y(TC_s^{EOQ} + C_s - TC_s^{JIT} + G_s - G_m) + G_m - C_s] \cdot (1-2y)[x(TC_m^{EOQ} + C_m - TC_m^{JIT} + G_m - G_s) + G_s - C_m] - x(1-x)(TC_s^{EOQ} + C_s - TC_s^{JIT} + G_s - G_m) \cdot y(1-y)(TC_m^{EOQ} + C_m - TC_m^{JIT} + G_m - G_s)$$

and

$$\text{tr } J = (1-2x)[y(TC_s^{EOQ} + C_s - TC_s^{JIT} + G_s - G_m) + G_m - C_s] + (1-2y)[x(TC_m^{EOQ} + C_m - TC_m^{JIT} + G_m - G_s) + G_s - C_m]$$

Proposition 3.

- (1) The steady-state (0,0) is an unstable point if $G_m > C_s$ and $G_s > C_m$.
- (2) If the penalty costs satisfy $G_s > C_s - \frac{H_s Q_E}{2} + \frac{H_s(-1+N-\frac{D(-2+N)}{P})Q_J}{2N} - \frac{DS_s}{Q_E} + \frac{DS_s}{Q_J}$ and $G_m > C_m + \frac{H_m(-NQ_E+Q_J)}{2N} + \frac{D(FNQ_E-FQ_J+Q_ES_m-Q_J S_m)}{Q_E Q_J}$, then (1, 1) is an ESS.

Proof.

At the steady-state (0, 0), if $G_m > C_s$ and $G_s > C_m$, we have $\det(J) > 0$ and $\text{tr}(J) > 0$. Therefore, (0, 0) is an

unstable point.

At the steady-state (1, 1), when $G_s > C_s - \frac{H_s Q_E}{2} + \frac{H_s(-1+N-\frac{D(-2+N)}{P})Q_J}{2N} - \frac{DS_s}{Q_E} + \frac{DS_s}{Q_J}$ and $G_m > C_m + \frac{H_m(-NQ_E+Q_J)}{2N} + \frac{D(FNQ_E-FQ_J+Q_ES_m-Q_J S_m)}{Q_E Q_J}$, we have $\det(J) > 0$ and $\text{tr}(J) < 0$. Hence, (1, 1) is the only ESS.

The analysis of local stability with penalty costs is shown in Table 4.

Table 4 The stability analysis with penalty costs.

Steady-state	$\det(J)$	$\text{tr}(J)$	Stability
(0,0)	+	+	Unstable
(0,1)	-		Unstable
(1,0)	-		Unstable
(1,1)	+	-	ESS

In the game model with penalty costs, the point (0, 0) is still a steady-state, but it is not an ESS. Steady-state (1, 1) is the only ESS in the model by imposing penalty costs. That is, with appropriate penalty costs, even most of the suppliers and manufacturers adopt the EOQ policy at the beginning, but eventually, all players will switch to the JIT policy.

3. Numerical experiments

In this section, the validity of the propositions is investigated by conducting simulation experiments. Also, sensitivity analysis is conducted to examine the influences of the main parameters on the behaviors of the supply chain members.

3.1 Case study

We conduct several numerical experiments to show how the parameters affect the evolutionary tendency. The parameters in example 1 appeared in Kim and Ha (2003) are modified and some of the additional parameters are introduced in this paper. The set of parameter values in example 1 are considered to be the baseline. To examine the impacts of different parameters in this model, the supplier's inventory cost and JIT investments take different values in example 2 and example 3. The parameter values and calculation results are shown in Tables 5 and 6.

Table 5 Parameter values.

Parameters	Example 1	Example 2	Example 3
D	4800	4800	4800
P	19200	19200	19200
H_m	7	7	7
H_s	6	13	6
F	50	50	50
S_m	25/order	25/order	25/order
S_s	600/order	600/order	600/order
C_m	500	500	250
C_s	500	500	250

Table 6 Computational results for three examples.

Parameters	Example 1	Example 2	Example 3
TC_s^{JIT}	5089.9346	8338.3672	4839.9346
TC_s^{EOQ}	6197.2699	8759.5091	6197.2699
TC_m^{JIT}	2520.7259	4235.4719	2270.7259
TC_m^{EOQ}	2980.9653	2624.6905	2980.9653
Q_E^*	706.0181	569.21	706.0181
N^*	3	2	3
q	346.4102	979.7959	346.4102
ESS	(0, 0), (1, 1)	(0, 0)	(0, 0), (1, 1)
Saddle point	(0.52, 0.31)		(0.26, 0.16)

Figure 3 is the phase diagram of these examples which was created by Dynamo (Franchetti and Sandholm, 2013). These diagrams depict how initial conditions impact the evolutionary process. The colors in the figures represent speeds of evolution under the dynamic: red is fast and blue is slow. The black dots represent stable points and white dots represent unstable points. The arrows represent the path of evolution. Note the phase diagram is only a tool to find a stable strategy in this paper, it cannot reveal the true strategy transformation in practice. It is helpful for us to analyze the decision-making of suppliers and manufacturers.

Fig. 3 The phase diagrams.

Figure 3 (a) shows that there are two ESSs, $(0, 0)$ and $(1, 1)$. With different initial conditions, the system converges to either $(0, 0)$ or $(1, 1)$. As Fig. 3 (a) depicts, the system converges to $(1, 1)$ if the initial point located in the upper area, which means that JIT purchasing policy is more preferable. However, the system converges to $(0, 0)$ if the initial point located in the lower area, which means that the EOQ policy is more preferable. Therefore, the result of this evolutionary game model is strongly impacted by the initial states. In example 2, the saddle point is not existing in the model, and the only ESS is $(0, 0)$. The manufacturer and the supplier tend to increase the order quantity and decrease the shipments so that the manufacturer will manage more inventory. From this example, we found that when the supplier's inventory at a higher level, the JIT purchasing is not cost-effective. In example 3, by decreasing the JIT investment, the saddle point is moved to the lower-left direction compare with example 1. Thus the area that will converge to JIT policy is increased.

As shown in Fig. 4 (a), we set the initial condition $x = 0.35, y = 0.35$ and the replicator dynamic system converges to the ESS $(0,0)$. The population of suppliers who are willing to adopt the JIT strategy increases slightly at first, but then

decreases and converges to 0. It is because the number of manufacturers who adopt the JIT purchasing system decreases continuously and the initial state is relatively low. Therefore, suppliers would rather take the EOQ policy.

Fig. 4 The influence of initial conditions on the evolutionary process

Then we set a different initial condition ($x = 0.45, y = 0.45$) for this example. As shown in Fig. 4 (b), the number of the manufacturers who adopt JIT purchasing decreases a little then increases rapidly and the system converges to ESS (1, 1) at the end. That is because the initial condition is higher than the example before and there are more manufacturers selected JIT system at the beginning. The initial condition is an important factor for the result of this evolutionary process.

In the classical JIT-EOQ model (Schniederjans and Cao, 2001; Wu et al., 2013), the strategy selection should be based on the cost difference. According to Eq. (10), the cost difference between JIT and EOQ purchasing system should be derived as $Z = -1567.6$. Adopting the JIT purchasing system is profitable compared with the EOQ system. This result indicates that the total profits can be increased from model transformation (Wang and Ye, 2019). Hence, it is important to encourage more companies to adopt JIT purchasing policy at the beginning. However, because of the bounded rationality of the suppliers and the manufacturers, it is still possible to converge to the EOQ strategy for the game model as shown in Fig. 4 (a). From this example, we found that the bounded rationality indeed impacts the strategy selection of the manufacturer and the supplier. Comparing with Schniederjans and Cao (2001) and Wu et al. (2013), we showed that calculating the profit difference is not enough to explain the companies' behaviors. To find a stable strategy, population decisions and bounded rationality should also be considered.

3.2 Examples with penalty costs

In the last example, we found that because of the bounded rationality of the manufacturers and the suppliers, it is possible for the supply chain to converge to the lower profit strategy. To solve this problem and improve supply chain performance, a contract method is proposed. In this example, we assume that companies will be punished for breaking the contract. Penalty costs, which satisfy Proposition 3, for manufacturers is $G_m = 750$, and for suppliers is $G_s = 750$. We set the initial conditions $x = 0.01, y = 0.01$. As shown in Fig. 5, although the initial conditions are much lower than the former example, the replicator dynamic system converges to ESS (1,1). That shows making a contract with penalty costs can promote the adoption of JIT policy in the market for both manufacturers and suppliers.

Fig. 5 Evolutionary process of example 1 with penalty costs ($x = 0.01, y = 0.01$)

According to Proposition 3, the only ESS is JIT strategy as shown in Fig. 3 (d). Therefore, if the penalty costs satisfy Proposition 3, this system converges to ESS (1, 1), which means all the companies will adopt the JIT purchasing policy. This suggested that the result of the evolution is not related to the initial states in this example. The initial conditions only influence the speed of convergence.

3.3 Sensitivity analysis

To examine the influences of the main parameters of the purchasing system on the behaviors of the supply chain members, we let these parameters take different values to observe the simulation results. The parameters' values of example 1 in Table 5 are considered to be the baseline.

3.3.1 Inventory cost

Inventory cost is an important factor that influences the decision-making of suppliers and manufacturers. If the manufacturer and the supplier are located in different locations, they may have different inventory costs. To investigate the impacts of inventory cost on the manufacturer's and supplier's behaviors, three values for the supplier's inventory cost are considered. We set the values as 6, 10, 13. And the value of inventory cost for the manufacturer keeps the same ($H_m = 7$).

As shown in Fig. 6, the inventory cost has an impact on the decision-making of suppliers and manufacturers. When the inventory cost for the supplier is 6, the JIT strategy is preferred. With the increasing inventory cost for the supplier, the manufacturers and suppliers will choose the EOQ purchasing system. The results show that the JIT system has an advantage when the inventory cost for the supplier is relatively low, and the EOQ system has an advantage when it is higher.

Fig. 6 The influence of inventory costs on the evolutionary process

By adopting the SSMD model, the supplier splits the order quantity into several shipments so that the manufacturer only needs to manage a small size of inventory. However, when the supplier has a higher inventory cost, this JIT strategy is no longer competitive compared with the EOQ strategy.

3.3.2 Investment costs

To examine the influences of investment costs on the strategy-selection, the values of investment cost for the manufacturer and the supplier is set as 250, 500, 750.

As shown in Fig. 7, the investment costs have effects on the behaviors of the manufacturer and the supplier. When the investment costs are low ($C_m = 250, C_s = 250$), the JIT strategy is preferred. The higher the investment costs, the supply chain members are more likely to choose the EOQ strategy.

3.3.3 Demand variations

Many researchers have focused on the impacts of demand on the selection of the JIT-EOQ model. Fazel (1997) illustrated that the EOQ is still competitive when the demand level is high. Wu et al. (2013) showed that EOQ purchasing is competitive with higher or lower levels of demand. In this paper, the demand is set as 500, 4800, 19000.

As shown in Fig. 8, in this example, the JIT policy is even more competitive with the increasing demand, as long as the demand is lower than the production capacity. When $D = 19000$, the system converges to JIT policy at a fast speed. However, EOQ is more cost-effective with lower demand.

Fig. 7 The influence of investment costs on the evolutionary process

Fig. 8 The influence of demand on the evolutionary process

3.3.4 Penalty costs

As illustrated in Proposition 3, the penalty costs have direct impacts on the decision-making process. We set $(G_s, G_m) \in \{(750, 750), (1000, 1000), (1250, 1250)\}$.

Figure 9 shows that by imposing the penalty cost, the system will eventually converge to the JIT strategy. The higher the penalty costs, the faster the supply chain members reach a stable state.

Fig. 9 The influence of penalty costs on the evolutionary process

4. Managerial insights

Several managerial insights are presented in this section.

- First, as shown in example 1, the total cost of an integrated supply under the JIT purchasing system is 7610.7, and the total cost under the EOQ purchasing system is 9178.2. The total cost of the JIT strategy is 1567.6 lower than the EOQ strategy. The results from the numerical experiments show that even when the JIT strategy is more cost-efficient compared with the EOQ strategy, the EOQ policy is still a stable strategy. The reason is that to implement the JIT strategy, the investment costs are necessary for both the manufacturer and the supplier. As a result, the EOQ policy is a safe choice.
- Second, it is also illustrated that which strategy will make the system converge depends on the initial distribution of population, when both JIT and EOQ strategies are stable. Fig. 4 shows that if 0.35% of the suppliers and the manufacturers adopt the JIT strategy, the EOQ strategy will be chosen. But if 0.45% of the suppliers and the manufacturers adopt the JIT strategy, eventually, the system will converge to the EOQ strategy. Hence, it is important to aware of the market situation. If most of the companies adopt JIT policy, it is likely that eventually the system will converge to the JIT purchasing strategy, and vice versa.
- Third, the effects of the supplier's inventory cost in our examples are investigated. As shown in Fig. 6, the system converges to the JIT strategy when $H_s = 6$, and the system converges to the EOQ strategy when $H_s = 10$ and $H_s = 13$. It is found the JIT strategy is preferred when the supplier's inventory cost is at a relatively low level, and the EOQ strategy is chosen when the supplier's inventory cost is at a higher level. By implementing the JIT strategy, the order quantity is divided into several small lot sizes, and the manufacturer could have less inventory. However, when the supplier's inventory cost is at a higher level, it is not worth to split the order quantity.
- Forth, the JIT investment costs are also a key factor for the successful implementation of JIT policy. As shown in Table 6 and Fig. 3, by decreasing the JIT investments from 500 to 250, the location of the saddle point is moved from (0.52, 0.31) to (0.26, 0.16). As a result, the area that will converge to JIT strategy is increased. Fig. 7 shows, assuming that 50% of the manufacturers and 50% of the suppliers adopt the JIT strategy, by increasing the JIT investments to 750, the system will converge to the EOQ strategy. The results are in line with managerial insight that, on the one hand, the JIT investments may improve the supply chain performance. On the other hand, investments also result in high switching costs (Buvik and Halskau, 2001). A JIT policy with low investment costs and high supply chain performance is preferred for the companies.
- Fifth, in our example, the higher the demand, the faster the system converges to JIT strategy. As shown in Fig. 8, when $D = 19000$, the system converges to the JIT strategy within $time = 0.005$. But when $D = 4800$, the manufacturer population spends around $time = 0.015$ to converge to the JIT strategy. The reason is that in our example, the supplier has a lower level of inventory cost. By dividing the order quantity into small lot sizes, the JIT strategy has more advantages. Also, when demand is extremely low, $D = 500$, because it is not worth to split the order quantity, the system converges to the EOQ strategy.
- Finally, when the JIT policy is cost-efficient compared with EOQ policy, a contract may promote the adoption of the JIT policy to improve the supply chain performance. It is also found that the higher the penalty costs, the faster the system converges to the JIT strategy. As shown in Fig. 9, by making the contract, all the system converges to the JIT system. When $G_s = 1250$ and $G_m = 1250$, the system could converge to the JIT strategy within $time = 0.01$. but when we set $G_s = 750$ and $G_m = 750$, the manufacturer population need more than $time = 0.015$ to converge to the JIT strategy. As in this example, we assume the JIT strategy is more profitable, the earlier the manufacturers and the suppliers adopt the JIT strategy, the better for the performance integrated supply chain.

5. Conclusions

In this paper, we have extended the basic JIT-EOQ model to a two-echelon supply chain model. Based on this model, an evolutionary game model has been developed and replicator dynamics equations have been obtained. By calculating the Jacobian matrix, the ESS is analyzed. Finally, numerical experiments have been provided to reach a further understanding of this system.

This paper provides a different perspective to understand the comparison between JIT purchasing strategy and EOQ purchasing strategy. The results suggest that, first, the bounded rationality impacts the strategy selection of the manufacturer and the supplier, which makes them choose the low-profit strategy in some situations. Second, in most of the situations, JIT is more cost-effective. However, the EOQ remains competitive when the supplier's inventory cost level is high. Third, the EOQ purchasing system has an advantage with a low level of demand. Finally, making a contrast with penalty costs can improve the adoption of JIT purchasing policy.

There are several future research directions. First, this paper discussed the cost functions in the JIT model and the EOQ model. However, the purchase prices in the JIT system and the EOQ system have not been investigated. Further research might explore the different pricing strategies in these two different systems. Second, because the JIT system and EOQ system have different inventory and deliver policies, the proposed model can be extended to compare the carbon emissions generated from these two systems. Third, in our evolutionary game model, the players update their strategies according to the replicator dynamics. Further research might examine other dynamics to study these problems. Finally, the propositions in this paper are theoretically proved and verified by conducting simulation experiments. However, experiments of real-world examples are not conducted. Future studies should investigate our model in practical examples.

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